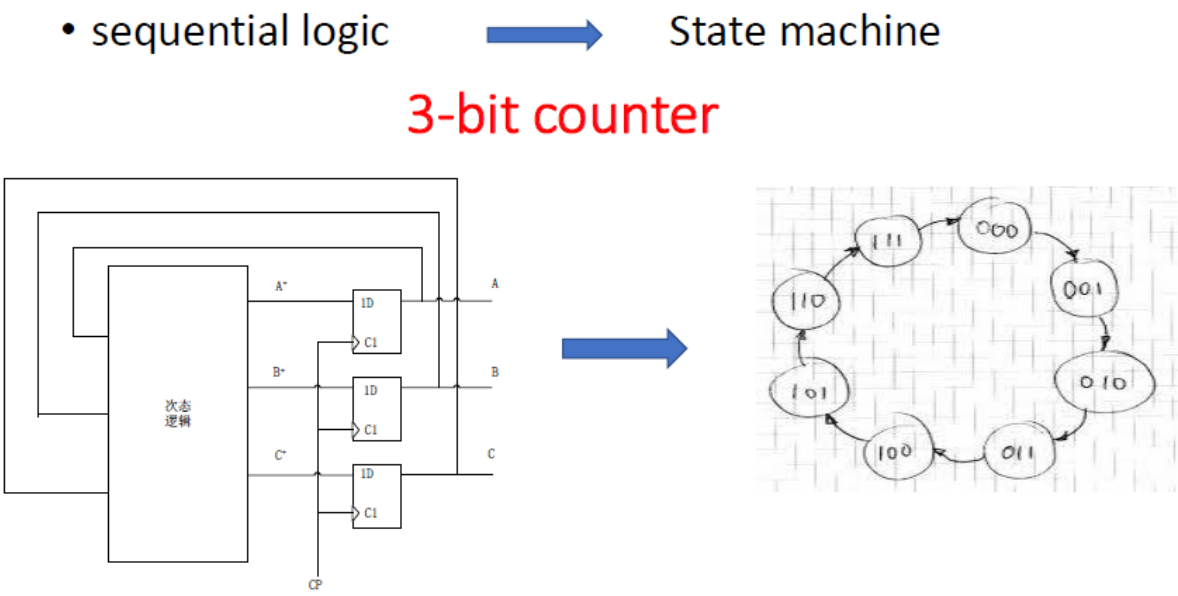
state machine

跟据sequential logic，我们可以将其转换成state machine



定义

- 状态机是用于描述计算机程序或**时序逻辑**的**数学模型。状态与事件**：

1. - **时序逻辑事件**可以被视为一个具有**有限状态**的抽象机器。
   - 在任意时间，状态机只能处于**一种状态。**
2. **状态转换 (State Transition)**：
   - 状态机从一个状态转换到另一个状态的过程由**触发事件**或**条件**引发。

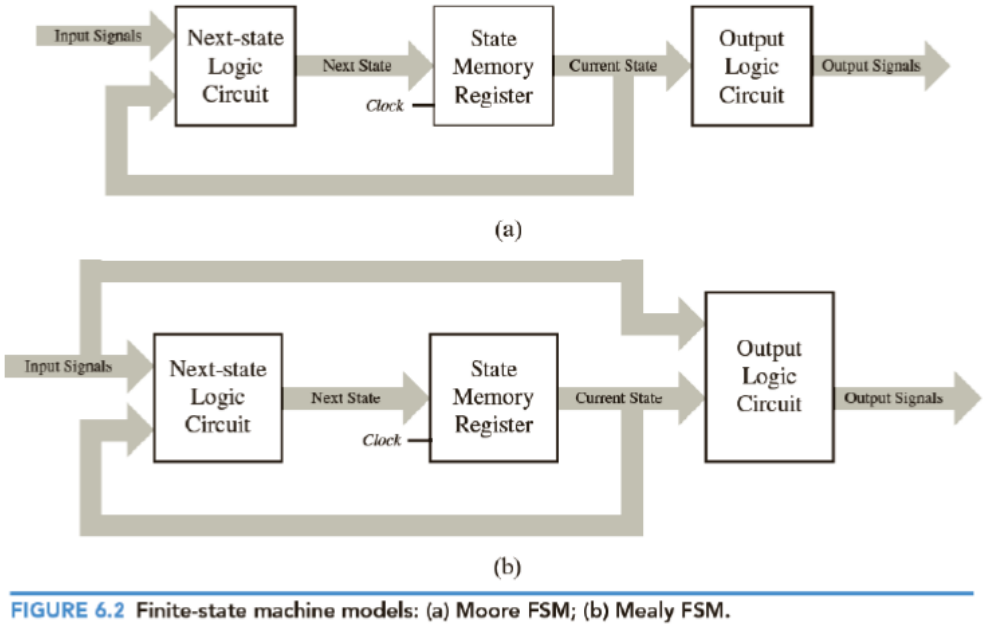
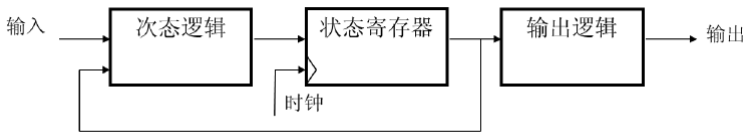
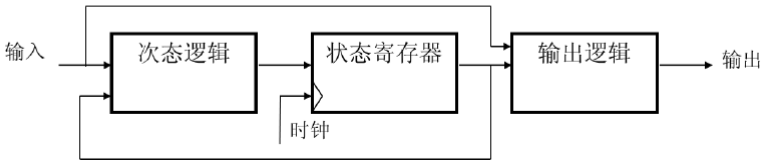


FIGURE 6.2 Finite-state machine models: (a) Moore FSM; (b) Mealy FSM.

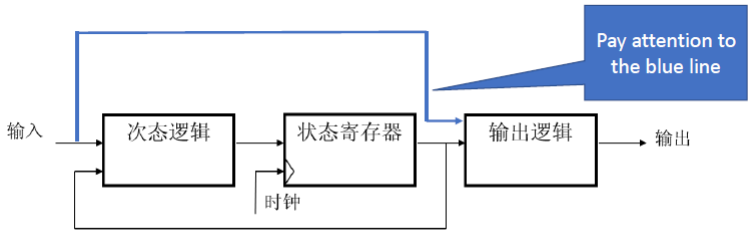
- Moore machine
  - The output depends on the present state only， the output is solely a function of the present state.
  - Diagram for Moore machine：



- Mealy Machine
  - The output depends on both the present state and the input， the output is a function of the present state and of input.
  - Mealy machine is more complicated than Moore machine.
  - Diagram for Mealy machine



- Moore machine V. S. Mealy machine：
  - Both machines are triggered by a single clock
  - The output of Moore machine is solely a function of present state, and independent of input;
  - The output of Mealy machine depends on both present state and input.



关键区别

特性	Moore 状态机	Mealy 状态机
输出依赖	仅依赖**当前状态**	依赖**当前状态和输入**
输出变化时机	在状态改变时更新	输入变化时输出也会立即变化
设计复杂度	较简单，输出与状态直接相关	较复杂，需考虑输入与状态的关系

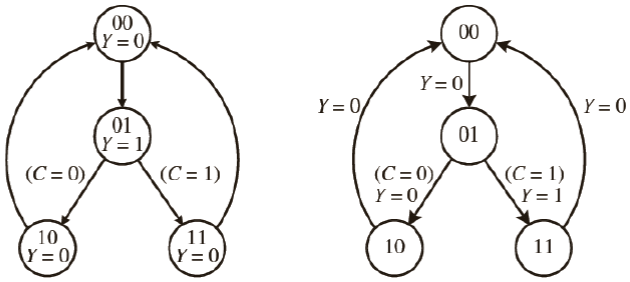
总结

- **Moore 机**：输出仅与当前状态相关，设计简单且稳定。
- **Mealy 机**：输出与当前状态和输入相关，反应更快但设计更复杂。

选择哪种状态机?

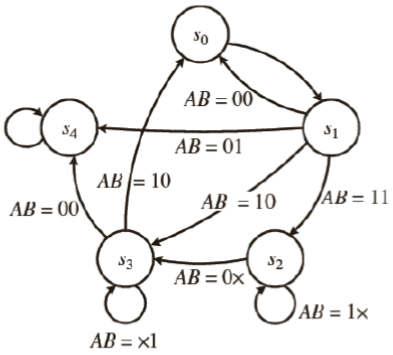
- **Moore 机**：设计简单，输出稳定，状态数量更多。
- **Mealy 机**：设计灵活，反应更快，状态数量更少。

- Sample state diagrams



Moore FSM　　Mealy FSM

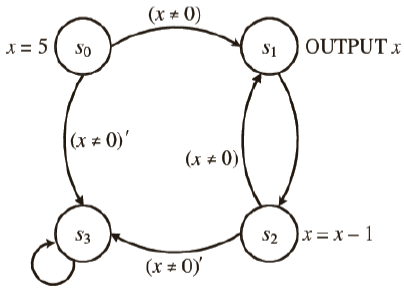
- Sample state diagrams



FSM with two input signals

- Example: Converting pseudocode to a state diagram

```
x = 5
WHILE (x ≠ 0){
   OUTPUT x
   x = x - 1
   }
```



s_0为初始状态，进入while循环，首先有个判断，$x \neq 0$，判断结束后来到s_1，之后output输出，之后来到s_2，此刻$x = x - 1$，这个时候正常情况下会一直进行循环，于是还有一个箭头指回s_1；当$x \neq 0$这个条件不被满足的时候，就会进入下一个状态，也就是s_3，这里s_3有个箭头指向其本身，而且没有从s_3出发的去往别的状态的箭头，则代表这个就是最终的状态，不可循环不可逆。最后s_0也有一个箭头指向s_3，这里就是为了迎接x在一开始就等于0的情况。

Steps for the analysis of FSM Machines

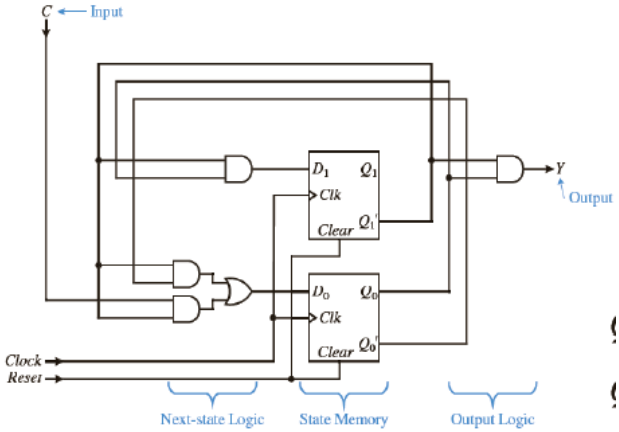
The steps for the analysis of FSM circuits:

1. **Derive the next-state equations**:

- From the combinational next-state logic circuit, determine the equations that describe the next state based on the current state and inputs.

2. **Derive the next-state table**:
   - Use the next-state equations to create the next-state table, showing transitions between states for all possible inputs.
3. **Derive the output equations**:
   - From the combinational output logic circuit, determine the equations that describe the outputs based on the current state (Moore) or the current state and inputs (Mealy).
4. **Derive the output table**:
   - Use the output equations to create a table describing the outputs for each state and input combination.
5. **Draw the state diagram**:
   - Using the next-state table and output table, create the state diagram to visually represent the FSM's behavior.

Next-State Equations



根据图片，我们可以发现：

$$D_1 = Q_1'Q_0$$
$$D_0 = Q_1'Q_0' + CQ_1'$$

于是，可以写出

$$Q_{1next} = D_1 = Q_1'Q_0$$
$$Q_{0next} = D_0 = Q_1'Q_0' + CQ_1'$$

Next-State Table

根据上面的Next-State Equations可以画出Next-State Table

CQ_1Q_0	$Q_{1next}Q_{0next}$
000	01
001	10
010	00
011	00
100	01
101	11
110	00
111	00

Truth table

Current State Q_1Q_0	Next State $Q_{1next}Q_{0next}$	
	$C = 0$	$C = 1$
00	01	01
01	10	11
10	00	00
11	00	00

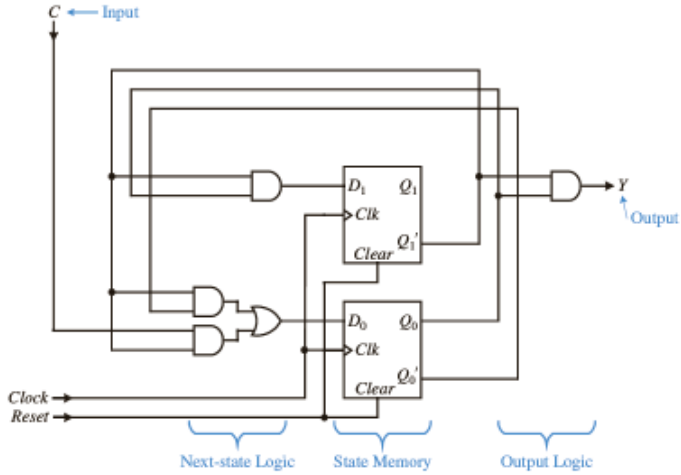
Next-state table

左边的图：左侧一列是所有涉及到的输入（当前），右边是涉及到的要变化的变量（next状态）

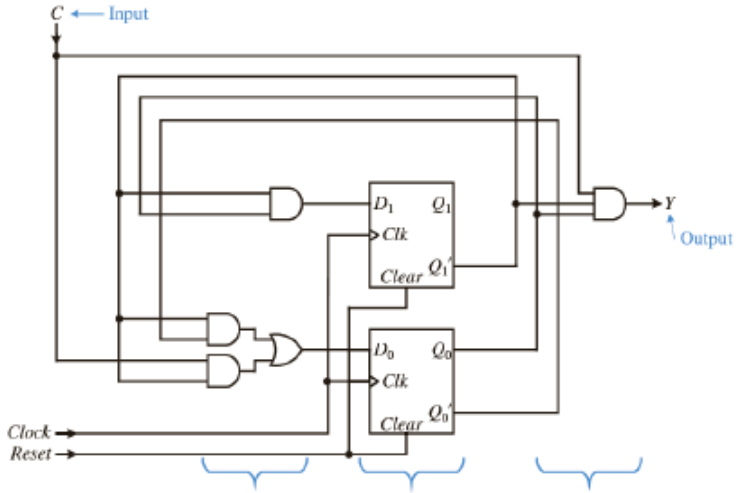
右边的图：左侧一列是当前状态，右侧由于有不随着状态机变化的变量C，于是分成两列两种情况

Output equations

这里就会分成两种情况，分别是Moore circuit和Mealy circuit，前者的输出不会和输入直接相关，后者会和输入相关。



Moore circuit



Mealy circuit

左: $Y = CQ_1'Q_0$

右: $Y = Q_1'Q_0$

Output Table

跟据上述两个output equations

$$Y = Q_1'Q_0 \qquad Y = CQ_1'Q_0$$

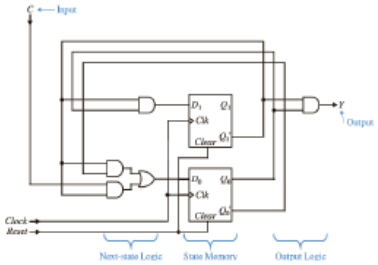
Current State Q_1Q_0	Output Y
00	0
01	1
10	0
11	0

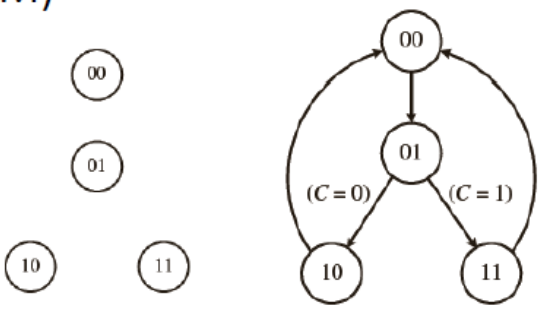
Current State Q_1Q_0	Output Y	
	$C = 0$	$C = 1$
00	0	0
01	0	1
10	0	0
11	0	0

State Diagram

跟据上述的两个Table，画出state diagram

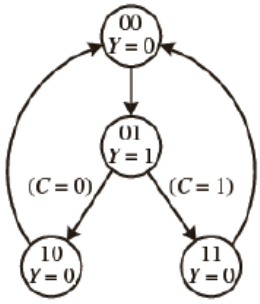
• State Diagram (Moore FSM)





Current State Q_1Q_0	Next State $Q_{1next}Q_{0next}$	
	$C = 0$	$C = 1$
00	01	01
01	10	11
10	00	00
11	00	00

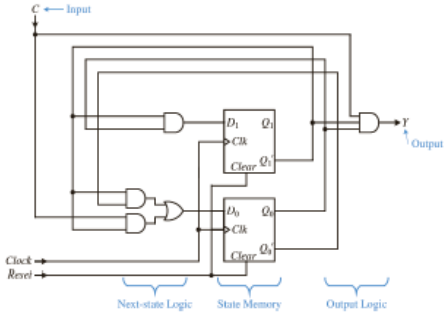
Current State Q_1Q_0	Output Y
00	0
01	1
10	0
11	0

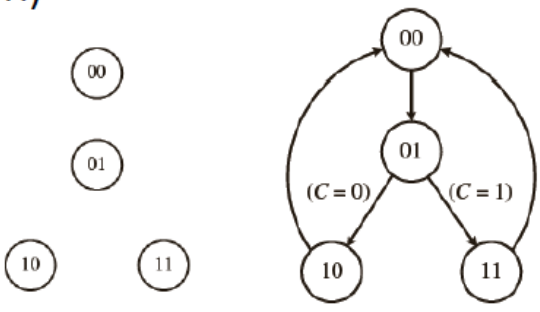


首先看到moore FSM，一共四种状态，就是四个圈，里面放Q_1Q_0，具体来说，里面放的是状态机的内部存储变量。之后，画出它们之间的转换顺序，用箭头表示，不同C的输入如果影响到Q_1Q_0则用不同的箭头分开来表示，然后表示一下C在不同情况下取得值。最后，将输出的Y也填写到圈中，整个过程就算完成。

Analysis of Finite-State Machines

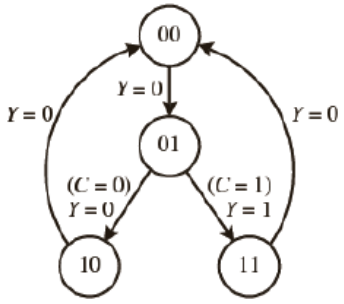
• State Diagram (Mealy FSM)





Current State Q_1Q_0	Next State $Q_{1next}Q_{0next}$	
	$C = 0$	$C = 1$
00	01	01
01	10	11
10	00	00
11	00	00

Current State Q_1Q_0	Output Y	
	$C = 0$	$C = 1$
00	0	0
01	0	1
10	0	0
11	0	0



Mealy机相比于Moore机的结果会多一个涉及到的变量，但是也是满足一个状态内对应一种输出的关系，与Moore机思路相似。

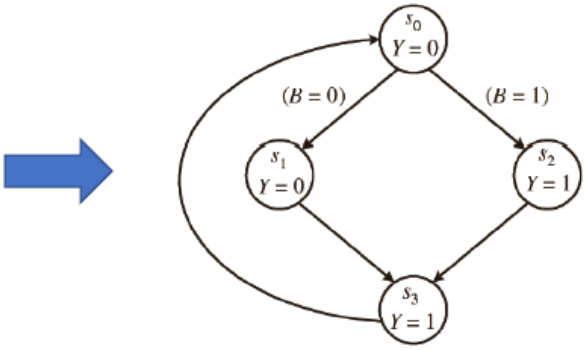
FSM电路综合的步骤(synthesis of circuits)

1. **根据功能描述绘制状态图**：
   - 从电路的功能描述出发，绘制出状态图，表示状态及其转换关系。
2. **从状态图推导下一状态表**：
   - 根据状态图，列出所有状态及输入组合下的下一状态表。
3. **从下一状态表推导下一状态方程**：
   - 使用下一状态表，推导出描述状态转换的布尔方程。
4. **推导输出**：
   - 从状态图中推导输出表，描述在不同状态和输入下的输出。
   - 从输出表中推导输出方程，得到输出逻辑的布尔表达式。
5. **绘制FSM电路**：
   - 根据推导出的**下一状态方程**和**输出方程**，绘制出完整的FSM电路。

这里相当于是和上面的步骤反过来，之前的是由电路图推出state diagram，现在通过伪代码先推到state diagram，然后再反推回电路图，后续需要通过卡诺图判断连接。

State diagram

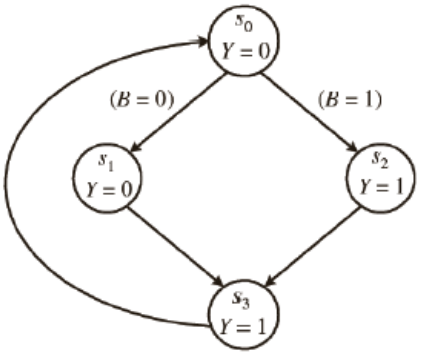
```
REPEAT {
   Y = 0                  // s0
   IF (B = 0) THEN
     Y = 0                // s1
   ELSE
     Y = 1                // s2
   END IF
   Y = 1                  // s3
}
```



跟据左侧的伪代码可以推理出，Y=0为最初始的状态，之后跟据B的值分出两个分支，得到s_1和s_2，分别代指Y=0和Y=1，之后收束到s_3，然后循环回到s_0

Next-State Table

- (2) Next-State Table
  - 2^n states -> n flip-flops for state encoding (4 states-> 2 FFs)



Current State Q_1Q_0	Next State $Q_{1next}Q_{0next}$	
	$B = 0$	$B = 1$
s_0 00	s_1 01	s_2 10
s_1 01	s_3 11	s_3 11
s_2 10	s_3 11	s_3 11
s_3 11	s_0 00	s_0 00

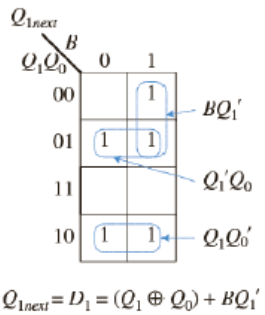
然后，注意总的state数，四个圈代表四个state，也就是说有2^2个状态，对应两个FF，则有Q_0Q_1两个存储变量。跟据$s_0 \rightarrow s_3$对应四种组合，从00到11，补充表格。

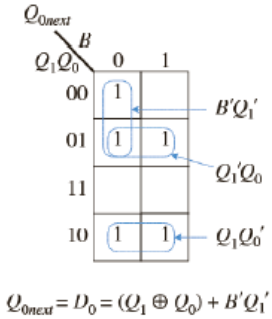
Next-State Equations

这一步需要通过卡诺图得到

- (3) Next-State Equations
  - n flip-flops -> n next-state equations
  - separate Q_{0next} and Q_{1next} and look at them individually as two separate truth tables.

Current State Q_1Q_0	Next State $Q_{1next}Q_{0next}$	
	$B = 0$	$B = 1$
s_0 00	s_1 01	s_2 10
s_1 01	s_3 11	s_3 11
s_2 10	s_3 11	s_3 11
s_3 11	s_0 00	s_0 00

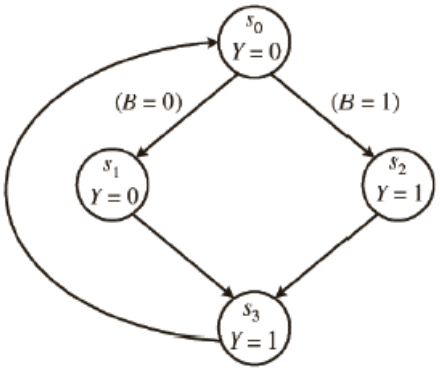




这里需要单独对两个存储变量进行卡诺图分析，即左边的卡诺图对应的是Q_{1next}的分析，里面的1代表在这个时刻$Q_{1next} = 1$，左侧一列Q_1Q_0则是current state，在通过current state的Q_1Q_0和外部输入B来分析这个对应Next state的Q_{1next}或者Q_{0next}的值。

Output Table and Output Equations

- (4) Output Table and Output Equations



Current State Q_1Q_0	Output Y
s_0 00	0
s_1 01	0
s_2 10	1
s_3 11	1

$$Y = Q_1$$

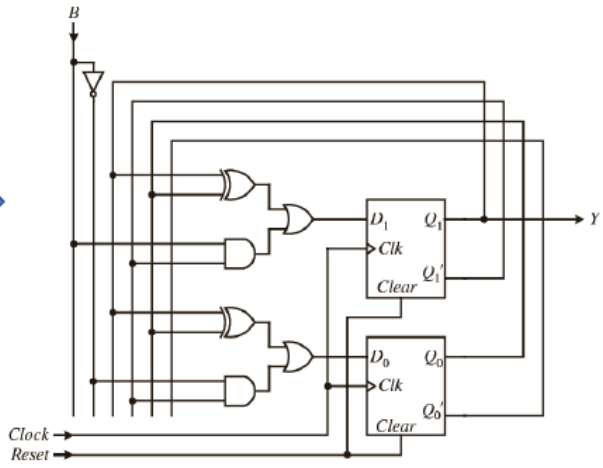
跟据原state diagram得到

FSM Circuit

- (5) FSM Circuit

Current State Q_1Q_0	Output Y
s_0 00	0
s_1 01	0
s_2 10	1
s_3 11	1

Current State Q_1Q_0	Next State $Q_{1next}Q_{0next}$	
	$B = 0$	$B = 1$
s_0 00	s_1 01	s_2 10
s_1 01	s_3 11	s_3 11
s_2 10	s_3 11	s_3 11
s_3 11	s_0 00	s_0 00



最后跟据两个equation进行接线，得到最终的FSM