

ASSIGNMENT 1

CS21BTECH11053

Abstract—From ICSE 2018 Class 12 Mathematics Examination

Problem (19.b). Find the coefficient of correlation from the regression lines

$$x - 2y + 3 = 0 \quad (1)$$

$$4x - 5y + 1 = 0 \quad (2)$$

Solution:

Given data as n ordered pairs

$$\left(\begin{matrix} x_1 \\ y_1 \end{matrix} \right), \left(\begin{matrix} x_2 \\ y_2 \end{matrix} \right), \dots, \left(\begin{matrix} x_n \\ y_n \end{matrix} \right) \quad (3)$$

Regression line of y on x in parametric form is

$$\begin{pmatrix} 0 \\ c_{yx} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ b_{yx} \end{pmatrix} \quad (4)$$

Regression line of x on y in parametric form is

$$\begin{pmatrix} c_{xy} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} b_{xy} \\ 1 \end{pmatrix} \quad (5)$$

We shall assume that (1) is the regression line of y on x and (2) is the regression line of x on y .

Note that any equation of the form

$$y = mx + c \quad (6)$$

can be expressed in vector form as

$$\begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix} \quad (7)$$

From (1) we have

$$y = \frac{x}{2} + \frac{3}{2} \quad (8)$$

From (2) we have

$$y = \frac{4x}{5} + \frac{1}{5} \quad (9)$$

Similar to conversion of (6) to (7) we can write vector form of (8) as

$$\begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \quad (10)$$

We can similarly write vector form of (9) as

$$\begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \quad (11)$$

$$= \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} - \frac{1}{4} \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} + \left(\lambda + \frac{1}{4} \right) \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \quad (12)$$

$$= \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} - \begin{pmatrix} \frac{1}{4} \\ \frac{1}{5} \end{pmatrix} + (\mu) \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \quad (13)$$

$$= \begin{pmatrix} \frac{-1}{4} \\ 0 \end{pmatrix} + \left(\frac{4}{5} \right) \left(\frac{5}{4} \right) (\mu) \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \quad (14)$$

$$= \begin{pmatrix} \frac{-1}{4} \\ 0 \end{pmatrix} + \left(\frac{4\mu}{5} \right) \begin{pmatrix} \frac{5}{4} \\ 1 \end{pmatrix} \quad (15)$$

$$= \begin{pmatrix} \frac{-1}{4} \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} \frac{5}{4} \\ 1 \end{pmatrix} \quad (16)$$

(Note that we have arbitrarily defined $\mu = \lambda + \frac{1}{4}$ and $\gamma = \frac{4\mu}{5}$)

Comparing (4) with (10) and (5) with (16), we evaluate regression coefficients b_{yx} and b_{xy} as

$$b_{yx} = \frac{1}{2} \quad (17)$$

$$b_{xy} = \frac{5}{4} \quad (18)$$

Given b_{yx} and b_{xy} , we can find the coefficient of correlation r as

$$r = \pm \sqrt{b_{yx} \times b_{xy}} \quad (19)$$

Note that b_{yx} , b_{xy} and r have the same sign and $|r| \leq 1$.

From (17), (18), (19)

$$r = \pm \sqrt{\frac{1}{2} \times \frac{5}{4}} = \pm \sqrt{\frac{5}{8}} \quad (20)$$

Since $b_{yx} > 0$ and $b_{xy} > 0$, $r > 0$. Also note that $|r| \leq 1$. Hence our initial assumption was correct.

$$\therefore r = \sqrt{\frac{5}{8}} \quad (21)$$

