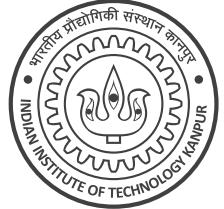


Name			<b>20 marks</b> Page 1 of 2
Roll No		Dept.	

**Instructions:**

1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ – such cases may get straight 0 marks.
5. Do not rush to fill in answers. You have enough time to solve this quiz.



**Q1. (Matrix Norm)**  $A \in \mathbb{R}^{m \times m}$  is a (possibly non-symmetric) square matrix with SVD  $A = U\Sigma V^T$ . The diagonal matrix  $\Sigma = \text{diag}(\sigma_1, \dots, \sigma_m) \in \mathbb{R}^{m \times m}$  has  $\sigma_1 > \sigma_2 > \dots > \sigma_m$  (all singular values are distinct),  $U = [\mathbf{u}_1, \dots, \mathbf{u}_m], V = [\mathbf{v}_1, \dots, \mathbf{v}_m] \in \mathbb{R}^{m \times m}$  are orthonormal. Let  $a^* \stackrel{\text{def}}{=} \max_{\|\mathbf{x}\|_2=1} \|A\mathbf{x}\|_2$ . Note

the constraint  $\|\mathbf{x}\|_2 = 1$ . Find the value of  $a^*$  in terms of  $\mathbf{u}_i, \mathbf{v}_i, \sigma_i, i \in [m]$ . Show brief derivation.

*Hint: try expressing  $\mathbf{x}$  as a linear combination of the right singular vectors  $\mathbf{v}_1, \dots, \mathbf{v}_m$  which is always possible as they form a basis over  $\mathbb{R}^m$ . You may also find  $VV^T = I = V^TV$  useful. (2+3 = 5 marks)*

$$a^* \stackrel{\text{def}}{=} \max_{\|\mathbf{x}\|_2=1} \|A\mathbf{x}\|_2 =$$

Give brief derivation below

**Q2. (Kernel Smash)**  $K_1, K_2: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  are Mercer kernels with  $K_i(x, y) = \langle \phi_i(x), \phi_i(y) \rangle$  for any  $x, y \in \mathbb{R}$  with the feature maps given below. Design a map  $\phi_3: \mathbb{R} \rightarrow \mathbb{R}^3$  for a kernel  $K_3$  defined as  $K_3(x, y) = K_1(x, y) \cdot K_2(x, y)$ . No derivation needed.  $\phi_3$  must not use more than 3 dimensions.

If your solution doesn't need 3 dimensions, fill remaining ones with zero. (6 marks)

$$\phi_1(x) = \left( \frac{1}{x}, \frac{1}{x^2} \right) \in \mathbb{R}^2$$

$$\phi_2(x) = (x, x^2) \in \mathbb{R}^2 \quad \phi_3(x) = \left( \boxed{\phantom{00}}, \boxed{\phantom{00}}, \boxed{\phantom{00}} \right)$$

**Q3. (True-False)** Write T or F for True/False in the **box on the right** and a **brief justification** in the space below (brief proof if T, counterexample if F). A square matrix is termed *diagonal* if all of its off-diagonal entries are zero (its diagonal entries can be zero/-ve/+ve).      (3 x (1+2) = 9 marks)

1	For a square symmetric matrix $A \in \mathbb{R}^{3 \times 3}$ having eigen decomposition $A = VLV^\top$ , we always have $\text{trace}(A) = \text{trace}(L)$ where $L$ is the diagonal matrix of eigenvalues.	
2	Given any two matrices $A, B \in \mathbb{R}^{3 \times 3}$ , the largest singular value of $A + B$ is always equal to the sum of the largest singular value of $A$ and largest singular value of $B$ .	
3	For a kernel $K: \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$ , $K(\mathbf{x}, \mathbf{y}) \stackrel{\text{def}}{=} \exp(\mathbf{x}^\top \mathbf{y})$ and any vectors $\mathbf{x}_1, \dots, \mathbf{x}_n \in \mathbb{R}^2$ , the matrix $G = [g_{ij}] \in \mathbb{R}^{n \times n}$ with $g_{ij} \stackrel{\text{def}}{=} K(\mathbf{x}_i, \mathbf{x}_j)$ is always full rank, $\text{rank}(G) = n$ .	