(1) ~~Of~~ Since, when the signal becomes zero, the process stops completely, hence we have a trap state here. ~~Outside~~ Each state can be represented by a number from $[0, \infty)$. And the trap state described above is state 0.

Since the initial signal is 1 initially it transmits one signal per second, so the starting state will be 1.

~~Probability of going from~~

~~Note, each state makes transition to~~

Also since any number of signals can be transmitted from the current state, hence there is a direct ~~ed~~ edge of transition from current state to all other states in the infinite state space.

For a given state s_t at time t the transition probabilities are same as out of it can be considered a poisson process with average rate $\lambda = s_t$

So $$P\{X_{t+1} = k \mid X_t = i\} = \frac{i^k e^{-i}}{k!}$$

This ~~is true for~~ statement about average rate is made based on the fact the if it amplifies x signals in the past it will amplify same number x. Therefore the average rate for that state to be x.

So the initial transition Matrix

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & \infty \\ e^{-1} & \frac{1e^{-1}}{1!} & \frac{e^{-1}}{2!} & \frac{e^{-3}}{3!} & \frac{e^{-4}}{4!} & \cdots & \infty \\ 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & & & & \\ \vdots & \vdots & \vdots & & & & \end{bmatrix}$$

All other rows are zero except $P_x[1][:]$

Also if you look then $\boxed{\sum_{k=0}^{\infty} \frac{e^{-1}}{k!} = 1}$

$$\because \quad e^{-1}\left(\frac{1}{0!} + \frac{1}{1!} \cdots \infty\right)$$

$$= e^{-1} \times e^{1} = \underline{1}$$

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(2) Let τ denote the RV modelling ~~the~~ stopping time of the process or Watt of Bazooka - 5000

$$\therefore \Pr\{\tau = n\} = \sum_{\lambda=1}^{\infty} \Pr\{X_{n-1} = \lambda\} \times \Pr\{X_n = 0 \mid X_{n-1}\}$$

$\therefore$ ~~Pr{X~~

$$\Pr\{X_n = 0 \mid X_{n-1} = \lambda\} = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda}$$

$$\Pr\{X_{n-1} = \lambda\} = \sum_{\lambda'=1}^{\infty} \Pr\{X_{n-2} = \lambda'\} \times \frac{\lambda'^{\lambda} e^{-\lambda'}}{\lambda!}$$

$$= \frac{1}{\lambda!} \sum_{\lambda'=1}^{\infty} \Pr\{X_{n-2} = \lambda'\} \underline{\underline{\lambda'^{\lambda} e^{-\lambda'}}}$$

$\therefore \Pr\{\tau = n\} = \Pr\{X_n = 0\} =$ ~~ΣA~~ Summation of entries in the first Column of $Q_0 P^{n-1}$

Where P is the transition Matrix and Q_0 is the initial state Matrix with ~~first~~ second row as Probability of transition from state 1 to other states.

So the Expected stopping time, $E[\tau]$

$$= \sum_{n=1}^{\infty} \Pr\{X_n = 0\} \times n$$

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(7) At any time t, Random Variable X_t defines the state of the system.

$$\therefore E(|X_n|) < \infty \text{ for any time } n.$$

So, Suppose if the signals being transmitted after time t are $X_t = i$ then Probability of $X_{t+1} = j$ is $Pr\{X_{t+1} = j \mid X_t = i\} = \frac{i^j e^{-i}}{j!}$

So the Expected value of X_{t+1} is given by

$$E[X_{t+1}] = \sum_{j=0}^{\infty} \frac{i^j e^{-i}}{j!} \times j$$

$$= e^{-i} \sum_{j=0}^{\infty} \frac{i^j}{j!} \times j$$

$$= e^{-i} \sum_{j=1}^{\infty} i \frac{i^{j-1}}{(j-1)!}$$

$$= i e^{-i} \times e^{i}$$

$$= i$$

$$\boxed{E[X_{t+1}] = X_t}$$

Hence this infinite random walk is of Martingale. The fact that average rate of signals transmitted in a state x are x, leads to conclusion that average transmission in state x must x.

(4) for a given time t, let X_t denote the number of transmitted signal from bazook-5000 and Y_t denote the number of ~~frequency~~ amplified signals transmitted by ~~by~~ second amplifier at time t. So Essentially Bazooka-5001 is taking in X_t signals and transmitting Y_t signals independently

~~So probability function measure can be defined as $Pr\{Y_{t+1} | X_1, X_2 \dots X_t\}$.~~

So essentially the system {Bazooka - 5000 + 5001} will act as a Martingale because Bazooka - 5001 is working independently and the behaviour will be same as that in case - 1. The transition b/w states are going to be Poisson and hence

$$Pr\{Y_{t+1} | Y_1 \dots Y_t\}$$

$$= \cancel{Pr} \; Y_t$$

$$= \underline{Pr\{Y_{t+1} | X_1, X_2 \dots X_t\}}$$