AI1110: Probability and Random Variables

Assignment 5

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Outline

Problem

Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

Solution: Defining Random Variables

We will define three random variables X, Y, Z such that $X, Y, Z \in 0, 1$. $X = 1$ is assigned an even card and $X = 0$ to odd cards. $Y = 1$ is assigned to cards which are greater than 3, $Y = 0$ otherwise. $Z = 1$ is assigned to cards which have $X = 1$ and $Y = 1$ and $Z = 0$ otherwise. In other words, any even card greater than 3 is assigned $Z = 1$. Based on the above definitions, we obtain the following table (1). (Shown in next slide)

Table of Random Variables

Card No	X	Y	$Z = X \wedge Y$
1	0	0	0
2	1	0	0
3	0	0	0
4	1	1	1
5	0	1	0
6	1	1	1
7	0	1	0
8	1	1	1
9	0	1	0
10	1	1	1

Table: Random Variables for Various Events

Probabilities for Y and Z

There are a total of 10 cards. Hence for the sample place S of the experiment,

$$n(S) = 10 \tag{1}$$

Using equation (1) and table (1) the probability that $Y = 1$ is given by

$$P(Y = 1) = \frac{n(Y = 1)}{n(S)} = \frac{7}{10} \tag{2}$$

Similarly the probability that $Z = 1$ is given by

$$P(Z = 1) = \frac{n(Z = 1)}{n(S)} = \frac{4}{10} \tag{3}$$

Conditional Probability

The Concept: The probability that event A occurs given that event B is given as

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} \tag{4}$$

Conditional Probability

Hence the probability that we get an even card given that the card is greater than 3 is given by

$$P((X=1)|(Y=1)) = \frac{P((X=1) \wedge (Y=1))}{P(Y=1)} \tag{5}$$

$$\implies P((X=1)|(Y=1)) = \frac{P(Z=1)}{P(Y=1)} \tag{6}$$

Using (2) and (3) in (6) we get

$$P((X=1)|(Y=1)) = \frac{\frac{4}{10}}{\frac{7}{10}} = \frac{4}{7} = \boxed{0.57143} \tag{7}$$

Python Code and Output

This problem has been solved using the following program (link below):
Click here for Python Code
The output of the above code has been shown below:

```
ravi@ravi-Legion-5-Pro-16ACH6H:~/Desktop/Python$ python3 cond.py
The required probability is 0.5714285714285715
ravi@ravi-Legion-5-Pro-16ACH6H:~/Desktop/Python$
```