All of Statistics: Expectation

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Selected Exercises and My Solutions

Exercise 3

Let $X_1, \ldots, X_n \sim \text{Uniform}(0, 1)$. Define $Y_n = \max\{X_1, \ldots, X_n\}$. Find $\mathbb{E}[Y_n]$.

$F_{Y_n}(y) = P(Y_n < y) = P(\max\{X_1, \ldots, X_n\} < y) = P(X_1 < y)P(X_2 < y)...P(X_n < y) = F_{X_1}(y)F_{X_2}(y)...F_{X_n}(y) = (F_{X_i}(y))^n$

$F_{X_i}(y) = \int_0^y dx = y$

$F_{Y_n}(y) = y^n$

$f_{Y_n}(y) = ny^{n-1}$

The expectation value of Y is: $\mathbb{E}[Y_n] = \int_0^1 yny^{n-1}dy = n\int_0^1 y^n dy = \frac{n}{n+1}$

Exercise 4

A particle starts at the origin of the real line and moves along the line in jumps of one unit. For each jump the probability is p that the particle will jump one unit to the left and the probability is $1-p$ that the particle will jump one unit to the right. Let X_n be the position of the particle after n jumps. Find $\mathbb{E}[X_n]$ and $\mathbb{V}[X_n]$.

Let X_n be the position of the particle after n jump, then$X_n = \Sigma_{i=1}^n X_i$ where X_i has the distribution:

x_i	$P(X_i = x_i)$
1	1-p
-1	p

$\therefore$

$\mathbb{E}[X_i] = (1-p) - p = 1 - 2p$

$\mathbb{E}[X_n] = \mathbb{E}[\Sigma_{i=1}^n X_i] = n(1-2p) = n - 2np$

$\mathbb{V}[X_i] = (1-1+2p)^2 * (1-p) + (-1-1+2p)^2 * p = 4p^2(1-p) + (4p^2 - 8p + 4)p = 4p - 4p^2$

$\mathbb{V}[X_n] = n(4p - 4p^2)$

Exercise 10

Let $X \sim \text{Normal}(0, 1)$. Define $Y = e^X$. Find $\mathbb{E}[Y]$ and $\mathbb{V}[Y]$. Your answer should be a function of e, not a decimal representation.