CS 771A: Intro to Machine Learning, IIT Kanpur				**Quiz I** (22 Aug 2025)
Name				**20 marks** Page **1** of **2**
Roll No		Dept.		

Instructions:

1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ – such cases may get straight 0 marks.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. (Subcalculus) Melba came across a function $f: \mathbb{R} \to \mathbb{R}$ described on the right and wants to analyse its properties. For parts d,e,f, **fill only one circle**. For parts a,b,c, **answer in the space provided**. No proofs/derivations needed in any part. **Note**: the subdifferential at a point is a set in general (singleton set if the function is differentiable at that point). e is the base of the natural logarithm. m and c are real numbers (could be +ve, -ve, zero) (**1 x 7 = 7 marks**)

$$f(x) = \begin{cases} e/2 & x \leq 0 \\ m \cdot x + c & 0 < x \leq 1 \\ e^{\sqrt{x}} & 1 < x \end{cases}$$

a.	Find out the values of m and c for which f is continuous and differentiable at $x = 1$	$m =$	$c =$
b.	For the correct values of m, c from part a., what is the subdifferential of f at $x = 0$?		
c.	For the correct values of m, c from part a., what is the subdifferential of f at $x = 1$?		
d.	For the correct values of m, c from part a., is f a continuous function over all $\mathbb{R}$?	True ◯	False ◯
e.	For the correct values of m, c from part a., is f a convex function over all $\mathbb{R}$?	True ◯	False ◯
f.	For the correct values of m, c from part a., is f differentiable at $x = 0$?	True ◯	False ◯

Q2. (**True-False**) Write **T** or **F** for True/False in the **box on the right** and a **brief justification** in the space below. **Note**: $L \in \mathbb{R}^{2\times 2}$ is not necessarily positive semidefinite. (**3 x (1+2) = 9 marks**)

1	For $\mathbf{w}, \mathbf{x} \in \mathbb{R}^2, b \in \mathbb{R}$, the classifier $\text{sign}\left(\left(\text{sign}(\mathbf{w}^\top \mathbf{x})\right) \cdot (\mathbf{w}^\top \mathbf{x} + b)\right)$ will always output either 1 or 0 (let $\text{sign}(0) = 0$). If **T**, give a brief proof. If **F**, give a counterexample with explicit values of $\mathbf{w}, \mathbf{x} \in \mathbb{R}^2, b \in \mathbb{R}$ where output is not 1 or 0.	

2	For any diagonal matrix $L \in \mathbb{R}^{2\times 2}$ with non-zero diagonal entries and any $\mathbf{a} \in \mathbb{R}^2$, the set $\mathcal{C} \stackrel{\text{def}}{=} \{\mathbf{x} \in \mathbb{R}^2 : \|L\mathbf{x} - \mathbf{a}\|_2 = 1\}$ always defines the boundary of an ellipse or a circle. If **T**, give a brief proof. If **F**, give counterexample.	
3	The solution to the opt. problem $\underset{x\in[3,4]}{\operatorname{argmax}}\{x^2 - 4x + 1\}$ is $x = 3$. Derive the solution	

Q3. (Infinite prototypes) Melba is learning an LwP model for a 2D problem with labels $+$ and $-$. All points on the unit circle centered at $(-1,1)$ are $-$ve prototypes. All points on the unit circle centered at $(1,-1)$ are $+$ve prototypes. Prototypes are only on the boundaries of the circles. Write the equation for the classifier decision boundary and give justification below. (**2 + 2 = 4 marks**)

Write equation of decision boundary here

Give justification here