ASSIGNMENT 1

CS21BTECH11053

***Abstract*—From ICSE 2018 Class 12 Mathematics Examination**

Problem (19.b)**.** *Find the coeffecient of correlation from the regression lines*

$$x - 2y + 3 = 0 \tag{1}$$
$$4x - 5y + 1 = 0 \tag{2}$$

Solution:

Given data as n ordered pairs

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, ..., \begin{pmatrix} x_n \\ y_n \end{pmatrix} \tag{3}$$

Regression line of y on x in parametric form is

$$\begin{pmatrix} 0 \\ c_{yx} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ b_{yx} \end{pmatrix} \tag{4}$$

Regression line of x on y in parametric form is

$$\begin{pmatrix} c_{xy} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} b_{xy} \\ 1 \end{pmatrix} \tag{5}$$

We shall assume that (1) is the regression line of y on x and (2) is the regression line of x on y.

Consider any equation of the form

$$y = mx + c \tag{6}$$

Substituting $x = 0$ in (6) we get $y = c$. This point can be represented in vector form as follows

$$\mathbf{P} = \begin{pmatrix} 0 \\ c \end{pmatrix} \tag{7}$$

Given slope m, the direction vector of the line is given as

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{8}$$

Hence from (7) and (8) we write (6) in vector form as

$$\begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{9}$$

From (1) we have

$$y = \frac{x}{2} + \frac{3}{2} \tag{10}$$

From (2) we have

$$y = \frac{4x}{5} + \frac{1}{5} \tag{11}$$

We can tabulate parameters $\mathbf{P}$, $\mathbf{m}$ from lines (10) and (11) as shown in Table (I)

Lines	$\mathbf{P}$	$\mathbf{m}$
(10)	$\begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}$	$\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$
(11)	$\begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix}$	$\begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix}$

TABLE I
TABLE OF PARAMETERS

Similar to conversion of (6) to (9) we can write vector form of (10) as

$$\begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{12}$$

We can similarly write vector form of (11) as

$$\begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \tag{13}$$
$$= \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} - \frac{1}{4} \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} + (\lambda + \frac{1}{4}) \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \tag{14}$$
$$= \begin{pmatrix} 0 \\ \frac{1}{5} \end{pmatrix} - \begin{pmatrix} \frac{1}{4} \\ \frac{1}{5} \end{pmatrix} + (\mu) \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \tag{15}$$
$$= \begin{pmatrix} \frac{-1}{4} \\ 0 \end{pmatrix} + (\frac{4}{5})(\frac{5}{4})(\mu) \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \tag{16}$$
$$= \begin{pmatrix} \frac{-1}{4} \\ 0 \end{pmatrix} + (\frac{4\mu}{5}) \begin{pmatrix} \frac{5}{4} \\ 1 \end{pmatrix} \tag{17}$$
$$= \begin{pmatrix} \frac{-1}{4} \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} \frac{5}{4} \\ 1 \end{pmatrix} \tag{18}$$

(Note that we have arbitrarily defined $\mu = \lambda + \frac{1}{4}$ and $\gamma = \frac{4\mu}{5}$)

Comparing (4) with (12) and (5) with (18), we evaluate regression coefficents b_{yx} and b_{xy} as

$$b_{yx} = \frac{1}{2} \tag{19}$$

$$b_{xy} = \frac{5}{4} \tag{20}$$

Given b_{yx} and b_{xy}, we can find the coefficient of correlation r as

$$r = \pm\sqrt{b_{yx} \times b_{xy}} \tag{21}$$

Note that b_{yx}, b_{xy} and r have the same sign and $|r| \leq 1$.

From (19), (20), (21)

$$r = \pm\sqrt{\frac{1}{2} \times \frac{5}{4}} = \pm\sqrt{\frac{5}{8}} \tag{22}$$

Since $b_{yx} > 0$ and $b_{xy} > 0$, $r > 0$. Also note that $|r| \leq 1$. Hence our initial assumption was correct.

$$\therefore r = \sqrt{\frac{5}{8}} \tag{23}$$