How Small Are Our Big Data:
Turning the 2016 Surprise into a 2020 Vision

Xiao-Li Meng
Department of Statistics, Harvard University

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- Meng (2018) **Statistical Paradises and Paradoxes in Big Data (I): Law of Large Populations, Big Data Paradox, and the 2016 US Election**. *Annals of Applied Statistics*

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- Many thanks to **Stephen Ansolabehere and Shiro Kuriwaki** for the CCES (**Cooperative Congressional Election Study**) data and analysis on 2016 US election.

Motivating questions

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- **But is an 80% non-random sample "better" than a 5% random sample in measurable terms? 90%? 95%? 99%?** (Wu, 2012, Seminar at Harvard Statistics)

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- **"Which one should we trust more: a 1% survey with 60% response rate or a non-probabilistic dataset covering 80% of the population?"** (Keiding and Louis, 2015, Joint Statistical Meetings; and *JRSSB*, 2016)

A Bit of History: Theory and Practice

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  Jakob Bernoulli (1713)

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- First implementation in US Census: 1940 led by Morris Hansen

No need to worry about the population size at all?

- Think about tasting soup ...

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- Stir it well, then a few bits are sufficient **regardless of the size of the container!**

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$\Longleftrightarrow$

So where were you on 29.02.18?

Menu 5
Xiao-Li Meng Department of Statistics, Harvard University
Motivation
Soup
Euler Identity
Derivation
Trio
LLP
Whats Big?
CCES
Assessing d.d.i
Paradox
Lessons

So where were you on 29.02.18?

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- 5 most fundamental numbers in mathematics:

$$0, 1, e, \pi, i = \sqrt{-1}$$

- The unexpected one: $i = \sqrt{-1}$

A statistical counterpart of the Euler's identity?

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What are the five most fundamental symbols in Statistics?

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The Most Beautiful Statistical Identity?

$$\hat{\mu}_n - \mu_N = \hat{\rho}\sigma\sqrt{\frac{N-n}{n}}$$

2016 US Presidential Election

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Estimatinng Trump's share: $\mu_N = \text{Ave}(X_j)$ by sample average:

$$\hat{\mu}_n = \frac{R_1X_1 + \ldots + R_NX_N}{n} = \frac{\text{Ave}(R_jX_j)}{\text{Ave}(R_j)}$$

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Actual estimation error

$$\begin{aligned}\hat{\mu}_n - \mu_N &= \frac{\text{Ave}(R_j X_j)}{\text{Ave}(R_j)} - \text{Ave}(X_j) \\ &= \left[\frac{\text{Ave}(R_j X_j) - \text{Ave}(R_j)\text{Ave}(X_j)}{\sigma_R \sigma_X}\right] \times \frac{\sigma_R}{\text{Ave}(R_j)} \times \sigma_X\end{aligned}$$

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Data quality, quantity, and uncertainty

Because $\sigma_R^2 = f(1-f)$, $f = \text{Ave}\{R_j\} = \frac{n}{N}$, we have

$$\text{Error} = \underbrace{\hat{\rho}_{R,X}}_{\textbf{Data Quality}} \times$$

Data quality, quantity, and uncertainty

Menu 8

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Data quality, quantity, and uncertainty

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Data Defect Index (d.d.i.)

Mean Squared Error (MSE)

$$\mathrm{MSE}(\hat{\mu}_n) = \mathsf{E}_R(\hat{\rho}^2) \times \frac{N-n}{n} \times \sigma_x^2$$

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Data Defect Index (d.d.i): $D_I = \mathsf{E}_R(\hat{\rho}^2)$

- For Simple Random Sample (SRS): $D_I = (N-1)^{-1}$
- For probabilistic samples in general: $D_I \propto N^{-1}$
- Deep trouble when D_I does not vanish with N^{-1} or equivalently $\hat{\rho}$ with $N^{-1/2}$ ...

A Law of Large Populations (LLP)

If $\rho = \mathrm{E}_R(\hat{\rho}) \neq 0$, then on average, the relative error $\uparrow \sqrt{N}$:

$$\frac{\text{Actual Error}}{\text{Benchmark SRS Standard Error}} = \sqrt{N-1}\hat{\rho}$$

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The (lack-of) design effect (Deff)

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The *Effective Sample Size* n_{eff} of a "Big Data" set

Equate its MSE to that from a SRS with size n_{eff}:

$$D_I \left[\frac{N-n}{n}\right]\sigma^2 = \frac{1}{N-1}\left[\frac{N-n_{\text{eff}}}{n_{\text{eff}}}\right]\sigma^2$$

What's Big? Relative Size or Absolute Size?

The *Effective Sample Size* of a "Big Data" in terms of SRS size

$$n_{\text{eff}} = \frac{n}{1 + (1-f)[(N-1)D_I - 1]} \approx \frac{f}{1-f}\frac{1}{\hat{\rho}^2},$$

where $f = n/N$ is the **relative size**.

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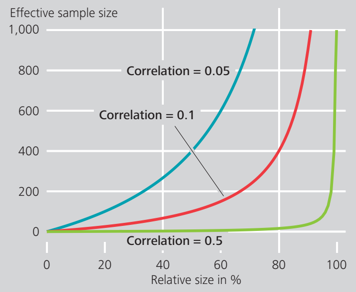

Deutsche Bundesbank

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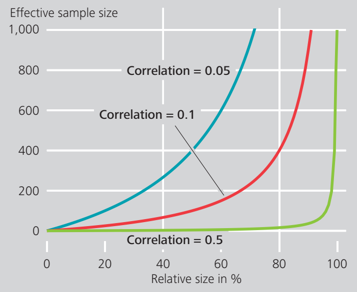

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- But $f = 1/2$ corresponds to $n \approx 160,000,000$ for the U.S. population;

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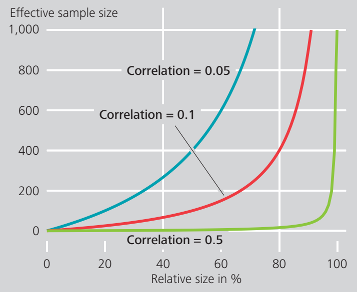

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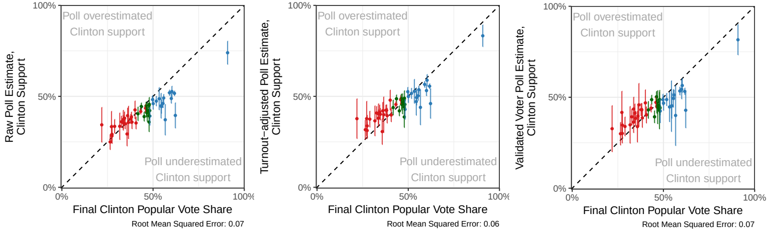
- But $f = 1/2$ corresponds to $n \approx 160,000,000$ for the U.S. population;
- Hence $\hat{\rho} = 0.05$ implies 99.99975% loss of sample size!

CCES: **Cooperative Congressional Election Study**

(Conducted by Stephen Ansolabehere, Brian Schaffner, Sam Luks, Douglas Rivers on **Oct 4 - Nov 6, 2016** (YouGov); Analysis assisted by Shiro Kuriwaki)



Raw Sample: 64,600   Voting Adj: 48,106   Validated: 34,156

Reasonable predictions for Clinton's Vote Share

Gross under-prediction/reporting of Trump's Share

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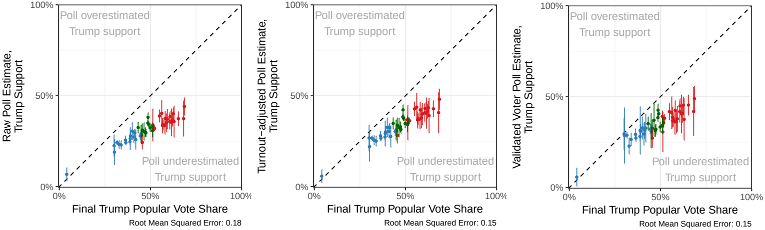

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There are many "undecided" ...

Assessing $\hat{\rho}$ using raw counts

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Let μ_N be the true share, and $\hat{\mu}_n$ the estimated share. Then

$$\hat{\rho} = \frac{\hat{\mu}_n - \mu_N}{\sqrt{\frac{N-n}{n}\sigma^2}}, \quad \& \quad \sigma^2 = \mu_N(1 - \mu_N)$$

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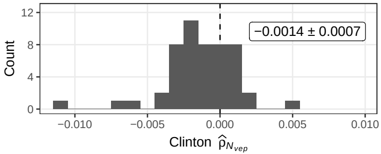

Clinton: $\hat{\rho} \approx -0.0014 \pm 0.0007$

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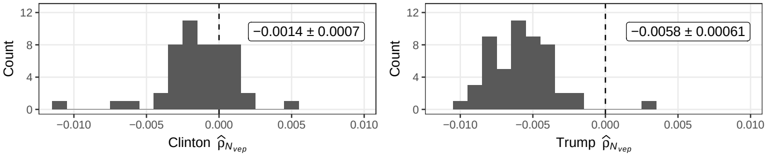

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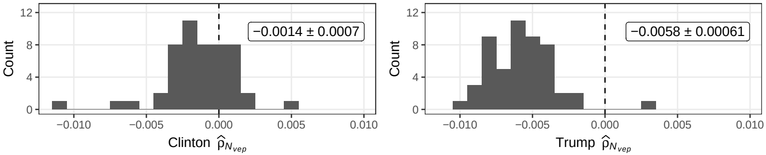

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- Problem: The mis-match of the *sampled population* and the *actual voting population*

Assessing $\hat{\rho}$ using voting propensity adjusted counts

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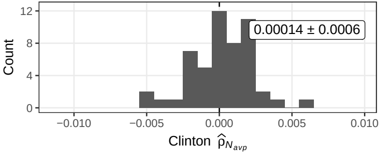

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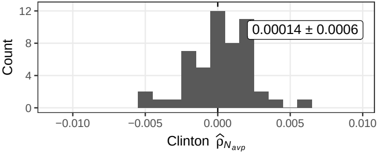

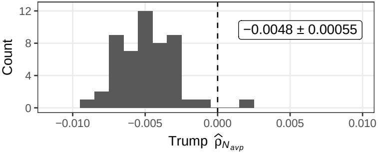

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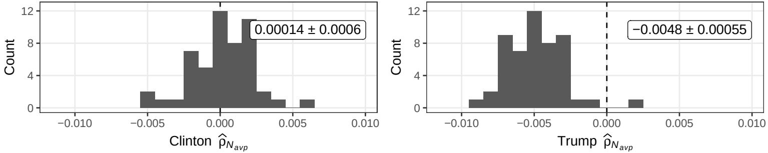

Clinton: $\hat{\rho} \approx 0.0001 \pm 0.0006$ Trump: $\hat{\rho} \approx -0.0048 \pm 0.0005$

- Problem: Estimating voting propensity (and N) is known to be unreliable; weighting may also introduce bias in assessing $\hat{\rho}$.

Assessing $\hat{\rho}_N$ using validated voter counts

Xiao-Li Meng
Department of Statistics, Harvard University

Let μ_N be the true share, and $\hat{\mu}_n$ the estimated share. Then

$$\hat{\rho} = \frac{\hat{\mu}_n - \mu_N}{\sqrt{\frac{N-n}{n}\sigma^2}}, \quad \& \quad \sigma^2 = \mu_N(1-\mu_N)$$

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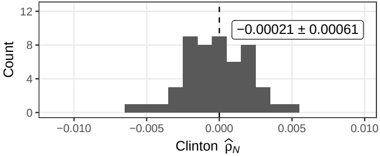

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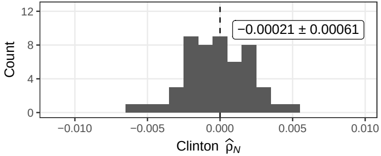

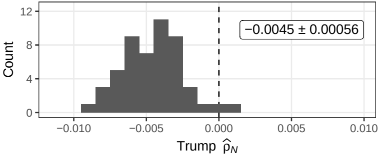

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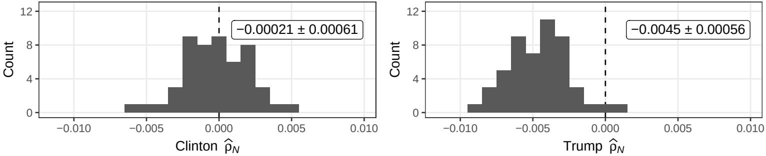

Clinton: $\hat{\rho} \approx -0.0002 \pm 0.0006$ Trump: $\hat{\rho} \approx -0.0045 \pm 0.0006$

- Problem: Voter validation is done through matching algorithms and it is not fool-proof, and it may introduce additional *selection bias*.

What's the implication of $\hat{\rho} = -0.005$?

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- **Butterfly Effect** due to Law of Large Populations (LLP)

$$\textbf{Relative Error} = \sqrt{\mathbf{N-1}}\,\hat{\rho}$$

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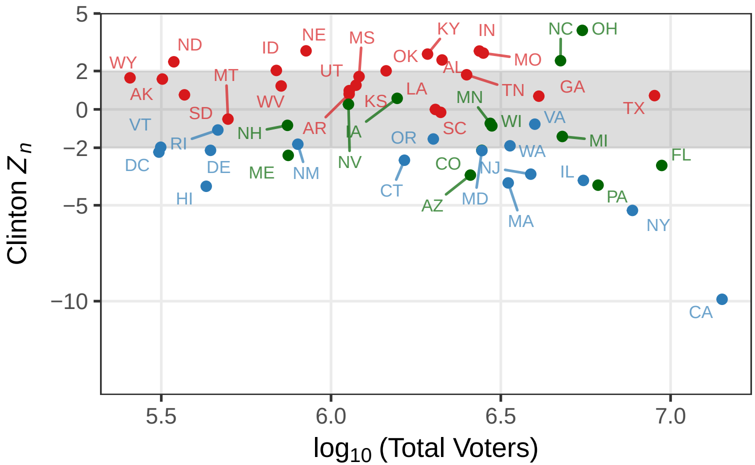
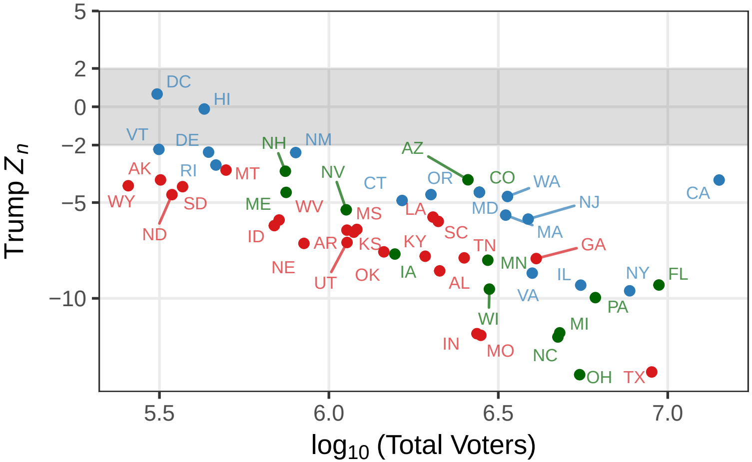


Visulizing LLP: Actual Coverage for Trump

Xiao-Li Meng
Department of Statistics, Harvard University



The Big Data Paradox:

If we do not pay attention to data quality, then

The bigger the data,

the surer we fool ourselves.

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Lessons Learned ...

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- Lesson 5: **We may all have had too much "confidence" in big size ...**

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… and learning from real experts …

19 things we learned from the 2016 election*

Andrew Gelman† Julia Azari‡

12 July 2017

We can all agree that the presidential election result was a shocker. According to news reports, even the Trump campaign team was stunned to come up a winner.

So now seems like a good time to go over various theories floating around in political science and political reporting and see where they stand, now that this turbulent political year has drawn to a close. In the present article, we go through several things that we as political observers and political scientists have learned from the election, and then discuss implications for the future.

The shock

Immediately following the election there was much talk about the failure of the polls: Hillary Clinton was seen as the clear favorite for several months straight, and then she lost. After all the votes were counted, though, the view is slightly different: by election eve, the national polls were giving Clinton 52 or 53% of the two-party vote, and she ended up receiving 51%. An error of 2 percentage points is no great embarrassment.

The errors in the polls were, however, not uniform. As Figures 1 and 2 show, the Republican candidate outperformed by about 5% in highly Republican states, 2% in swing states, and not at all, on average, in highly Democratic states. This was unexpected in part because, in other recent elections, the errors in poll-based forecasts did not have this sort of structure. In 2016, though, Donald Trump won from his better-than-expected performance in Wisconsin, Michigan, North Carolina, Pennsylvania, and several other swing states.

Trump's win in the general election, and the corresponding success of Republican candidates for the U.S. Senate, then raises two questions: (1) What did the polls get wrong in these key states?, (2) How did Trump and his fellow Republicans do so well? The first is a question about survey respondents, the second a question about voters.

Going backward in time from the election-day shocker, there is the question of how Trump, as a