All of Statistics: Random Variables

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Selected Exercises and My Solutions

Exercise 16

Let $X \sim \text{Poisson}(\lambda)$ and $Y \sim \text{Poisson}(\mu)$ and assume that X and Y are independent. Find the distribution of X given that $X + Y = n$.

My solution:

$X + Y \sim \text{Poisson}(\lambda + \mu)$ because X and Y are independent.

$$f_{X|X+Y=n}(x|x+y=n) = \frac{f_{X,X+Y=n}(x, x+y=n)}{f_{X+Y}(x+y)} = \frac{f_{X,Y}(x, n-x)}{f_{X+Y}(x+y)} = \frac{f_X(x)f_Y(y=n-x)}{f_{X+Y}(x+y)} = \frac{\frac{\lambda^x}{e^\lambda x!}\frac{\mu^{n-x}}{e^\mu(n-x)!}}{\frac{(\mu+\lambda)^{x+y}}{e^\mu e^\lambda(x+y)!}} = \frac{\frac{\lambda^x}{x!}\frac{\mu^{n-x}}{(n-x)!}}{\frac{(\mu+\lambda)^{x+y}}{(x+y)!}}$$

$$= \frac{\frac{\lambda^x}{x!}\frac{\mu^{n-x}}{(n-x)!}}{\frac{(\mu+\lambda)^n}{n!}} = \binom{n}{x}\frac{\lambda^x\mu^{n-x}}{(\mu+\lambda)^n} = \binom{n}{x}(\frac{\lambda}{\lambda+\mu})^x(1-\frac{\lambda}{\lambda+\mu})^{n-x}$$

This shows that X given X+Y=n $\sim Binomial(n, \frac{\lambda}{\mu+\lambda})$

Exercise 17

Let

$$f_{X,Y}(x,y) = \begin{cases} c(x+y)^2 & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X < 0.5 \mid Y = 0.5)$.

My solution:

First, find $f_Y(y)$ by integrating over x. $f_Y(y) = \int_0^1 c(x+y)^2dx = c(\frac{1}{3} + y + y^2)$

So, $f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{c(x+y)^2}{c/3+cy+cy^2} = \frac{(x+y)^2}{1/3+y+y^2}$

$P(X < 0.5 \mid Y = 0.5) = \int_0^{0.5} \frac{(x+0.5)^2}{1/3+0.5+0.5^2}dx \approx 0.2692$