Assignment 1

CS21BTECH11053

Problem 6c, ICSE Math Paper (2017):
If $\frac{7m+2n}{7m-2n} = \frac{5}{3}$, use properties of proportion to find
i $\frac{m}{n}$
ii $\frac{m^2+n^2}{m^2-n^2}$

Solution:
We are given,

$$\frac{7m+2n}{7m-2n} = \frac{5}{3} \tag{1}$$

From componendo - dividendo, we know

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d} \tag{2}$$

Hence, we have, from equations 1 and 2,

$$\frac{(7m+2n)+(7m-2n)}{(7m+2n)-(7m-2n)} = \frac{5+3}{5-3}$$

$$\Rightarrow \frac{14m}{4n} = \frac{8}{2}$$

$$\Rightarrow \frac{7m}{2n} = \frac{4}{1}$$

$$\Rightarrow \frac{m}{n} = \boxed{\frac{8}{7}} \tag{3}$$

From the equation 3, we see that

$$\left(\frac{m}{n}\right)^2 = \left(\frac{8}{7}\right)^2$$

$$\Rightarrow \frac{m^2}{n^2} = \frac{8^2}{7^2} = \frac{64}{49} \tag{4}$$

Using componendo-dividendo again on equation 4, we have

$$\Rightarrow \frac{m^2+n^2}{m^2-n^2} = \frac{64+49}{64-49} = \boxed{\frac{113}{15}} \tag{5}$$