

ASSIGNMENT 1

CS21BTECH11053

Problem. 6c, ICSE Math Paper (2017):

Using componendo-dividendo again on equation 4, we have

If $\frac{7m+2n}{7m-2n} = \frac{5}{3}$ use properties of proportion to find: $\Rightarrow \frac{m^2 + n^2}{m^2 - n^2} = \frac{64 + 49}{64 - 49} = \frac{\mathbf{113}}{\mathbf{15}}$ (5)

- (i) $\frac{m}{n}$
- (ii) $\frac{m^2+n^2}{m^2-n^2}$

Solution:

We are given,

$$\frac{7m + 2n}{7m - 2n} = \frac{5}{3} \quad (1)$$

From componendo - dividendo, we know

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a + b}{a - b} = \frac{c + d}{c - d} \quad (2)$$

Hence, we have, from equations 1 and 2,

$$\begin{aligned} \frac{(7m + 2n) + (7m - 2n)}{(7m + 2n) - (7m - 2n)} &= \frac{5 + 3}{5 - 3} \\ \Rightarrow \frac{14m}{4n} &= \frac{8}{2} \\ \Rightarrow \frac{7m}{2n} &= \frac{4}{1} \\ \Rightarrow \frac{m}{n} &= \frac{\mathbf{8}}{\mathbf{7}} \end{aligned} \quad (3)$$

From the equation 3, we see that

$$\begin{aligned} \left(\frac{m}{n}\right)^2 &= \left(\frac{8}{7}\right)^2 \\ \Rightarrow \frac{m^2}{n^2} &= \frac{8^2}{7^2} = \frac{64}{49} \end{aligned} \quad (4)$$