

ASSIGNMENT 3

CS21BTECH11053

Abstract—From NCERT Mathematics Class 11, Chapter 16 (Probability), Exercise 16.3

Problem (Ex:16.3, Question:2). A coin is tossed twice, what is the probability that atleast one tail occurs?

Solution:

1 SOLUTION

We will use binomial distribution here as we are repeating a Bernoulli trial with two possible outcomes *Heads* or *Tails*. We will consider getting *Tails* as a success and *Heads* as a failure. We will define random variable X representing the number of successes. Given we have a total of $n = 2$ trials, $X \in \{0, 1, 2\}$.

Assuming a fair coin, the probability of success (getting *Tails*) in a single trial $p = 0.5$.

The probability that $X = i$ is given by

$$P(X = i) = {}^nC_i \times p^i \times (1 - p)^{n-i} \quad (1)$$

One can define cumulative probability $P(X \leq i)$ as

$$P(X \leq i) = \sum_{r=i}^n {}^nC_r \times p^r \times q^{n-r} \quad (2)$$

Let us define another random variable Y where $Y = 0$ when $X < 1$ and $Y = 1$ when $X \geq 1$. Random variable Y represents a Bernoulli distribution. Hence

$$P(Y = 0) + P(Y = 1) = 1 \quad (3)$$

Note that

$$P(Y = 0) = P(X < 1) = P(X = 0) \quad (4)$$

From (1) and (4)

$$P(Y = 0) = P(X = 0) = {}^nC_0 \times p^0 \times (1 - p)^n \quad (5)$$

Substituting $n = 2$ and $p = 0.5$ into (5)

$$P(Y = 0) = {}^2C_0(0.5)^0(1 - 0.5)^2 = 0.25 \quad (6)$$

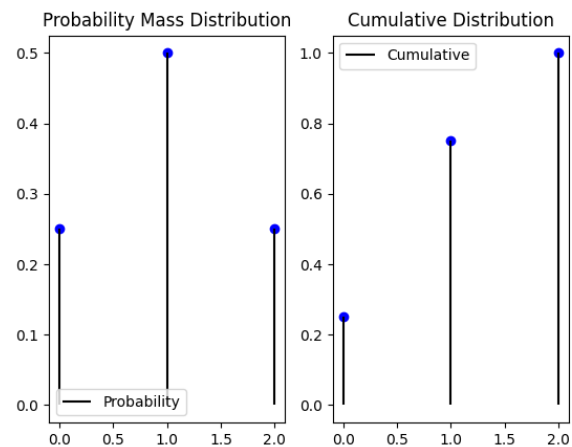
Using (4) and (6)

$$P(Y = 1) = 1 - P(Y = 0) = 1 - 0.25 = \boxed{0.75} \quad (7)$$

\therefore The probability of getting atleast one Tail while tossing a coin twice is **0.75**.

2 GRAPH AND CODE OUTPUT

Graph: The probability mass and cumulative distribution are plotted below.



Code Output:

```
ravi@ravi-Legion-5-Pro-16ACH6H:~/Desktop/Python$ python3 binomplot.py
Required Probability = 0.75
ravi@ravi-Legion-5-Pro-16ACH6H:~/Desktop/Python$
```