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Ass → PARAMETER ESTIMATION

Solⁿ 1

Given sample $(x_1 \rightarrow x_n)$ from normal Dist.

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)}$$

Taking log of fxⁿ

$$\Rightarrow \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2) \right)$$

To find MLEs, differentiating the fnⁿ with respect to θ_1, θ_2

θ_1

$$\frac{\delta}{\delta\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma^2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0.$$

$$\Rightarrow \mu = \frac{1}{n} \sum_{i=1}^{n} x_i$$

OR

$$\boxed{\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i}$$

θ_2

$$\frac{\delta}{\delta\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\theta_1)^2}{2\theta_2^2} + \frac{1}{2\theta_2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \frac{(x_i - \theta_1)^2}{\theta_2^2} - \frac{n}{\theta_2} = 0$$

$$\Rightarrow \frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

Sol^n 2

MLE of parameter θ for Binomial Dist $B(m, \theta)$

wher $m \longrightarrow$ Known +ve integer.

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking natural log

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left[\ln\binom{m}{x_i} + x_i \ln(\theta) + (m - x_i) \ln(1-\theta) \right]$$

$$\frac{\partial}{\partial \theta} \ln L(\theta) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m - x_i}{1-\theta}$$

$$\Rightarrow \sum_{i=1}^{n} x_i(1-\theta) = \sum_{i=1}^{n} (m - x_i)\,\theta$$

$$\Rightarrow \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\Rightarrow \boxed{\theta = \frac{1}{m} \sum_{i=1}^{n} x_i}$$

MLE of θ = sample mean of observations