CS 771A: Intro to Machine Learning, IIT Kanpur				**Quiz I** (22 Aug 2025)
Name	MELBO			**20 marks**
Roll No	250007	Dept.	AWSM	Page **1** of **2**

Instructions:
1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ – such cases may get straight 0 marks.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. (Subcalculus) Melba came across a function $f: \mathbb{R} \to \mathbb{R}$ described on the right and wants to analyse its properties. For parts d,e,f, **fill only one circle**. For parts a,b,c, **answer in the space provided**. No proofs/derivations needed in any part. **Note**: the subdifferential at a point is a set in general (singleton set if the function is differentiable at that point). e is the base of the natural logarithm. m and c are real numbers (could be +ve, -ve, zero) **(1 x 7 = 7 marks)**

$$f(x) = \begin{cases} e/2 & x \leq 0 \\ m \cdot x + c & 0 < x \leq 1 \\ e^{\sqrt{x}} & 1 < x \end{cases}$$

a.	Find out the values of m and c for which f is continuous and differentiable at $x = 1$	$m = e/2$	$c = e/2$
b.	For the correct values of m, c from part a., what is the subdifferential of f at $x = 0$?	$[0, e/2]$	
c.	For the correct values of m, c from part a., what is the subdifferential of f at $x = 1$?	$\{e/2\}$	
d.	For the correct values of m, c from part a., is f a continuous function over all $\mathbb{R}$?	True ●	False ○
e.	For the correct values of m, c from part a., is f a convex function over all $\mathbb{R}$?	True ●	False ○
f.	For the correct values of m, c from part a., is f differentiable at $x = 0$?	True ○	False ●

Q2. (**True-False**) Write **T** or **F** for True/False in the **box on the right** and a **brief justification** in the space below. **Note**: $L \in \mathbb{R}^{2\times2}$ is not necessarily positive semidefinite. **(3 x (1+2) = 9 marks)**

1	For $\mathbf{w}, \mathbf{x} \in \mathbb{R}^2, b \in \mathbb{R}$, the classifier $\text{sign}\left((\text{sign}(\mathbf{w}^\top\mathbf{x})) \cdot (\mathbf{w}^\top\mathbf{x} + b)\right)$ will always output either 1 or 0 (let $\text{sign}(0) = 0$). If **T**, give a brief proof. If **F**, give a counterexample with explicit values of $\mathbf{w}, \mathbf{x} \in \mathbb{R}^2, b \in \mathbb{R}$ where output is not 1 or 0.	F

Let $\mathbf{w} = (1,1), \mathbf{x} = (1,1), b = -10$. Then we have $\mathbf{w}^\top\mathbf{x} = 2, \mathbf{w}^\top\mathbf{x} + b = -8$. Thus, the output would be $\text{sign}\left((\text{sign}(2)) \cdot (-8)\right) = \text{sign}(1 \cdot (-8)) = \text{sign}(-8) = -1$.

2	For any diagonal matrix $L \in \mathbb{R}^{2\times 2}$ with non-zero diagonal entries and any $\mathbf{a} \in \mathbb{R}^2$, the set $\mathcal{C} \stackrel{\text{def}}{=} \{\mathbf{x} \in \mathbb{R}^2 : \|L\mathbf{x} - \mathbf{a}\|_2 = 1\}$ always defines the boundary of an ellipse or a circle. If **T**, give a brief proof. If **F**, give counterexample.	T

Let $L = \text{diag}(p, q)$. Also let $\mathbf{x} = (x, y), \mathbf{a} = (a, b)$. This gives us

$$\|L\mathbf{x} - \mathbf{a}\|_2 = 1 \Leftrightarrow (px - a)^2 + (qy - b)^2 = 1$$

Since $p \neq 0, q \neq 0$, we get, for $\tilde{a} \stackrel{\text{def}}{=} a/p, \tilde{b} \stackrel{\text{def}}{=} b/q$

$$p^2 \cdot (x - \tilde{a})^2 + q^2 \cdot \left(y - \tilde{b}\right)^2 = 1$$

which is the standard form of an axis-aligned ellipse. Note: this works even if $p < 0$ or $q < 0$.

3	The solution to the opt. problem $\underset{x\in[3,4]}{\text{argmax}}\{x^2 - 4x + 1\}$ is $x = 3$. Derive the solution	F

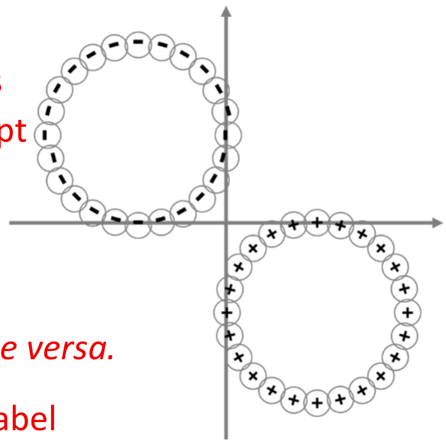
The objective function is $(x - 2)^2 - 3$. To maximize its value, the value of $|x - 2|$ must be maximized i.e., x must be "farthest" from 2. The maximum value achieved by $|x - 2|$ in the interval $[3,4]$ is 2 and this occurs at $x = 4$. Thus, the solution to the optimization problem is achieved at $x = 4$.

Q3. (Infinite prototypes) Melba is learning an LwP model for a 2D problem with labels $+$ and $-$. All points on the unit circle centered at $(-1,1)$ are $-$ve prototypes. All points on the unit circle centered at $(1,-1)$ are $+$ve prototypes. Prototypes are only on the boundaries of the circles. Write the equation for the classifier decision boundary and give justification below. **(2 + 2 = 4 marks)**

Write equation of decision boundary here

$$x = y$$

Give justification here



Using symmetry simplifies the proof. Note that swapping the axes i.e. $(x, y) \leftrightarrow (y, x)$ leaves the placement of the points intact except that all $+$ve prototypes are swapped for $-$ve prototypes and vice versa. This gives us the following interesting result:

The LwP algorithm for this arrangement of prototypes will classify a point (x, y) as $+$ve iff it classifies the point (y, x) as $-$ve and vice versa.

Since the decision boundary is composed of points where either label could have been assigned (the model is confused), this means the decision boundary comprises exactly those points for which $(x, y) = (y, x)$ i.e. $x = y$.