$$
\begin{aligned}
P_N\left(t+\Delta t\right) = P_N\left(t\right) \times Prob\left[\text{no birth in } \left(t,t+\Delta t\right)\right] + \\
P_{N-1}(t) \times Prob\left[\text{no birth in } \left(t,t+\Delta t\right)\right]
\end{aligned}
$$

where:

$$
p_N\left(t\right) = Prob\left[\text{population size is N at time t}\right]
$$

$$
\begin{aligned}
\frac{dN_t}{dt} &= \frac{N(t+\Delta t)-N(t)}{\Delta t} \\
&= \frac{rN(t)\Delta t}{\Delta t}
\end{aligned}
$$

thus the solution is:

$$
N_t = N_0 e^{rt}
$$

where:

$$
\begin{aligned}
N_0 &= \text{the initial population size} \\
N_t &= \text{the population size at time t} \\
r &= \text{birth rate[1/time step]}
\end{aligned}
$$

$$
\begin{aligned}
P_N\left(t+\Delta t\right) &= P_N\left(t\right) \times Prob\left[\text{no birth in } \left(t,t+\Delta t\right)\right] + \\
&\quad P_{N-1}(t) \times Prob\left[\text{1 birth in } \left(t,t+\Delta t\right)\right] \\
&= P_N\left(t\right)\left(1-rN\Delta t+o(\Delta t)\right) + P_{N-1}\left(t\right)\left(r\left(N-1\right)\Delta t+o(\Delta t)\right)
\end{aligned}
$$

thus:

$$
\begin{aligned}
\frac{dP_N}{dt} &= \frac{P_N\left(t+\Delta t\right)-P_N\left(t\right)}{\Delta t} \\
&= -rNP_N\left(t\right) + P_{N-1}\left(t\right) r\left(N-1\right)
\end{aligned}
$$

whose solution is:

$$
P_N(t) = \binom{N-1}{N_t-N_0} e^{-rN_0 t}(1-e^{-rt})^{N_t-N_0}
$$

where:

$$
\begin{aligned}
P_N\left(t\right) &= P\{N_t = n\} = Prob\left[\text{population size is N at time t}\right] \\
o(\Delta t) &= Prob\left[\text{more than 1 birth in } \left(t,t+\Delta t\right)\right] \\
r &= \text{birth rate[1/time step]}
\end{aligned}
$$