Foundation of Data Science
Lecture 2, Module 1
Fall 2022

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Getting to Know the Data

- Statistical Descriptions of Data
- Measuring Data Similarity and Dissimilarity
- Summary

Descriptive vs. Inferential Statistics

Refresher!

- **Descriptive:**
  - Describes data you have but can't be generalized beyond that (e.g., median)
  - Scope: Exploratory Data Analysis
- **Inferential:**
  - Enables inference about a certain *population* beyond the data you have, based on *samples of this population* (e.g., t-test)
  - Adequate sampling techniques are key
  - Scope: Machine Learning and Prediction

Descriptive vs. Inferential Statistics

- **Descriptive:** **Focus today!**
  - Describes data you have but can't be generalized beyond that (e.g., median)
  - Scope: Exploratory Data Analysis
- **Inferential:**
  - Enables inference about a certain population beyond the data you have (e.g., t-test)
  - Scope: Machine Learning and Prediction

Basic Statistical Descriptions of Data

- Motivation
  - To better understand the data: central tendency, variation and spread
- Measures of central tendency
  - mean, median, mode
- Measures of dispersion (variation)
  - *max*, *min*, variance, standard deviation

Measuring Central Tendency

- Mean (algebraic measure) (sample vs. population):

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{\sum x}{N}$$

random sample population

Note: *n* is sample size and *N* is population size!

- Weighted arithmetic mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

- Trimmed mean: chopping extreme values

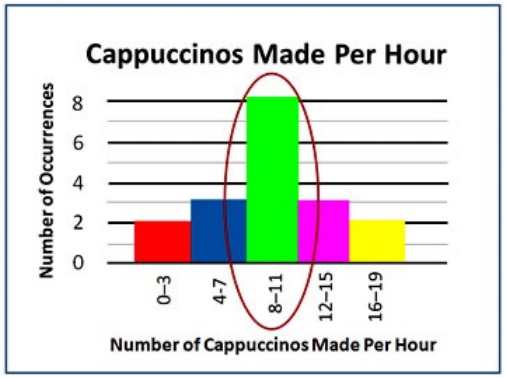
Measuring Central Tendency

- Median:
  - Middle value if odd number of values; average of the middle two values otherwise
  - **Estimated by interpolation (for *grouped data*)**

    Estimated Median class is halfway through groups

    Estimated Median = L + (((n/2) − B)/G) × w
    - **L** is the lower class boundary of the group containing the median
    - **n** is the total number of values
    - **B** is the cumulative frequency of the groups before the median group
    - **G** is the frequency of the median group
    - **w** is the group width

What is the estimated median here?



Measuring Central Tendency

- Mode
  - Value that occurs most frequently in the data
  - Unimodal, bimodal, trimodal, multimodal
  - Empirical formula

$$mean - mode = 3 \times (mean - median)$$

Approximation in the case of a normal distribution with a large sample size

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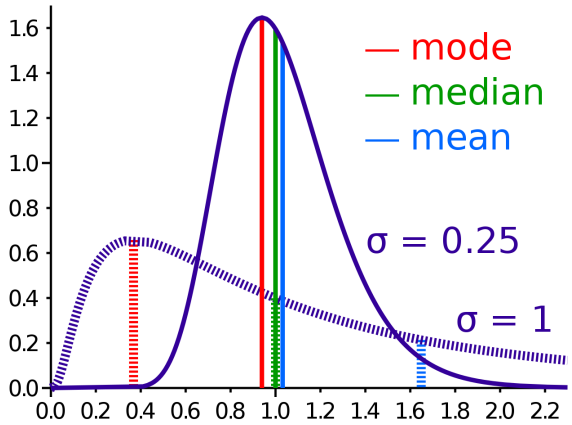


How do mean and median compare?

- When should we use the mean? What are some of its drawbacks?
- And how about the median?

Summary Stats: Limitations

Set A		Set B		Set C		Set D	
X	Y	X	Y	X	Y	X	Y
10	8.04	10	9.14	10	7.46	8	6.58
8	6.95	8	8.14	8	6.77	8	5.76
13	7.58	13	8.74	13	12.74	8	7.71
9	8.81	9	8.77	9	7.11	8	8.84
11	8.33	11	9.26	11	7.81	8	8.47
14	9.96	14	8.1	14	8.84	8	7.04
6	7.24	6	6.13	6	6.08	8	5.25
4	4.26	4	3.1	4	5.39	19	12.5
12	10.84	12	9.11	12	8.15	8	5.56
7	4.82	7	7.26	7	6.42	8	7.91
5	5.68	5	4.74	5	5.73	8	6.89

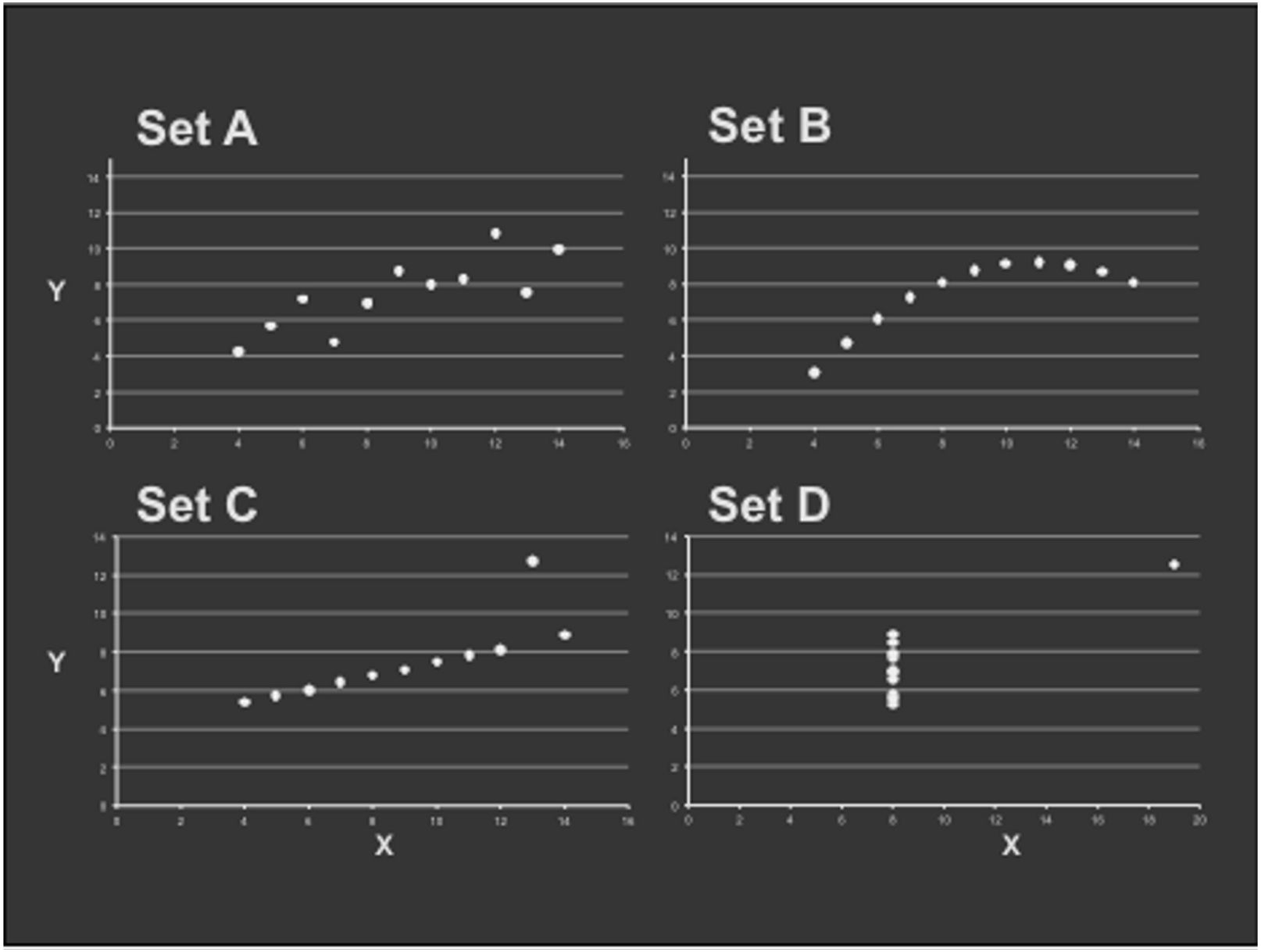
Summary Statistics Linear Regression

$u_X = 9.0$ $\sigma_X = 3.317$ $Y = 3 + 0.5\ X$

$u_Y = 7.5$ $\sigma_Y = 2.03$ $R^2 = 0.67$

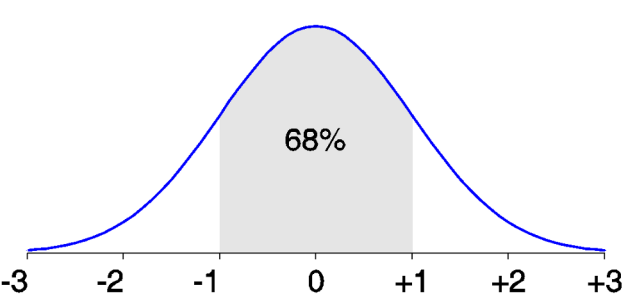
[Anscombe 73]

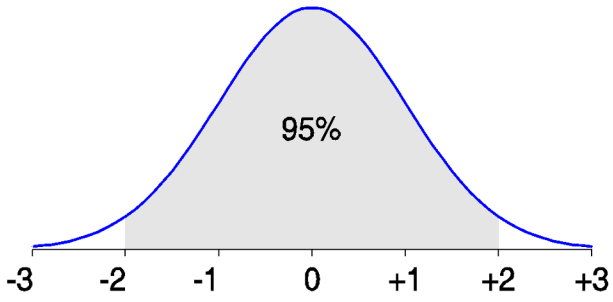
Looking at Data

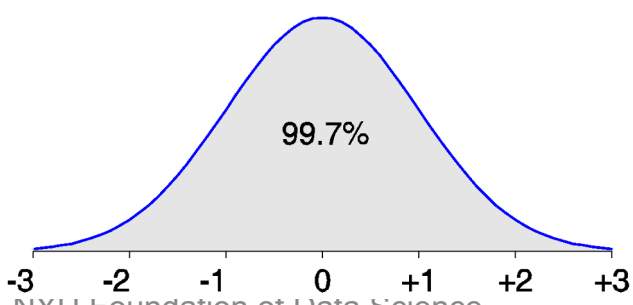


Properties of Normal Distribution Curve

- The normal (distribution) curve
- mean = median = mode
  - From μ–σ to μ+σ: contains about 68% of the measurements (μ: mean, σ: standard deviation)
  - From μ–2σ to μ+2σ: contains about 95% of it
  - From μ–3σ to μ+3σ: contains about 99.7% of it







Measuring the Dispersion of Data

- Variance and standard deviation (*sample vs population)*
  - **Variance** (algebraic, scalable computation)
    - How far a set of numbers is spread out from their mean value

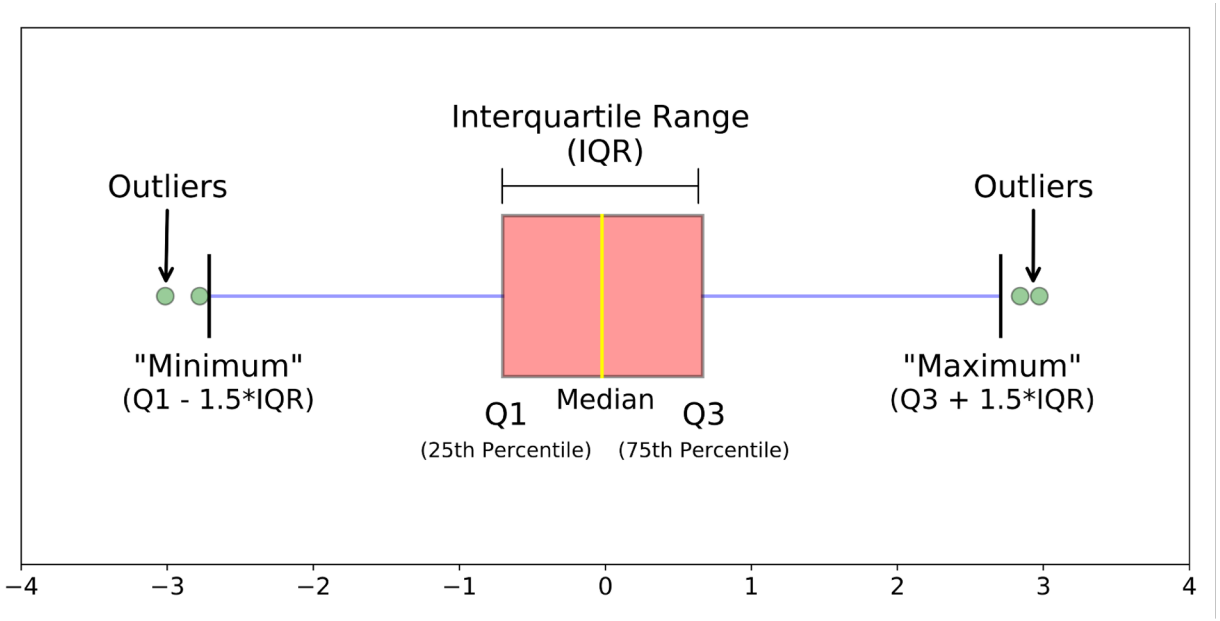
$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \mu)^2$$

  - **Standard deviation** *s (or σ)* is the square root of variance
    - May give more clarity about the deviation of data from a mean

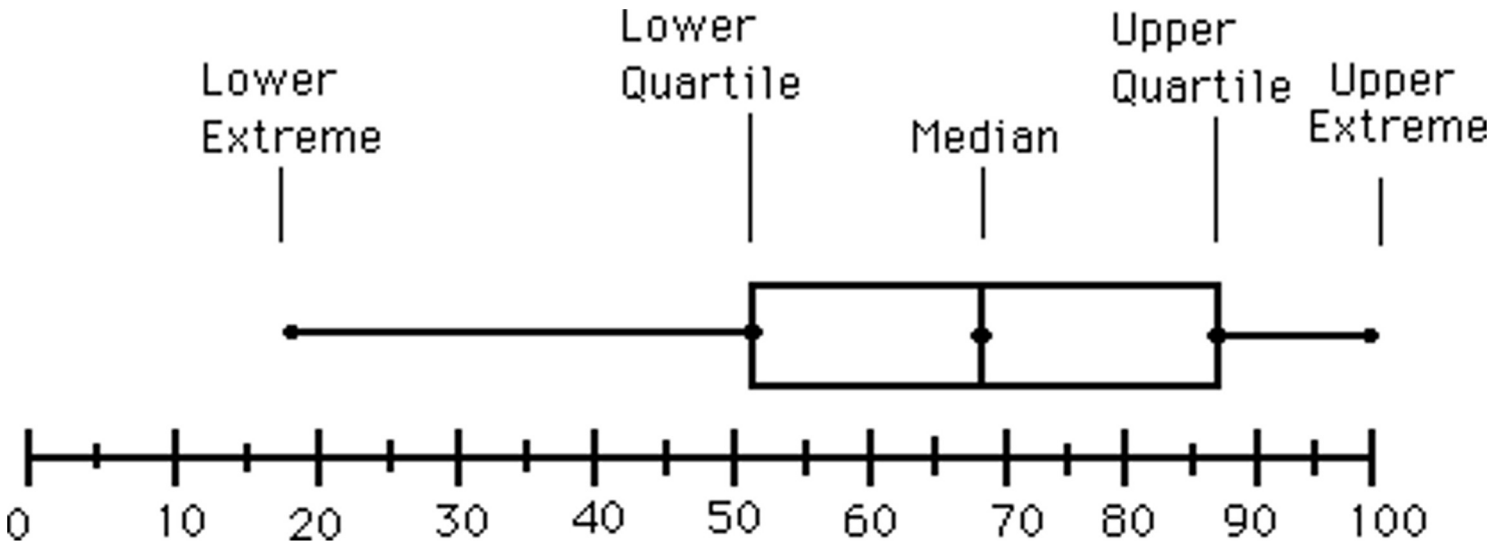
Measuring the Dispersion of Data

- Quartiles, outliers and boxplots
  - **Quartiles**: Q_1 (25th percentile), Q_3 (75th percentile)
  - **Five number summary**: min, Q_1, median, Q_3, max
  - **Boxplot**: ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually



Is the data normally distributed?

- Many statistical tools (mean, t-test, Pearson's correlation etc.) **assume data is normally distributed**.
  - Often not true!
  - Box-and-whisker plot is a good clue



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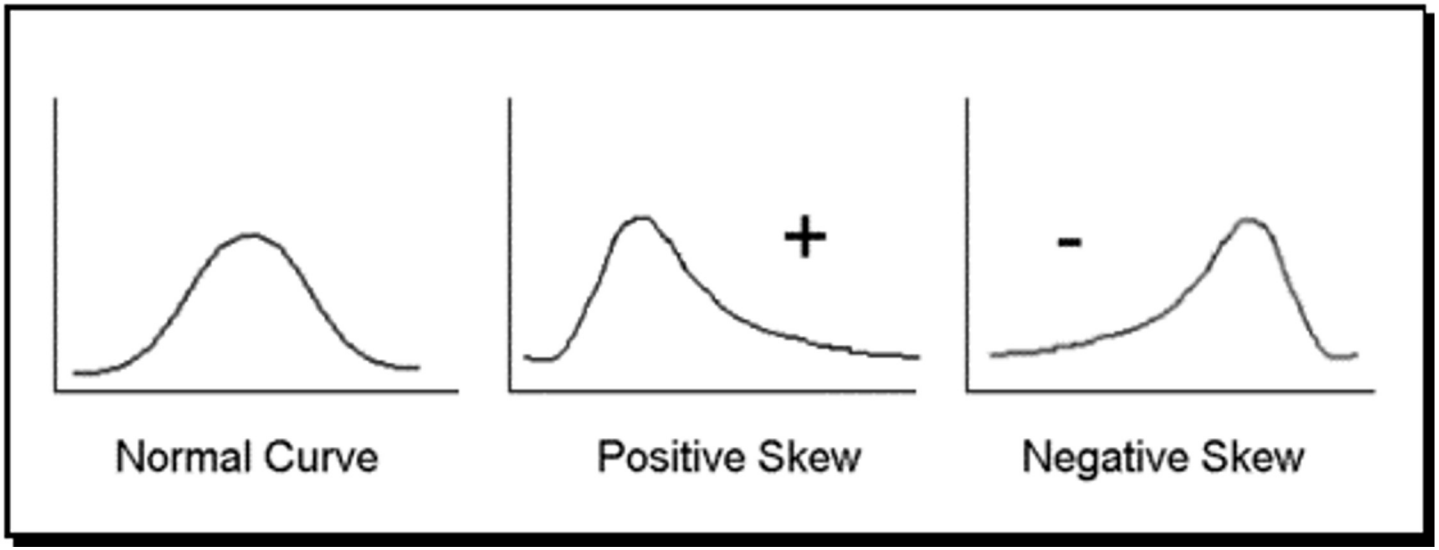
- Whenever it is asymmetric, the data cannot be normal
  - The histogram gives even more information.



Distributions

Some other important distributions:

- **Poisson:** the distribution of counts that occur at a certain *rate.*
  - Number of visits to a website in a fixed time interval.
  - Number of website clicks in an hour.
- **Zipf/Pareto/Yule distributions:** govern the frequencies of different terms in a document, or website visits
  - Long-tails.
- **Binomial/Multinomial:** The number of counts of events out of n trials
  - Coin flips = 6, how many heads did I see?

You should understand the distribution of your data before applying any model!