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22/9/22

Assignment No.1

Que.1 Explain ASP and DSP systems. (10 marks)

Que.2 Write advantages of DSP systems over ASP systems (5 marks)

Que.3 Explain FIR system in detail. (5m)

Que.4 Explain IIR system in detail (5m)

FIR System:

FIR filter is defined by z-transform z(t) of impulse function.

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$y(n) = \sum_{n=0}^{N-1} b_k \, x(n-k)$$

ZT of above eqn

$$Y(z) = \sum_{n=0}^{N-1} b_k z^{-k} X(z)$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} b_k z^{-k}}$$

FIR filter Structure Realization :

1) Direct form Realization of FIR Filter

Que.1 Realize the direct form of FIR filter, if the difference equation is given by

$$y(n) = x(n)\,a_0 + a_1 x(n-1) + a_2 x(n-2) - b_2\, y(n-2) - b_1\, y(n-1)$$

Solⁿ: By applying ZT on above difference eqⁿ

$$Y(z) = a_0 X(z) + a_1 z^{-1} X(z) + a_2 z^{-2} X(z) - b_2 z^{-2} Y(z) - b_1 z^{-1} Y(z)$$

$$Y(z) + b_1 z^{-1} Y(z) + b_2 z^{-2} Y(z) = (a_0 + a_1 z^{-1} + a_2 z^{-2}) X(z)$$

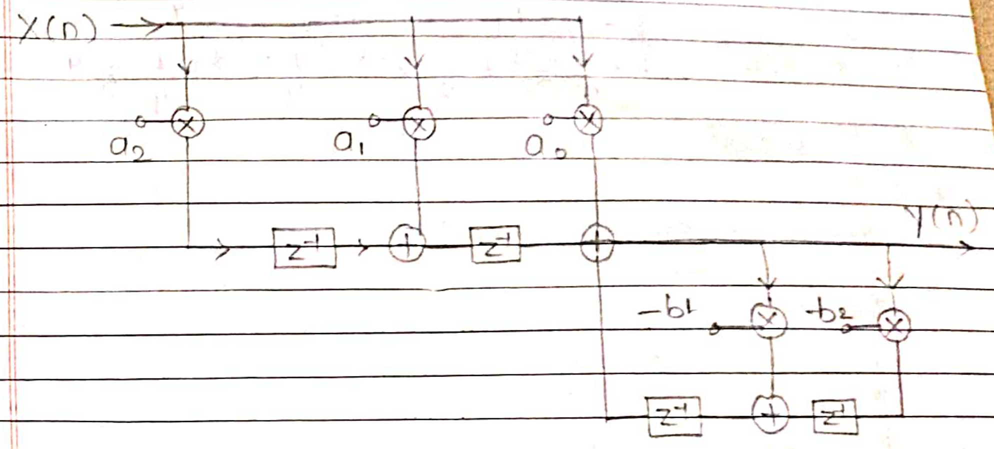
$$(1 + b_1 z^{-1} + b_2 z^{-2}) Y(z) = (a_0 + a_1 z^{-1} + a_2 z^{-2}) X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

** Standard format for FIR structure

$$\text{Numerator} \quad \frac{N(z)}{D(z)} = \frac{N(z)}{1 + D'(z)} \quad \text{Denominator}$$

Coefficient of numerator is always +ve
Coefficient of Denominator is always -ve



Que. $H(z) = \dfrac{4(z-1)^4}{4z^4+3z^3+2z^2+z+1}$ realize the given transfer function into direct form of FIR

Solⁿ $H(z) = \dfrac{4(z-1)^2 (z-1)^2}{4z^4+3z^3+2z^2+z+1}$

$$= \frac{4(z^2-2z+1)(z^2-2z+1)}{4z^4+3z^3+2z^2+z+1}$$

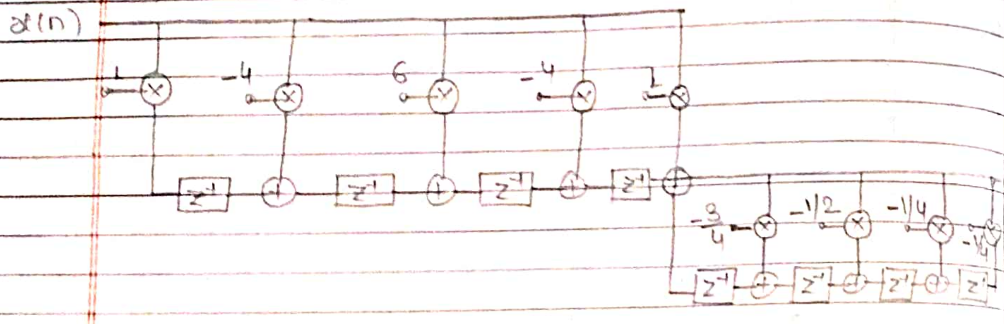
$$= \frac{4(z^4-2z^3+z^2-2z^3+4z^2-2z+z^2-2z+1)}{4z^4+3z^3+2z^2+z+1}$$

$$= \frac{4(z^4-4z^3+6z^2-4z+1)}{4z^4+3z^3+2z^2+z+1} \times \frac{z^{-4}}{z^{-4}}$$

$$= \frac{4(1-4z^{-1}+6z^{-2}-4z^{-3}+z^{-4})}{4+3z^{-1}+2z^{-2}+z^{-3}+z^{-4}}$$

multiply both by 1/4

$$= \frac{1 - 4z^{-1} + 6z^{-2} - 4z^{-3} + z^{-4}}{1 + \frac{3}{4}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{4}z^{-4}}$$



Que.5 Obtain direct form realization of an FIR filter

$$H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right)$$

Ans $$H(z) = 1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2} - \frac{1}{4}z^{-1} + \frac{1}{32}z^{-2} + \frac{1}{8}z^{-3} + \frac{3}{8}z^{-2} - \frac{3}{64}z^{-3} - \frac{3}{16}z^{-4}$$

$$= 1 - \left(\frac{1}{8} + \frac{1}{4}\right)z^{-1} - \left(\frac{1}{2} - \frac{1}{32} - \frac{3}{8}\right)z^{-2} + \left(\frac{1}{8} - \frac{3}{64}\right)z^{-3} - \frac{3}{16}z^{-4}$$

$$H(z) = 1 - \frac{3}{8}z^{-1} - \frac{5}{32}z^{-2} + \frac{5}{64}z^{-3} - \frac{3}{16}z^{-4}$$

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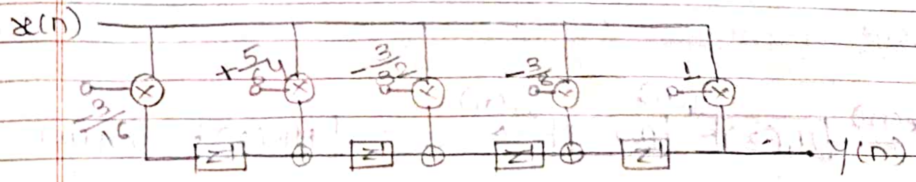


Que.4 Realize the given FIR filter in direct form

$$\frac{Y(z)}{X(z)} = \frac{z^{-1}+1}{z^{-1}+2} + \frac{z^{-1}+3}{z^{-1}-4}$$

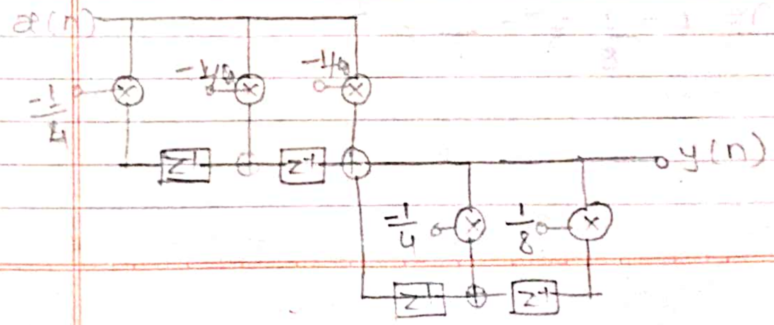
Ans $$H(z) = \frac{(z^{-1}+1)(z^{-1}-4) + (z^{-1}+3)(z^{-1}+2)}{(z^{-1}+2)(z^{-1}-4)}$$

$$= \frac{z^{-2} - 3z^{-1} - 4 + z^{-2} + 5z^{-1} + 6}{z^{-2} - 2z^{-1} - 8}$$

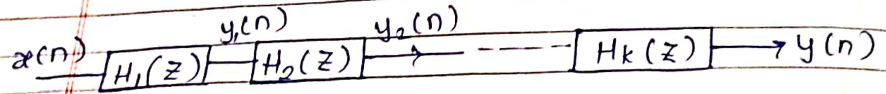
$$= \frac{2z^{-2} + 2z^{-1} + 2}{z^{-2} - 2z^{-1} - 8}$$

$$= \frac{(2z^{-2} + 2z^{-1} + 2)}{-8 - 2z^{-1} + z^{-2}}$$

$$= \frac{(z^{-2} + z^{-1} + 1)(-1/8)}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$



Cascade form of FIR Filter:



$$H(z) = H_1(z) \quad H_2(z) \; ------ \; H_k(z)$$

$$= \frac{y_1(z)}{x_2(z)} \quad \frac{y_2(z)}{x_2(z)} \; ------ \; \frac{Y_k(z)}{X_k(z)}$$

Realise each block

$$H_1(z) = \frac{N}{D} = \frac{N}{1+D^1}$$

Que.1 System function is given by

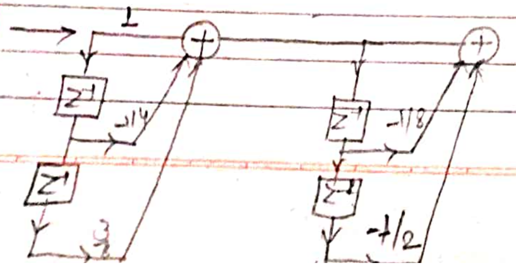
$$H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right)$$

Draw cascade realization of FIR filter.

Ans $H(z) = 1 - \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} + \frac{5}{64}z^{-3} - \frac{3}{16}z^{-4}$

$$H_1(z) = 1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}$$

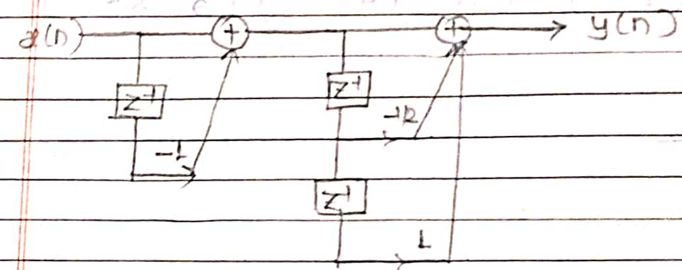
$$H_2(z) = 1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}$$



Que.2 Transfer function is given. Draw cascade realization

$$H(z) = 1 - \frac{3}{2}z^{-1} + \frac{3}{2}z^{-2} - z^{-3}$$

$$= (1 - z^{-1})\left(1 - \frac{1}{2}z^{-1} + z^{-2}\right)$$



$(1-z^{-1})$	1	$-\frac{3}{2}z^{-1}$	$\frac{3}{2}z^{-2}$	$-z^{-3}$
z^{-1}		z^{-1}	$-\frac{1}{2}z^{-2}$	z^{-3}
	1	$-\frac{1}{2}z^{-1}$	z^{-2}	0

Que.3 $H(z) = \frac{4(z-1)^4}{4z^4 + 3z^3 + 2z^2 + z + 1}$

Transfer function is given, Draw cascade realization of FIR filter.

Ans $H(z) = \frac{4(z-1)^2(z-1)^2}{4z^4 + 3z^3 + 2z^2 + z + 1}$

$$= \frac{4(z^4 - 4z^3 + 6z^2 - 4z + 1)}{4z^4 + 3z^3 + 2z^2 + z + 1} \times \frac{z^{-4}}{z^{-4}}$$

$$= \frac{4(1 - 4z^{-1} + 6z^{-2} - 4z^{-3} + z^{-4})}{4 + 3z^{-1} + 2z^{-2} + z^{-3} + z^{-4}}$$

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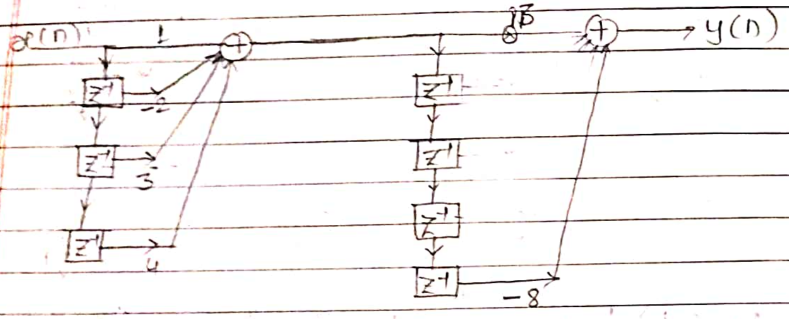
$$= \frac{1 - 4z^{-1} + 6z^{-2} - 4z^{-3} + z^{-4}}{1 + \frac{3}{4}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{4}z^{-4}}$$

Que.3 Cascade form of FIR filter

$$H(z) = (1 - 2z^{-1} + 3z^{-2} + 4z^{-3})(3 - 8z^{-4})$$

Ans



IIR Filter Design:

1) Direct form-I Realization of IIR filter

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \frac{W(z)}{X(z)} = H_1(z) H_2(z)$$

$$H_1(z) = \sum_{k=0}^{M} b_k z^{-k} \text{ ——— (Numerator/zeros) — (1)}$$

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \text{ ——— (denominator/Poles) — (2)}$$

$$H_1(z) = \frac{W(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$W(z) = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

IZT,

$$W(n) = \sum_{k=0}^{M} b_k x(n-k) \text{ ——— (I)}$$

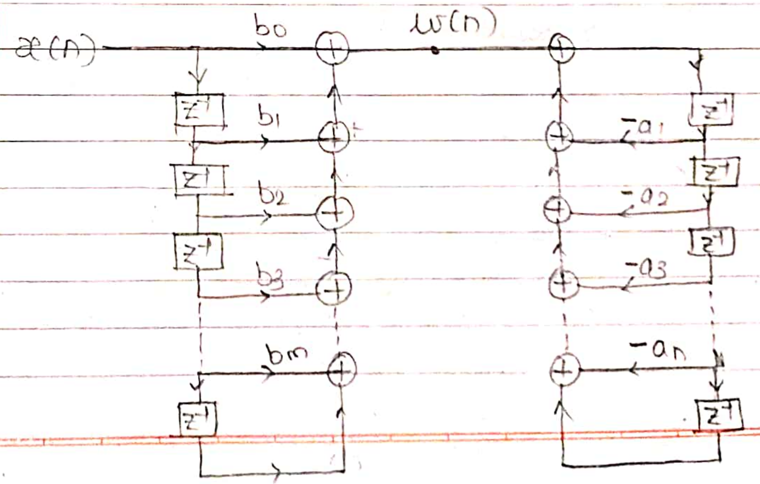
Now $H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$

$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = W(z)$$

IZT

$$y(n) + \sum_{k=1}^{N} a_k y(n-k) = w(n)$$

$$y(n) = w(n) - \sum_{k=1}^{N} a_k y(n-k) \text{ ——— (II)}$$



from eq^n (II) feedback value is there that's why have to take coefficients of denominator is -ve.

Que.1 The 2nd order filter is given by

$$y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + x(n) - r\cos\omega_0\, x(n-1)$$

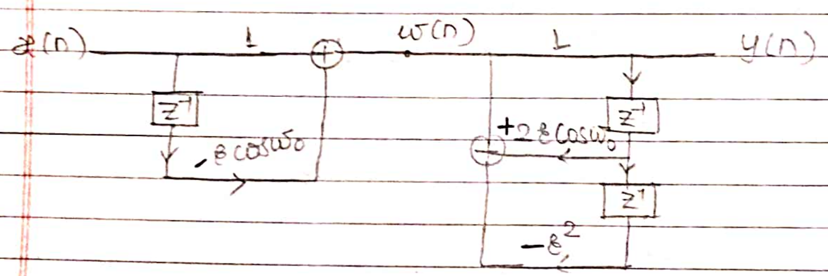
Implement the given filter in Direct form I realization.

Ans By taking Z-Transform of given difference eq^n

$$Y(z) = 2r\cos\omega_0 z^{-1} Y(z) - r^2 z^{-2} Y(z) + X(z) - r\cos\omega_0 z^{-1} X(z)$$

$$Y(z)(1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}) = X(z)(1 - r\cos\omega_0 z^{-1})$$

$$\frac{Y(z)}{X(z)} = \frac{1 - r\cos\omega_0 z^{-1}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}}$$



Que.2 The 2nd order filter is given by

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) + \frac{1}{3} x(n-1)$$

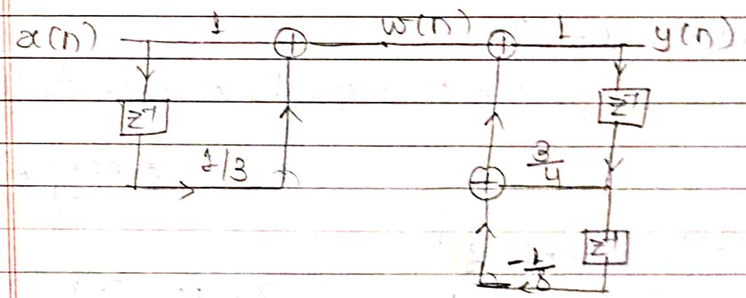
Implement the given filter in Direct form I realization.

Ans $\cancel{\dfrac{Y(z)}{X(z)} = \dfrac{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}{1 + \frac{1}{3}z^{-1}}}$

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = X(z) + \frac{1}{3} z^{-1} X(z)$$

$$Y(z)\left(1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right) = \left(1 + \frac{1}{3} z^{-1}\right) X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$



Que.3 $H(z) = \dfrac{\frac{1}{2} z^{-1} + \frac{1}{3} z^{-2} + \frac{1}{4} z^{-3}}{1 + 2z^{-1} + 3z^{-2} + 4z^{-3}}$ Realize the given filter using Direct form - I.

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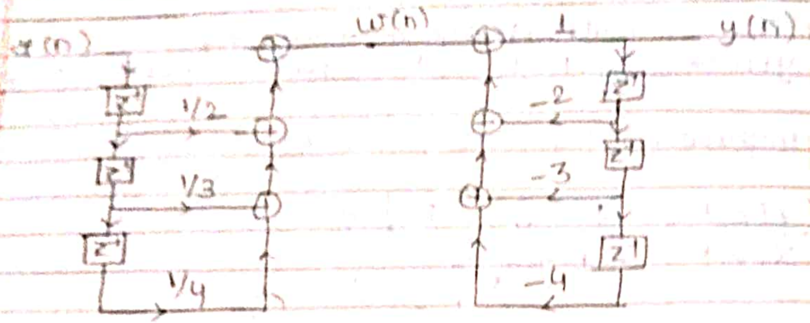
Ans



Que. 4 Consider a causal LTI system with system function

$$H(z) = \frac{1+\frac{1}{5}z^{-1}}{(1-0.5z^{-1}+\frac{1}{3}z^{-2})(1+0.25z^{-1})}$$

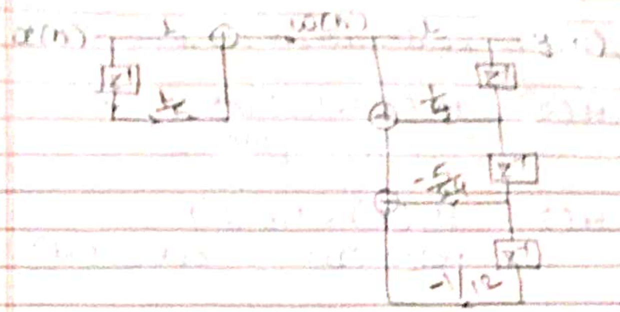
Realize the filter by using direct form I.

Ans

$$H(z) = \frac{1+\frac{1}{5}z^{-1}}{1+0.25z^{-1}-0.5z^{-1}-0.125z^{-2}+\frac{1}{3}z^{-2}+\cancel{0.0833}\,\frac{1}{12}z^{-3}}$$

$$= \frac{1+\frac{1}{5}z^{-1}}{1-0.25z^{-1}-5/24\,z^{-2}+1/12\,z^{-3}}$$

$$= \frac{1+\frac{1}{5}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}}$$



Que. 5 Realize the given IIR filter by using direct form I

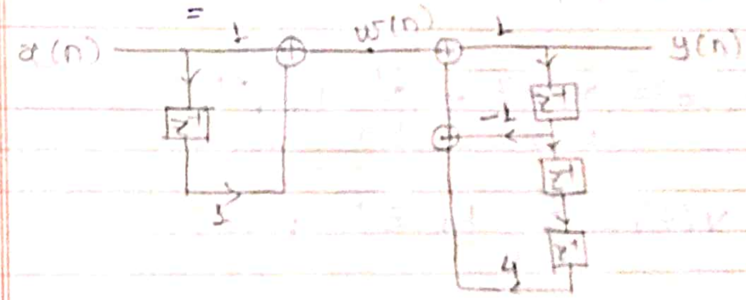
$$H(z) = \frac{z^2(z+1)}{(z^3+z^2-4)}$$

Ans

$$H(z) = \frac{z^3+z^2}{z^3+z^2-4} \times \frac{z^{-3}}{z^{-3}}$$

$$= \frac{1+z^{-1}}{1+z^{-1}-4z^{-3}}$$

=



27) Direct form – II Realization of IIR Filter.

DF-I $\quad H(z) = H_1(z) \cdot H_2(z)$

zeros $\quad$ poles

DF-II $\quad H(z) = H_1(z) \cdot H_2(z)$

poles (D) $\quad$ zeros (N)

$$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

$$W(z)\left(1+\sum_{k=1}^{N} a_k z^{-k}\right) = X(z)$$

$$W(z) + \sum_{k=1}^{N} a_k z^{-k} W(z) = X(z)$$

IZT,

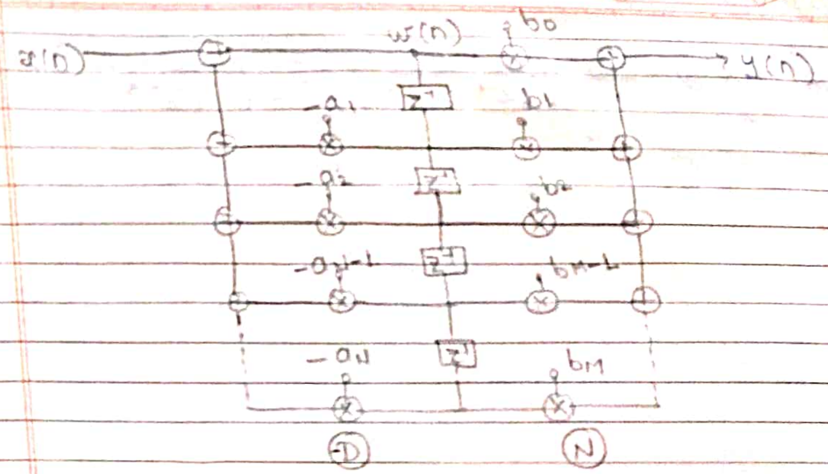
$$w(n) + \sum_{k=1}^{N} a_k w(n-k) = x(n) \quad \text{——— (1)}$$

$$H_2(z) = \frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$Y(z) = \sum_{k=0}^{M} b_k z^{-k} W(z)$$

IZT,

$$y(n) = \sum_{k=0}^{M} b_k w(n-k) \quad \text{——— (2)}$$



Que.1 $y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + x(n) - r\cos\omega_0\, x(n-1)$

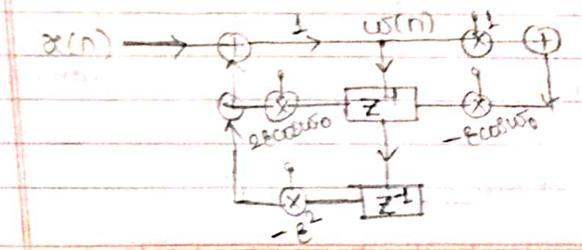
realize the given filter in Direct form – II.

Ans $\quad$ By Z-T of given difference equation

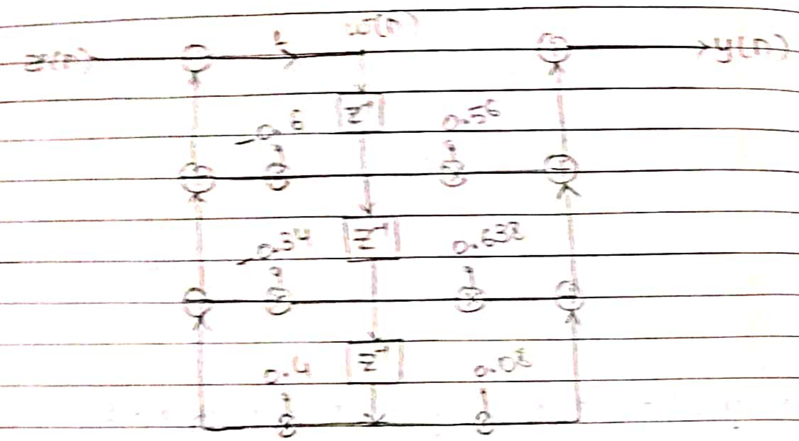
$$Y(z) = 2r\cos\omega_0 z^{-1} Y(z) - r^2 z^{-2} Y(z) + X(z) - r\cos\omega_0 z^{-1} X(z)$$

$$Y(z)\left[1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}\right] = X(z)\left(1 - r\cos\omega_0 z^{-1}\right)$$

$$\frac{Y(z)}{X(z)} = \frac{1 - r\cos\omega_0 z^{-1}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}}$$



Ques.2 $H(z) = \dfrac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$



Ques.3 A system has an impulse response

$$h(n) = (0.5)^n u(n) + n(0.2)^n u(n)$$

Show the direct form I realization

Soln: We know that,

$$\alpha^n u(n) \xrightarrow{ZT} \frac{z}{z-\alpha} \quad \text{——— (1)}$$

$$n\beta^n u(n) \xrightarrow{ZT}$$

$$(0.5)^n u(n) \xrightarrow{ZT} \frac{z}{z-0.5} \quad \text{——— from (1)}$$

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$$n\beta^n u(n) \longrightarrow n x(n) \longrightarrow -z \frac{d}{dz} X(z)$$

$$n x(n) \longrightarrow -z \frac{d}{dz} X(z) = -z \cdot \frac{d}{dz}\left[\frac{z}{z-0.2}\right]$$

$$= -z \cdot \left(\frac{+0.2z}{(z-0.2)^2}\right)$$

$$= -z \left[\frac{(z-0.2)\frac{d}{dz}(z) - z\frac{d}{dz}(z-0.2)}{(z-0.2)^2}\right]$$

$$= \frac{0.2z}{(z-0.2)^2}$$

$$H(z) = \frac{z}{z-0.5} + \frac{0.2z}{(z-0.2)^2}$$

$$= \frac{z(z^2 - 0.4z + 0.04) + 0.2z^2 - 0.1z}{(z-0.5)(z^2 - 0.4z + 0.04)}$$

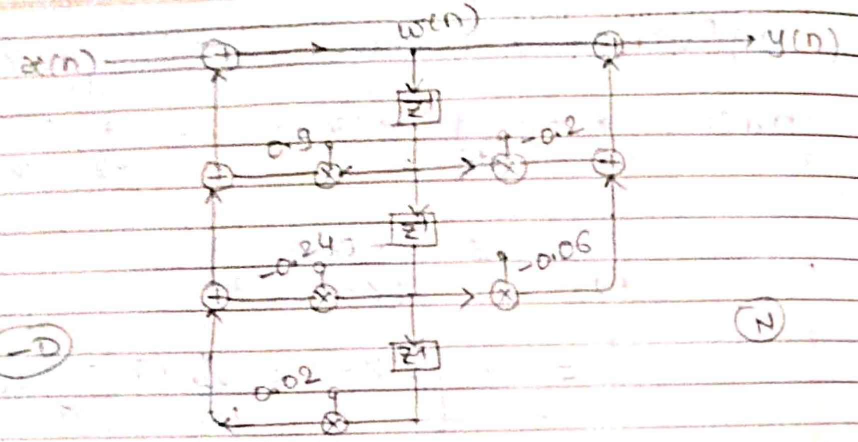
$$= \frac{z^3 - 0.4z^2 + 0.04z + 0.2z^2 - 0.1z}{z^3 - 0.4z^2 + 0.04z - 0.5z^2 + 0.2z - 0.02}$$

0.04
×0.5
0.020

0.1
−0.04
0.06

$$= \frac{z^3 - 0.2z^2 - 0.06z}{z^3 - 0.9z^2 + 0.24z - 0.02}$$

$$= \frac{1 - 0.2z^{-1} - 0.06z^{-2}}{1 - 0.9z^{-1} + 0.24z^{-2} - 0.02z^{-3}}$$



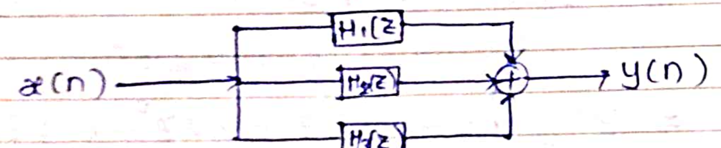
(D)

(N)

3) Parallel IIR Filter Realization

As we require less no. of z^{-1} blocks we prefer to realize parallel IIR filter by using Direct form II.

$$H(z) = H_1(z) + H_2(z) + H_3(z) \cdots\cdots$$



we have to solve $H_1(z)$, $H_2(z)$ and $H_3(z)$ by Direct form II realization.

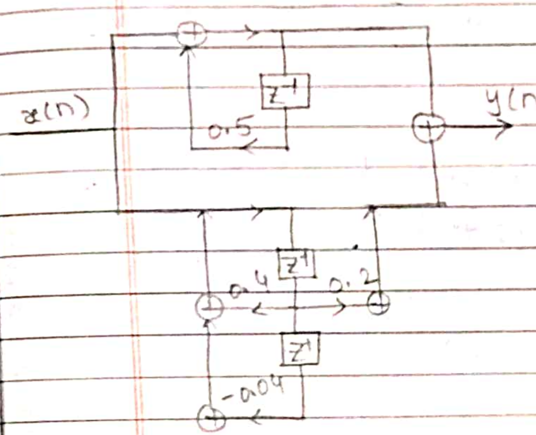
Que. 1

$$h(n) = 0.5^n u(n) + n\, 0.2^n\, u(n)$$

Sol^n:

$$H(z) = \frac{z}{z-0.5} + \frac{0.2z}{(z-0.2)^2}$$

$$= \frac{1}{1-0.5z^{-1}} + \frac{0.2z^{-1}}{1-0.4z^{-1}+0.04z^{-2}}$$



Que. 2 Realize the given filter in Parallel IIR form

$$H(z) = \frac{3(2z^2+5z+4)}{(2z+1)(z+2)}$$

Sol^n:

$$H(z) = \frac{3(2z^2+5z+4)}{(2z+1)(z+2)} = \frac{A}{(2z+1)} + \frac{B}{(z+2)}$$

$$A = \frac{3(2z^2+5z+4)\,\cancel{(2z+1)}}{\cancel{(2z+1)}(z+2)}\Bigg|_{z=\frac{-1}{2}}$$

$$A = \frac{3\left(2\left(\frac{1}{4}\right) - \frac{5}{2} + 4\right)}{\left(-\frac{1}{2}+2\right)}$$

$$= \frac{3\left[\frac{1}{2} - \frac{5}{2} + 4\right]}{\left(\frac{3}{2}\right)}$$

$$= \frac{3\left(\frac{-4}{2}+4\right)}{\frac{3}{2}}$$

$$= \cancel{3}\left(\frac{4}{\cancel{2}}\right)\left(\frac{\cancel{2}}{\cancel{3}}\right)$$

$$= 4$$

$$B = \cancel{(z+2)} \left.\frac{3\left(2z^2+5z+4\right)}{(2z+1)\cancel{(z+2)}}\right|_{z=-2}$$

$$= \frac{3\left(2\times 4 + 5(-2)+4\right)}{-4+1}$$

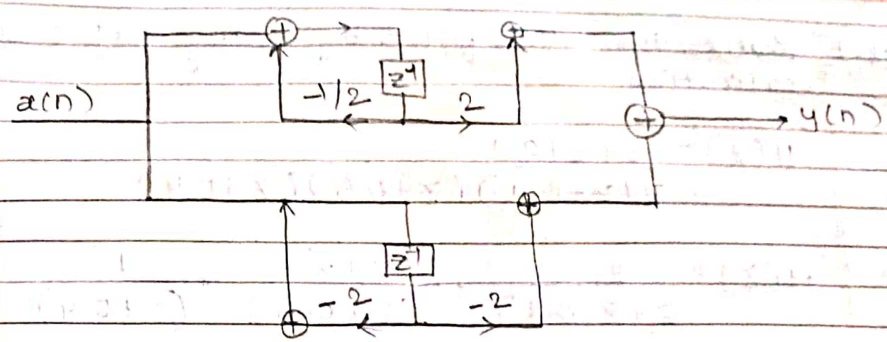
$$= \frac{\cancel{3}(8-10+4)}{-\cancel{3}}$$

$$= -2$$

$$H(z) = \frac{4}{(2z+1)} + \frac{-2}{(z+2)}$$

$$= \frac{4z^{-1}}{2+z^{-1}} + \frac{-2z^{-1}}{1+2z^{-1}}$$

$$= \frac{2z^{-1}}{1+z^{-1}/2} + \frac{-2z^{-1}}{1+2z^{-1}}$$



Que. 3

4) Cascade IIR Filter Realization

As we require less no. of z^{-1} blocks we prefer to realize cascade IIR filter by using direct form II

$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z)$ ------

$x(n)$ —— $\boxed{H_1(z)}$ —— $\boxed{H_2(z)}$ —— $\boxed{H_3(z)}$ —— $y(n)$

we have to solve $H_1(z)$, $H_2(z)$ and $H_3(z)$ by Direct form II realization.

Que.1 Realize the given filter by using IIR cascade realization

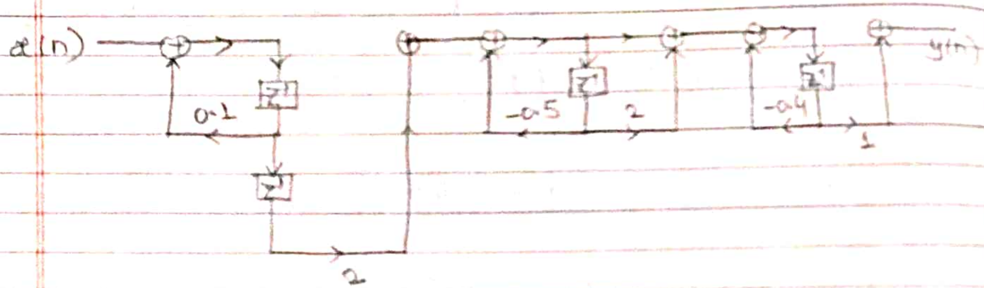
$$H(Z) = \frac{2(Z+2)}{Z(Z-0.1)(Z+0.5)(Z+0.4)}$$

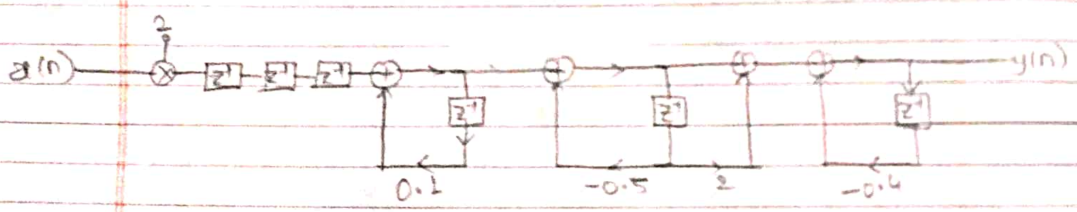
Solⁿ

$$H(Z) = \frac{2}{Z(Z-0.1)} \cdot \frac{(Z+2)}{(Z+0.5)} \cdot \frac{1}{(Z+0.4)}$$

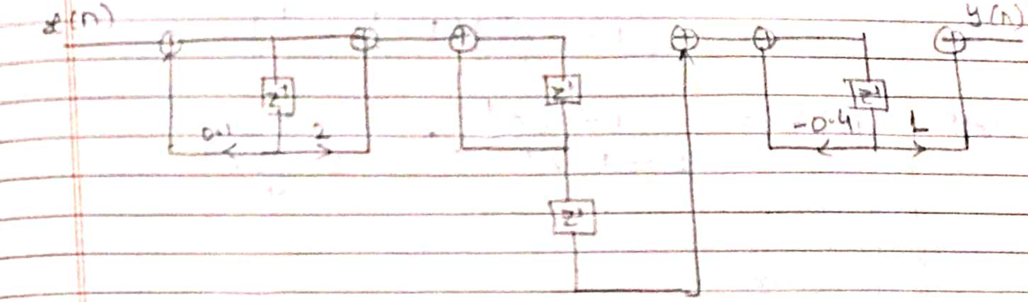
$$= \frac{Z^{-2}}{Z^{-2}} \frac{2}{Z(Z-0.1)} \frac{Z^{-1}}{Z^{-1}} \frac{(Z+2)}{(Z+0.5)} \frac{Z^{-1}}{Z^{-1}} \frac{1}{(Z+0.4)}$$

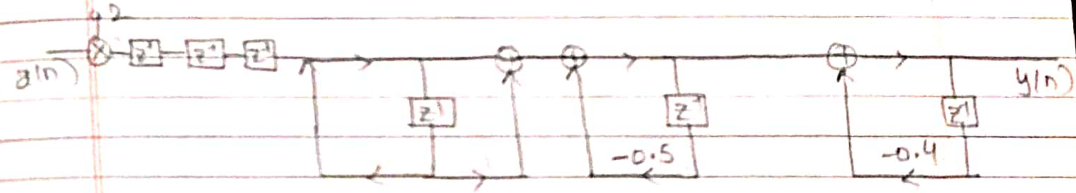
$$= \frac{2Z^{-2}}{1-0.1Z^{-1}} \cdot \frac{1+2Z^{-1}}{1+0.5Z^{-1}} \cdot \frac{Z^{-1}}{1+0.4Z^{-1}}$$





$$X(Z) = \frac{2(1+2Z^{-1})}{1-0.1Z^{-1}} \cdot \frac{Z^{-2}}{1+0.5Z^{-1}} \cdot \frac{Z^{-1}}{1+0.4Z^{-1}}$$





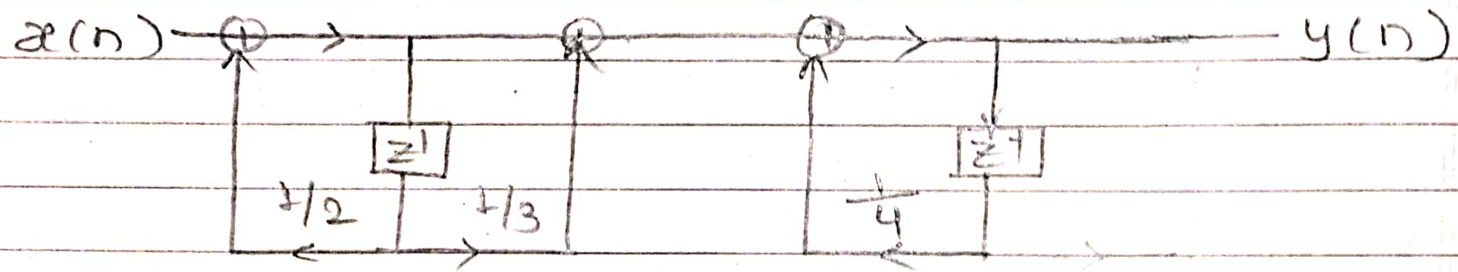
- Filter
  - FIR
    - Direct
    - Cascade
  - IIR
    - Direct form I
    - Direct form II
      - Parallel
      - Cascade

Que. 2 Realize the filter in cascade IIR Realization

$$H(Z) = \frac{1 + \frac{1}{3} Z^{-1}}{\left(1 - \frac{1}{2} Z^{-1}\right)\left(1 - \frac{1}{4} Z^{-1}\right)}$$

Solⁿ

$$H(Z) = \frac{1 + \frac{1}{3} Z^{-1}}{1 - \frac{1}{2} Z^{-1}} \cdot \frac{1}{1 - \frac{1}{4} Z^{-1}}$$



Unit 2 : Discrete Fourier Transform

Date: 3/10/22

* Discrete Fourier Transform (DFT) :

let $x(n)$ is the finite duration sequence so N point DFT of the sequence $x(n)$ is expressed as

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N}$$

$$x(n) \underset{IDFT}{\overset{DFT}{\rightleftharpoons}} X(k)$$

The given summation is for 0 to N-1 which gives total samples as 'N' nos. hence it is called as 'N' point DFT.

* Inverse Discrete Fourier Transform (IDFT) :

$$X(k) \xrightarrow{IDFT} x(n)$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}$$

$n = 0, 1, 2 --- N-1$ = N point

Que. 1 Plot magnitude and phase response of sampled data sequence $\{2, 0, 0, 1\}$ which was obtained using a sampling frequency of 20 kHz. Consider $N = 4$.

Soln: Given data : $x(n) = \{2, 0, 0, 1\}$

freq = 20 kHz

$N = 4$.

$e^{-j\theta} = \cos\theta - j\sin\theta$ 4/10/22

DFT is given by.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$N = 4$, $n = 0$ to $N-1$
$= 0$ to 3

$k = 0$ to 3

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi kn/4}$$

$$= x(0)\, e^{-j2\pi k0/4} + x(1)\, e^{-j2\pi k/4} + x(2)\, e^{-j2\pi k2/4} + x(3)\, e^{-j2\pi k3/4}$$

$$= 2e^{0} + 0\, e^{-j2\pi k/2} + 0\, e^{-j\pi k} + 1\, e^{-j3\pi k/2}$$

$$X(k) = 2 + e^{-j\frac{3\pi}{2}k} \qquad \text{——— (II)}$$

when $k=0$, $X(k) = 2 + e^{-j\frac{3\pi}{2}0} = 3 = 3\angle 0^\circ$

$k=1$, $X(k) = 2 + e^{-j\frac{3\pi}{2}} = 2 + j = 2.236\angle 26.57^\circ$

$k=2$, $X(k) = 2 + e^{-j3\pi} = 1 = 1\angle 0^\circ$

$k=3$, $X(k) = 2 + e^{-j\frac{9\pi}{2}} = 2 - j = 2.236\angle -26.57^\circ$

$X(k) = \{3, (2+j), 1, (2-j)\}$

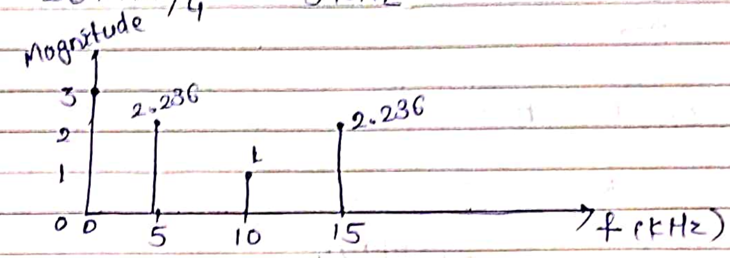
Note : for even no of N point
1^{st} term is always constant
then from remaining terms middle term is also constant

$a + jb$
magnitude $= \sqrt{a^2 + b^2}$, Angle (phase) $= \tan^{-1}\frac{b}{a}$

Suppose $N = 8$
$\{1, 2, 3, 4, 5, 6, 7, 8\}$

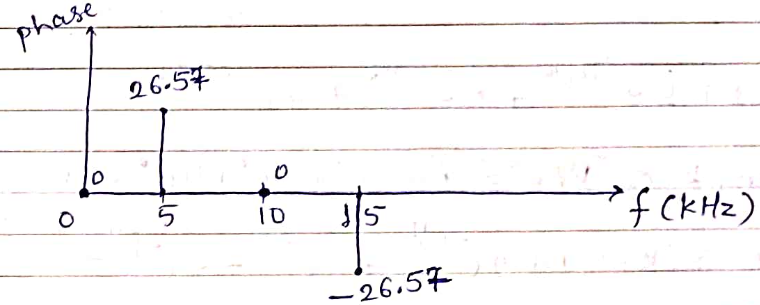
1 & 5 always constant
remainings are complex conjugate of each other

$20\,kHz/4 = 5\,kHz$



Magnitude spectrum of 4 point DFT

$x(k) = \{3, (2+j), 1, (2-j)\}$



Phase spectrum of 4 point DFT

$x(k) = \{3, (2+j), 1, (2-j)\}$

Que.2 Find 4 point DFT of $x(n) = \{1, 2, 3, 4\}$

Ans Given: $N = 4$

$x(n) = \{1, 2, 3, 4\}$

DFT is given by.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$N = 4, \quad n = 0 \text{ to } 3$

$k = 0 \text{ to } 3$

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j2\pi kn/4}$$

$$= x(0)e^{0} + x(1)e^{-j2\pi k/4} + x(2)e^{-j2\pi k2/4} + x(3)e^{-j2\pi k3/4}$$

$$= 1 + 2e^{-j\pi/2 k} + 3e^{-j\pi k} + 4e^{-j\frac{3\pi}{2}k}.$$

when, $k=0$, $X(k) = 1 + 2e^{0} + 3e^{0} + 4e^{0} = 10$

$k=1$, $X(k) = 1 + 2e^{-j\frac{\pi}{2}} + 3e^{-j\pi} + 4e^{-j\frac{3\pi}{2}}$

$= -2 + j2$

$k=2$, $X(k) = 1 + 2e^{-j\pi} + 3e^{-j2\pi} + 4e^{-j3\pi}$

$= -2$

$k=3$, $X(k) = 1 + 2e^{-j\frac{3\pi}{2}} + 3e^{-j3\pi} + 4e^{-j\frac{9\pi}{2}}$

$= -2 - j2$

$x(k) = \{10, (-2+j2), -2, (-2-j2)\}$

★ Twiddle Factor

DFT is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0 \text{ to } N-1$$

let, $W = e^{-j2\pi}$

W is called as 'Twiddle factor' it helps to compute DFT easier and faster.

$$e^{-j2\pi kn/N} = (e^{-j2\pi})^{kn/N}$$

$$= (W)^{kn/N}$$

$$= W^{kn/N}$$

$$= W_N^{kn}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}, \quad k = 0, 1, 2, \dots (N-1)$$

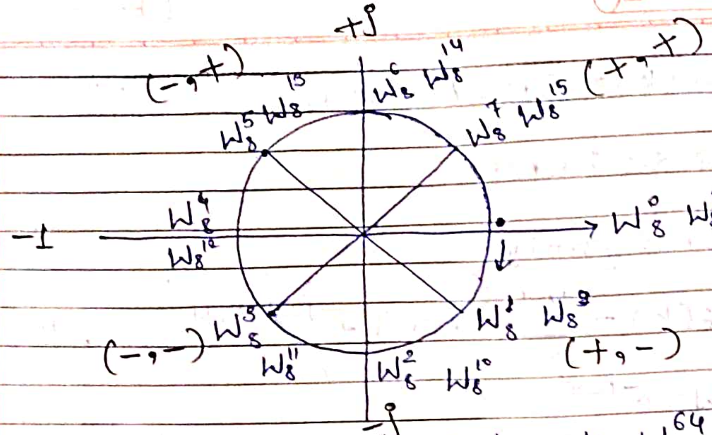
for 8pt DFT $N=8$

$kn=0$, $W_N^0 = W_8^0 = e^{-j2\pi 0/8} = e^0 = 1$

$kn=1$, $W_N^1 = W_8^1 = e^{-j2\pi 1/8} = 0.707 - j0.707$

$kn=2$, $W_N^2 = W_8^2 = e^{-j2\pi 2/8} = -j$

$kn=3$, $W_N^3 = W_8^3 = e^{-j2\pi 3/8} = -0.707 - j0.707$



we have to complete this circle upto W_8^{64}

Properties of DFT :

1) DFT as linear Transformation

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}, \quad 0 \le k \le N-1$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad \text{——— (1)}$$

$\boxed{X_N = W_N x_N}$ When $\left.\begin{matrix} X_N \\ W_N \\ x_N \end{matrix}\right\}$ are in Matrix form

for IDFT,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{+j2\pi kn}{N}}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} X(k)\, e^{-j2\pi\left(\frac{-kn}{N}\right)}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} X(k)\, W_N^{-kn} \qquad x_N = \frac{1}{N} X_N W_N^{-kn}$$

$$= \frac{1}{N} X_N W_N^{*kn} \quad \text{——— (*)}$$

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$$x_N = \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} \qquad X_N = \begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix}$$

$$W_N = \begin{bmatrix}
W_N^0 & W_N^0 & W_N^0 & W_N^0 & W_N^0 & W_N^0 & W_N^0 & W_N^0 \\
W_N^0 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 & W_N^6 & W_N^7 \\
W_N^0 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} & W_N^{12} & W_N^{14} \\
W_N^0 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} & W_N^{18} & W_N^{21} \\
W_N^0 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} & W_N^{24} & W_N^{28} \\
W_N^0 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} & W_N^{30} & W_N^{35} \\
W_N^0 & W_N^6 & W_N^{12} & W_N^{18} & W_N^{24} & W_N^{30} & W_N^{36} & W_N^{42} \\
W_N^0 & W_N^7 & W_N^{14} & W_N^{21} & W_N^{28} & W_N^{35} & W_N^{42} & W_N^{49}
\end{bmatrix}$$

In (*) eqn W_N^{kn} & W_N^{-kn} are complex conjugate of each other

- Padding of Zero : If $x(n) = \{1, 2\}$ & $N = 4$

we have to make it of same length, so $x(n)$ becomes

$x(n) = \{1, 2, 0, 0\}$

* **Two-Point Twiddle Factor Matrix:**

DFT is given by.

$$X_2 = W_2 x_2$$

$$W_2 = \begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{bmatrix}_{2\times 2}$$

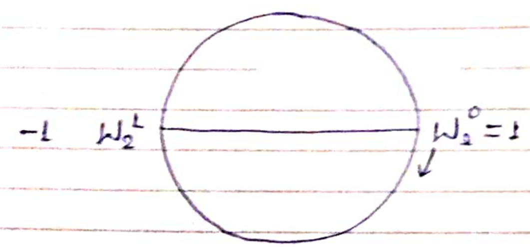
$$W_2^0 = e^{-j2\pi 0/2} = e^0 = 1$$

$$W_2^1 = e^{-j2\pi 1/2} = e^{-j\pi} = -1$$

$$\therefore W_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

In 2×2 matrix there is no 'j' term present

$N=2$ (circle in two equal parts)



* **Four-Point Twiddle Factor Matrix**

DFT is given by.

$$X_4 = W_4 x_4$$

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}_{4\times 4}$$

$$W_4^0 = e^{-j2\pi 0/2} = e^0 = 1$$

$$W_4^1 = e^{-j\pi/2} = -j$$

$$W_4^2 = -1$$

$$W_4^3 = j$$

$$W_4^4 = 1$$

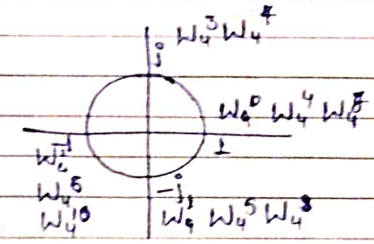
$$W_4^5 = -j$$

$$W_4^6 = -1$$

$$W_4^7 = j$$

$$W_4^8 = 1$$

$$W_4^9 = -j$$



$$\therefore W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Que.1 $x(n) = \{2, 0, 0, 1\}$
solve by using matrix method

Ans

The DFT is given by.

$$X_4 = W_4 x_4$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}_{4\times4} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} 2+0+0+1 \\ 2+0+0+j \\ 2+0+0+-1 \\ 2+0+0-j \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} 3 \\ 2+j \\ 1 \\ 2-j \end{bmatrix}_{4\times1}$$

$\therefore X_4 = \{3, 2+j, 1, 2-j\}$

Que.2 $x(n) = \{1, 2, 3, 4\}$
solve by using matrix method

Ans

The DFT is given by

$$X_4 = W_4 x_4$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}_{4\times4} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}_{4\times1}$$

$\therefore X_4 = \{10, (-2+2j), -2, (-2-2j)\}$

Que.3 Calculate 2-pt DFT
$x(n) = \{1, 2\}$

Ans The DFT is given by

$$X_2 = W_2 x_2$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}_{2\times2} \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times1}$$

$$= \begin{bmatrix} 1+2 \\ 1-2 \end{bmatrix}_{2\times1}$$

$$= \begin{bmatrix} 3 \\ -1 \end{bmatrix}_{2\times1}$$

$\therefore X_2 = \{3, -1\}$

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Que. 4 Calculate 4-pt DFT
$x(n) = \{1, 2\}$ as $N = 4$

Ans DFT is given by, as $N=4$
$x(n) = \{1, 2, 0, 0\}$

$$X_4 = W_4\, x_4$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}_{4\times4} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} 1+2+0+0 \\ 1-j2+0+0 \\ 1 \mp 2+0+0 \\ 1+j2+0+0 \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} 3 \\ 1-j2 \\ -1 \\ 1+j2 \end{bmatrix}_{4\times1}$$

$$\therefore X_4 = \{3, 1-j2, -1, 1+j2\}$$

* 8 by 8 Matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (+,-) & -j & (-,-) & -1 & (-,+) & j & (+,+) \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & (-,-) & j & (+,-) & -1 & (+,+) & -j & (-,+) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & (-,+) & -j & (+,+) & -1 & (+,-) & j & (-,-) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & (+,+) & j & (-,+) & -1 & (-,-) & -j & (+,-) \end{bmatrix}$$

Que.1 Find 8-point DFT of $x(n)=\{1,0,0,1,0,0,0,0\}$

Ans The DFT is given by

$$X_8 = W_8\, x_8$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (+,-) & -j & (-,-) & -1 & (-,+) & j & (+,+) \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & (-,-) & j & (+,-) & -1 & (+,+) & -j & (-,+) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & (-,+) & -j & (+,+) & -1 & (+,\mp) & j & (\mp,\mp) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & (+,+) & j & (-,+) & -1 & (-,-) & -j & (+,\mp) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 1\times1 \\ 1\times1 - 0.707 \mp 0.707j \\ 1\times1 + j\times1 \\ 1\times1 + 0.707 - 0.707j \\ 1\times1 + 1\times-1 \\ 1\times1 + 0.707 + 0.707j \\ 1\times1 + 1\times-j \\ 1\times1 \mp 0.707 + 0.707j \end{bmatrix}$$

$$\therefore X_8 = \begin{bmatrix} 2 \\ 0.293-0.707j \\ 1+j \\ 1.707-0.707j \\ 0 \\ 1.707+0.707j \\ 1-j \\ 0.293+0.707j \end{bmatrix}$$

$\therefore X_8 = \{2, (0.293-0.707j), (1+j), (1.707-0.707j), 0, (1.707-0.707j), (1-j), (0.293+0.707j)\}$

Que.3 Find 8-point DFT of $x(n)=\{1,2,3,4,5,6,7,8\}$

Ans The DFT is given by

$$X_8 = W_8\, x_8$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (+,-) & -j & (-,-) & -1 & (-,+) & j & (+,+) \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & (-,-) & j & (+,-) & -1 & (+,+) & -j & (-,+) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & (-,+) & -j & (+,+) & -1 & (+,-) & j & (-,-) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & (+,+) & j & (-,+) & -1 & (-,-) & -j & (+,-) \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+3+4+5+6+7+8 \\ 1+1.414-1.414j-3j-2.828-2.828j-5-4.242+4.242j+7j+5.656j+5.656 \\ 1-2j-3+4j+5-6j-7+8j \\ 1-1.414-1.414j+3j+2.828-2.828j-5+4.242+4.242j-7j-5.656+5.656j \\ 1-2+3-4+5-6+7-8 \\ 1-1.414+1.414j-3j+2.828+2.828j-5+4.242-4.242j+7j-5.656-5.656j \\ 1+2j-3-4j+5+6j-7-8j \\ 1+1.414+1.414j+3j-2.828+2.828j-5-4.242-4.242j-7j+5.656-5.656j \end{bmatrix}$$

$$= \begin{bmatrix} 36 \\ -4+9.656j \\ -4+4j \\ -4+1.656j \\ -4 \\ -4-1.656j \\ -4-4j \\ -4-9.656j \end{bmatrix}$$

$\therefore X_8 = \{36, (-4+9.656j), (-4+4j), (-4+1.656j), -4, (-4-1.656j), (-4-4j), (-4-9.656j)\}$

Que. 2 Find the 8-point DFT of $x(n) = \{1,1,1,1,1,1,1,1\}$

Ans DFT is given by,

$X_8 = W_8 \, x_8$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (+,-) & -j & (-,-) & -1 & (-,+) & j & (+,+) \\ 1 & -j & -1 & j & +1 & -j & -1 & j \\ 1 & (-,-) & j & (+,-) & -1 & (+,+) & -j & (-,+) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & (-,+) & -j & (+,+) & -1 & (+,-) & j & (-,-) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & (+,+) & j & (-,+) & -1 & (-,-) & -j & (+,-) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1+1+1+1+1+1 \\ 1+0.707-0.707j-j-0.707-0.707j-1-0.707+0.707j+j+0.707+0.707j \\ 1-j-1+j+1-j-1+j \\ 1-0.707-0.707j+j+0.707-0.707j-1+0.707+0.707j-j-0.707+0.707j \\ 1-1+1-1+1-1+1-1 \\ 1-0.707+0.707j-j+0.707+0.707j-1+0.707-0.707j+j-0.707-0.707j \\ 1+j-1-j+1+j-1-j \\ 1+0.707+0.707j+j-0.707+0.707j-1-0.707-0.707j-j+0.707-0.707j \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore X_8 = \{8,0,0,0,0,0,0,0\}$

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- Peoblems on IDFT :

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi kn}{N}}$$

$$W_2^* = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$W_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

Que. 1 Calculate IDFT of $X(k) = \{2, (1+j), 0, (1-j)\}$

Ans IDFT is given by

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi kn}{N}}$$

$N = 4$,

$$x(n) = \frac{1}{4}\sum_{k=0}^{3} X(k)\, e^{\frac{j2\pi kn}{4}}$$

$$x(n) = \frac{1}{4}\sum_{k=0}^{3} X(k)\, e^{\frac{j\pi kn}{2}}$$

$$= \frac{1}{4}\left[X(0)e^{0} + X(1)e^{\frac{j\pi n}{2}} + X(2)e^{j\pi n} + X(3)e^{\frac{j3\pi n}{2}}\right]$$

$$= \frac{1}{4}\left[2 + (1+j)e^{\frac{j\pi n}{2}} + (1-j)e^{\frac{j3\pi n}{2}}\right] \quad \text{——— (1)}$$

Substitute $n=0$ in eq^n (1)

$$x(0) = \frac{1}{4}\left[2 + (1+j)e^{0} + (1-j)e^{0}\right]$$

$$= \frac{1}{4}\left[2 + 1 + j + 1 - j\right]$$

$$= \frac{4}{4}$$

$$= 1$$

$$x(0) = 1$$

$$x(1) = \frac{1}{4}\left[2 + (1+j)e^{j\frac{\pi}{2}} + (1-j)e^{j\frac{3\pi}{2}}\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(j) + (1-j)(-j)\right]$$

$$= \frac{1}{4}\left[2 + \cancel{j} + (-1) \cancel{-j} + 1\right]$$

$$= \frac{1}{4}(2-2)$$

$$= 0$$

$\therefore x(1) = 0$

$$x(2) = \frac{1}{4}\left[2 + (1+j)e^{j\pi} + (1-j)e^{j3\pi}\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(-1) + (1-j)(-1)\right]$$

$$= \frac{1}{4}\left[2 - 1 \cancel{-j} - 1 \cancel{+j}\right]$$

$\therefore x(2) = 0$

$$x(3) = \frac{1}{4}\left[2 + (1+j)e^{j\frac{3\pi}{2}} + (1-j)e^{j\frac{9\pi}{2}}\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(-j) + (1-j)(j)\right]$$

$$= \frac{1}{4}\left[2 \cancel{-j} + 1 \cancel{+j} + 1\right]$$

$\therefore x(3) = 1$

$\therefore x(n) = \{1, 0, 0, 1\}$

$$x_4 = \frac{1}{N} W_4^{*} X_4$$

IDFT by matrix Method

$$x_4 = \frac{1}{N} W_4^{*} X_4$$

$$= \frac{1}{4} \underset{4\times4}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 2 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}}$$

$$= \frac{1}{4}\begin{bmatrix} 2+1+\cancel{j}+0+1\cancel{-j} \\ 2+\cancel{j}-1+0\cancel{-j}-1 \\ 2-1\cancel{-j}+0-1\cancel{+j} \\ 2\cancel{-j}+1+0\cancel{+j}+1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore x(n) = \{1, 0, 0, 1\}$

Que.2 Find DTS of $X(k) = \{4, 1-j, -2, 1+j\}$

Ans As DTS is calculated ~~from~~ from $X(k)$, IDFT method is used.

IDFT is given by

$$x_4 = \frac{1}{N} W_4^{*} X_4$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4+1-\cancel{j}-2+1+\cancel{j} \\ 4+\cancel{j}+1+2-\cancel{j}+1 \\ 4-1+\cancel{j}-2-1-\cancel{j} \\ 4-\cancel{j}-1+2+\cancel{j}-1 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4 \\ 8 \\ 0 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore x(n) = \{1, 2, 0, 1\}$

IDFT is given by

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}$$

$N = 4$,

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(k) e^{\frac{j\pi kn}{2}}$$

$$= \frac{1}{4} \left[X(0) e^{0} + X(1) e^{\frac{j\pi n}{2}} + X(2) e^{j\pi n} + X(3) e^{\frac{j3\pi n}{2}} \right]$$

$$= \frac{1}{4} \left[4 + (1-j) e^{\frac{j\pi n}{2}} + (-2) e^{j\pi n} + (1+j) e^{\frac{j3\pi n}{2}} \right] \quad \text{--- (1)}$$

$$x(0) = \frac{1}{4} \left[4 + (1-j) e^{0} + (-2) e^{0} + (1+j) e^{0} \right]$$

$$= \frac{1}{4} \left[4 + 1 - \cancel{j} - 2 + 1 + \cancel{j} \right]$$

$$= 1$$

$$x(1) = \frac{1}{4} \left[4 + (1-j) e^{\frac{j\pi}{2}} + (-2) e^{j\pi} + (1+j) e^{\frac{j3\pi}{2}} \right]$$

$$= \frac{1}{4} \left[4 + (1-j) j + (-2)(-1) + (1+j)(-j) \right]$$

$$= \frac{1}{4} \left[4 + \cancel{j} + 1 + 2 - \cancel{j} + 1 \right]$$

$$= \frac{8}{4}$$

$$= 2$$

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$$x(2) = \frac{1}{4}\left[4 + (1-j)e^{j\pi} + (-2)e^{j2\pi} + (1+j)e^{j3\pi}\right]$$

Que.3 Find IDFT of given DFT

$X(k) = \{4, (2+j), 1, (2-j)\}$

Ans IDFT is given by

$$x_4 = \frac{1}{N} W_4^{*} X_4$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & +j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix}\begin{bmatrix} 4 \\ 2+j \\ 1 \\ 2-j \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4+2+j+1+2-j \\ 4+\cancel{2j}-1-1-\cancel{2j}\mp 1 \\ 4-2-j+1\mp 2+j \\ 4-2j+1-1+2j+1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 9 \\ 1 \\ 1 \\ 5 \end{bmatrix}$$

$$\therefore x(n) = \left\{\frac{9}{4}, \frac{1}{4}, \frac{1}{4}, \frac{5}{4}\right\}$$

2-Point IDFT :

Que.1 Find DFT & then the IDFT of DFT of $x(n) = \{3, 4\}$

Ans DFT is given by

$$X_N = W_N x_N$$

$$X_2 = W_2 x_2$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4 \\ 3-4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$X(k) = \{7, -1\}$

IDFT is given by,

$$x_N = \frac{1}{N} W_N^{*} X_N$$

$$x_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 7-1 \\ 7+1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad \therefore x(n) = \{3, 4\}$$

Que Find 8-point IDFT of given DFT

i) $X(k) = \{8, 0, 0, 0, 0, 0, 0, 0\}$

Ans IDFT is given by

$$x_8 = \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (+,+) & j & (-,+) & -1 & (-,-) & -j & (+,-) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & (-,+) & -j & (+,+) & -1 & (+,-) & j & (-,-) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & (-,-) & j & (+,-) & -1 & (+,+) & -j & (-,+) \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & (+,-) & -j & (-,-) & -1 & (-,+) & j & (+,+) \end{bmatrix}\begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 8 \\ 8 \\ 8 \\ 8 \\ 8 \\ 8 \\ 8 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$x_8 = \{1, 1, 1, 1, 1, 1, 1, 1\}$

ii) $X(k) = \{40, 0, 0, 0, 0, 0, 0, 0\}$

Ans IDFT is given by

$$x_n = \frac{1}{N} W_N^* X_n$$

$$x_8 = \frac{1}{8} W_8^* X_8$$

$$= \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (+,+) & j & (-,+) & -1 & (-,-) & -j & (+,-) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & (-,+) & -j & (+,+) & -1 & (+,-) & j & (-,-) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & (-,-) & j & (+,-) & -1 & (+,+) & -j & (-,+) \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & (+,-) & -j & (-,-) & -1 & (-,+) & j & (+,+) \end{bmatrix}\begin{bmatrix} 40 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 40 \\ 40 \\ 40 \\ 40 \\ 40 \\ 40 \\ 40 \\ 40 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$$

$\therefore x_8 = \{5, 5, 5, 5, 5, 5, 5, 5\}$

(iii) $X(K) = \{36, -4+j9.656, -4+j4, -4+j1.656, -4,$
$-4-j1.656, -4-j4, -4-j9.656\}$

Ans IDFT is given by

$$x_8 = \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (+,+) & j & (-,+) & -1 & (-,-) & -j & (+,-) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & (-,+) & -j & (+,+) & -1 & (+,-) & j & (-,-) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & (-,-) & j & (+,-) & -1 & (+,+) & -j & (-,+) \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & (+,-) & -j & (-,-) & -1 & (-,+) & j & (+,+) \end{bmatrix} \begin{bmatrix} 36 \\ -4+j9.656 \\ -4+j4 \\ -4+j1.656 \\ -4 \\ -4-j1.656 \\ -4-j4 \\ -4-j9.656 \end{bmatrix}$$

$= \frac{1}{8}$ [$36 - 4 + j9.656 - 4 + j4 - 4 + j1.656 - 4 - 4 - j1.656$
$-4 - j4 - 4 - j9.656$

$36 + (0.707 - j0.707)(-4+j9.656) - 4j - 4 +$
$(-0.707 + j0.707)(-4+j1.656) + 4 +$
$(-0.707 \mp j0.707)(-4-j1.656) + 4j - 4 +$
$(0.707 - j0.707)(-4-j9.656)$

$36 \mp 4j - j9.656 + 4 - j4 \mp j4 + j4 - 1.656 + j1.656$
$-4 + 4 + j4 + j4 - 9.656$

$36 + (-0.707 + j0.707)(-4+j9.656) + j4 + 4 +$
$(0.707 + j0.707)(-4+j1.656) + 4 + (0.707 - j0.707)$
$(-4-j1.656) - j4 + 4 + (-0.707 - j0.707)$
$(-4-j9.656)$

$36 + 4 - j9.656 \mp 4 + j4 + 4 - j1.656 - 4 + 4 +$
$j1.656 - 4 - j4 + 4 + j9.656$

$36 + (-0.707 - j0.707)(-4+j8.656) - j4 - 4 +$
$(0.707 - j0.707)(-4+j1.656) + 4 + (0.707 + j0.707)$
$(-4-j1.656) + j4 - 4 + (-0.707 + j0.707)(-4-j9.656)$

$86 + j4 + 9.656 + 4 - j4 - j4 - 1.656 - 4 + j4 - 1.656$
$4 + j4 - j4 + 9.656$

$36 + (0.707 - j0.707)(-4+j9.656) + j4 + 4 +$
$(0.707 - j0.707)(-4+j1.656) + 4 + (-0.707 + j0.707)$
$(-4-j1.656) - j4 + 4 + (0.707 + j0.707)(-4-j9.656)$]

$$= \frac{1}{8}\begin{bmatrix} 8 \\ 16 \\ 24 \\ 32 \\ 40 \\ 48 \\ 56 \\ 64 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$\therefore x_8 = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Properties of DFT:

2) Linearity

If $x_1(n) \xleftrightarrow[N]{DFT} X_1(k)$ & $x_2(n) \xleftrightarrow[N]{DFT} X_2(k)$

$\therefore b_1 x_1(n) + b_2 x_2(n) \xleftrightarrow[N]{DFT} b_1 X_1(k) + b_2 X_2(k)$

→ DFT is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} [b_1 x_1(n) + b_2 x_2(n)] e^{-j2\pi kn/N}]$$

$$= \sum_{n=0}^{N-1} b_1 x_1(n) e^{-j2\pi kn/N} +$$

$$\sum_{n=0}^{N-1} b_2 x_2(n) e^{-j2\pi kn/N}$$

$$= b_1 \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi kn/N} +$$

$$b_2 \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi kn/N}$$

$$= b_1 X_1(k) + b_2 X_2(k)$$

Hence proved.

3) Periodicity

If $x(n) \xleftrightarrow[N]{DFT} X(k)$

If $x(n+N) = x(n)$, $X(k+N) = X(k)$

The periodicity is the result of complex exponential periodic property.

→ $$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

let $k \Rightarrow k+N$

$$X(k+N) = \sum_{n=0}^{N-1} x(n) W_N^{(k+N)n}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{kn} W_N^{Nn}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} e^{-j2\pi n}$$

$e^{-j2\pi n} \Rightarrow 1$ always

$$= \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$$= X(k)$$

Hence proved.

4) Circular Symmetry of a Sequence

Let consider the sequence $x(n)$ of length N

for periodic sequence

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$$

If we shift we get another periodic sequence

let $x_p'(n)$

$$\therefore x_p'(n) = x_p(m-k)_N$$

where $x_p'(n)$ is basically

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Imp 5-10 marks

5) Symmetry Property

let $x(n)$ is a complex value function

$$x(n) = x_R(n) + j x_I(n), \quad 0 \leq n \leq N-1 \quad \text{---} ①$$

$$X(k) = X_R(k) + j X_I(k), \quad 0 \leq k \leq N-1 \quad \text{---} ②$$

DFT is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$$X_R(k) + j X_I(k) = \sum_{n=0}^{N-1} [x_R(n) + j x_I(n)] \left[\cos\frac{2\pi kn}{N} - j\sin\frac{2\pi kn}{N}\right]$$

$$= \sum_{n=0}^{N-1} \left\{ x_R(n)\cos\frac{2\pi kn}{N} + x_I(n)\sin\frac{2\pi kn}{N} \right\}$$

$$+ j\left\{ x_I(n)\cos\frac{2\pi kn}{N} - x_R(n)\sin\frac{2\pi kn}{N} \right\}$$

Equating real and imaginary parts

$$X_R(k) = \sum_{n=0}^{N-1} \left\{ x_R(n)\cos\frac{2\pi kn}{N} + x_I(n)\sin\frac{2\pi kn}{N} \right\}$$

$$X_I(k) = \sum_{n=0}^{N-1} \left\{ x_I(n)\cos\frac{2\pi kn}{N} - x_R(n)\sin\frac{2\pi kn}{N} \right\}$$

①

Case. I When $x(n)$ is purely Imaginary

$$X_R(k) = \sum_{n=0}^{N-1} x_I(n) \sin \frac{2\pi kn}{N}$$

$$X_I(k) = \sum_{n=0}^{N-1} x_I(n) \cos \frac{2\pi kn}{N}$$

Case. II When $x(n)$ is real & odd

$$X_R(k) = \sum_{n=0}^{N-1} x_R(n) \cos \frac{2\pi kn}{N} = 0$$ as cos is even funct^n

$$X_I(k) = \sum_{n=0}^{N-1} -x_R(n) \sin \frac{2\pi kn}{N}$$

$\therefore X_R(k) = 0$

$$\therefore X(k) = -j \sum_{n=0}^{N-1} -x_R(n) \sin \frac{2\pi kn}{N}$$

Case III When $x(n)$ is real & even

$$X_R(k) = \sum_{n=0}^{N-1} x_R(n) \cos \frac{2\pi kn}{N}$$

$$X_I(k) = \sum_{n=0}^{N-1} -x_R(n) \sin \frac{2\pi kn}{N}$$

as sine is odd function

$\therefore X_I(k) = 0$

$$\therefore X(k) = \sum_{n=0}^{N-1} x_R(n) \cos \frac{2\pi kn}{N}$$

Case IV When $x(n)$ is real value function then

$$X(N-k) = X(-k) = X^*(k)$$

$\rightarrow$ let $k = N-k$ in DFT eq^n.

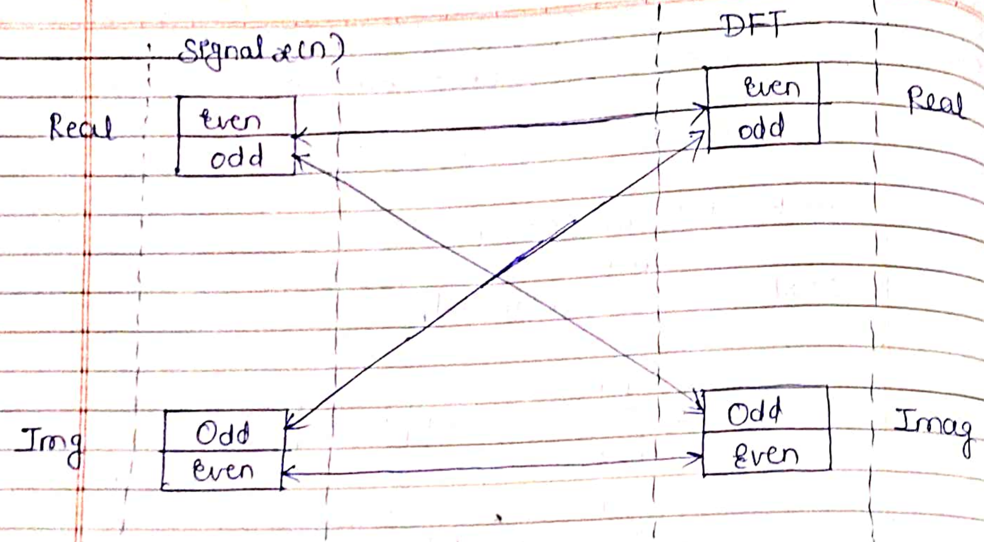
$$X(N-k) = \sum_{n=0}^{N-1} x(n) W_N^{(N-k)n}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{nN} \cdot W_N^{-kn}$$ $W_N^{nN} = 1$

$$= \sum_{n=0}^{N-1} x(n) W_N^{(-k)n}$$

$$= X(-k)$$

$$= X^*(k)$$



Unit III :

IIR Filter Design

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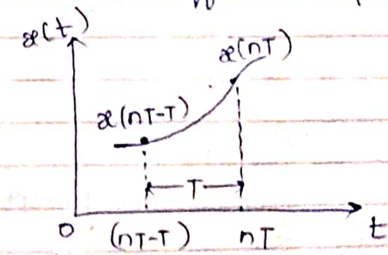
IIR filter design → Application of Derivatives
→ Impulse invariance method (IIM)
→ Bilinear ZT
→ Matched ZT

IIR, FIR → Digital filters

Stable Analog filter $H(S)$ → Digital IIR filter $H(z)$

Imp 1) **Approximation of Derivatives**

This is the simplest method of designing an analog filter to a digital filter in which the differential eq^n is approximated by an equivalent backward difference eq^n.



$$y(nT) = \frac{x(nT) - x(nT-T)}{T}$$

$$y(n) = \frac{x(n) - x(n-1)}{T}$$

$$ZT \Rightarrow \; Y(z) = \frac{X(z) - z^{-1} X(z)}{T}$$

$$\frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{T}$$

$$H(z) = \frac{1 - z^{-1}}{T} \quad \text{—— (1)}$$

LP → derivative $\frac{d}{dt}$
integration $\int dt = \frac{1}{S}$

and $y(t) = \frac{d}{dt} x(t)$

by laplace transform

$y(S) = S \cdot X(S)$

$\frac{y(S)}{X(S)} = S$

$H(S) = S$ ——— ②

equating X for funct^n ① from ZT to ② from Laplace funct^n

$$\boxed{H(z) = H_a(S) \Big|_{S = \frac{1-z^{-1}}{T}}}$$

eq^{n}① = eq^{n}②

$S = \frac{1 - z^{-1}}{T}$

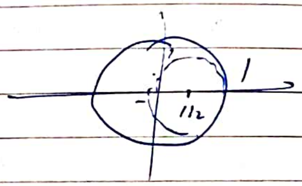
$|a + jb| = \sqrt{a^2+b^2}$

$\sqrt{a^2 + (-b)^2}$

$z = \frac{1}{1 - j\Omega T}$

$z - \frac{1}{2} = \frac{1 + j\Omega T}{2(1 - j\Omega T)}$

$|z| = \frac{1}{\sqrt{1 + \Omega^2 T^2}}$



Doubt

$\left|z - \frac{1}{2}\right| = \frac{1}{1 - j\Omega T} - \frac{1}{2}$

$= \frac{\sqrt{1 + (\Omega T)^2}}{2\sqrt{1 + (\Omega T)^2}}$

$= \frac{1}{2}$

$\frac{2 - 1 + j\Omega T}{2(1 - j\Omega T)}$

$\frac{1 + j\Omega T}{2(1 - j\Omega T)} = \frac{\sqrt{1 + \Omega^2 T^2}}{2\sqrt{1 + \Omega^2 T^2}}$

Analog → S domain

$\sigma + j\Omega = \frac{1 - z^{-1}}{T}$

$= \frac{z - 1}{zT}$

$z\sigma T + j\Omega z T = z - 1$

if $\sigma = 0$,

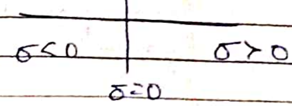
$j\Omega z T = z - 1$

$1 = z - j\Omega z T$

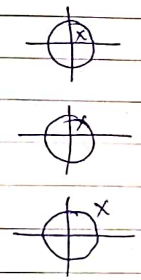
$1 = z(1 - j\Omega T)$

$\therefore z = \frac{1}{1 - j\Omega T}$

$\sigma = 0$ min cond^n to have stable nature in s domain



stability of z-plane (unit circle)

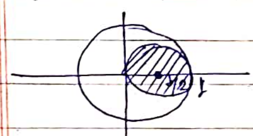


→ stable

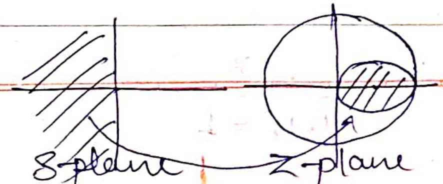
marginally stable

unstable

min cond^n to have stable in z domain



circle having radius 1/2



Stability of of S-plane converted into z-plane

$T = 0.1$ sec if not given

This method implies that imaginary axis in s-plane gets mapped to a circle of radius 1/2 in z-plane

The stability of s-plane (analog filter) has been transferred to the stability in z-plane (digital IIR filter)

Que.9 Use the backward difference method to convert analog LPF into digital IIR filter if

$$H(S) = \frac{1}{S+1}$$

Sol^n: backward difference method = approximation of derivative

$$H(z) = H_a(S)\Big|_{S=\frac{1-z^{-1}}{T}}$$

$$= \frac{1}{S+1}\Big|_{S=\frac{1-z^{-1}}{T}}$$

$$= \frac{1}{\frac{1-z^{-1}}{T}+1}$$

$$= \frac{T}{1-z^{-1}+T}$$

$$= \frac{TZ}{Z-1+TZ}$$

$$= \frac{TZ}{Z(1+T)-1} = \frac{0.1Z}{1.1Z-1}$$

* methods proof: 2 marks

The transfer funct^n of analog filter is

$$H(S) = \frac{1}{S+1}$$

The mapping of approx. of derivation method for the designing of FIR filter is

$$H(Z) = H(S)\Big|_{S=\frac{1-z^{-1}}{T}} = \frac{Z-1}{ZT}$$

$$H(Z) = H(S)\Big|_{S=\frac{Z-1}{0.1Z}} = \frac{10Z-10}{Z}$$

$$= \frac{1}{S+1}\Big|_{S=\frac{10Z-10}{Z}}$$

$$= \frac{1}{\frac{10Z-10}{Z}+1}$$

$$= \frac{Z}{10Z-10+Z}$$

$$\therefore H(Z) = \frac{Z}{11Z-10}$$

Que. 2 Convert the analog bandpass filter with system function

$$H(s) = \frac{1}{(s+0.1)^2+9}$$

into a digital IIR filter by using backward difference for derivatives

sol^n

$$H(s) = \frac{1}{s^2+0.2s+0.01+9} \Bigg|_{s = \frac{1-z^{-1}}{T} = \frac{10z-10}{z}}$$

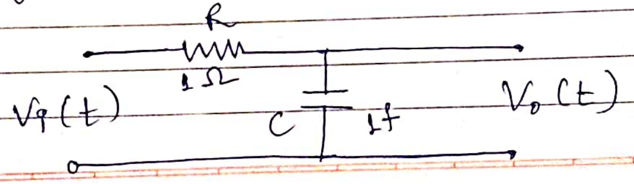
$$= \frac{1}{\left(\frac{10z-10}{z}\right)^2 + 0.2\left(\frac{10z-10}{z}\right) + 0.01+9}$$

$$= \frac{1}{\frac{100z^2-200z+100}{z^2} + \frac{2z-2}{z} + 9.01}$$

$$= \frac{z^2}{100z^2-200z+100+2z^2-2z+9.01z^2}$$

$$\therefore H(s) = \frac{z^2}{111.01z^2-202z+100}$$

Que. 3 Transform analog low pass filter into digital by using backward difference method.



sol^n:

$V_i(t) - R - C = 0$ input eq^n —— ①

$V_o(t) - C = 0$ o/p eq^n —— ②

$V_i(t) - 1 - 1 = 0$ $V_o(t) - 1 = 0$

$V_i(t) - 2 = 0$ $V_o(t) = 1$ —— ②

$V_i(t) = 2$ —— ①

$$V_i(S) - R\,i(S) - \frac{1}{CS}\,i(S) = 0 \quad \text{—— ①}$$

$$V_o(S) - \frac{1}{CS}\,i(S) \quad \text{—— ②}$$

$$V_i(S) = \left(R + \frac{1}{CS}\right) i(S) \quad \text{—— ③}$$

$$V_o(S) = \frac{1}{CS}\,i(S) \quad \text{—— ④}$$

divide eq^n ④ by eq^n ③

$$\frac{V_o(S)}{V_i(S)} = \frac{1/CS\ \cancel{i(S)}}{R+\frac{1}{CS}\ \cancel{i(S)}}$$

$$H(S) = \frac{1}{RCS+1} \quad \text{as } R = 1\Omega,\ C = 1f$$

$$= \frac{1}{S+1}$$

$$H(Z) = \frac{1}{S+1}\Bigg|_{S = \frac{10z-10}{z}}$$

pole at s=-1

$\therefore H(z) = \dfrac{z}{11z-10}$

2) Impulse Invariance Method

The design starts with the specifications of analog filter.
Impulse response of digital filter resembles with impulse response of analog filter

$h(t) \rightarrow$ Impulse response in time domain
$H_a(s) \rightarrow$ TF of analog filter

$h(nT_s) \rightarrow$ sampled version of $h(t)$ obtained by replacing $t \rightarrow nT_s$

$H(z) \rightarrow$ TF of digital filter, response of digital filter
ZT of $h(nT_s)$

$\Omega \rightarrow$ analog frequency

$\omega \rightarrow$ digital frequency

$$H_a(s) = \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2} + \frac{A_3}{s-p_3} + \ldots\ldots$$

$$H_a(s) = \sum_{k=1}^{N} \frac{A_k}{s-p_k}$$

$A_k = A_1, A_2, \ldots\ldots$ Coeff of PFE
$P_k = P_1, P_2, \ldots\ldots$ poles

By Laplace inverse transform

$$h(t) = \sum_{k=1}^{N} A_k e^{p_k t}$$

The DTS for $t \rightarrow nT_s$

$$h(n) = \sum_{k=1}^{N} A_k e^{p_k \cdot nT_s}$$

The ZT is given by

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left[\sum_{k=1}^{N} A_k e^{p_k n T_s} \right] z^{-n}$$

$$= \sum_{k=1}^{N} A_k \sum_{n=0}^{\infty} \left(e^{p_k T_s} \cdot z^{-1}\right)^n$$

$$\boxed{H(z) = \sum_{k=1}^{N} A_k \frac{1}{1 - e^{p_k T_s} z^{-1}}} \qquad \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

Transfer function of Digital filter

$$\boxed{H_a(s) = \sum_{k=1}^{N} A_k \frac{1}{s - p_k}}$$

Transfer funct^n of analog filter

By comparing two equations

$$\sum_{k=1}^{N} A_k \frac{1}{S-P_k} \Longrightarrow \sum_{k=1}^{N} A_k \frac{1}{1-e^{P_k T_s} z^{-1}}$$

$$\boxed{\frac{1}{S-P_k} \longrightarrow \frac{1}{1-e^{P_k T_s} z^{-1}}} \quad \text{Mapping}$$

$$\therefore S = P_k \quad \& \quad \frac{z}{z - e^{P_k T_s}}$$

$$z = e^{P_k T_s}$$

As, $S = P_k$ & $z = e^{P_k T_s}$

$$\therefore \boxed{z = e^{S T_s}} \quad \text{———} \quad (1)$$

Now, $z = r e^{j\omega}$, $S = \sigma + j\Omega$

$\therefore$ eq^n (1) becomes

$$r e^{j\omega} = e^{(\sigma + j\Omega) T_s}$$

$$r e^{j\omega} = e^{\sigma T_s} \cdot e^{j\Omega T_s}$$

by equating real and imaginary parts

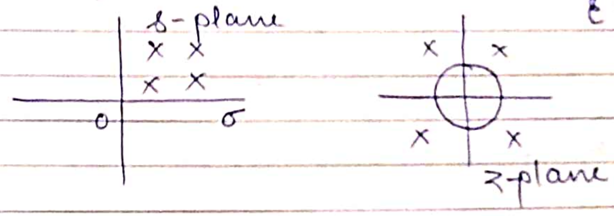
$$\boxed{r = e^{\sigma T_s}} \cdot \quad \boxed{\omega = \Omega T_s}$$

Minimum Condition for stability

1) $\sigma = 0$, $r = e^{\sigma T_s} = e^0 = 1$ Marginally stable

2) $\sigma < 0$, stable condition $r = e^{\sigma T_s} < 1$

3) $\sigma > 0$, unstable condition $r = e^{\sigma T_s} > 1$



Que.) Find out $H(z)$ by using impulse invariance method at 5Hz sampling freq, the analog transfer function is given by

$$H(S) = \frac{2}{(S+1)(S+2)}$$

Sol^n

$$H(S) = \frac{2}{(S+1)(S+2)} = \frac{A}{S+1} + \frac{B}{S+2}$$

$$A = \left.\frac{2\cancel{(S+1)}}{\cancel{(S+1)}(S+2)}\right|_{S=-1}$$

$$= \frac{2}{(-1+2)}$$

$\therefore A = 2$

$$B = \left.\frac{2\cancel{(S+2)}}{(S+1)\cancel{(S+2)}}\right|_{S=-2}$$

$$= \frac{2}{S+1}$$

$$= \frac{2}{-2+1}$$

$\therefore B = -2$

Mapping of IIM is given by

$$\frac{1}{S-P_k} \longrightarrow \frac{1}{1-e^{P_k T_S} z^{-1}}$$

$f_s = 5\text{Hz}$ ——— given

$$T_S = \frac{1}{f_s} = \frac{1}{5} = 0.2 \text{ sec}$$

$$H(S) = \frac{2}{S+1} - \frac{2}{S+2}$$

$$= \frac{2}{\boxed{S-(-1)}} - \frac{2}{S-(-2)} \qquad \frac{1}{S-P_k}$$

$P_k = -1$ $\quad$ $P_k = -2$

$t_S = 0.2$ $\quad$ $t_S = 0.2$

$$H(z) = H(S) \qquad \frac{1}{S-P_k} \longrightarrow \frac{1}{1-e^{P_k T_S} z^{-1}}$$

$$H(z) = \frac{2}{1-e^{-1\times0.2}z^{-1}} - \frac{2}{1-e^{-2\times0.2}z^{-1}}$$

$$= \frac{2z}{z-e^{-0.2}} - \frac{2z}{z-e^{-0.4}}$$

$$H(z) = \frac{2z}{z-0.818} - \frac{2z}{z-0.67}$$

$$= \frac{2z(z-0.67) - 2z(z-0.82)}{(z-0.82)(z-0.67)}$$

$$= \frac{\cancel{2z^2} - 1.34z - \cancel{2z^2} + 1.64z}{z^2 - 0.67z - 0.82z + 0.55}$$

$$\therefore H(z) = \frac{0.3z}{z^2 - 1.49z + 0.55}$$

3) Bilinear ZT Method

- This is important method for obtaining IIR filter coefficient.
- The BZT is transformation from analog s-plane to z-plane.
- The analog poles and zeros are transfer to digital poles and zeros respectively.

The mapping of BZT is

$$\boxed{S = \frac{2}{T_s}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]} = \frac{2}{T_s}\left[\frac{z-1}{z+1}\right]$$

Relationship of S & z-plane

$S = \sigma + j\Omega \longrightarrow \Omega$ analog freq

$z = re^{j\omega} \longrightarrow \omega$ digital freq

$$S = \frac{2}{T_s}\left[\frac{z-1}{z+1}\right]$$

$$= \frac{2}{T_s}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right]$$

$$= \frac{2}{T_s}\left[\frac{r(\cos\omega + j\sin\omega) - 1}{r(\cos\omega + j\sin\omega) + 1}\right]$$

$$= \frac{2}{T_s}\left[\frac{(r\cos\omega - 1) + jr\sin\omega}{(r\cos\omega + 1) + jr\sin\omega}\right] \times \frac{a-jb}{a-jb}$$

By solving

$$S = \frac{2}{T_s}\left[\frac{r^2-1}{1+r^2+2\cos\omega} + j\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right]$$

As $S = \sigma + j\Omega$

equating real and imaginary

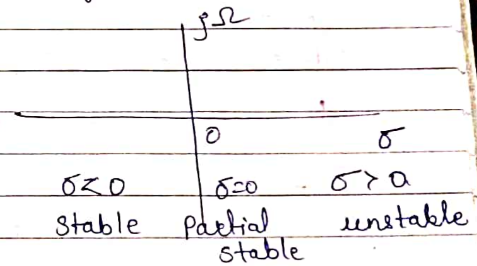
$$\sigma = \frac{2}{T_s}\left[\frac{r^2-1}{1+r^2+2\cos\omega}\right]$$

checking the stability criteria for r & σ

filter is stable 1) If $r<1$, $\sigma<0$

Marginally stable 2) If $r=1$, $\sigma=0$

unstable 3) If $r>1$, $\sigma>0$



Hence it proves that the stability of S-domain has been completely transform to the stability of z-domain by using BZT mapping

Now, considering analog frequency Ω

$$\Omega = \frac{2}{T_s}\frac{2r\sin\omega}{1+r^2+2r\cos\omega}$$

If $r = 1$,

$$\Omega = \frac{2}{T_s} \frac{2 \sin \omega}{1+1+2\cos\omega}$$

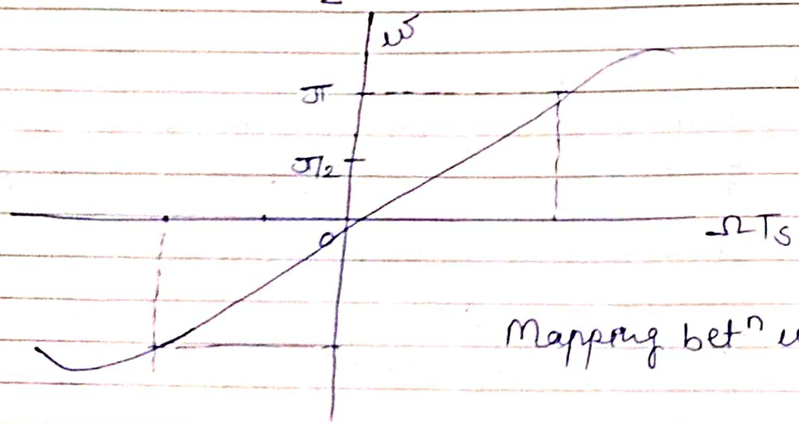
$$= \frac{2}{T_s} \frac{\cancel{2} \sin\omega}{\cancel{2}(1+\cos\omega)}$$

$$= \frac{2}{T_s} \frac{\cancel{2} \sin \omega/2 \cos \omega/2}{\cancel{2} \cos^2 \omega/2}$$

$$= \frac{2}{T_s} \frac{\sin \omega/2}{\cos \omega/2}$$

$$\therefore \boxed{\Omega = \frac{2}{T_s} \tan \omega/2}$$

$$\omega = 2 \tan^{-1} \frac{\Omega T_s}{2}$$



Mapping betⁿ ω & Ω

With this graphical representation of analog and digital frequency we observe a freq compression or freq warping due to non linearity of arctangent function.

digital value upto π, there is 1 to 1 relationship

- Pre warping effect:

The mapping of BZT is

$$S = \frac{2}{T_s}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = \frac{2}{T_s}\left[\frac{z-1}{z+1}\right]$$

By solving

$$S = \frac{2}{T_s}\left[\frac{r^2-1}{1+r^2+2r\cos\omega} + j\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right]$$

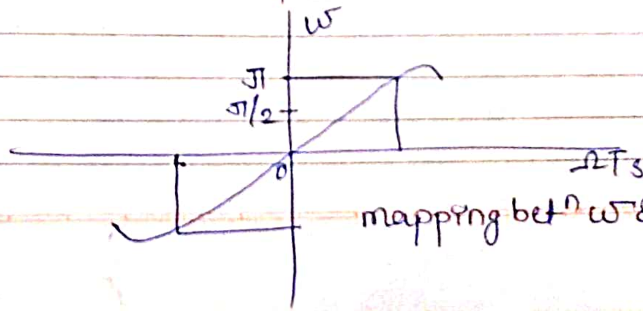
If $r=1$,

$$\Omega = \frac{2}{T_s} \frac{2\sin\omega}{2+2\cos\omega}$$

$$= \frac{2}{T_s} \frac{\sin\omega}{1+\cos\omega} = \frac{2}{T_s} \frac{\cancel{2} \sin \omega/2 \cos\omega/2}{\cancel{2}\cos^2\omega/2}$$

$$\therefore \Omega = \frac{2}{T_s} \tan \omega/2$$

$$\therefore \omega = 2\tan^{-1} \Omega T_s/2$$



mapping betⁿ ω & Ω

- because of the non-linearity of arctangent function we get freq warping effect
- This compression is compensated by introducing suitable pre-scaling or pre-warping to frequency scale

For BZT Ω scale changed to Ω^* scale (Prewarped)

$$\Omega^* = \frac{2}{T_s} \tan \frac{\omega T_s}{2}$$

&

$$\omega = \frac{2}{T_s} \arctan \frac{\Omega^* T_s}{2}$$

If $\omega < \pi$ → No prewarping is required

$\omega > \pi$ → prewarping is required

Que. 1 The given analog filter system function is

$$H(S) = \frac{S+0.1}{(S+0.1)^2+16}$$

into digital IIR filter by means of BZT, The resonance frequency $(\omega_r) = \pi/2$

Ans As ω_r is $\frac{\pi}{2}$ that is less than π so, no pre-warping procedure is required.

when $H(S) = \frac{S+a}{(S+a)^2+\Omega_r^2}$

$\Omega_r^2 = 16$; $\Omega = 4$

$$\Omega = \frac{2}{T_s} \tan \omega/2$$

$$= \frac{2}{T_s} \tan \frac{\pi/2}{2}$$

$$\Omega = \frac{2}{T_s} \tan \frac{\pi}{4}$$

$$T_s = \frac{2}{4}$$

$\therefore T_s = 0.5$

The mapping of BZT is

$$S = \frac{2}{T_s}\left[\frac{Z-1}{Z+1}\right]$$

$$S = \frac{2}{0.5}\left[\frac{Z-1}{Z+1}\right]$$

$$\therefore S = 4\left[\frac{Z-1}{Z+1}\right]$$

$$H(Z) = H(S)\Big|_{S=4\left[\frac{Z-1}{Z+1}\right]}$$

$$= \frac{S+0.1}{(S+0.1)^2+16}\Bigg|_{S=4\left[\frac{Z-1}{Z+1}\right]}$$

$$H(z) = \frac{s+0.1}{s^2+0.2s+0.01+16}\Bigg|_{s=4\left[\frac{z-1}{z+1}\right]}$$

$$= \frac{4\left(\frac{z-1}{z+1}\right)+0.1}{\left[4\left(\frac{z-1}{z+1}\right)\right]^2+0.2\left[\frac{z-1}{z+1}\right]4+16.01}$$

$$= \frac{4\left(\frac{z-1}{z+1}\right)+0.1}{16\left(\frac{z-1}{z+1}\right)^2+0.2\left(\frac{z-1}{z+1}\right)4+16.01}$$

$$= \frac{4\left(\frac{z-1}{z+1}\right)+0.1}{16\frac{(z-1)^2}{(z+1)^2}+0.2\frac{(z-1)(z+1)}{(z+1)^2}\times 4+16.01\frac{(z+1)^2}{(z+1)^2}}$$

$$= \frac{(z+1)^2\left[4\left(\frac{z-1}{z+1}\right)+0.1\right]}{16(z-1)^2+0.8(z-1)(z+1)+16.01(z+1)^2}$$

$$= \frac{4(z-1)(z+1)+0.1(z+1)^2}{16(z-1)^2+0.8(z-1)(z+1)+16.01(z+1)^2}$$

$$= \frac{4[z^2-1]+0.1(z^2+2z+1)}{16(z^2-2z+1)+0.8(z^2-1)+16.01(z^2+2z+1)}$$

$$= \frac{4z^2-4+0.1z^2+0.2z+0.1}{16z^2-32z+16+0.8z^2-0.8+16.01z^2+32.02z+16.01}$$

$$= \frac{4.1z^2+0.2z-3.9}{z^2(16+0.8+16.01)z^2-(32z-32.02)z+(16-0.8+16.01)}$$

$$= \frac{4.1z^2+0.2z-3.9}{32.81z^2+0.02z+31.21}$$

Que. 2 The RC LPF is $H(s) = \frac{1}{s+1}$, BW = 1 rad/sec design digital filter whose BW is 20 Hz and digital freq $f_s = 60$ sps use BZT.

Ans The given data is,

$\omega > \pi$ prewarping

Ω^*

$H(s)^* = H(s)\Big|_{s\to\frac{s}{\Omega^*}}$

Analog BW = 1 rad/sec
digital BW = 20 Hz
digital f_s = 60 sps

$H(z) = H^*(s)\Big|_{s=\frac{2}{T_s}\frac{z-1}{z+1}}$

$\omega_p = 2\pi(20)$ rad/sec $= 40\pi > \pi$

pre-warping procedure is required

1) Modified freq

$$\Omega^*_p = \frac{2}{T_s}\tan\left(\frac{\omega_p T_s}{2}\right)$$

$$= \frac{2}{1/60}\tan\left(\frac{40\pi\times 1/60}{2}\right)$$

$$= 120\tan\pi/3 \quad = 120\sqrt{3}$$

$$= 207.85$$

2) Modified transfer funct?

$$H^*(S) = H(S)\Big|_{S=\frac{S}{\Omega_p^*}} = \frac{S}{120\sqrt{3}}$$

$$= \frac{1}{S+1}\Bigg|_{S=\frac{S}{120\sqrt{3}}}$$

$$H^*(S) = \frac{1}{\left(\frac{S}{120\sqrt{3}}\right)+1}$$

$$= \frac{207.85}{S+207.85}$$

3) $$H(Z) = H^*(S)\Big|_{S=\frac{2}{T_s}\frac{Z-1}{Z+1}} = 120\left[\frac{Z-1}{Z+1}\right]$$

$$= \frac{207.85}{S+207.85}\Bigg|_{S=120\left[\frac{Z-1}{Z+1}\right]}$$

$$= \frac{207.85}{120\left[\frac{Z-1}{Z+1}\right]+207.85}$$

$$= \frac{207.85\,(Z+1)}{120(Z-1)+207.85(Z+1)}$$

Que. 3 Design a single pole low pass filter with 3db bandwidth of 0.2π by using BZT $H(S) = \frac{\Omega_c}{S+\Omega_c}$

Ans $\omega_c = 0.2\pi < \pi$ pre-warping is not required

$$\Omega_c = \frac{2}{T_s}\left(\tan\frac{\omega}{2}\right)$$

$$= \frac{2}{T_s}\tan\frac{0.2\pi}{2}$$

$$\therefore \Omega_c = \frac{0.65}{T_s}$$

$$H(Z) = H(S)\Big|_{S=\frac{2}{T_s}\frac{Z-1}{Z+1}}$$

$$= \frac{0.65/T_s}{S+0.65/T_s}\Bigg|_{S=\frac{2}{T_s}\frac{Z-1}{Z+1}}$$

$$= \frac{0.65/T_s}{\frac{2}{T_s}\frac{Z-1}{Z+1}+\frac{0.65}{T_s}}$$

$$= \frac{0.65}{2\frac{Z-1}{Z+1}+0.65}$$

$$= \frac{0.65\,(Z+1)}{\cancel{26.5Z}\ 2.65Z - 1.35}$$

10/11/22

4) Matched ZT

$$H(S) = \frac{\prod_{k=1}^{M} (S - Z_k)}{\prod_{k=1}^{N} (S - P_k)}$$

$$H(z) = \frac{\prod_{k=1}^{m} (1 - e^{Z_k T_s} \cdot z^{-1})}{\prod_{k=1}^{m} (1 - e^{P_k \cdot T_s} \cdot z^{-1})}$$

Direct mapping of poles & zeros by matched ZT

Q.1. for the give H(S) convert it into z-domain using matched z-Transform.

$$H(S) = \frac{S+2}{(S+1)(S+3)}$$

Ans

The mapping of matched z-Transform is given by

If $$H(S) = \frac{\prod_{k=1}^{m} (S - Z_k)}{\prod_{k=1}^{m} (S - P_k)}$$

for Matched ZT

$$H(Z) = \frac{\prod_{k=1}^{M} (1 - e^{Z_k T_s} \cdot z^{-1})}{\prod_{k=1}^{m} (1 - e^{P_k \cdot T_s} \cdot z^{-1})}$$

Q.1. $$H(S) = \frac{S+2}{(S+1)(S+3)}$$

$$= \frac{(S-(-2))}{[S-(-1)][S-(-3)]}$$

$$H(Z) = \frac{1 - e^{-2T_s} z^{-1}}{(1 - e^{-1T_s} z^{-1})(1 - e^{-3T_s} z^{-1})} \qquad T_s = 1$$

$$= \frac{z^2}{z^2} \frac{(1 - e^{-2} z^{-1})}{(1 - e^{-1} z^{-1})(1 - e^{-3} z^{-1})}$$

$$= \frac{z(z - 0.135)}{(z - 0.36)(z - 0.049)}$$

Q.2 $$H(S) = \frac{S^2 - 2S + 1}{S^2 - S - 6}$$

design IIR filter by using matched ZT

Sol^n $$H(S) = \frac{(S-1)(S-1)}{(S-3)(S+2)}$$

$$H(Z) = \frac{(1 - e^{1} z^{-1})(1 - e^{1} z^{-1})}{(1 - e^{3} z^{-1})(1 - e^{-2} z^{-1})}$$

$$= \frac{(Z - 2.72)(Z - 2.72)}{(Z - 20.08)(Z - 0.135)}$$

$$= \frac{Z^2 - 5.42 Z - 7.344}{Z^2 - 21.215 Z - 2.710}$$

The mapping of matched z-Transform is given by

If $H(S) = \dfrac{\prod_{k=1}^{m}(S - Z_k)}{\prod_{k=1}^{m}(S - P_k)}$

for matched ZT

$$H(z) = \frac{\prod_{k=1}^{m}(1 - e^{Z_k T_s} \cdot z^{-1})}{\prod_{k=1}^{m}(1 - e^{P_k \cdot T_s} \cdot z^{-1})}$$

$$H(S) = \frac{(S-1)(S-1)}{(S-3)(S+2)}$$

$$H(z) = \frac{(1 - e^{1} z^{-1})(1 - e^{1} z^{-1})}{(1 - e^{3} z^{-1})(1 - e^{-2} z^{-1})}$$

$$= \frac{(z - 2.72)(z - 2.72)}{(z - 20.08)(z - 0.135)}$$

$$= \frac{z^2 - 5.42z - 7.344}{z^2 - 21.215z - 2.710}$$

Q.3 Design the digital IIR filter for sampling time 0.01 sec. The transfer function is given by

$$H(S) = \frac{10}{S+2}$$

Ans The mapping of matched z-Transform is given by

If $H(S) = \dfrac{\prod_{k=1}^{m}(S - Z_k)}{\prod_{k=1}^{m}(S - P_k)}$

for matched ZT

$$H(z) = \frac{\prod_{k=1}^{m}(1 - e^{Z_k T_s} \cdot z^{-1})}{\prod_{k=1}^{m}(1 - e^{P_k \cdot T_s} \cdot z^{-1})}$$

$$H(S) = \frac{10}{S+2}$$

$$= \frac{10}{1 - e^{-2 \times 0.01} z^{-1}}$$

$$= \frac{10}{1 - 0.98 z^{-1}} \times \frac{z^{+1}}{z^{+1}}$$

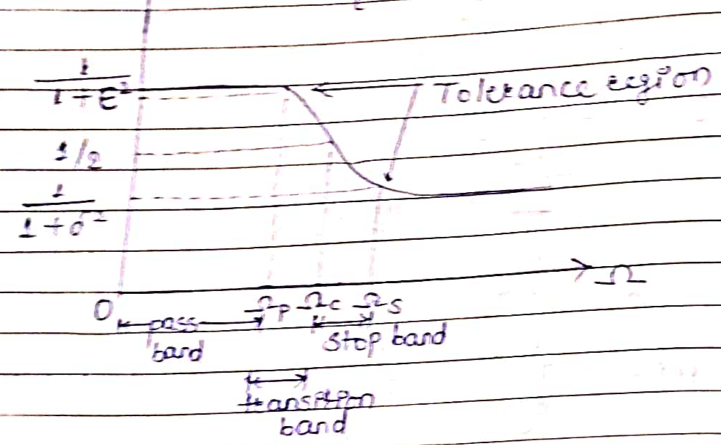
$$\therefore H(z) = \frac{10z}{z - 0.98}$$

* DSP Processors
DSP Processors Architecture
Addressing modes of DSP processors
Write the architecture of DSP TMS320C67XX processor

6/11/22

★ Butterworth Filter

$$|H(j\Omega)| = \frac{A}{\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{0.5}}$$



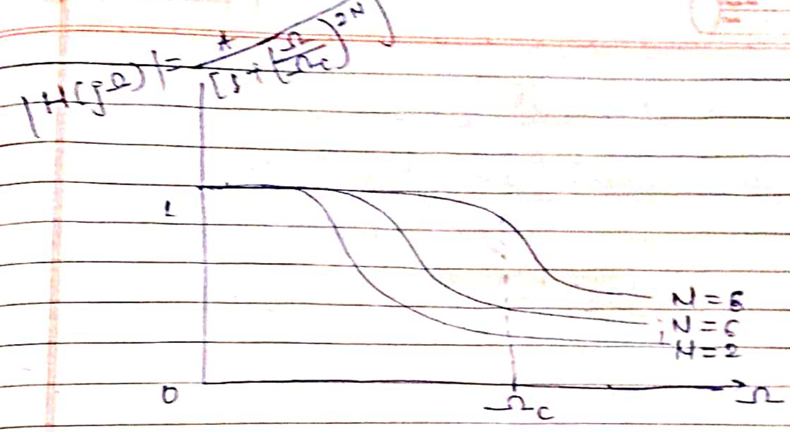
$A \rightarrow$ Filter gain
$\Omega_c \rightarrow$ 3db cutoff frequency
$N \rightarrow$ Order of the filter
$1+\epsilon^2 \rightarrow$ Pass band edge value
$1+\delta^2 \rightarrow$ stop band edge value

Here pass band is maximly flat hence called as Butterworth that is there are no variations in the pass band

$\epsilon \rightarrow$ parameter related to repulse in pass band
$\delta \rightarrow$ parameter related to repulse in stop band.

As the order of filter increases, the response of the filter is more close to ideal response.

$$|H(j\Omega)| = \frac{A}{\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{0.5}}$$



The max gain occurs at Ω_c

★★ Designing of Butterworth filter

$A_p \rightarrow$ Attenuation in pass pass band
$A_s \rightarrow$ Attenuation in stop band
$\Omega_p \rightarrow$ pass band edge frequency
$\Omega_s \rightarrow$ stop band edge frequency
$\Omega_c \rightarrow$ cutoff frequency

Step 1: Obtain equation of Analog filter

a) For IIM, $\Omega = \frac{\omega}{T_s}$

b) For BZT, $\Omega = \frac{2}{T_s} \tan\left(\frac{\omega}{2}\right)$

If ω_c is not given,

i) $\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_p^2} - 1\right)^{1/2N}}$ ii) $\Omega_c = \frac{\Omega_p}{10^{0.1 A_p(db)} - 1}$

Step 2: Calculate 'Order 'N' of filter

$$N = \frac{\frac{1}{2}\log\left[\dfrac{\left(\frac{1}{A_s^2}-1\right)}{\left(\frac{1}{A_p^2}-1\right)}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

OR

$$N = \frac{\log\left(\frac{1}{\delta^2}-1\right)}{2\log\left[\frac{\Omega}{\Omega_s}\right]}$$

OR

$$N = \frac{\frac{1}{2}\log\left[\dfrac{\left(10^{0.1 A_s(db)}-1\right)}{\left(10^{0.1 A_p(db)}-1\right)}\right]}{\frac{1}{2}\log\left[\frac{\Omega_s}{\Omega_p}\right]}$$

Step 3: Calculation of Poles

$$P_k = \pm\,\Omega_c\, e^{j(N+2k+1)\pi/2N}$$

$k = 0, 1, 2, \ldots\ldots N-1$

If poles are complex conjugate S_1 & S_1^*

S_2 & S_2^*

Step 4: Calculate Transfer function

$$H(S) = \frac{\Omega_c^N}{(S-P_1)(S-P_2)}$$

for complex poles

$$H(S) = \frac{\Omega_c^N}{(S-S_1)(S-S_1^*)(S-S_2)(S-S_2^*)}$$

Step 5: Calculate Digital Transfer function

$$H(Z) = H(S)\Big|_{S \to}$$

→ IIM mapping

→ BZT

Q.1 A digital filter has freq specifications
passband freq = $\omega_p = 0.2\pi$
stopband freq = $\omega_s = 0.3\pi$
What are the corresponding specifications for passband & stop band freq in analoge domain if
1) we use IIM for designing
2) BZT for designing

Ans Given: $\omega_p = 0.2\pi$
$\omega_s = 0.3\pi$

Step 1: Ω_c

i) for IIM $\Omega_p = \frac{\omega_p}{T_s} = \frac{0.2\pi}{1} = 0.2\pi$ rad/sec $= 0.628$

$\Omega_s = \frac{\omega_s}{T_s} = \frac{0.3\pi}{1} = 0.3\pi$ rad/sec $= 0.942$

ii) for BZT $\Omega_p = \frac{2}{T_s}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{1}\tan\left(\frac{\overset{0.1}{\cancel{0.2}}\pi}{\cancel{2}}\right)$

$= 2\tan(0.1\pi)$
$= 2 \times 0.325$
$= 0.65$ rad/sec

$\Omega_s = \frac{2}{T_s}\tan\left(\frac{\omega_s}{2}\right) = \frac{2}{1}\tan\left(\frac{0.3\pi}{2}\right)$
$= 2\tan(0.471)$
$= 2 \times 0.51$
$= 1.02$ rad/sec

Q.2 Design a 2^{nd} order Butterworth filter with cutoff freq of 1kHz & sampling freq 10^4 samples/sec by BZT.

Soln: Given: $N = 2$
$F_c = 1\,kHz$
$F_s = 10^4$ sample/sec

$f_c = \frac{F_c}{F_s} = \frac{1k}{10^4} = 0.1$ cycle/sample

$\omega_c = 2\pi f_c = 2 \times \overset{\pi}{3.14} \times 0.1 = 0.2\pi$ rad/sec

Step.I: Ω_c calculation by BZT

$\Omega_c = \frac{2}{T_s}\tan\frac{\omega_c}{2}$

$= \frac{2}{T_s}\tan\left(\frac{\overset{0.1}{\cancel{0.2}}\pi}{\cancel{2}}\right)$ $\quad T_s = \frac{1}{F_s}$

$= 2 \times 10^4 \, \cancel{\tan} \, (0.325)$

$\Omega_c = 6498.39$ rad/sec

$\therefore \Omega_c = 6500$

Step2: $N = 2$ given

Step3: $P_k = \pm \Omega_c e^{j(N+2k+1)\pi/2N}$

$k = 0$ to $N-1$ $\quad k = 0$ & 1.
$= 0$ to $2-1$
$= 0$ to 1

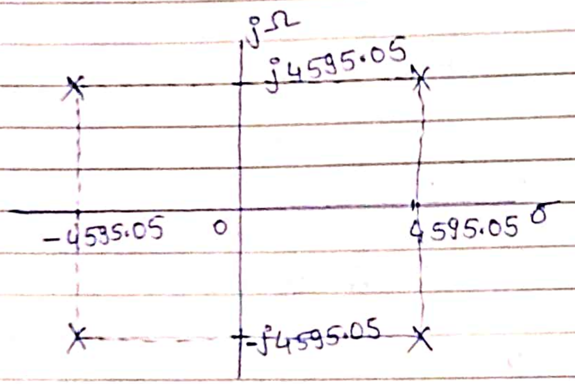
$$P_0 = \pm 6498.39\, e^{j\frac{3\pi}{4}} = \pm 6498.39\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right)$$

$$= \pm 6498.39\,($$

$$= \mp 4595.05 \pm j4595.05$$

$$P_1 = \pm 6498.39\, e^{j\frac{5\pi}{4}} = \pm 6498.39\left(\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4}\right)$$

$$= \mp 4595.05 \mp j4595.05$$



The stable poles are $-4595.05 + j4595.05$

$-4595.05 - j4595.05$

Step 4: Analog transfer function

$$H(S) = \frac{\Omega_c^N}{(S-P_0)(S-P_0^*)}$$

$$= \frac{(6498.39)^2}{(S+4595.05-j4595.05)(S+4595.05+j4595.05)}$$

- ω_c $0.2\pi < \pi \rightarrow$ pre-warping procedure is not required

Step 5: $H(Z) = H(S)\Big|_{S = \frac{2}{T_s}\left[\frac{Z-1}{Z+1}\right]} = 2\times10^4\left[\frac{Z-1}{Z+1}\right]$

$$\therefore H(Z) = \frac{(6498.39)^2}{\left(2\times10^4\left[\frac{Z-1}{Z+1}\right] + 4595.05 - j4595.05\right) \times \left(2\times10^4\left[\frac{Z-1}{Z+1}\right] + 4595.05 + j459.05\right)}$$

Q.2 Using BZT design a butterworth filter which satisfies the following conditions

$$0.8 \leq |H(e^{j\omega})| \leq 1 \quad , \quad 0 \leq \omega \leq 0.2\pi$$

$$|H(e^{j\omega})| \leq 0.2 \quad , \quad 0.6\pi \leq \omega \leq \pi$$

draw its poles.

Ans As.

$$\left.\begin{aligned} A_p \leq H(e^{j\omega}) \leq 1 \quad , \quad 0 \leq \omega \leq \omega_p \\ H(e^{j\omega}) \leq A_s, \quad \omega_s \leq \omega \leq \pi \end{aligned}\right\} \text{Logic}$$

$\therefore$ $A_p = 0.8$, $\omega_p = 0.2\pi$

$A_s = 0.2$, $\omega_s = 0.6\pi$

Step1 $\Omega_p = \frac{2}{T_s} \tan \frac{\omega_p}{2}$

$= \frac{2}{1} \tan\left(\frac{0.2\pi}{2}\right)$

$\therefore \Omega_p = 0.65$ rad/sec

$\Omega_s = \frac{2}{T_s} \tan \frac{\omega_s}{2}$

$= \frac{2}{1} \tan\left(\frac{0.6\pi}{2}\right)$

$= 2 \tan(0.3\pi)$

$= 2 \tan(0.942)$

$= 2 \times 1.375$

$\therefore \Omega_s = 2.75$ rad/sec

Step2: $N = \frac{1}{2} \dfrac{\log\left[\dfrac{1/A_s^2 - 1}{1/A_p^2 - 1}\right]}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$

$= \frac{1}{2} \dfrac{\log\left[\dfrac{\frac{1}{0.04} - 1}{\frac{1}{0.64} - 1}\right]}{\log\left(\dfrac{2.75}{0.65}\right)}$

$= \frac{1}{2} \dfrac{\log\left[\dfrac{\frac{1-0.04}{0.04}}{\frac{1-0.64}{0.64}}\right]}{\log(4.23)}$

$= \frac{1}{2} \dfrac{\log\left(\frac{24}{0.56}\right)}{\log(4.23)}$

$= \frac{1}{2} \dfrac{\log(42.86)}{\log(4.23)}$

$= \frac{1}{2} \left(\dfrac{1.682}{0.626}\right)$

$= \frac{1}{2} (2.6)$

$N = 1.3$

$\therefore N = 1.3 \approx 2$

for order of filter always go with higher no.

step3: $P_k = \pm \Omega_c e^{j(N+2k+1)\pi/2N}$ $\quad k = 0$ to $N-1$

$= 0$ to $2-1$

$= 0 \,\&\, 1$

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_p^2} - 1\right)^{1/2N}}$$

$$= \frac{0.65}{\left(\frac{1}{0.64} - 1\right)^{1/4}}$$

$$= \frac{0.65}{(0.5624)^{1/4}}$$

$$= \underline{0.65}$$

$\therefore \Omega_c = 0.75$ rad/sec

$$P_k = \pm 0.75\, e^{j(2+2k+1)\pi/2*2}$$

$$= \pm 0.75\, e^{j(3+2k)\pi/4}$$

$$P_0 = \pm 0.75\, e^{j3\pi/4}$$

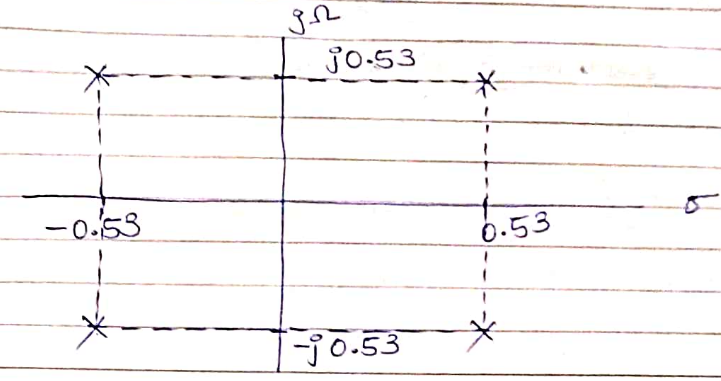
$$= \pm 0.75\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right)$$

$$= \mp 0.53 \pm j0.53$$

$$P_1 = \pm 0.75\, e^{j5\pi/4}$$

$$= \pm 0.75\left(\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4}\right)$$

$$= \mp 0.53 \mp j0.53$$



$P_a = -0.53 + j0.53$ are the stable poles

$P_b = -0.53 - j0.53$

$$H(S) = \frac{\Omega_c^N}{(S-P_a)(S-P_b)}$$

$$H(S) = \frac{(0.75)^2}{(S+0.53-j0.53)(S+0.53+j0.53)}$$

$$H(z) = H(S)\Big|_{S=\frac{2}{T_S}\left[\frac{z-1}{z+1}\right]} = 2\left[\frac{z-1}{z+1}\right]$$

$$H(z) = \frac{(0.75)^2}{\left(2\left[\frac{z-1}{z+1}\right] + 0.53 - j0.53\right)\left(2\left[\frac{z-1}{z+1}\right] + 0.53 + j0.53\right)}$$

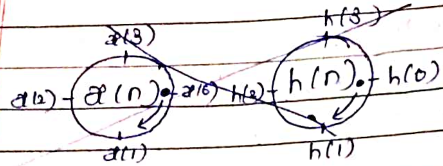
* Circular Convolution

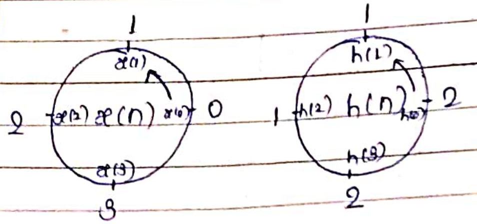
Circular Convolution →
- Circular representation
- Matrix method
- DFT & IDFT method

Q.1 Find circular convolution of if

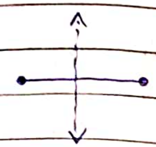
$x(n) = \{0, 1, 2, 3\}$ $\quad h(n) = \{2, 1, 1, 2\}$

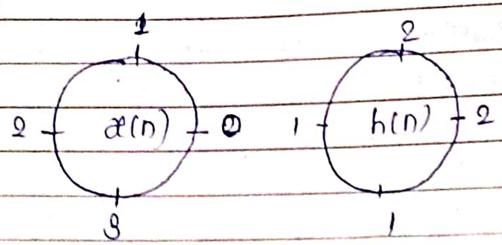
Soln:





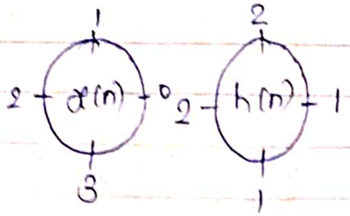
$h(n)$





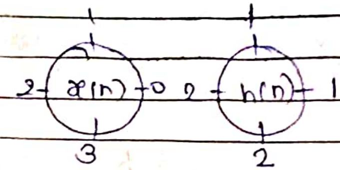
$y(0) = x(n)\,h(-n)$
$= 0\times2 + 1\times2 + 2\times1 + 3\times1$
$= 7$

$y(1) = x(n)\,h(1-n)$



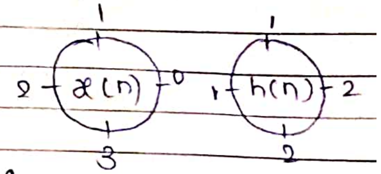
$= 0\times1 + 1\times2 + 2\times2 + 3\times1$
$= 2+4+3$
$= 9$

$y(2) = x(n)\,h(2-n)$



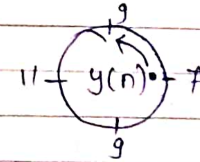
$= 0\times1 + 1\times1 + 2\times2 + 3\times2$
$= 0+1+4+6$
$= 11$

$y(3) = x(n)\,h(3-n)$



$= 0\times2 + 1\times1 + 2\times1 + 3\times2$
$= 0+1+2+6$
$= 9$

$\therefore y(n) = \{7, 9, 11, 9\}$



* Matrix Method

$$y(n) = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \end{bmatrix}_{4\times4} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} 0+2+2+3 \\ 0+2+4+3 \\ 0+1+4+6 \\ 0+1+2+6 \end{bmatrix}$$

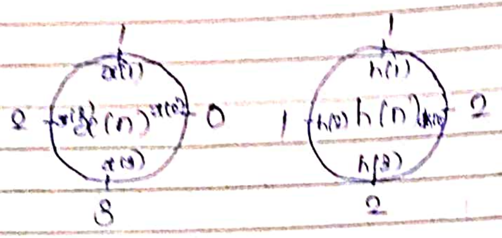
$$= \begin{bmatrix} 7 \\ 9 \\ 11 \\ 9 \end{bmatrix}$$

$\therefore y(n) = \{7, 9, 11, 9\}$

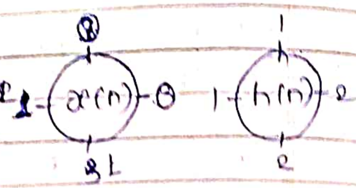
Q.2 Calculate circular convolution of
$x(n) = \{0, 1, 2, 3\}$, $h(n) = \{2, 1, 1, 2\}$

Keep $h(n)$ fixed & $x(n)$ is rotating

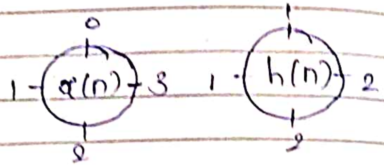
Sol^n



$$y(0) = x(-n)\,h(n)$$
$$= 2\times 0 + 1\times 3 + 1\times 2 + 2\times 2$$
$$= 3+2+2$$
$$= 7$$



$$y(0) = x(-n)\,h(n)$$
$$=$$



Matrix Method:

$$y(n) = \begin{bmatrix} 0 & 3 & 2 & 1 \\ 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3+2+2 \\ 2+0+3+4 \\ 4+1+0+6 \\ 6+2+1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ 9 \\ 11 \\ 9 \end{bmatrix}$$

$\therefore y(n) = \{7, 9, 11, 9\}$

Q. 3 find the circular convolution & linear convolution by using DFT & IDFT method if $x(n) = \{1, 2\}$ & $h(n) = \{2, 1\}$

$\Rightarrow$ Circular Convolution

$$x(n) \xrightarrow{DFT} X(k)$$
$$h(n) \xrightarrow{DFT} H(k)$$
$$\circledast \Rightarrow Y(k) \xrightarrow{IDFT} y(n)$$

$X(k) = W_2 x_2$

$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 \\ 1-2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$\therefore X(k) = \{3, -1\}$

$H(k) = W_2 h_2$

$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 \\ 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\therefore X(k) = \{3, 1\}$

$Y(k) = X(k) \cdot H(k)$

$= \{3, -1\} \cdot \{3, 1\}$

$= \{9, -1\}$

IDFT of $Y(k)$

$$y(n) = \frac{1}{N} W_2^* y_2$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 9-1 \\ 9+1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 8 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$\therefore y(n) = \{4, 5\}$

Matrix Method

$$y(n) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 \\ 4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$\therefore y(n) = \{4, 5\}$

* Linear Convolution by DFT – IDFT method

$L + M - 1$

$L \rightarrow$ length of $x(n)$

$M \rightarrow$ length of $x(n)$

$L + M - 1 = 2 + 2 - 1 = 3$

DFT is always power of 2 (2^n)

so $3 \approx 4$

$x(n) = \{1, 2, 0, 0\}$

$X(k) = W_4 x_4$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 \\ 1-j2 \\ 1-2 \\ 1+j2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1-j2 \\ -1 \\ 1+j2 \end{bmatrix}$$

$\therefore X(k) = \{3, 1-j2, -1, 1+j2\}$

$H(k) = W_4 h_4$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$Y(k) = X(k)\,H(k)$

$= \{3 \times 3, (1-j2)(2-j), 1 \times -1, (1+j2)(2+j)\}$

$= \{9, (2-j-4j \mp 2), -1, (2+j+4j \mp 2)\}$

$= \{9, (0-5j), -1, (5j)\}$

$\therefore Y(k) = \{9, -5j, -1, 5j\}$

$y(n) = \frac{1}{N} W_4^* Y_4$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 9 \\ -j5 \\ -1 \\ 5j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 9-5j-1+5j \\ 9+5+1+5 \\ 9+5j-1-5j \\ 9-5+1\mp5 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ 20 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$\therefore y(n) = \{2, 5, 2\}$

21/11/22

* Radix 2 - ~~DFT~~ FFT Algorithm

FFT → fast fourier transform
DIT → Decimation in time

The N-pt DFT is $N = r^V$

4 pt DFT → $N = r^V = 2^2 = 4$

8 pt DFT → $N = r^V = 2^3 = 8$

16 pt DFT → $N = r^V = 2^4 = 16$

r is radix = 2
&
$V = 2, 3, 4 \ldots$

There are two types of Radix 2 FFT Algorithm
1) DIT - FFT algorithm

DIT - Decimation in time
FFT - fast fourier transfor

2) DIF - FFT algorithm

DIF - Decimation in frequency

1> Radix 2 - DIT FFT Algorithm

DFT ⇒ $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$ $\quad 0 \leq k \leq N-1$

$x(n) = \{x(0), x(1), x(2), x(3), x(4), x(5), x(6), x(7)\}$
even ——
odd - - - -

Considering $x(n)$ as even & odd

even → $x(2n)$, $0 \leq m \leq \left(\frac{N}{2} - 1\right)$

Odd → $x(2n+1)$, $0 \leq n \leq \left(\frac{N}{2} - 1\right)$

$$X(k) = \sum_{n\ even} x(n) W_N^{nk} + \sum_{n\ odd} x(n) W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)k} \text{ —— (I)}$$

$$W_N^{2nk} = e^{-j2\pi \frac{2nk}{N}} = e^{-j2\pi \frac{nk}{N/2}} = W_{N/2}^{nk} \text{ —— (a)}$$

$$W_N^{(2n+1)k} = e^{-j2\pi \frac{(2n+1)k}{N}} = W_{N/2}^{nk} \cdot W_N^{k} \text{ —— (b)}$$

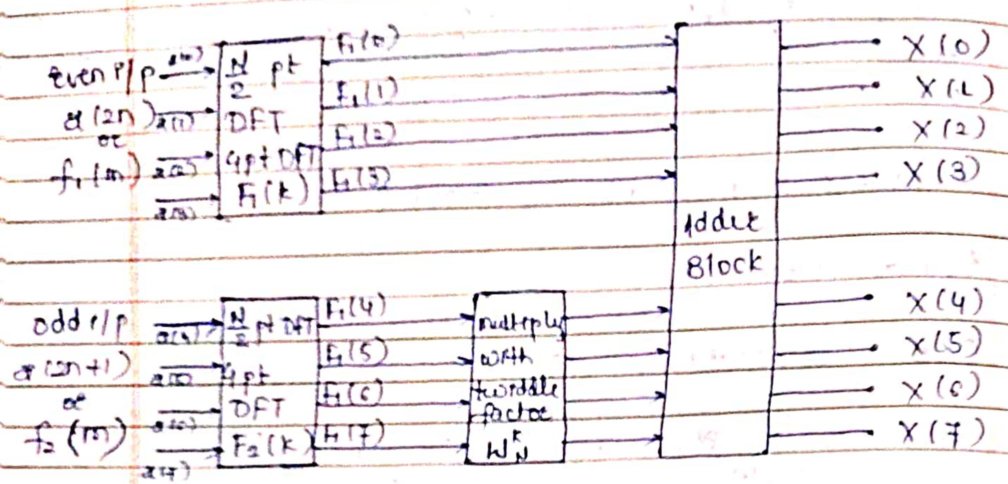
put eqn (a) & (b) in eqn (I)

$$X(k) = \sum_{n=0}^{N/2-1} x(2n) W_{N/2}^{nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_{N/2}^{nk}$$

let $x(2n) = f_1(m)$ —— even
$x(2n+1) = f_2(m)$ —— odd

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} f_1(m) W_{N/2}^{nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} f_2(m) W_{N/2}^{nk}$$

∴ $\boxed{X(k) = F_1(k) + W_N^k F_2(k) \quad 0 \leq k \leq N-1}$



$X(k) = F_1(k) + W_N^k F_2(k)$

$X(0) = F_1(0) + W_N^0 F_2(0)$

$X(1) = F_1(1) + W_N^1 F_2(1)$

$X(2) = F_1(2) + W_N^2 F_2(2)$

$X(3) = F_1(3) + W_N^3 F_2(3)$

$X(4) = F_1(4) + W_N^4 F_2(4)$

$X(5) = F_1(5) + W_N^5 F_2(5)$

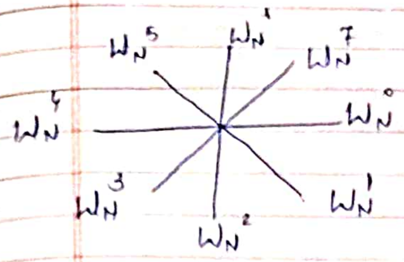
$X(6) = F_1(6) + W_N^6 F_2(6)$

$X(7) = F_1(7) + W_N^7 F_2(7)$

According to periodicity property of DFT

$\cancel{f(0)}$ $\cancel{f(1)}$ $\cancel{f(2)}$ $\cancel{f(3)}$

$f_1(0) = f_1(4)$ similarly $f_2(0) = f_2(4)$
$f_1(1) = f_1(5)$ $f_2(1) = f_2(5)$
$f_1(2) = f_1(6)$ $f_2(2) = f_2(6)$
$f_1(3) = f_1(7)$ $f_2(3) = f_2(7)$



$W_N^4 = -W_N^0$
$W_N^5 = -W_N^1$
$W_N^6 = -W_N^2$
$W_N^7 = -W_N^3$

Hence, $X(0) = F_1(0) + W_N^0 F_2(0)$

$X(1) = F_1(1) + W_N^1 F_2(1)$

$X(2) = F_1(2) + W_N^2 F_2(2)$

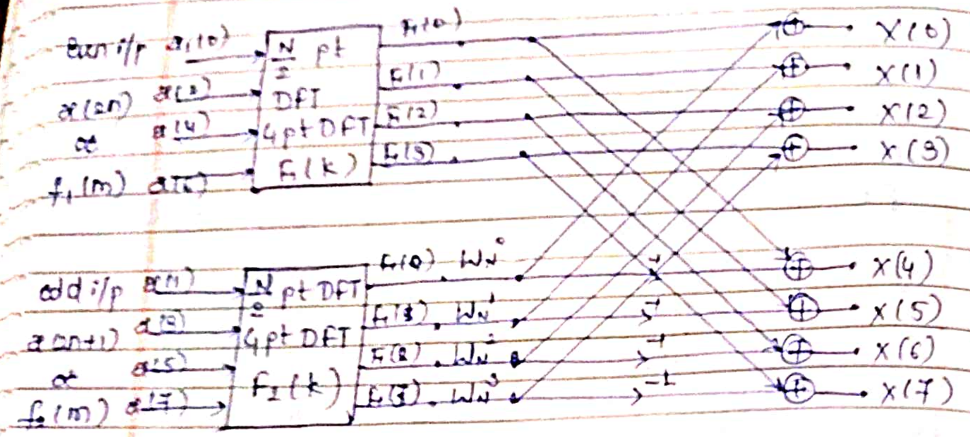
$X(3) = F_1(3) + W_N^3 F_2(3)$

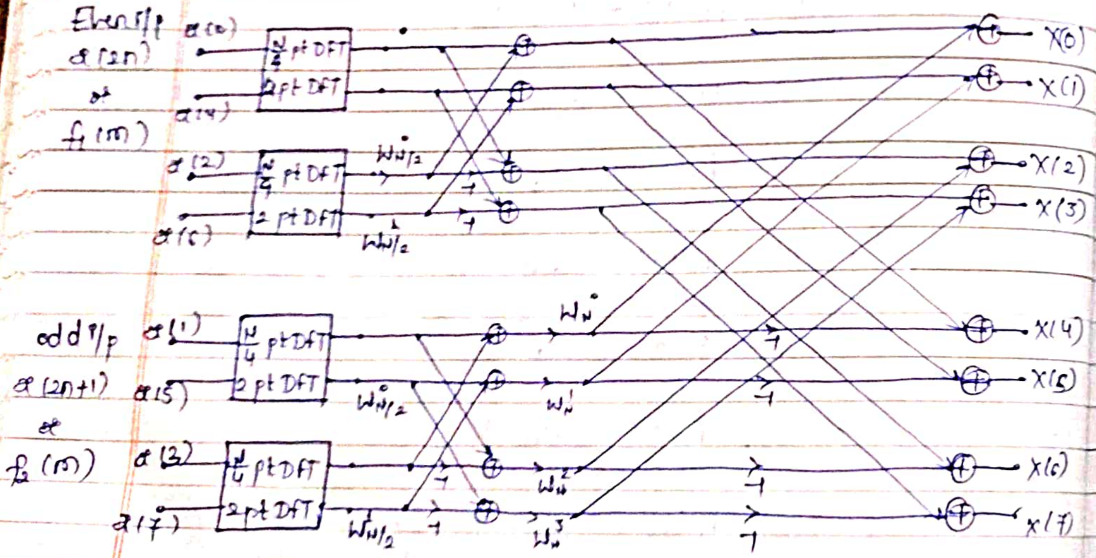
$X(4) = F_1(0) - W_N^0 F_2(0)$

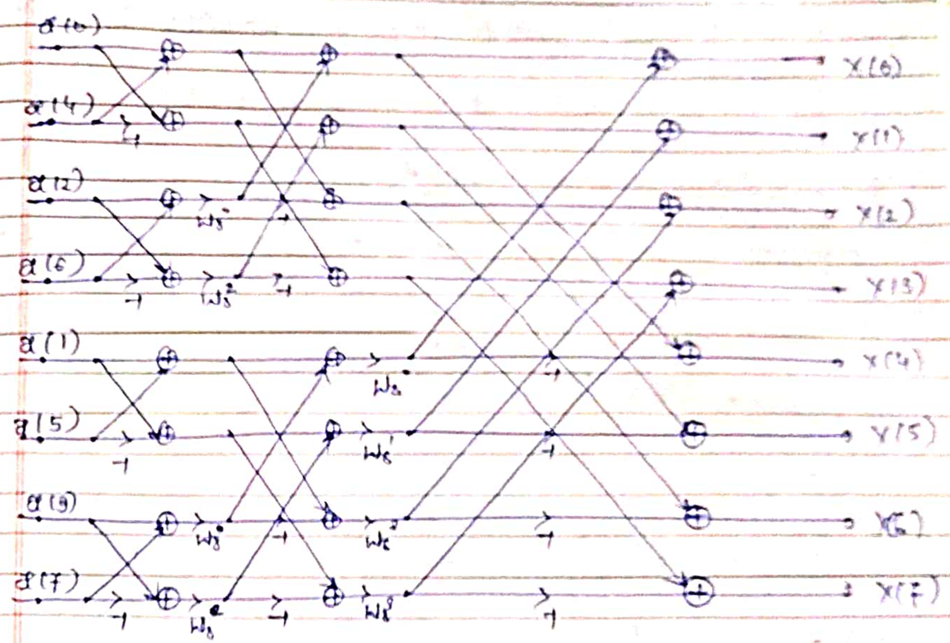
$X(5) = F_1(1) - W_N^1 F_2(1)$

$X(6) = F_1(2) - W_N^2 F_2(2)$

$X(7) = F_1(3) - W_N^3 F_2(3)$







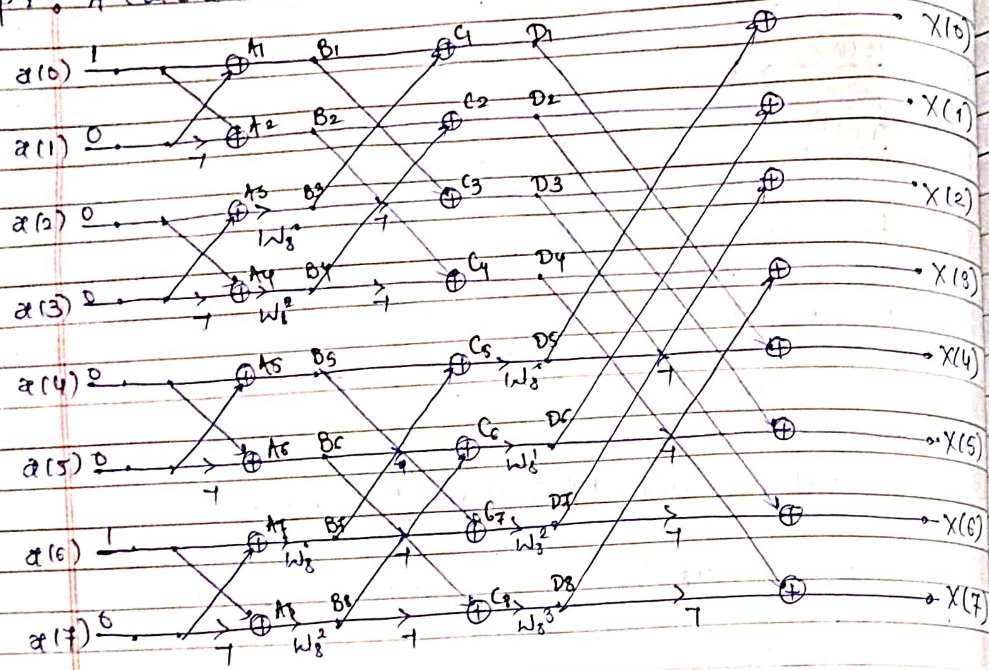
Radix 2 DIT - FFT Butterfly structure

$W_8^0 = 1$, $W_8^1 = (+,-)$, $W_8^2 = -j$, $W_8^3 = (-,-)$

Que. Calculate 8-point DFT by Radix 2-DIT-FFT algorithm

$x(n) = \{1, 0, 0, 1, 0, 0, 0, 0\}$

step 1: A calculations ($A_1 - A_8$)



step 1: A calculations

$A_1 = x(0) + x(4) = 1 + 0 = 1$

$A_2 = x(0) - x(4) = 1 - 0 = 1$

$A_3 = x(2) + x(6) = 0 + 0 = 0$

$A_4 = x(2) - x(6) = 0 - 0 = 0$

$A_5 = x(1) + x(5) = 0 + 0 = 0$

$A_6 = x(1) - x(5) = 0 - 0 = 0$

$A_7 = x(3) + x(7) = 1 + 0 = 1$

$A_8 = x(3) - x(7) = 1 - 0 = 1$

step 2: B calculation ($B_1 - B_8$)

$B_1 = A_1 = 1$

$B_2 = A_2 = 1$

$B_3 = W_8^0 A_3 = A_3 = 0$

$B_4 = W_8^2 A_4 = A_4 = 0$

$B_5 = A_5 = 0$

$B_6 = A_6 = 0$

$B_7 = W_8^0 A_7 = 1 \times 1 = 1$

$B_8 = W_8^2 A_8 = -j \times 1 = -j$

step 3: C Calculation ($C_1 - C_8$)

$C_1 = B_1 + B_3 = 1 + 0 = 1$

$C_2 = B_2 + B_4 = 1 + 0 = 1$

$C_3 = B_1 - B_3 = 1 - 0 = 1$

$C_4 = B_2 - B_4 = 1 - 0 = 1$

$C_5 = B_5 + B_7 = 0 + 1 = 1$

$C_6 = B_6 + B_8 = 0 - j = -j$

$C_7 = B_5 - B_7 = 0 - 1 = -1$

$C_8 = B_6 - B_8 = 0 + j = j$

step 4: D Calculation ($D_1 - D_8$)

$D_1 = C_1 = 1$

$D_2 = C_2 = 1$

$D_3 = C_3 = 1$

$D_4 = C_4 = 1$

$D_5 = C_5 W_8^0 = 1 \times 1 = 1$

$D_6 = C_6 W_8^1 = -j(0.707 - j0.707) = -0.707 - 0.707j$

$D_7 = C_7 W_8^2 = -1(-j) = j$

$D_8 = C_8 W_8^3 = j(-0.707 - 0.707j) = 0.707 - 0.707j$

Step5: $X_0 = D_1 + D_5 = 1+1 = 2$

$X_1 = D_2 + D_6 = 1 + (-0.707 - 0.707j) = 0.293 - 0.707j$

$X_2 = D_3 + D_7 = 1 + j = 1+j$

$X_3 = D_4 + D_8 = 1 + (0.707 - 0.707j) = 1.707 - 0.707j$

$X_4 = D_1 - D_5 = 1 - 1 = 0$

$X_5 = D_2 - D_6 = 1 - (-0.707 - 0.707j) = 1.707 + 0.707j$

$X_6 = D_3 - D_7 = 1 - j = 1 - j$

$X_7 = D_4 - D_8 = 1 - (0.707 - 0.707j) = 0.293 + 0.707j$

* Overlap Add Method

Linear Convolution
- → eq^n method
- → graphical
- → tabular
- → Sectional method (Overlap Add method)

To solve linear convolution by overlap Add method.

Que. Find the linear convolution by using overlap Add method

$x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$, $h(n) = \{-1, 1\}$

Sol^n: $x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$

$x_1(n) = \{1, -1\}$ | $x_2(n) = \{2, -2\}$ | $x_3(n) = \{3, -3\}$ | $x_4(n) = \{4, -4\}$

	m	-1	0	1	2
	$x_1(m)$		1	-1	
	$h(m)$		-1	1	
h_0	$h(-m)$	1	-1		
h_1	$h(1-m)$		1	-1	
h_2	$h(2-m)$			1	-1

$n=0$, $y_1(0) = \sum x_1(m) h_0(m) = 0\times1 + 1\times-1 + 1\times0 = -1$

$=1$ $y_1(1) = \sum x_1(m) h_1(m) = 1\times1 + (-1)\times(-1) = 2$

$=2$ $y_1(2) = \sum x_2(m) h_2(m) = 1\times0 + (-1)\times1 + 0\times(-1) = -1$

	m	-1	0	1	2	3	4
	$x_2(m)$				2	-2	
	$h(m)$		-1	1			
h_0	$h(-m)$	1	-1				
~~h_1~~	$h(1-m)$		1	-1			
h_2	$h(2-m)$			1	-1		
h_3	$h(3-m)$				1	-1	
h_4	$h(4-m)$					1	-1

$y_2(2) = \sum x_2(m)\, h_2(m) = -2$

$y_2(3) = \sum x_2(m)\, h_3(m) = 2\times1 + (-2)(-1) = 4$

$y_2(4) = \sum x_2(m)\, h_4(m) = -2$

	m	-1	0	1	2	3	4	5	6
	$x_3(m)$						3	-3	
	$h(m)$		-1	1					
	$h(-m)$	1	-1						
h_4	$h(4-m)$					1	-1		
h_5	$h(5-m)$						1	-1	
h_6	$h(6-m)$							1	-1

$y_3(4) = \sum x_3(m)\, h_4(m) = 3\times -1 = -3$

$y_3(5) = \sum x_3(m)\, h_5(m) = 3\times1 + (-3)(-1) = 6$

$y_3(6) = \sum x_3(m)\, h_6(m) = 3\times -1 = -3$

m	-1	0	1	2	3	4	5	6	7	8
$x_4(m)$								4	-4	
$h(m)$		-1	1							
$h(-m)$	1	-1								
$h(6-m)$							1	-1		
$h(7-m)$								1	-1	
$h(8-m)$									1	-1

$y_4(6) = \sum x_4(m)\, h_6(m) = -4$

$y_4(7) = \sum x_4(m)\, h_7(m) = 8$

$y_4(8) = \sum x_4(m)\, h_8(m) = -4$

- final Combined output calculation

n	0	1	2	3	4	5	6	7	8
$y_1(n)$	-1	2	-1						
$y_2(n)$			-2	4	-2				
$y_3(n)$					-3	6	-3		
$y_4(n)$							-4	8	-4
$y(n)$	-1	2	-3	4	-5	6	-7	8	-4

* Relationship betⁿ DFT & ZT

ZT ⇒ $$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} \quad \text{--- (1)}$$

DFT ⇒ $$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} x(n) \left(e^{\frac{j2\pi k}{N}}\right)^{-n} \quad \text{--- (2)}$$

Comparing (1) & (2)

$$\boxed{X(k) = X(z)\Big|_{z = e^{\frac{j2\pi k}{N}}}}$$

* Relation betⁿ FT & ZT

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$\frac{zeros}{poles}$

FIR filters are also called as non-recursive / feed forward / transverse filter.
because it does not use the feedback system.
sometimes it is also called as all zeros filter.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{N(z)}{D(z)} = \frac{zeros}{poles}$$

Zero filter means Denominator term does not exist only numerator term is present.

FIR filters are always stable filter cause is does not have poles.

* **Properties of FIR filter**

1) FIR filters are inherently stable

The LSI (linear shift invariant) system is said to be stable if the bounded i/p produces bounded o/p (BIBO)

The FIR filter is given by

$$y(n) = \sum_{k=0}^{N-1} b_k \cdot x(n-k) \quad \text{——— ①}$$

ZT,

$$Y(z) = \sum_{k=0}^{N-1} b_k z^{-k} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{N-1} b_k z^{-k}$$

IZT is

$$h(n) = \begin{cases} b_n & \text{for } 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases} \quad \text{——— ②}$$

(I) $y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_{N-1} x(n-M+1)$

(II) $y(n) = h_0 x(n) + h_1 x(n-1) + \dots + h(M-1) x(n-M+1)$

where $h(0), h(1), \dots$ are constant

∴ It is bounded

BI produces BO

∴ FIR filters are stable.

2) Symmetric & Anti Symmetric FIR filters:

Unit sample response of FIR filter is symmetric if it satisfies the condition.

$$h(n) = h(M-1-n) \quad \text{———} \quad n = 0, 1, \dots, M-1$$

M = no. of samples

if $M = 8$, for $n = 0$, $h(0) = h(7)$
$n = 1$, $h(1) = h(6)$
$n = 2$, $h(2) = h(5)$
$n = 3$, $h(3) = h(4)$

$h(n) = \{h(0), h(1), h(2), h(3), h(4), h(5), h(6), h(7)\}$

hence, it is symmetric

- Now, unit sample response of FIR filter is Anti-symmetric if it satisfies the condition

$h(n) = -h(M-1-n)$, $n = 0, 1, 2, \dots, M-1$

if $M=8$, for $n=0$, $h(0) = -h(7)$
$n=1$, $h(1) = -h(6)$
$n=2$, $h(2) = -h(5)$
$n=3$, $h(3) = -h(4)$

$h(n) = \{h(0)$

hence, it is Anti symmetric

The phase of FIR filter is given by

$$\angle H(\omega) = \begin{cases} -\omega\left(\frac{M-1}{2}\right) & , \text{for } |H(\omega)| \geq 0 \\ -\omega\left(\frac{M-1}{2}\right) + \pi & \text{for } |H(\omega)| < 0 \end{cases} \quad \text{—} ①$$

$$h(n) = \pm h(M-1-n) \quad \text{—} ②$$

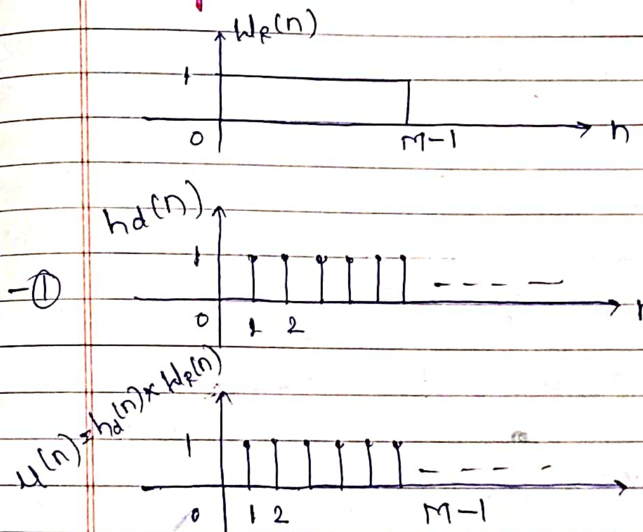
The eq^n ① is represent that the FIR filter is peace wise symmetric hence for symmetric & Anti-symmetric FIR filter. The condition for linear phase is given by eq^n ②.

Design of FIR filter using Windows

for the design of FIR filter, various types of windows are used

1) Rectangular
2) hamming
3) hanning window
4) Triangular / Bartlet window
5) Blackman
6) Kaiser

1) Rectangular Window



The rectangular window has magnitude 1

let $h_d(n)$ is the impulse response having infinite duration.

After multiplication of rectangular window with infinite unit sample response.

We get the truncated i/p sequence by using a window. This process is called as Truncation process.

As the shape of the window funct^n is rectangular it is called as Rectangular Window.

- Magnitude response of Rectangular Window:

The rectangular window is defined as

$$W_R(n) = \begin{cases} 1, & \text{for } n = 0,1,2 \ldots M-1 \\ 0 & \text{elsewhere} \end{cases} \quad \text{—— (1)}$$

$$h(n) = h_d(n) \cdot W_R(n) \quad \text{—— (2)}$$

The FT of $W_R(n)$ is $W_R(\omega)$

$$W_R(\omega) = \sum_{n=0}^{M-1} W_R(n)\, e^{-j\omega n} \quad \text{—— (3)}$$

$$= \sum_{n=0}^{M-1} e^{-j\omega n}$$

Window sequence can be represented as

$$W_R(n) = u(n) - u(n-M) \quad \text{for } 0 \text{ to } \infty$$

$$\therefore W_R(\omega) = \sum_{n=0}^{\infty} [u(n) - u(n-M)]\, e^{-j\omega n} \quad \text{—— (4)}$$

- $$FT[u(n)] = \sum_{n=0}^{\infty} u(n) e^{-j\omega n} = \sum_{n=0}^{\infty} e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (e^{-j\omega})^n = \frac{1}{1 - e^{-j\omega}} \quad \text{—— (a)}$$

- $$FT[u(n-M)] = e^{-j\omega M} F[u(n)] = e^{-j\omega n} \frac{1}{1 - e^{-j\omega}} \quad \text{—— (b)}$$

(a) & (b) in (4) gives

$$W_R(\omega) = \frac{1}{1 - e^{-j\omega}} - e^{-j\omega M} \frac{1}{1 - e^{-j\omega}}$$

$$= \frac{1 - e^{-j\omega M}}{1 - e^{-j\omega}}$$

- Rearranging the term

$$W_R(\omega) = \frac{e^{-j\omega\frac{M}{2}} e^{j\omega\frac{M}{2}} - e^{-j\omega\frac{M}{2}} e^{-j\omega\frac{M}{2}}}{e^{-\frac{j\omega}{2}} e^{\frac{j\omega}{2}} - e^{-\frac{j\omega}{2}} e^{-\frac{j\omega}{2}}}$$

$$= \frac{e^{-j\omega\frac{M}{2}}\left[e^{j\omega\frac{M}{2}} - e^{-j\omega\frac{M}{2}}\right]}{e^{-j\frac{\omega}{2}}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}$$

$$= e^{-j\omega\left[\frac{M-1}{2}\right]} \frac{\cancel{2j}\sin\left(\frac{\omega M}{2}\right)}{\cancel{2j}\sin\frac{\omega}{2}}$$

** $$W_R(\omega) = e^{-j\omega\left(\frac{M-1}{2}\right)} \frac{\sin\left(\frac{\omega M}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

** $$\therefore |W_R(\omega)| = \left|\frac{\sin\left(\frac{\omega M}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right|$$

Magnitude of $e^{-j\omega\left(\frac{M-1}{2}\right)}$ is 1.

Inverse fourier transform formula.

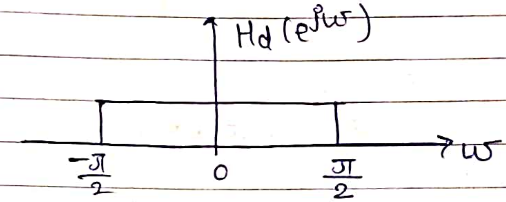
6/12/22

Q.1 An ideal low pass filter with a freq response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad -\pi/2 \leq \omega \leq \pi/2$$
$$= 0 \quad \text{for} \quad \pi/2 \leq \omega < \pi$$

find $h(n)$ for $N=11$,
find $H(z)$
plot magnitude response

sol^n: The freq response of LPF is



IFT is given by

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{+j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\, e^{+j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{+j\omega n}}{+jn}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2\pi jn}\left[e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}\right]$$

$$= \frac{1}{\cancel{2}\pi \cancel{j}n} \cancel{2j}\sin\frac{n\pi}{2}$$

$e^{2\pi x}\, dx$

$\frac{e^{2\pi x}}{2\pi}$

$$h_d(n) = \frac{\sin n\pi/2}{n\pi}$$

As, $\boxed{h(n) = h_d(n), \quad \text{for } |n| < \frac{N-1}{2}}$

$$h(n) = \frac{\sin n\pi/2}{n\pi}, \quad |n| < \frac{11-1}{5}$$

$$< 5$$

for $n=0$, $h(0) = \lim_{n\to 0} \frac{\sin n\pi/2}{n\pi}$

$$= \lim_{n\to 0} \frac{\sin n\pi/2}{n\pi \times 2 \times 1/2}$$

$$= \frac{1}{2} \lim_{n\to 0} \frac{\sin n\pi/2}{n\pi/2}$$

$$= \frac{1}{2} \qquad \left(\lim_{n\to 0} \frac{\sin\theta}{\theta} = 1\right)$$

for antisymmetric

$$h(1) = h(-1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.3183$$

$$h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0$$

$$h(3) = h(-3) = \frac{\sin \frac{3\pi}{2}}{3\pi} = \frac{-1}{9.42} = -0.1061$$

$$h(4) = h(-4) = \frac{\sin 2\pi}{4\pi} = 0.$$

$$h(5) = h(-5) = \frac{\sin 5\pi/2}{5\pi} = \frac{1}{15.7} = 0.0636.$$

The transfer funct^n of filter is given

Imp formula $\boxed{H(z) = h(0) + \sum_{n=1}^{N-1/2} [h(n)(z^n + z^{-n})]}$

$$= 0.5 + \sum_{n=1}^{5} [h(n)(z^n + z^{-n})]$$

$$= 0.5 + h(1)(z^1 + z^{-1}) + h(2)(z^2 + z^{-2})$$
$$h(3)(z^3 + z^{-3}) + h(4)(z^4 z^{-4}) + h(5)(z^5 + z^{-5})$$

$$= 0.5 + 0.818(z^1 + z^{-1}) + (-0.106)(z^{+3} + z^{-3}) +$$
$$(0.063)(z^5 + z^{-5})$$

The transfer funct^n of Realizable filter is

$$\boxed{H(z) = z^{-\left(\frac{N-1}{2}\right)} \quad H(z)}$$

$$H'(z) = z^{-5}(0.5 + 0.3183 z + 0.318 z^{-1} - 0.106 z^3 -$$
$$0.106 z^{-3} + 0.063 z^5 + 0.063 z^{-5})$$

$$= 0.5 z^{-5} + 0.318 z^{-4} + 0.318 z^{-6} - 0.106 z^{-2} -$$
$$0.106 z^{-8} + 0.063 z^0 + 0.063 z^{-10}.$$

$h(n) = \{0.063z^{-10}, 0, (-1.06)z^{-8}, 0, 0.3183z^{-6}, 0, (-1.06)z^{-2}, 0.5z^{-5}, 0.3183z^{-4}, 0, 0.063, 0, (-1.06)z^{2}, 0, 0.3183z^{4}, 0, 0.5z^{5}, 0.3183z^{6}, 0, (-1.06)z^{8}, 0, 0.063z^{10}\}$

The filter coefficient of causal filter is

$h(0) = h(10) = 0.063$ $\qquad h(3) = h(7) = 0$

$h(1) = h(9) = 0$ $\qquad h(4) = h(6) = 0.3183$

$h(2) = h(8) = -1.06$ $\qquad h(5) = 0.5$

- The frequency response is given by

$$\boxed{H(e^{j\omega}) = \sum_{n=0}^{5} a_n \cos\omega n \qquad a_n = 2h\left[\frac{N-1}{2} - n\right]}$$

$$a(0) = 2h\left(\frac{N-1}{2}\right)$$

$a(0) = h\left(\frac{11-1}{2}\right) = h(5) = 0.5$

$a(1) = 2h(4) = 2 \times 0.3183 = 0.6366$

$a(2) = 2h(3) = 2 \times 0 = 0$

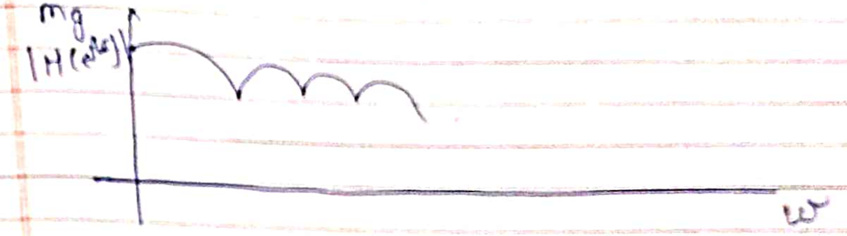
$a(3) = 2h(2) = 2 \times (-1.06) = -2.12$

$a(4) = 2h(1) = 2 \times 0 = 0$

$a(5) = 2h(0) = 2 \times 0.063 = 0.126$

$H(e^{j\omega}) = 0.5 + 0.636\cos\omega - 0.212\cos 3\omega + 0.126\cos 5\omega$

ω degree	0	10	20	30	40	50	60	70	80	90
$\lvert H(e^{j\omega})\rvert$ dB										



Q. 2 An ideal low pass filter with a freq response

$H_d(e^{j\omega}) = 1 \quad$ for $\quad \pi/4 \le \omega < \pi$

$= 0$

find $h(n)$ for $N = 11$.

find $H(z)$

plot magnitude response