Evaluation of risk to the schedule using z-values

Table 7.6 provides additional activity duration estimates for the network shown in Figure 6.29. There are new estimates for a and b and the original activity duration estimates have been used as the most likely times, m. Calculate the expected duration, t_e, for each activity.

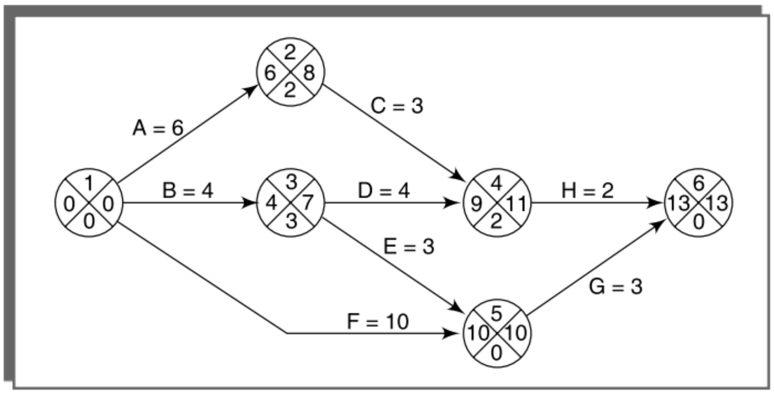
TABLE 7.6 PERT activity time estimates

Activity	Optimistic (a)	Activity durations (weeks). Most likely (m)	Pessimistic (b)
A	5	6	8
B	3	4	5
C	2	3	3
D	3.5	4	5
E	1	3	4
F	8	10	15
G	2	3	4
H	2	2	2.5

PERT then combines these three estimates to form a single expected duration, t_e, using the formula

$$t_e = \frac{a + 4m + b}{6}$$

(Figure 6.29).



The critical path is the longest path through the network.

FIGURE 6.29 The critical path

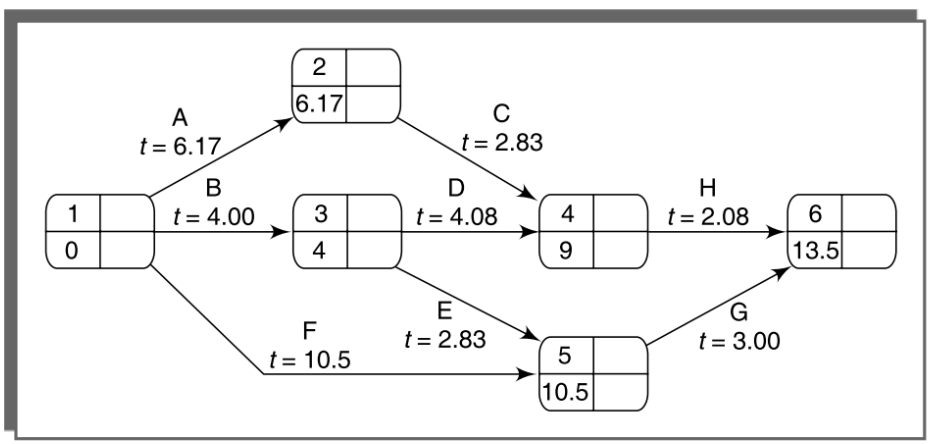


FIGURE 7.6 The PERT network after the forward pass

Before reading further, use your calculated expected activity durations to carry out a forward pass through the network (Figure 6.29) and verify that the project duration is 13.5 weeks. What does an expected duration of 13.5 weeks mean in terms of the completion date for the project?

7.7 The forward pass to calculate expected completion date

The expected duration and the expected dates for the other project events are shown in Figure 7.6. An expected duration of 13.5 weeks means that we expect the project to be completed halfway through week 14, although since this is only an expected value it could finish earlier or later.

TABLE C.10 Calculating expected activity durations

Activity	Activity durations (weeks)			
	Optimistic (a)	Most likely (m)	Pessimistic (b)	Expected (t_e)
A	5	6	8	6.17
B	3	4	5	4.00
C	2	3	3	2.83
D	3.5	4	5	4.08
E	1	3	4	2.83
F	8	10	15	10.50
G	2	3	4	3.00
H	2	2	2.5	2.08

Activity standard deviations

A quantitative measure of the degree of uncertainty of an activity duration estimate may be obtained by calculating the standard deviation s of an activity time, using the formula

This standard deviation formula is based on the rationale that there are approximately six standard deviations between the extreme tails of many statistical distributions.

$$s = \frac{b - a}{6}$$

The activity standard deviation is proportional to the difference between the optimistic and pessimistic estimates, and can be used as a ranking measure of the degree of uncertainty or risk for each activity. The activity expected durations and standard deviations for our sample project are shown in Table 7.7.

expected times and standard deviations

$S = \frac{b-a}{6}$

Activity	Activity duration (weeks)				
	optimistic (a)	most likely (m)	pessimistic (b)	Expect (te)	SD (σ)
A	5	6	8	6.17	0.50
B	3	4	5	4.00	0.33
C	2	3	3	2.83	0.17
D	3.5	4	5	4.08	0.25
E	1	3	4	2.83	0.50
F	8	10	15	10.50	1.17
G	2	3	4	3.00	0.33
H	2	2	2.5	2.08	0.08

The standard deviation for event 3 depends solely on that of activity B. The standard deviation for event 3 is therefore 0.33.

For event 5 there are two possible paths, B + E or F. The total standard deviation for path B + E is $\sqrt{(0.33^2 + 0.50^2)} = 0.6$ and that for path F is 1.17; the standard deviation for event 5 is therefore the greater of the two, 1.17.

Verify that the standard deviations for each of the other events in the project are as shown in Figure 7.7.

7.8 Calculating standard deviations

The correct values are shown in Figure 7.7. Brief calculations for events 4 and 6 are given here.

Event 4: Path A + C has a standard deviation of $\sqrt{(0.50^2 + 0.17^2)} = 0.53$

Path B + D has a standard deviation of $\sqrt{(0.33^2 + 0.25^2)} = 0.41$

Node 4 therefore has a standard deviation of 0.53.

Event 6: Path 4 + H has a standard deviation of $\sqrt{(0.53^2 + 0.08^2)} = 0.54$

Path 5 + G has a standard deviation of $\sqrt{(1.17^2 + 0.33^2)} = 1.22$

Node 6 therefore has a standard deviation of 1.22.

Suppose that we must complete the project within 15 weeks at the outside. We expect it will take 13.5 weeks but it could take more or, perhaps, less. In addition, suppose that activity C must be completed by week 10, as it is to be carried out by a member of staff who is scheduled to be working on another project, and that event 5 represents the delivery of intermediate products to the customer, which must take place by week 10. These three target dates are shown on the PERT network in Figure 7.7.

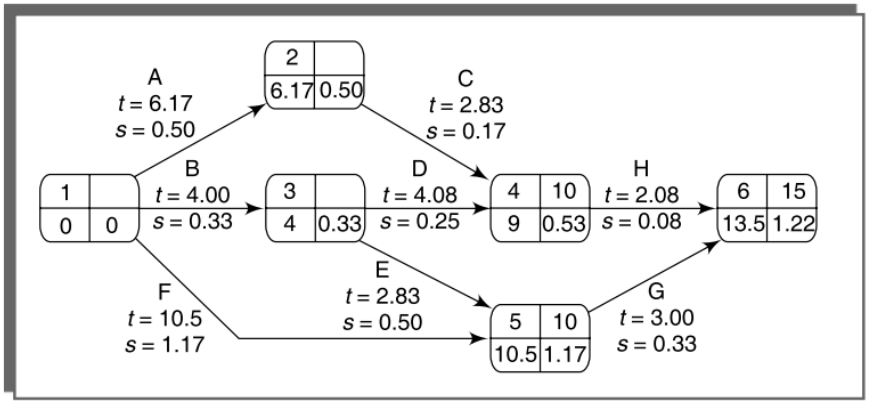


FIGURE 7.7 The PERT network with three target dates and calculated event standard deviations

Calculating the *z* values

The z value is calculated for each node that has a target date. It is equivalent to the number of standard deviations between the node's expected and target dates. It is calculated using the formula

$$z = \frac{T - t_e}{s}$$

where t_e is the expected date and T the target date.

EXERCISE 7.9

The z value for event 4 is (10 – 9.00)/0.53 = 1.8867.

Calculate the z values for the other events with target dates in the network shown in Figure 7.7.

7.9 Calculating *z* values

The z value for event 5 is $\frac{10 - 10.5}{1.17} = -0.43$, for event 6 it is $\frac{15 - 13.5}{1.22} = 1.23$.

EXERCISE 7.10

The z value for the project completion (event 6) is 1.23. Using Figure 7.8 we can see that this equates to a probability of approximately 11%, that is, there is an 11% risk of not meeting the target date of the end of week 15.

7.10 Obtaining probabilities

Event 4: The z value is 1.89 which equates to a probability of approximately 3%. There is therefore only a 3% chance that we will not achieve this event by the target date of the end of week 10.

Event 5: The z value is −0.43 which equates to a probability of approximately 67%. There is therefore a 67% chance that we will not achieve this event by the target date of the end of week 10.

To calculate the probability of completing the project by week 14 we need to calculate a new z value for event 6 using a target date of 14. This new z value is

$$z = \frac{14 - 13.5}{1.22} = 0.41$$

This equates to a probability of approximately 35%. This is the probability of not meeting the target date. The probability of meeting the target date is therefore 65% (100% − 35%).