Foundation of Data Science
Lecture 8, Module 1
Fall 2022

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Today

- Time-series Analyses
- Regression and lagged data

Time Series Discussions

- Overview
- Basic definitions
- Time domain

Why Time Series Analysis?

- Sometimes the concept we want to learn is the relationship between points in time

What is a time series?

Time series:

a sequence of measurements over time

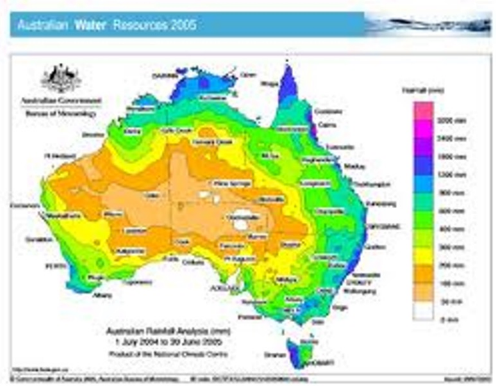
A sequence of random variables

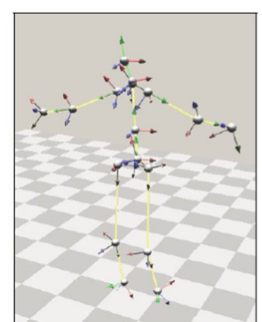
$x_1, x_2, x_3, \ldots$

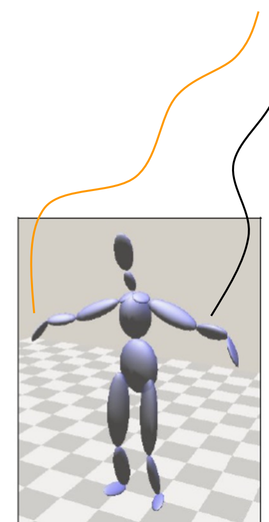
Time Series Examples

Definition: *A sequence of measurements over time*

- ***Finance***
- ***Social science***
- ***Epidemiology***
- ***Medicine***
- ***Meteorology***
- ***Speech***
- ***Geophysics***
- ***Seismology***
- ***Robotics***



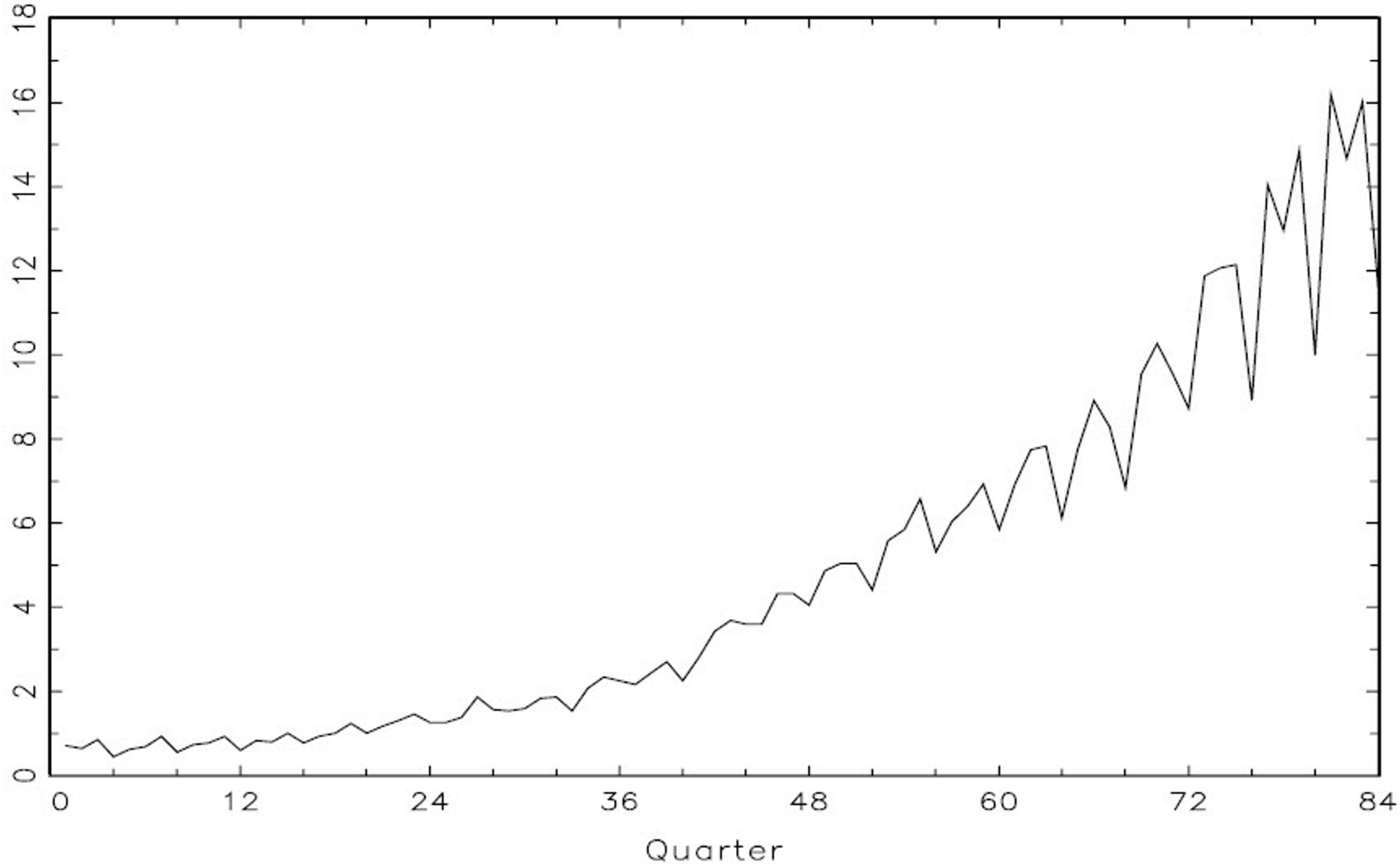




Three Approaches

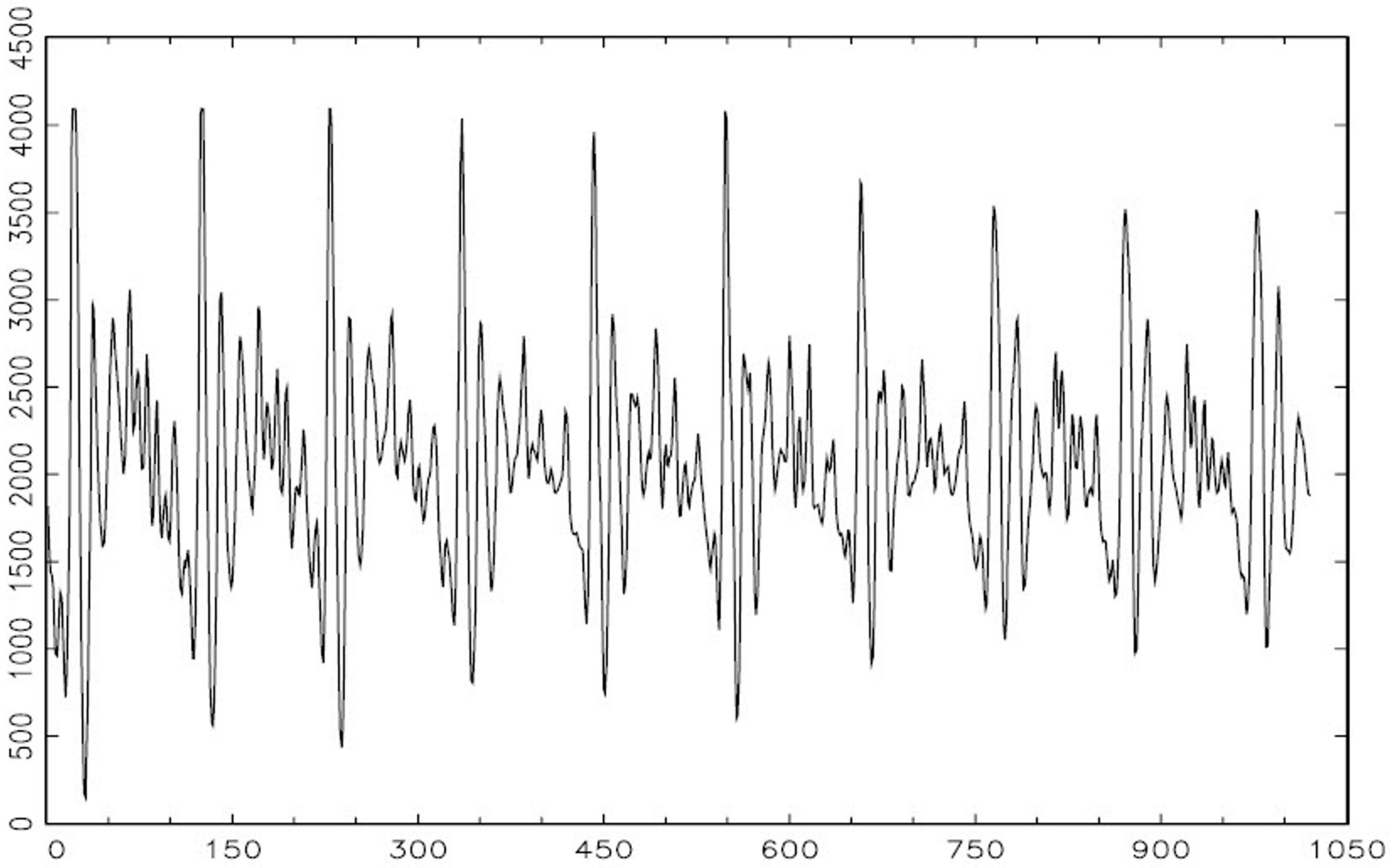
- Time domain approach
  - Analyze dependence of current value on past values
- Frequency domain approach
  - Analyze periodic sinusoidal variation (sine wave)
- State space models
  - Represent state as collection of variable values
  - Model transition between states

Sample Time Series Data



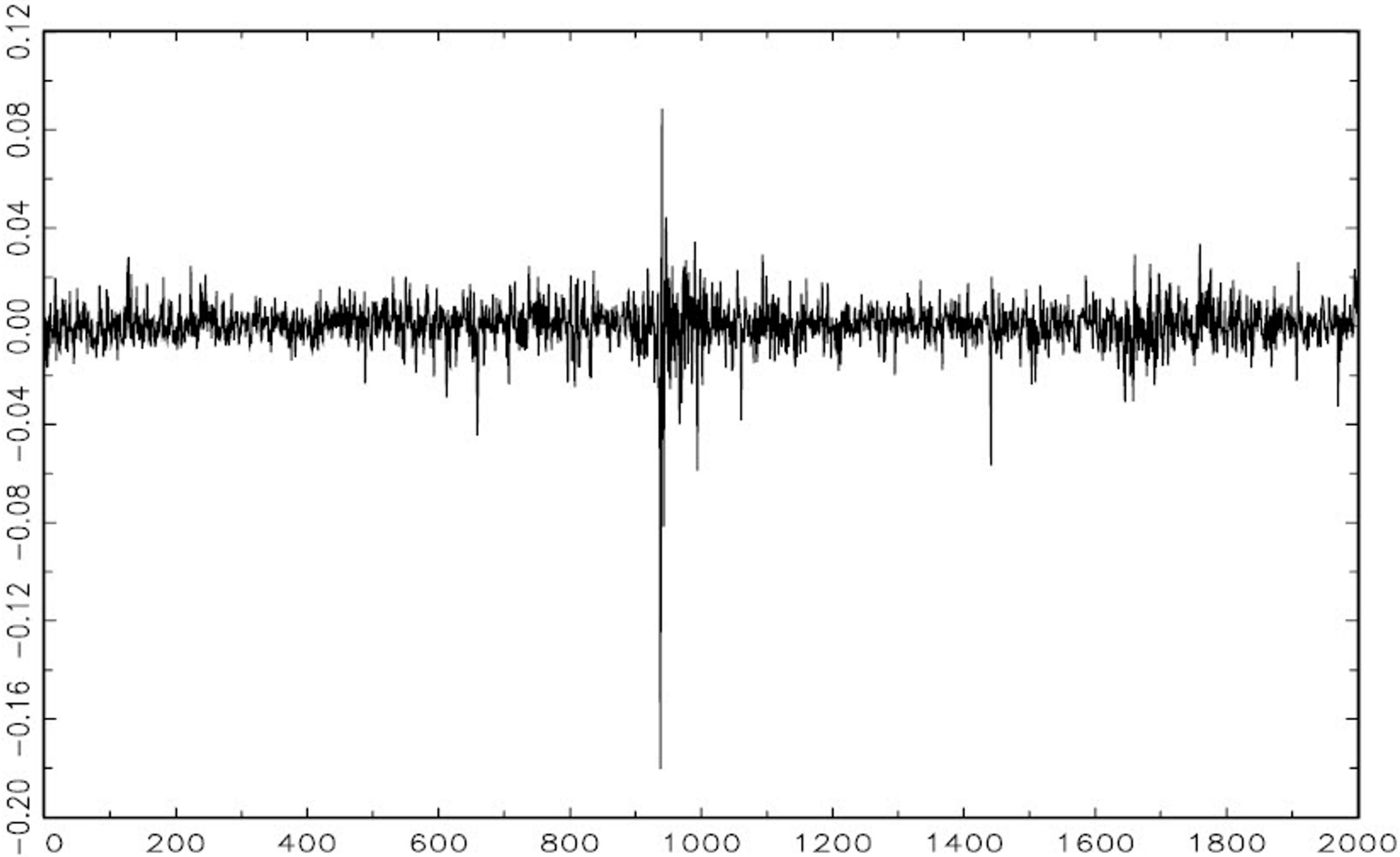
Johnson & Johnson quarterly earnings/share, 1960-1980

Sample Time Series Data



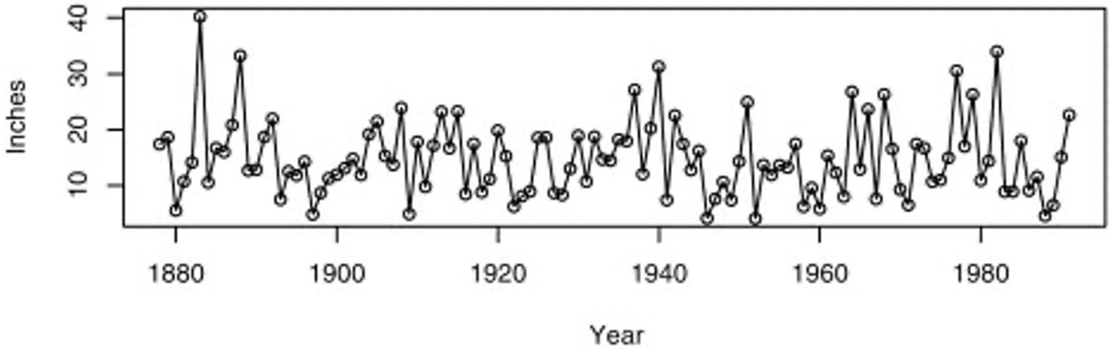
Speech recording of “aaa…hhh”, 10k pps

Sample Time Series Data

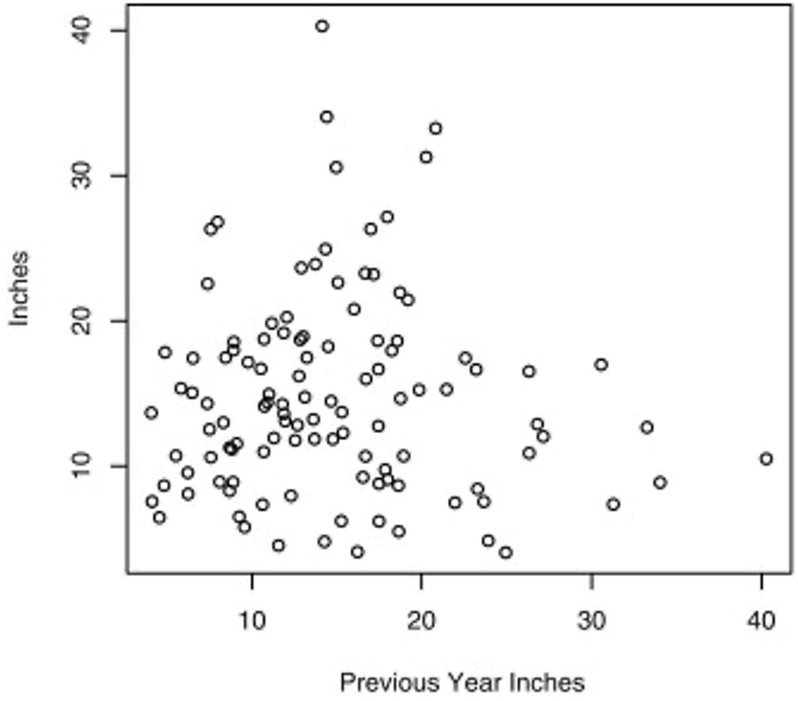


NYSE daily weighted market returns 2/2/84 - 12/31/92

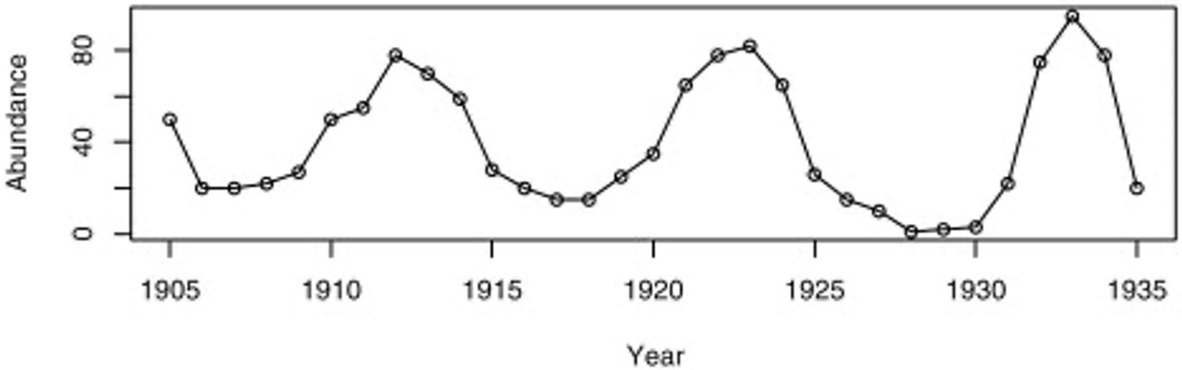
Not all time data will exhibit strong patterns



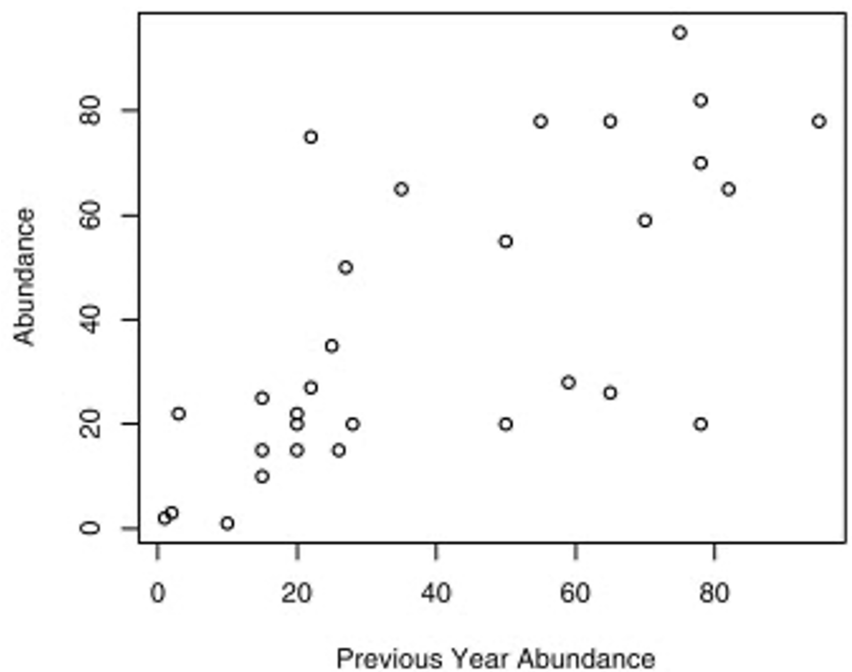
LA annual rainfall



…and others will be apparent



Canadian Hare counts



Time Series Discussions

- Overview
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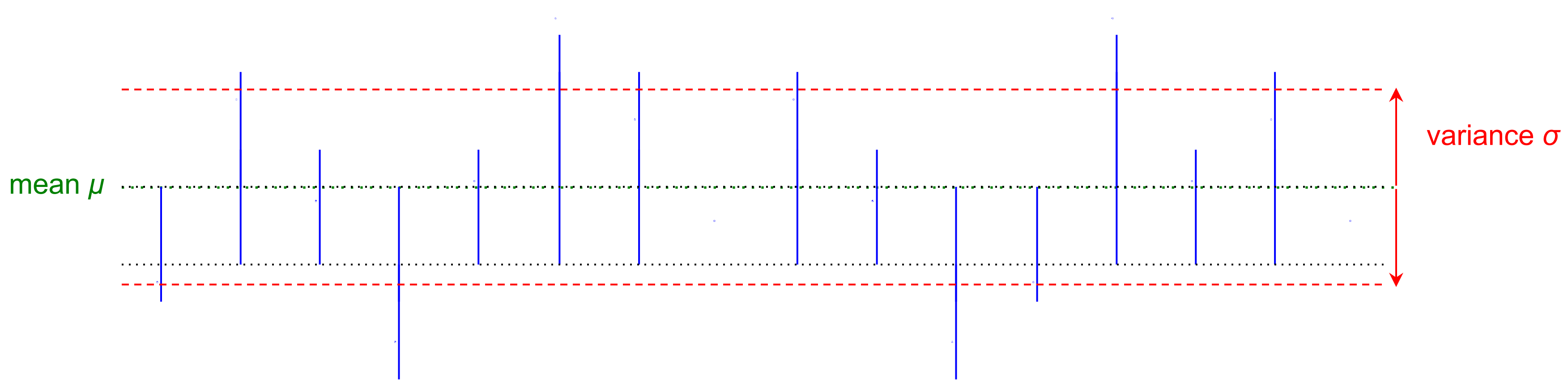
Original Definitions - Random Variables

- Mean $\mu \equiv \mathsf{E}[x_t] := \frac{1}{N}\sum_{t=1}^{N} x_t$

- Variance $\sigma^2 \equiv \mathsf{Var}[x_t] := \frac{1}{N}\sum_{t=1}^{N}(x_t - \mu)^2$



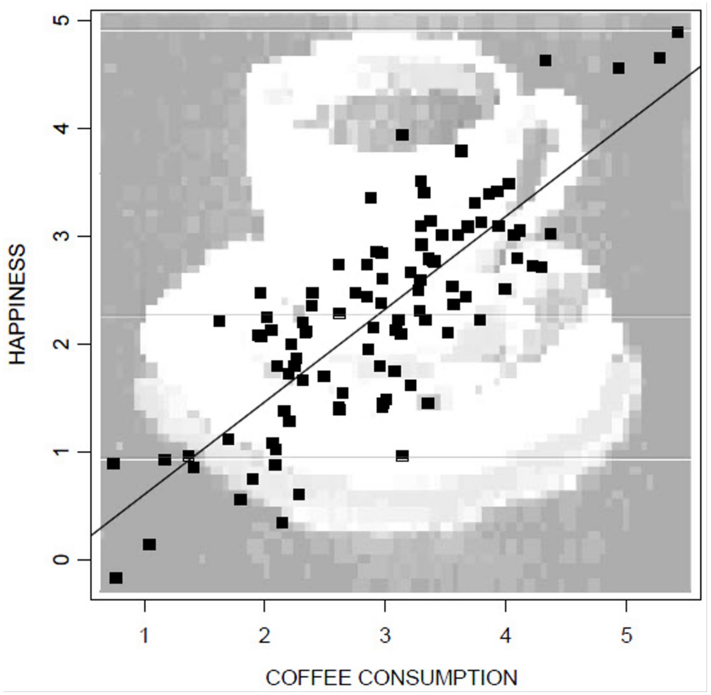
Original Definitions - Random Variables

- Covariance

$$Cov(X,Y) = \sum_{i=1}^{N} \frac{(x_i - \mu_x)(y_i - \mu_y)}{N}$$

- Correlation

$$Cor(X,Y) = r = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$



Redefined for Time

Mean function

$$\mu_X(t) = E(X_t) \quad for\, t = 0, \pm 1, \pm 2, \ldots$$

- Ergodic process: if the mean computed over any time *t* is the same as the ensemble mean, then the process is ergodic
  - Example: tossing a fair coin multiple times (random variable X_t can be 0 or 1)

Redefined for Time

- Autocovariance - how X_t relates to its previous values

$$\gamma_X(h) = Cov(X_{t+h}, X_t)$$

lag

- Autocorrelation

$$\rho_X(h) \equiv \frac{\gamma_X(h)}{\gamma_X(0)} = Cor(X_{t+h}, X_t)$$

Durbin-Watson test provides measures of significance for autocorrelation.

Metrics for Unknown Distributions

Sample Mean function

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t$$

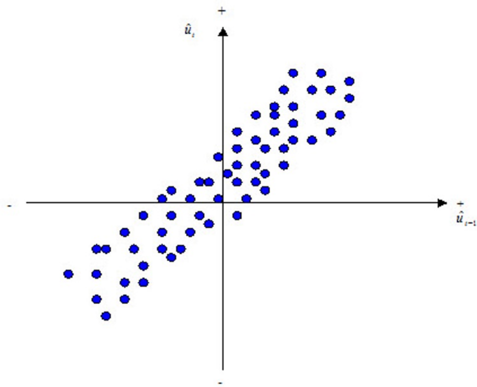
Sample Autocovariance

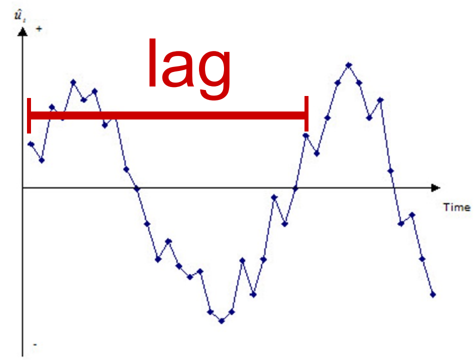
$$\hat{\gamma}(h) := n^{-1}\sum_{t=1}^{n-|h|}(x_{t+|h|} - \bar{x})(x_t - \bar{x}), \quad -n < h < n$$

Sample Autocorrelation

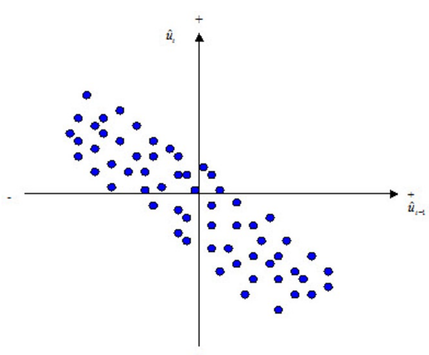
$$\hat{\rho}(h) = \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)}, \quad -n < h < n$$

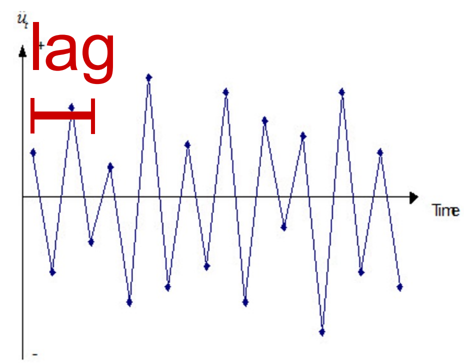
Autocorrelation Examples





Positive



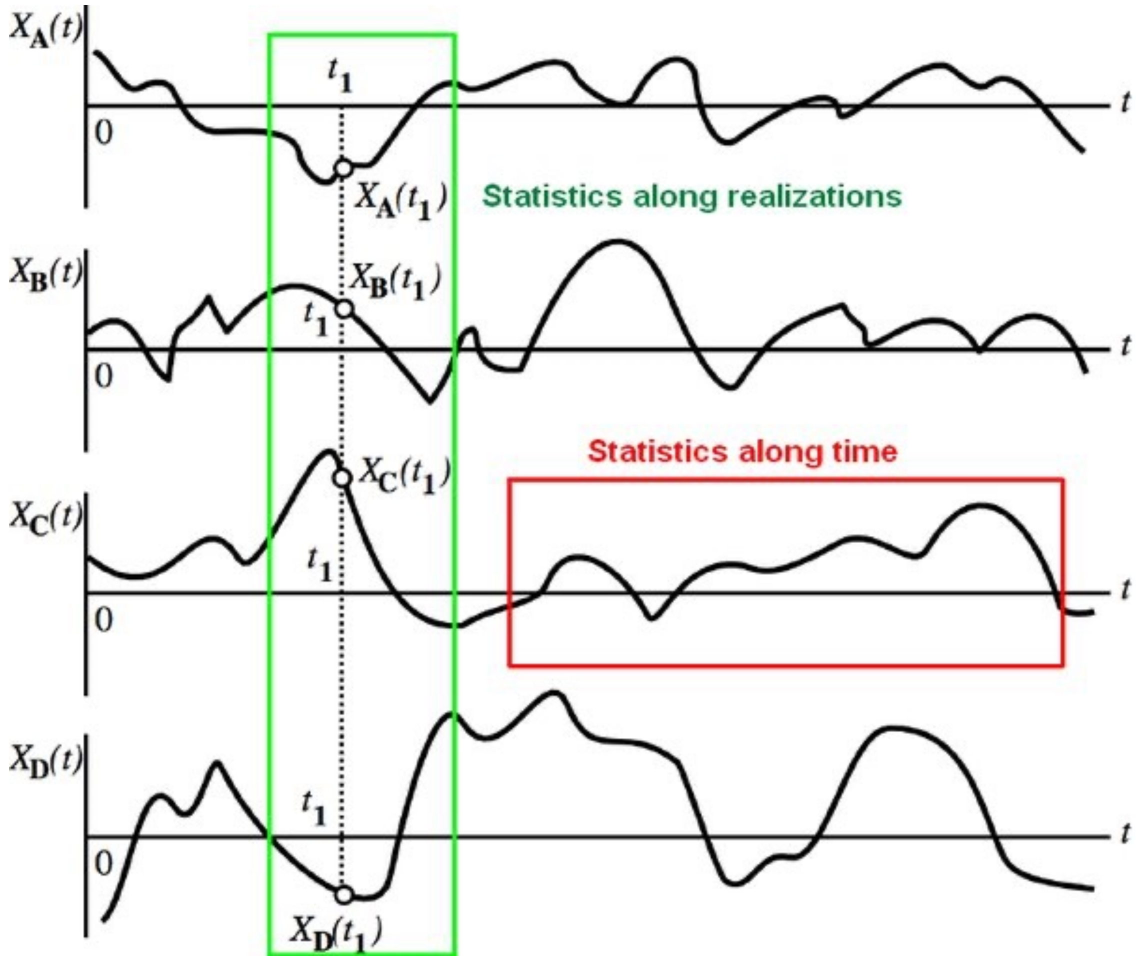


Negative

Stationarity Time Series

- $\{X_t\}$ is stationary if
  - $\mu_X(t)$ is independent of t
  - $\gamma_X(t+h,t)$ is independent of t for each h
- Special case: white noise
  - $\{X_t\}$ is a sequence of **uncorrelated** random variables, each with constant mean and variance
- Stationary series are much easier to forecast with
  - Much of time series analysis involves trying to reduce a complicated series to a stationary one

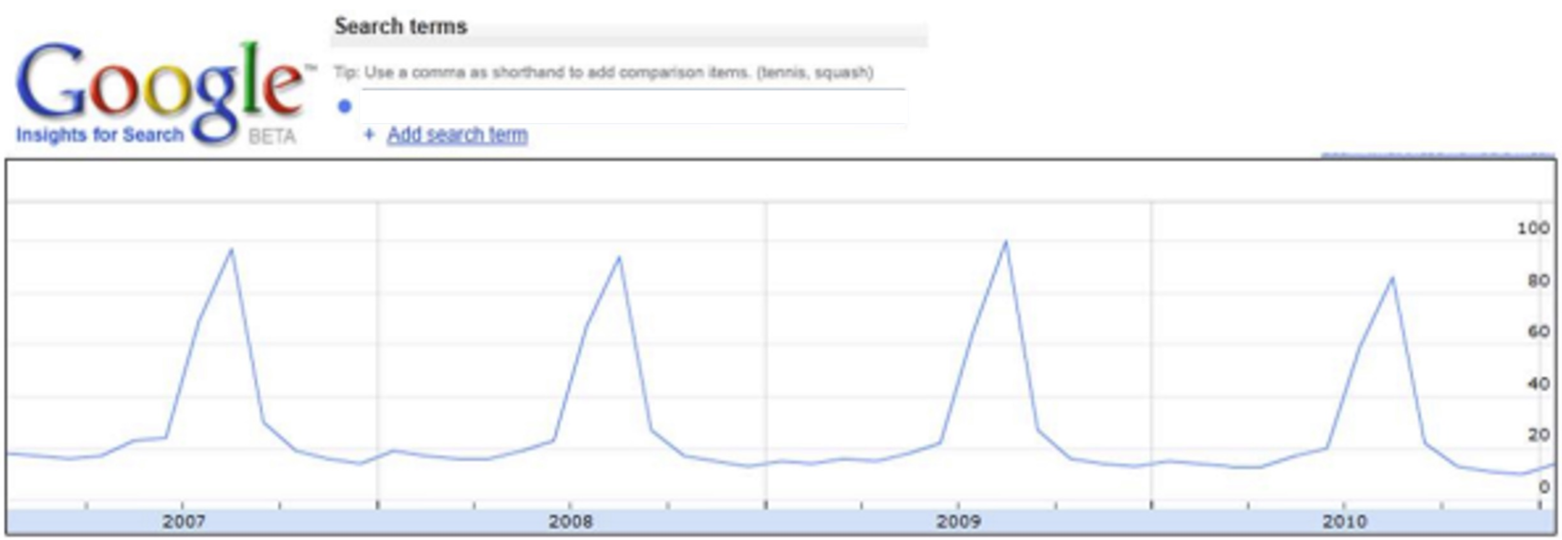
Stationary vs. Ergodic



https://www.quora.com/What-is-the-difference-between-a-stationary-ergodic-and-a-stationary-non-ergodic-process#:~:text=Stationary%20non%2Dergodic%20processes%20do,between%20temporal%20and%20ensemble%20statistics.&text=For%20an%20ergodic%20process%2C%20you,representative%20of%20the%20random%20process.

What if a linear trend does not fit my data?

- Could be no relationship
- Could be too much local variation. In that case:
  - Look at longer-term trends
  - Smooth the data
  - Check for nonlinear relationships



Moving Average

- Compute an average of the last *m* consecutive data points
  - 4-point moving average is

$$\bar{x}_{MA(4)} = \frac{(x_t + x_{t-1} + x_{t-2} + x_{t-3})}{4} \qquad m_t = \sum_{j=-k}^{k} a_j x_{t-j}$$

- Smooths white noise
- Exponential smoothing
  - Higher weights to more recent times

Power Load Data