

① Or since, when the signal becomes zero, the process stops completely, hence we have a trap state here. Observe Each state can be represented by a number from $[0, \infty)$. And the trap state described above is state 0.

Since the initial signal is 1 initially it transmits one signal per second, so the starting state will be 1.

Probability of going from

Note, each state makes transition to

Also since any number of signals can be transmitted from the current state, hence there is a direct edge of transition from current state to all other states in the infinite state space.

For a given state S_i at time t the transition probabilities are same as out of it can be considered a poisson process, with average rate $\lambda = g_i$

$$P(X_{t+h} = k \mid X_t = i) = \frac{i^k e^{-i}}{k!}$$

This is true for statement about average rate λ made based on the fact that if it amplifies x signals in the past it will amplify same number x . Therefore the average rate for that state to be λ .

Go the mital transition Matrix

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ e^{-1} & 1e^{-1} & e^{-1} & e^{-3} & e^{-4} & \dots & 0 \\ & 1! & 2! & 3! & 4! & & \\ 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & & & & \\ \vdots & \vdots & \vdots & & & & \\ \vdots & \vdots & \vdots & & & & \end{bmatrix}$$

All other rows are zero except $P_x[1][C:]$

Also if you look then

$$\sum_{k=0}^{\infty} \frac{e^{-1}}{k!} = 1$$

$$= e^{-1} \left(\frac{1}{0!} + \frac{1}{1!} + \dots + \infty \right)$$

$$= e^{-1} \times e^1 = \underline{\underline{1}}$$

② At any time t , Random Variable X_t defines the state of the system.

$$\therefore E(|X_n|) < \infty \text{ for any time } n.$$

So, suppose if the signals being transmitted after time t are $X_t = i$ then probability of $X_{t+1} = j$ is $\Pr(X_{t+1} = j | X_t = i) = \frac{i^j e^{-i}}{j!}$

So the Expected value of X_{t+1} is given by

$$E[X_{t+1}] = \sum_{j=0}^{\infty} \frac{i^j e^{-i}}{j!} x_j$$

$$= e^{-i} \sum_{j=0}^{\infty} \frac{i^j}{j!} x_j$$

$$= e^{-i} \sum_{j=1}^{\infty} i \frac{i^{j-1}}{(j-1)!}$$

$$= i e^{-i} \times e^i$$

$$= i$$

$$\boxed{E[X_{t+1}] = X_t}$$

Hence this infinite random walk is a Martingale. The fact that average rate of signals transmitted in a state x are x , lead to conclusion that average transmission in state x must be x .

④ for a given time t , let X_t denote the number of transmitted signal from bazooka-5000 and Y_t denote the number of ~~frequency~~ amplified signals transmitted by second amplifier at time t . So essentially Bazooka-5001 is taking in X_t signals and transmitting Y_t signals independently.

So ~~probability function measure~~ can be defined on $\text{Pr}\{Y_{t+1} \mid X_1, X_2, \dots, X_t\}$.

So essentially the system of Bazooka-5000 + 5001 will act as a Martingale because Bazooka-5001 is working independently and the behaviour will be same as that in case-1. The transition b/w states are going to be Poisson and hence

$$\text{Pr}\{Y_{t+1} \mid Y_1, \dots, Y_t\}$$

$$= \text{Pr}\{Y_t\}$$

$$= \text{Pr}\{Y_{t+1} \mid X_1, X_2, \dots, X_t\}$$