Problem 1: Linear Regression from Scratch (30 points)

In [56]:
```
# import the necessary packages
import numpy as np
from matplotlib import pyplot as plt
np.random.seed(100)
```

Let's generate some data points first, by the equation $y = x - 3$.

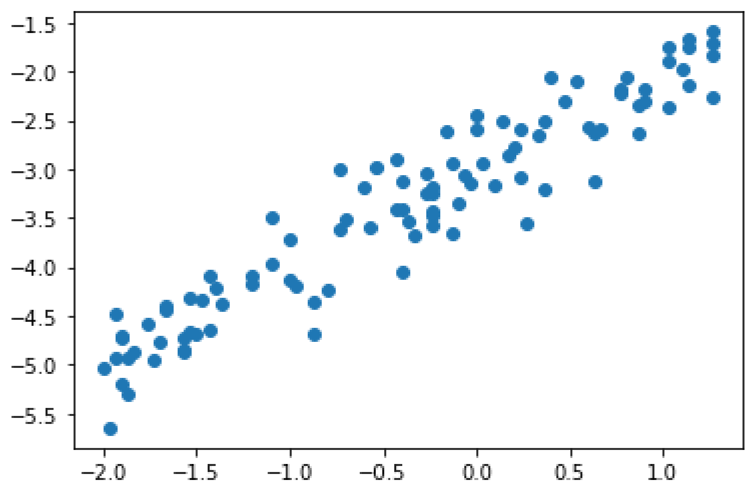
In [57]:
```
x = np.random.randint(100, size=100)/30 - 2
X = x.reshape(-1, 1)

y = x + -3 + 0.3*np.random.randn(100)
```

Let's then visualize the data points we just created.

In [58]:
```
plt.scatter(X, y)
```

Out[58]: <matplotlib.collections.PathCollection at 0x1c5de2c6640>



1.1 Gradient of vanilla linear regression model (5 points)

In the lecture, we learn that the cost function of a linear regression model can be expressed as **Equation 1**:

$$J(\theta) = \frac{1}{2m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

The gredient of it can be written as **Equation 2**:

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{m}[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}]$$

1.2 Gradient of vanilla regularized regression model (5 points)

After adding the L2 regularization term, the linear regression model can be expressed as **Equation 3**:

$$J(\theta) = \frac{1}{2m}\sum_{i}^{m}\left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j}^{n}(\theta_j)^2$$

The gredient of it can be written as **Equation 4**:

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{m}[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} + \lambda\theta_j]$$

1.3 Implement the cost function of a regularized regression model (5 points)

Please implement the cost function of a regularized regression model according to the above equations.

1.4 Implement the gradient of the cost function of a regularized regression model (5 points)

Please implement the gradient of the cost function of a regularized regression model according to the above equations.

In [78]:

```
def regularized_linear_regression(X, y, alpha=0.01, lambda_value=1, epochs=30):
    """
    :param x: feature matrix
    :param y: target vector
    :param alpha: learning rate (default:0.01)
    :param lambda_value: lambda (default:1)
    :param epochs: maximum number of iterations of the
           linear regression algorithm for a single run (default=30)
    :return: weights, list of the cost function changing overtime
    """

    m = np.shape(X)[0]  # total number of samples
    n = np.shape(X)[1]  # total number of features

    X = np.concatenate((np.ones((m, 1)), X), axis=1)
    W = np.random.randn(n + 1, )

    # stores the updates on the cost function (loss function)
    cost_history_list = []

    # iterate until the maximum number of epochs
    for current_iteration in np.arange(epochs):  # begin the process

        # compute the dot product between our feature 'X' and weight 'W'
        y_estimated = X.dot(W)
```

```
        # calculate the difference between the actual and predicted value
        error = y_estimated - y


################################################################################################
######################################## Begin of Question 1.3 ##########################
################################################################################################

        ##### Please write down your code here:####

        # calculate the cost (MSE) (Equation 1)
        cost_without_regularization = (1 / (2 * m)) * np.sum(np.square(error))


        ##### Please write down your code here:####

        # regularization term
        reg_term = (lambda_value / (2 * m)) * np.sum(np.square(W))


        # calculate the cost (MSE) + regularization term (Equation 3)
        cost_with_regularization = cost_without_regularization + reg_term

################################################################################################
######################################## End of Question 1.3 ############################
################################################################################################


################################################################################################
######################################## Begin of Question 1.4 ##########################
################################################################################################

        ##### Please write down your code here:####

        # calculate the gradient of the cost function with regularization term (Equatio
        gradient = (1 / m) * (X.T.dot(error) + (lambda_value * W))

        # Now we have to update our weights
        W = W - alpha * gradient

################################################################################################
######################################## End of Question 1.4 ############################
################################################################################################


        # keep track the cost as it changes in each iteration
        cost_history_list.append(cost_with_regularization)

    # Let's print out the cost
    print(f"Cost with regularization: {cost_with_regularization}")
    print(f"Mean square error: {cost_without_regularization}")

    return W, cost_history_list
```

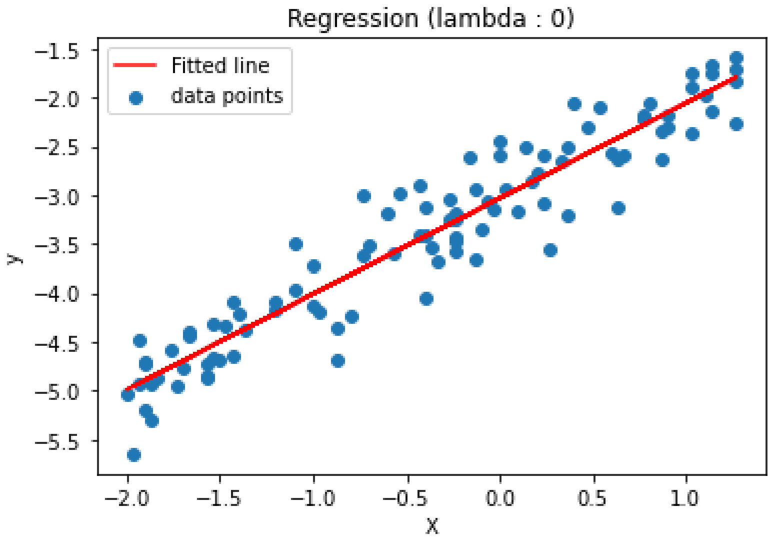
Run the following code to train your model.

Hint: If you have the correct code written above, the cost should be 0.5181222986588751 when $\lambda = 10$.

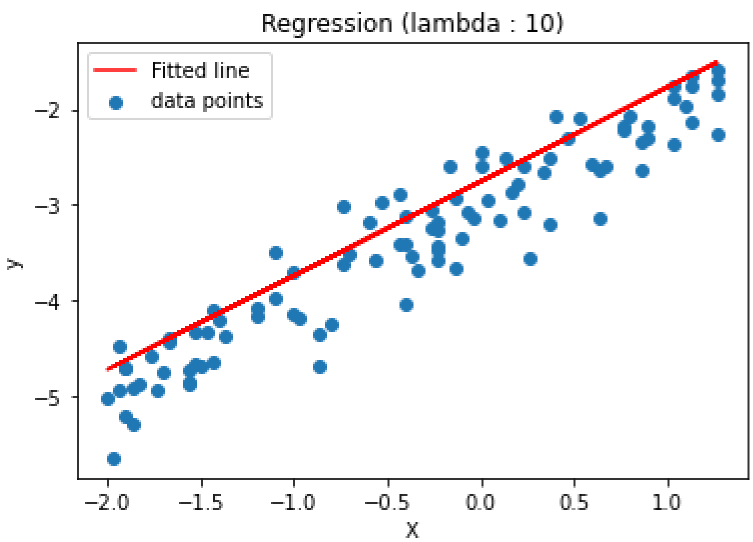
In [79]:
```
lambda_list = [0, 10, 100, 1000, 10000]
for lambda_ in lambda_list:
    # calls regression function with different values of lambda
    weight, _ = regularized_linear_regression(X, y, alpha=0.01,
                                              lambda_value=lambda_, epochs=1000)

    fitted_line = np.dot(X, weight[1]) + weight[0]
    plt.scatter(X, y, label='data points')
    plt.plot(X, fitted_line, color='r', label='Fitted line')
    plt.xlabel("X")
    plt.ylabel("y")
    plt.title(f"Regression (lambda : {lambda_})")
    plt.legend()
    plt.show()
```

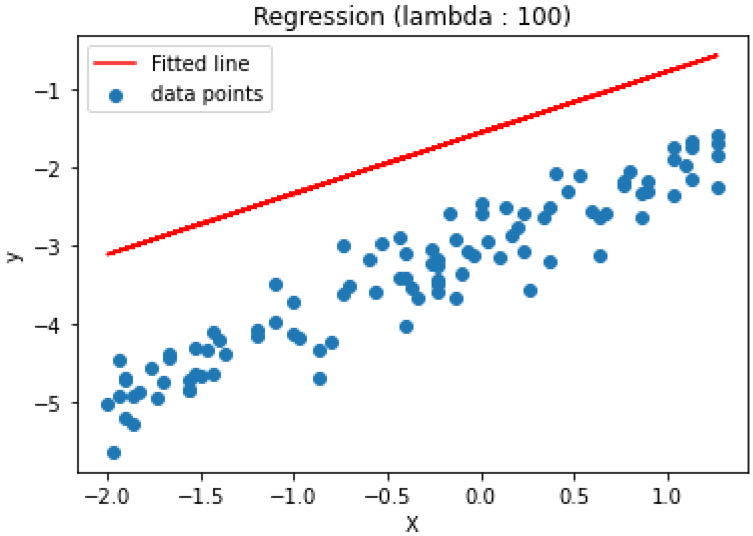
```
Cost with regularization: 0.05166213955813263
Mean square error: 0.05166213955813263
```



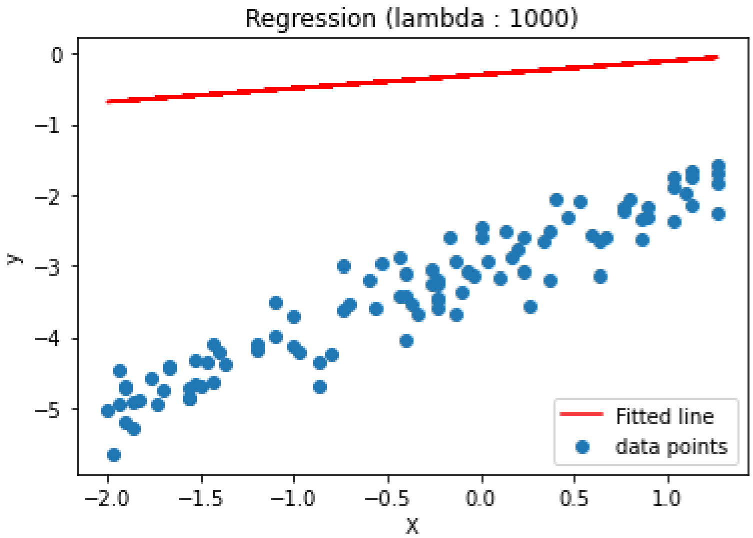

```
Cost with regularization: 0.5181226342691986
Mean square error: 0.08984723994262132
```



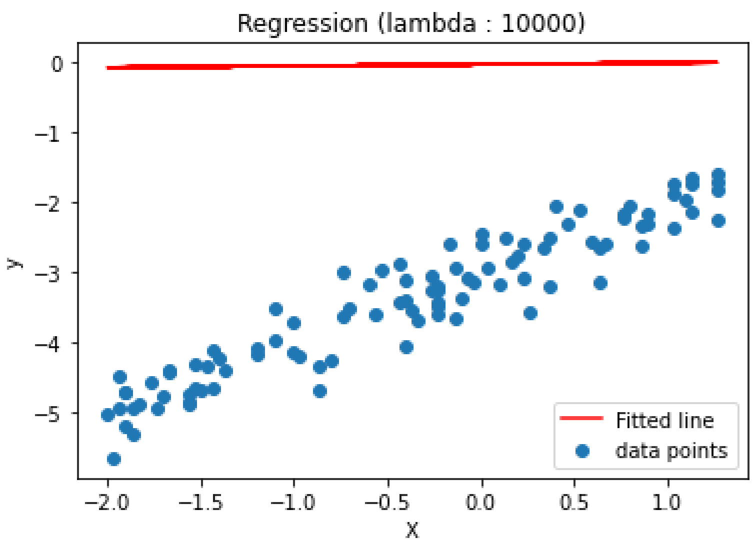

```
Cost with regularization: 2.7931724887400255
Mean square error: 1.278510707593800 7
```




```
Cost with regularization: 5.591464362606628
Mean square error: 4.946888025066498
```




```
Cost with regularization: 6.2426956269339735
Mean square error: 6.1614425833558135
```



1.5 Analyze your results (10 points)

According to the above figures, what's the best choice of λ?

λ = 0.

Why the regressed line turns to be flat as we increase λ?

As λ goes large, our cost increases. We increase the penality for weight, in order to reduce the cost. As a result, the model gradually under-fit the data as λ goes large.

Problem 2: Getting familiar with PyTorch (30 points)

In [5]:

```
# Your code:
import numpy as np
import mltools as ml
data = np.genfromtxt("data/curve80.txt")
X = data[:,0]
X = np.atleast_2d(X).T # code expects shape (M,N) so make sure i t s 2-dimensional
Y = data[:,1] # d o e s n t matter for Y
Xtr,Xte,Ytr,Yte = ml.splitData(X,Y,0.75) # split data set 75/25

degree = 5
XtrP = ml.transforms.fpoly(Xtr, degree=degree, bias=False)
XtrP,params = ml.transforms.rescale(XtrP)
```

In [3]:

```
import torch
```

In [6]:

```
XtrP_tensor = torch.from_numpy(XtrP)
Ytr_tensor = torch.from_numpy(Ytr)

XtrP_tensor = XtrP_tensor.float()
Ytr_tensor = Ytr_tensor.float()
```

In [7]:

```
XtrP.shape
```

Out[7]: (60, 5)

In [8]:

```
Ytr.shape
```

Out[8]: (60,)

In [9]:

```
linear_regressor = torch.nn.Linear(in_features=5, out_features=1)
```

In [10]:

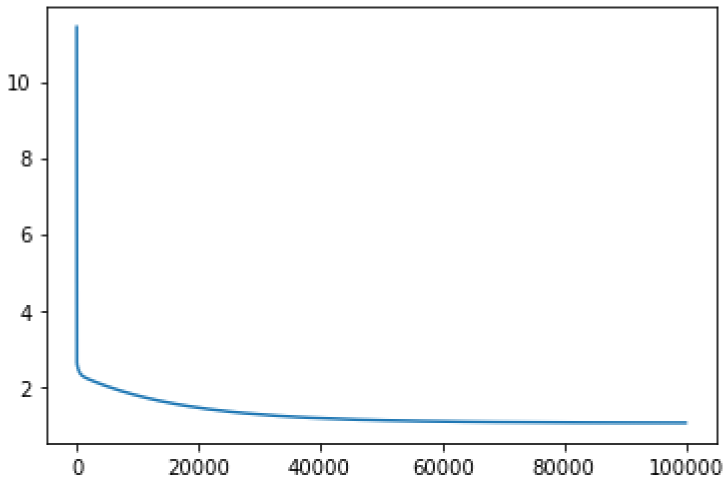
```
criterion = torch.nn.MSELoss()
optimizer = torch.optim.SGD(linear_regressor.parameters(), lr=0.1)
epochs = 100000
```

In [18]:
```
loss_record = []
for _ in range(epochs):
    optimizer.zero_grad() # set gradient to zero
    pred_y = linear_regressor(XtrP_tensor)
    pred_y = pred_y.flatten()
    loss = criterion(pred_y, Ytr_tensor) # calculate loss function
    loss.backward() # backpropagate gradient
    loss_record.append(loss.item())
    optimizer.step() # update the parameters in the linear regressor
```

In [19]:
```
import os
os.environ["KMP_DUPLICATE_LIB_OK"]="TRUE"
```

In [20]:
```
import matplotlib.pyplot as plt
plt.plot(range(epochs), loss_record)
```

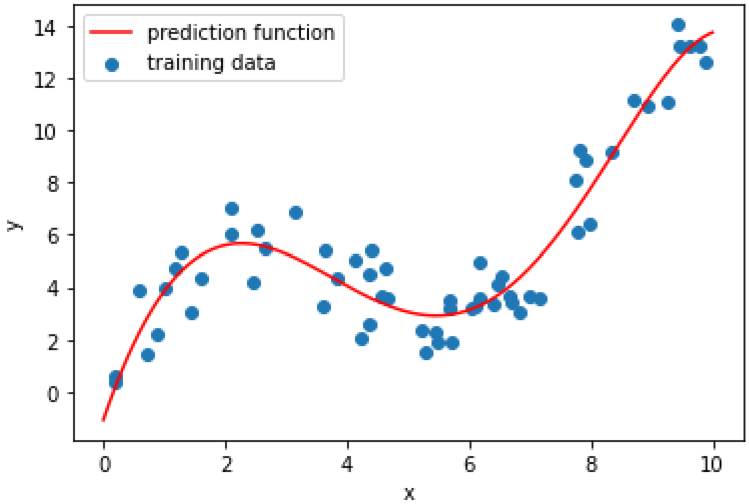
Out[20]:
```
[<matplotlib.lines.Line2D at 0x28a671ab6d0>]
```



In [21]:
```
xs = np.linspace(0,10,200)
xs = xs[:,np.newaxis]
xsP, _ = ml.transforms.rescale(ml.transforms.fpoly(xs,degree=degree,bias=False), params
xsP_tensor = torch.from_numpy(xsP).float()
ys = linear_regressor(xsP_tensor)

plt.scatter(Xtr,Ytr,label="training data")
plt.plot(xs,ys.detach().numpy(),label="prediction function",color ='red')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
```

Out[21]:
```
<matplotlib.legend.Legend at 0x28a6768beb0>
```



Statement of Collaboration (5 points)

Personal work. Referred to Google when getting the cost wrong.