

ASSIGNMENT 1

CS21BTECH11053

Abstract—From ICSE 2018 Class 12 Mathematics Examination

Problem (19.b). Find the coefficient of correlation from the regression lines

$$x - 2y + 3 = 0 \quad (1)$$

$$4x - 5y + 1 = 0 \quad (2)$$

Solution:

Given data as n ordered pairs

$$\left(\begin{matrix} x_1 \\ y_1 \end{matrix} \right), \left(\begin{matrix} x_2 \\ y_2 \end{matrix} \right), \dots, \left(\begin{matrix} x_n \\ y_n \end{matrix} \right) \quad (3)$$

Regression line of y on x in parametric form is

$$\left(\begin{matrix} 0 \\ c_{yx} \end{matrix} \right) + \lambda \left(\begin{matrix} 1 \\ b_{yx} \end{matrix} \right) \quad (4)$$

Regression line of x on y in parametric form is

$$\left(\begin{matrix} c_{xy} \\ 0 \end{matrix} \right) + \lambda \left(\begin{matrix} b_{xy} \\ 1 \end{matrix} \right) \quad (5)$$

We shall assume that (1) is the regression line of y on x and express it as

$$\left(\begin{matrix} 0 \\ 6 \end{matrix} \right) + \lambda \left(\begin{matrix} 1 \\ 2 \end{matrix} \right) \quad (6)$$

And (2) is the regression line of x on y and express it as

$$\left(\begin{matrix} -\frac{1}{4} \\ 0 \end{matrix} \right) + \lambda \left(\begin{matrix} \frac{5}{4} \\ 1 \end{matrix} \right) \quad (7)$$

Comparing (4) with (6) and (5) with (7), we evaluate regression coefficients b_{yx} and b_{xy} as

$$b_{yx} = \frac{1}{2} \quad (8)$$

$$b_{xy} = \frac{5}{4} \quad (9)$$

Given b_{yx} and b_{xy} , we can find the coefficient of correlation r as

$$r = \pm \sqrt{b_{yx} \times b_{xy}} \quad (10)$$

Note that b_{yx} , b_{xy} and r have the same sign and $|r| \leq 1$.

From (8), (9), (10)

$$r = \pm \sqrt{\frac{1}{2} \times \frac{5}{4}} = \pm \sqrt{\frac{5}{8}} \quad (11)$$

Since $b_{yx} > 0$ and $b_{xy} > 0$, $r > 0$. Also note that $|r| \leq 1$. Hence our initial assumption was correct.

$$\therefore r = \sqrt{\frac{5}{8}} \quad (12)$$

