

Expected Utility using Linear Programming

1. 'A' matrix is written transposed for ease of work.
2. 'X' vector is the solution (Number of times action 'a' is taken in state 'i').
3. 'AX' is the matrix multiplication which is compared for equality with vector 'Alpha' in LP.
4. 'R' is reward vector, which depends upon the action, that is,
-> $-X/10$ for all steps = -4 ($X=40$)
-> For the terminal states it is based upon functions of X ($X/10$, X and $-X/5$).
5. Required Utility Value is the maximized value of the vector 'RX', which is acquired after solving the LP.
6. To solve the LP, we use the Simplex Solver, to maximize the value of RX following that $AX = \text{Alpha}$ and $X \geq 0$.

After solving LP, answer is 8.79679409 which is optimal utility of start state.

In Value Iteration,

utility of start state is 8.796 for Error Rate $\leq 0.01\%$ (Iterations = 23, Gamma = 1)

The difference is less than delta. Hence, answers obtained from Linear Programming and Value Iteration match.