AI1110: Probability and Random Variables

Assignment 6

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Outline

Problem

Find the probability distribution of

1. number of heads in two tosses of a coin.
2. number of tails in the simultaneous tosses of three coins.
3. number of heads in four tosses of a coin.

Binomial Distributions

Take a Bernoulli trial with parameter p represented by random variable X, where probability that $X = 1$ is p and $X = 0$ is $1 - p$. If this trial is repeated n times, we obtain a binomial distribution. The probability that $X = i$, that is, $X = 1$ occurs exactly i times out of n trials is given by

Binomial Distribution for $X = i$

$$\Pr(X = i) = \binom{n}{i} \times p^i \times (1 - p)^{n-i} \tag{1}$$

Number of *heads* in 2 tosses of a coin

Assume X_1 to be a random variable representing the number of *heads* in $n = 2$ trials. Using (1) we can define the probability mass function of the random variable X_1 as follows

$$\Pr(X_1 = k) = \begin{cases} \frac{1}{4}, & k = 0 \\ \frac{1}{2}, & k = 1 \\ \frac{1}{4}, & k = 2 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Refer next slide for graph (1)

Graph for Part 1

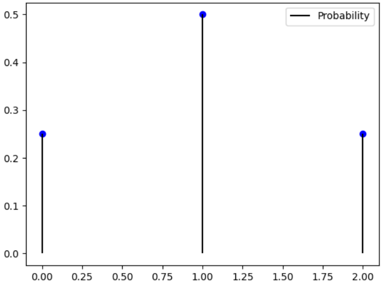

Figure: PMF for Part 1

Number of *tails* in simultaneous tosses of 3 coins

This is the same as tossing a coin 3 times. Assume X_2 to be a random variable representing the number of *tails* in $n = 3$ trials. Using (1) we can define the probability mass function of the random variable X_2 as follows

$$\Pr(X_2 = k) = \begin{cases} \frac{1}{8}, & k = 0 \\ \frac{3}{8}, & k = 1 \\ \frac{3}{8}, & k = 2 \\ \frac{1}{8}, & k = 3 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Refer next slide for graph (2)

Graph for Part 2

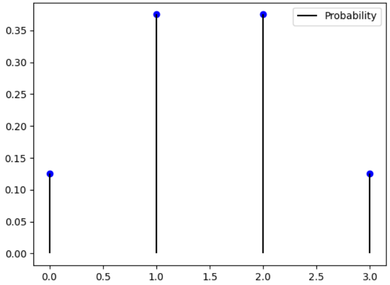

Figure: PMF for Part 2

Number of *heads* in 4 tosses of a coin

Assume X_3 to be a random variable representing the number of *heads* in $n = 2$ trials. Using (1) we can define the probability mass function of the random variable X_3 as follows

$$\Pr(X_3 = k) = \begin{cases} \frac{1}{16}, & k = 0 \\ \frac{1}{4}, & k = 1 \\ \frac{3}{8}, & k = 2 \\ \frac{1}{4}, & k = 3 \\ \frac{1}{16}, & k = 4 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Refer next slide for graph (3)

Graph for Part 3

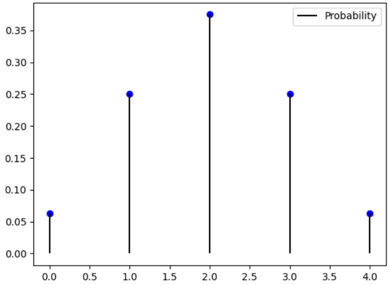

Figure: PMF for Part 3