$$\frac{dN_t}{dt} = \frac{N(t+\Delta t) - N(t)}{\Delta t}$$

$$= rN(t) - \mu N(t)$$

thus the solution is:

$$N_t = N_0 e^{(r-\mu)t}$$

where:

$$\begin{aligned} N_0 &= \text{the initial population size} \\ N_t &= \text{the population size at time t} \\ r &= \text{birth rate[1/time step]} \\ \mu &= \text{death rate[1/time step]} \end{aligned}$$

$$\text{Probability: } r(N-1)\Delta t P_{N-1}(t)$$

$$\text{Probability: } \mu(N+1)\Delta t P_{N+1}(t)$$

$$\text{Probability: } \left[1 - \mu - r\right] N \Delta t P_N(t)$$

$$P_N(t+\Delta t) = r(N-1)\Delta t P_{N-1}(t) + \mu(N+1)\Delta t P_{N+1}(t) + \left[1-\mu-r\right] N\Delta t P_N(t)$$

$$\frac{P_N(t)}{dt} = -N(r+\mu)P_N(t) + r(N-1)P_{N-1}(t) + \mu(N+1)P_{N+1}(t)$$

whose solution is:

$$P_N(t) = \sum_{j=0}^{min(N_0,N)} \binom{N_0}{j} \binom{N_0+N-j-1}{N_0-1} \alpha^{N_0-j} \beta^{N-j} (1-\alpha-\beta)^j$$

where:

$$\begin{aligned} \alpha &= \frac{\mu\left(e^{(r-\mu)t}-1\right)}{re^{(r-\mu)t}-\mu} \\ \beta &= \frac{r\left(e^{(r-\mu)t}-1\right)}{re^{(r-\mu)t}-\mu} \end{aligned} \tag{1}$$