CS 374 : Computational and Numerical Methods

Set 5

The Bisection Method

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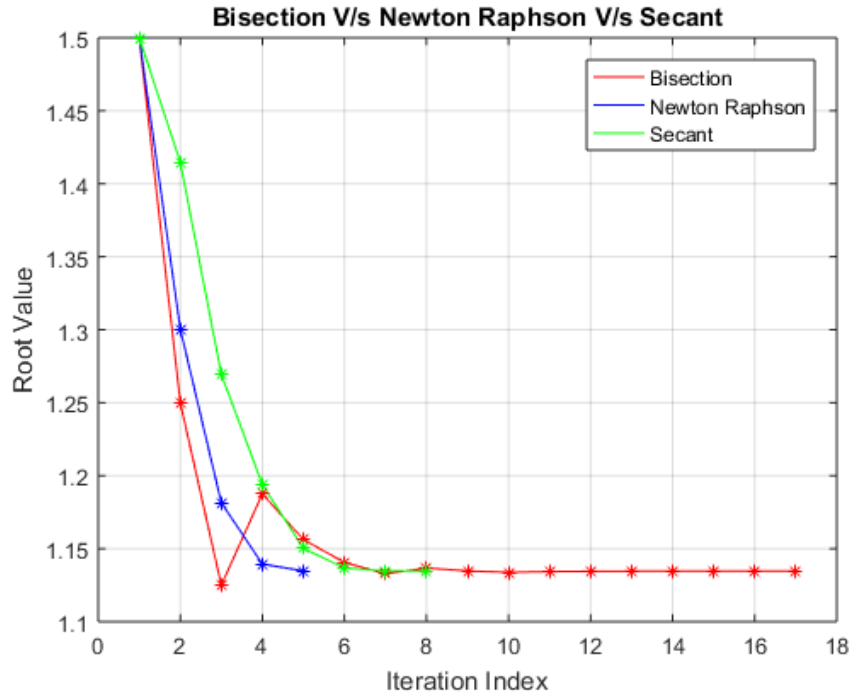
Contents

1 $f(x) = x^6 - x - 1$

1.1 Plots



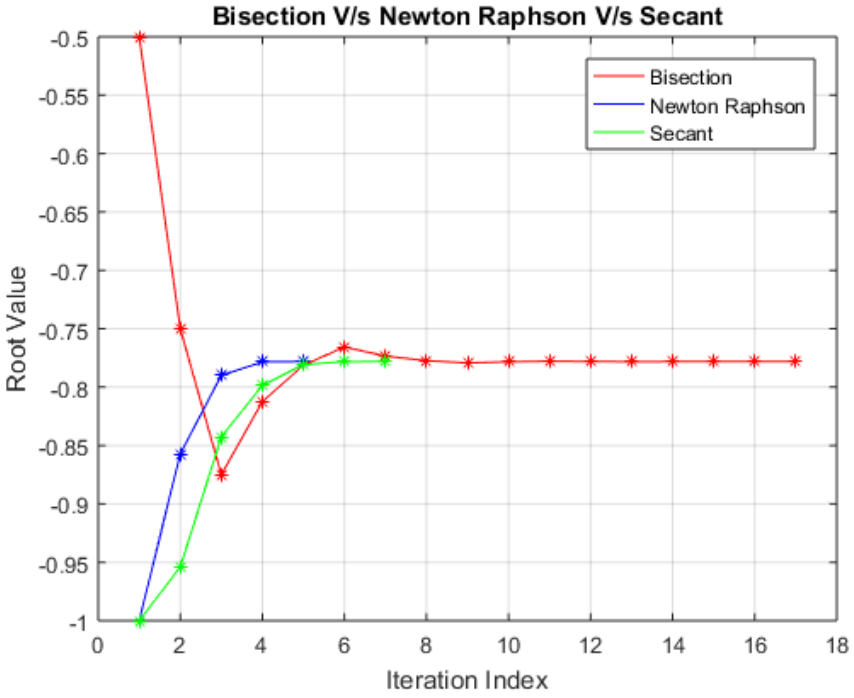


Figure 1: Fig 1.1: Positive root of $x^6 - x - 1 = 0$ || Fig 1.2: Negative root of $x^6 - x - 1 = 0$

1.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n)-f(x_{n-1})$	x_n-x_{n-1}	x_{n+1}	$x_{n+1}-x_n$
1	2	61	0	0	1.5	0
2	1.5	8.8906	-0.5	-52.109	1.4147	-0.08531
3	1.4147	5.6016	-0.08531	-3.289	1.2694	-0.14529
4	1.2694	1.9147	-0.14529	-3.6869	1.194	-0.07545
5	1.194	0.70289	-0.07545	-1.2118	1.1502	-0.04376
6	1.1502	0.16516	-0.04376	-0.53774	1.1367	-0.01344
7	1.1367	0.02092	-0.01344	-0.14424	1.1348	-0.00195
8	1.1348	0.00076	-0.00195	-0.02016	1.1347	-7e-05
9	1.1347	0	-7e-05	-0.00076	1.1347	0

Table 1: Positive Root of $x^6 - x - 1 = 0$

ItrNo	x_n	$f(x_n)$	$f(x_n)-f(x_{n-1})$	x_n-x_{n-1}	x_{n+1}	$x_{n+1}-x_n$
1	-1	1	0	0	-1.5	0
2	-1.5	11.891	-0.5	10.891	-0.95409	0.54591
3	-0.95409	0.70837	0.54591	-11.182	-0.91951	0.03458
4	-0.91951	0.52391	0.03458	-0.18446	-0.82128	0.09822
5	-0.82128	0.12815	0.09822	-0.39576	-0.78948	0.03181
6	-0.78948	0.0316	0.03181	-0.09656	-0.77907	0.01041
7	-0.77907	0.00266	0.01041	-0.02894	-0.77811	0.00096
8	-0.77811	6e-05	0.00096	-0.0026	-0.77809	2e-05
9	-0.77809	0	2e-05	-6e-05	-0.77809	0

Table 2: Negative Root of $x^6 - x - 1 = 0$

2 $f(x) = x^3 - x^2 - x - 1$

2.1 Plots

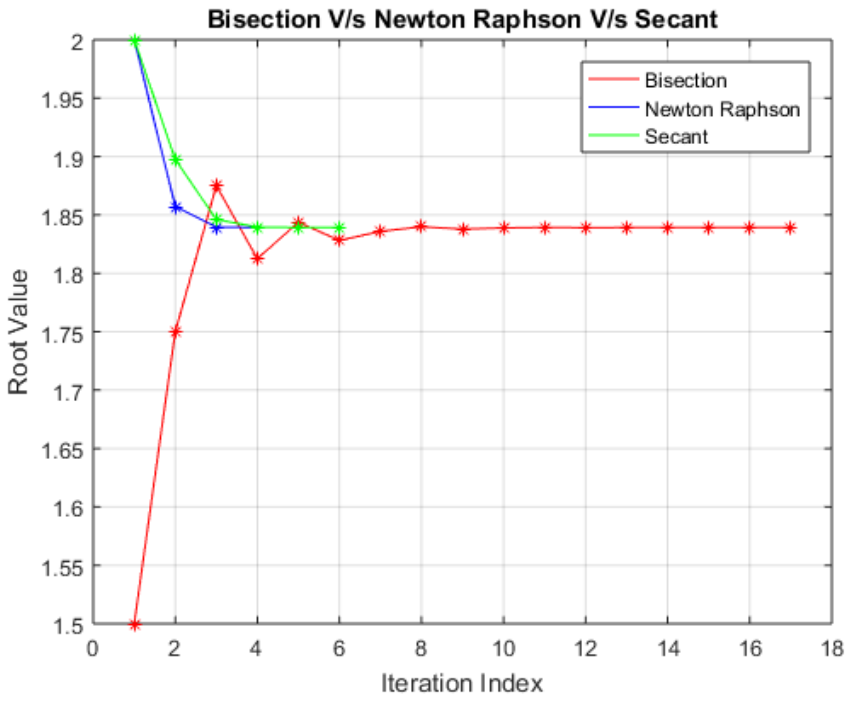


Figure 2: Positive root of $x^3 - x^2 - x - 1 = 0$

2.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n) - f(x_{n-1})$	$x_n - x_{n-1}$	x_{n+1}	$x_{n+1} - x_n$
1	2.5	5.875	0	0	2	0
2	2	1	-0.5	-4.875	1.8974	-0.10256
3	1.8974	0.33357	-0.10256	-0.66643	1.8461	-0.05134
4	1.8461	0.03748	-0.05134	-0.29609	1.8396	-0.0065
5	1.8396	0.00172	-0.0065	-0.03576	1.8393	-0.00031
6	1.8393	1e-05	-0.00031	-0.00171	1.8393	0
7	1.8393	0	0	-1e-05	1.8393	0

Table 3: Positive Root of $x^3 - x^2 - x - 1 = 0$

3 $f(x) = 1 + 0.3cos(x) - x$

3.1 Plots

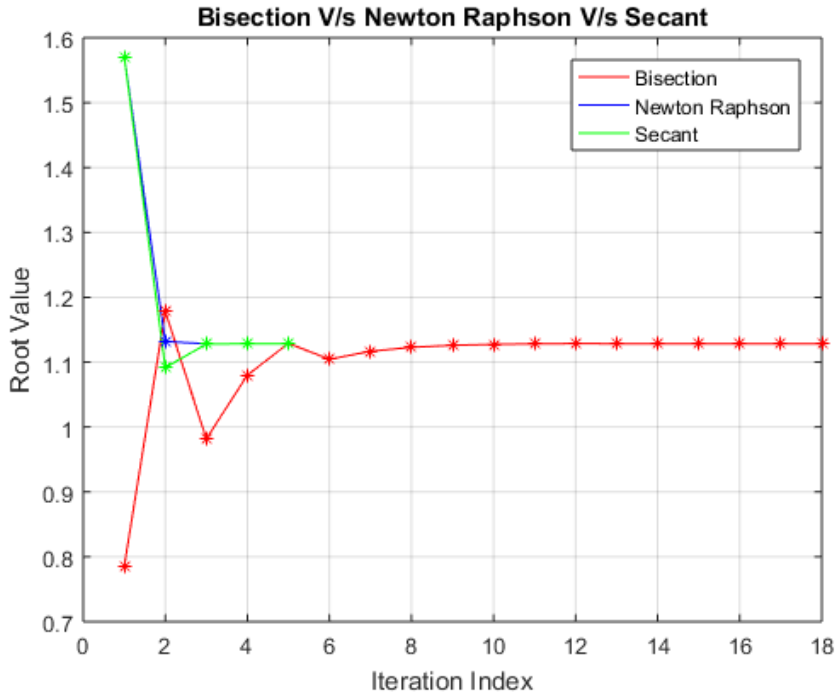


Figure 3: Fig 1.1,1.2 : Positive root of $f(x) = 1 + 0.3cos(x) - x$

3.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n) - f(x_{n-1})$	$x_n - x_{n-1}$	x_{n+1}	$x_{n+1} - x_n$
1	3.1416	-2.4416	0	0	1.5708	0
2	1.5708	-0.5708	-1.5708	1.8708	1.0915	-0.47926
3	1.0915	0.04681	-0.47926	0.6176	1.1279	0.03632
4	1.1279	0.00073	0.03632	-0.04608	1.1284	0.00057
5	1.1284	0	0.00057	-0.00073	1.1284	0
6	1.1284	0	0	0	1.1284	0

Table 4: Positive Root of $f(x) = 1 + 0.3cos(x) - x$

4 $f(x) = 0.5 + \sin x - \cos x$

4.1 Plots

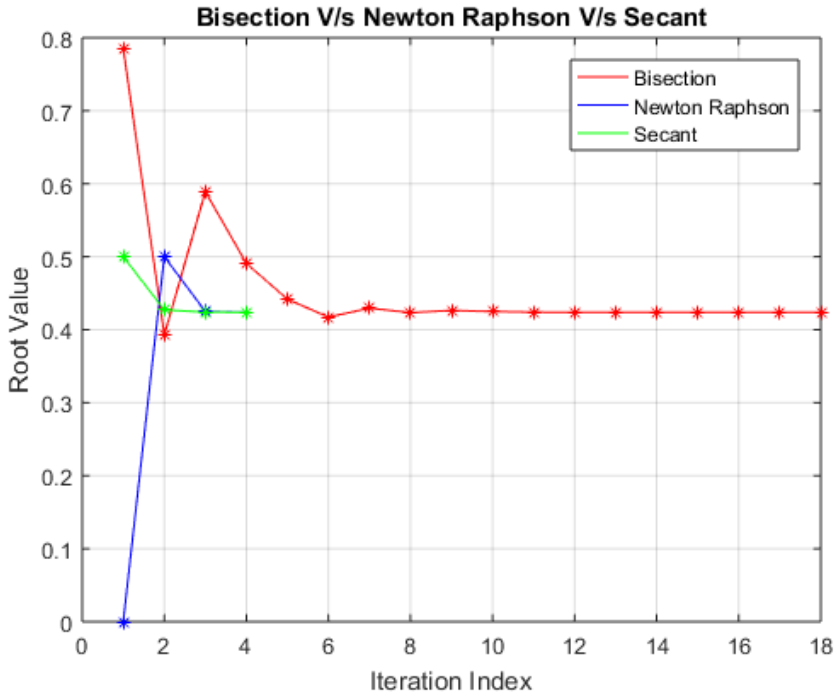


Figure 4: Fig 1.1,1.2 : Positive root of $f(x) = 0.5 + \sin x - \cos x$

4.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n) - f(x_{n-1})$	$x_n - x_{n-1}$	x_{n+1}	$x_{n+1} - x_n$
1	1	0.80117	0	0	0.5	0
2	0.5	0.10184	-0.5	-0.69933	0.42718	-0.07282
3	0.42718	0.00417	-0.07282	-0.09767	0.42407	-0.00311
4	0.42407	5e-05	-0.00311	-0.00412	0.42403	-4e-05
5	0.42403	0	-4e-05	-5e-05	0.42403	0

Table 5: The smallest Positive Root of $f(x) = 0.5 + \sin x - \cos x$

5 $f(x) = x - e^{-x}$

5.1 Plots

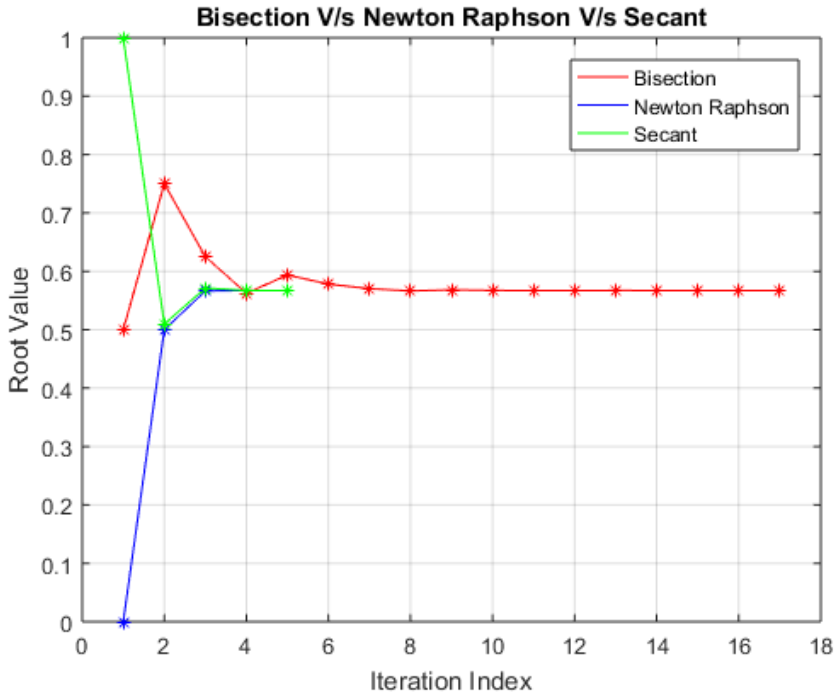


Figure 5: Positive root of $f(x) = x - -e^{-x}$

5.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n) - f(x_{n-1})$	$x_n - x_{n-1}$	x_{n+1}	$x_{n+1} - x_n$
1	1.5	-1.2769	0	0	1	0
2	1	-0.63212	-0.5	0.64475	0.50979	-0.49021
3	0.50979	0.09083	-0.49021	0.72295	0.57138	0.06159
4	0.57138	-0.00663	0.06159	-0.09746	0.56719	-0.00419
5	0.56719	-7e-05	-0.00419	0.00656	0.56714	-4e-05
6	0.56714	0	-4e-05	7e-05	0.56714	0

Table 6: The smallest Positive Root of $f(x) = x - e^{-x}$

6 $f(x) = \sin x - e^{-x}$

6.1 Plots

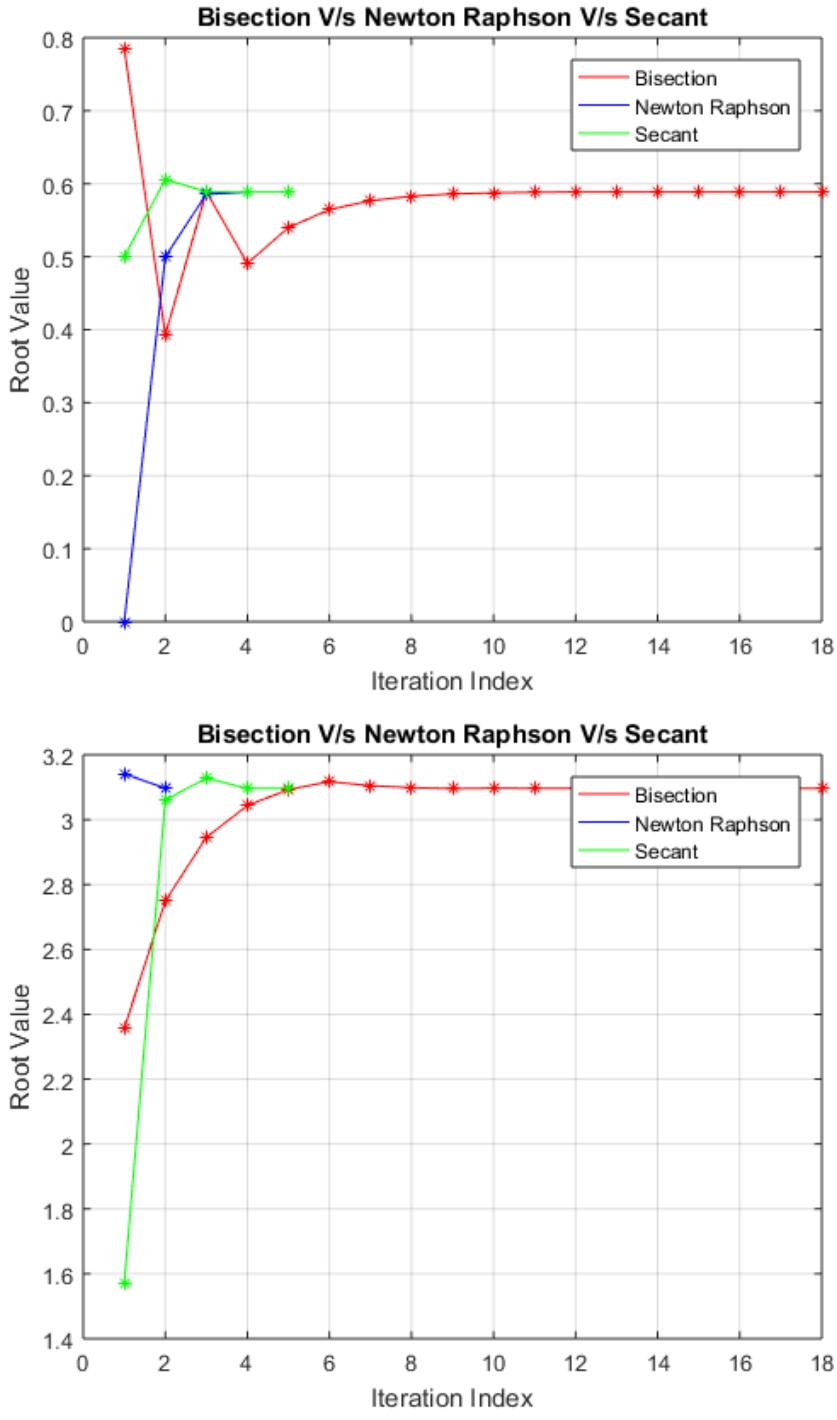


Figure 6: 1.1 : First smallest Positive root of $f(x) = \sin x - e^{-x}$ || 1.2 : Second Smallest Positive root of $f(x) = \sin x - e^{-x}$

6.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n)-f(x_{n-1})$	x_n-x_{n-1}	x_{n+1}	$x_{n+1}-x_n$
1	1	-0.47359	0	0	0.5	0
2	0.5	0.12711	-0.5	0.6007	0.6058	0.1058
3	0.6058	-0.02378	0.1058	-0.15088	0.58912	-0.01667
4	0.58912	-0.00082	-0.01667	0.02296	0.58853	-0.0006
5	0.58853	1e-05	-0.0006	0.00083	0.58853	0
6	0.58853	0	0	-1e-05	0.58853	0

Table 7: The first smallest Positive Root of $f(x) = \sin x - e^{-x}$

ItrNo	x_n	$f(x_n)$	$f(x_n)-f(x_{n-1})$	x_n-x_{n-1}	x_{n+1}	$x_{n+1}-x_n$
1	3.1416	0.04321	0	0	1.5708	0
2	1.5708	-0.79212	-1.5708	-0.83533	3.0603	1.4895
3	3.0603	-0.0343	1.4895	0.75782	3.1277	0.06742
4	3.1277	0.02997	0.06742	0.06427	3.0963	-0.03144
5	3.0963	-5e-05	-0.03144	-0.03003	3.0964	5e-05
6	3.0964	0	5e-05	5e-05	3.0964	0

Table 8: The Second smallest Positive Root of $f(x) = \sin x - e^{-x}$

7 $f(x) = x^3 - 2x - 2$

7.1 Plots

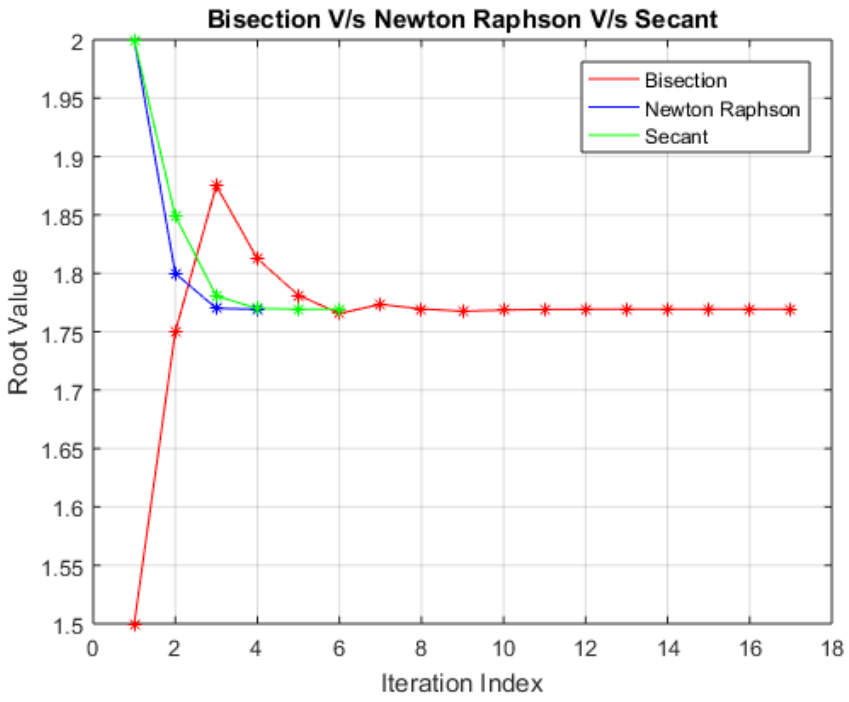


Figure 7: Positive root of $f(x) = x^3 - 2x - 2$

7.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n) - f(x_{n-1})$	$x_n - x_{n-1}$	x_{n+1}	$x_{n+1} - x_n$
1	2.5	8.625	0	0	2	0
2	2	2	-0.5	-6.625	1.8491	-0.15094
3	1.8491	0.62383	-0.15094	-1.3762	1.7806	-0.06842
4	1.7806	0.0845	-0.06842	-0.53933	1.7699	-0.01072
5	1.7699	0.00458	-0.01072	-0.07992	1.7693	-0.00061
6	1.7693	4e-05	-0.00061	-0.00454	1.7693	-1e-05
7	1.7693	0	-1e-05	-4e-05	1.7693	0

Table 9: The Positive Root of $f(x) = x^3 - 2x - 2$

8 $f(x) = x^4 - x - 1$

8.1 Plots

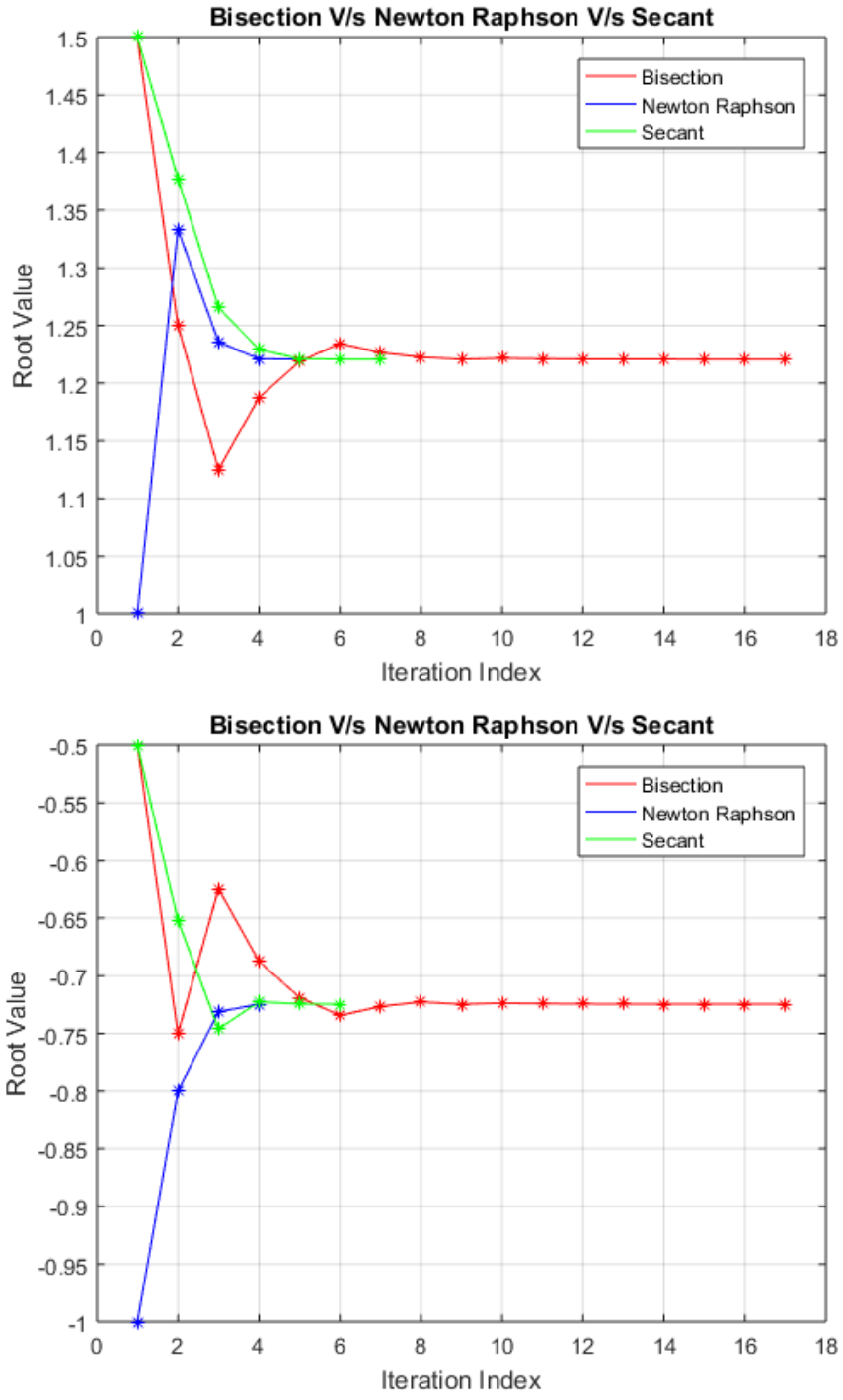


Figure 8: 1.1 : Positive root of $f(x) = x^4 - x - 1$ | 1.2 : Negative Root of $f(x) = x^4 - x - 1$

8.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n)-f(x_{n-1})$	x_n-x_{n-1}	x_{n+1}	$x_{n+1}-x_n$
1	2	13	0	0	1.5	0
2	1.5	2.5625	-0.5	-10.438	1.3773	-0.12275
3	1.3773	1.2206	-0.12275	-1.3419	1.2656	-0.11166
4	1.2656	0.29986	-0.11166	-0.92076	1.2292	-0.03636
5	1.2292	0.05384	-0.03636	-0.24603	1.2213	-0.00796
6	1.2213	0.00325	-0.00796	-0.05059	1.2208	-0.00051
7	1.2208	4e-05	-0.00051	-0.00321	1.2207	-1e-05
8	1.2207	0	-1e-05	-4e-05	1.2207	0

Table 10: The Positive Root of $f(x) = x^4 - x - 1$

ItrNo	x_n	$f(x_n)$	$f(x_n)-f(x_{n-1})$	x_n-x_{n-1}	x_{n+1}	$x_{n+1}-x_n$
1	-1	1	0	0	-0.5	0
2	-0.5	-0.4375	0.5	-1.4375	-0.65217	-0.15217
3	-0.65217	-0.16692	-0.15217	0.27058	-0.74605	-0.09388
4	-0.74605	0.05584	-0.09388	0.22276	-0.72252	0.02353
5	-0.72252	-0.00497	0.02353	-0.06081	-0.72444	-0.00192
6	-0.72444	-0.00013	-0.00192	0.00483	-0.72449	-5e-05
7	-0.72449	0	-5e-05	0.00013	-0.72449	0

Table 11: The Negative Root of $f(x) = x^4 - x - 1$

9 $f(x) = e^x - x - 2$

9.1 Plots

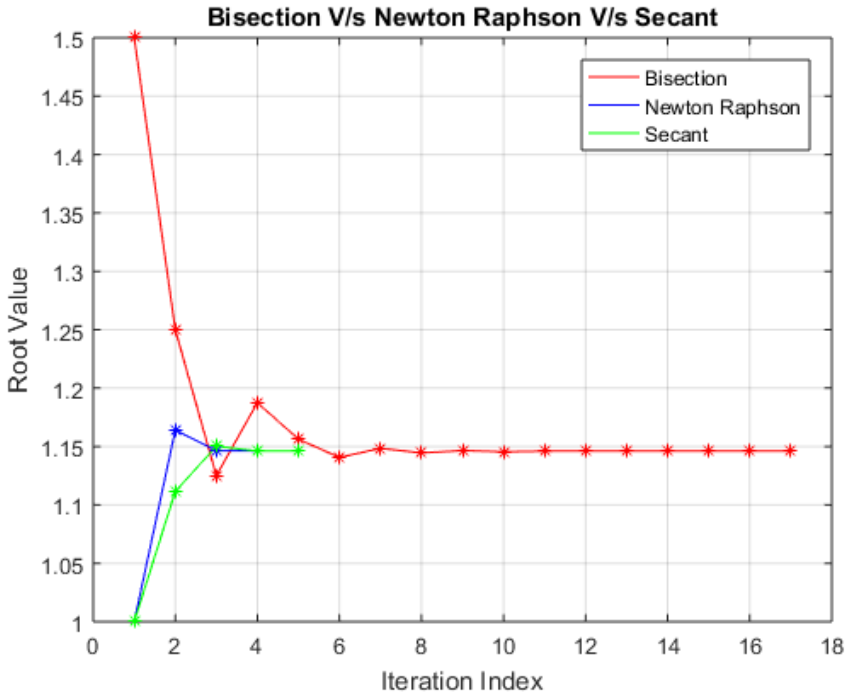


Figure 9: Positive root of $f(x) = e^x - x - 2$

9.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n) - f(x_{n-1})$	$x_n - x_{n-1}$	x_{n+1}	$x_{n+1} - x_n$
1	1.5	0.98169	0	0	1	0
2	1	-0.28172	-0.5	-1.2634	1.1115	0.11149
3	1.1115	-0.0726	0.11149	0.20911	1.1502	0.03871
4	1.1502	0.00863	0.03871	0.08123	1.1461	-0.00411
5	1.1461	-0.00022	-0.00411	-0.00885	1.1462	0.0001
6	1.1462	0	0.0001	0.00022	1.1462	0

Table 12: The Positive Root of $f(x) = e^x - x - 2$

10 $f(x) = 1 - x + \sin x$

10.1 Plots

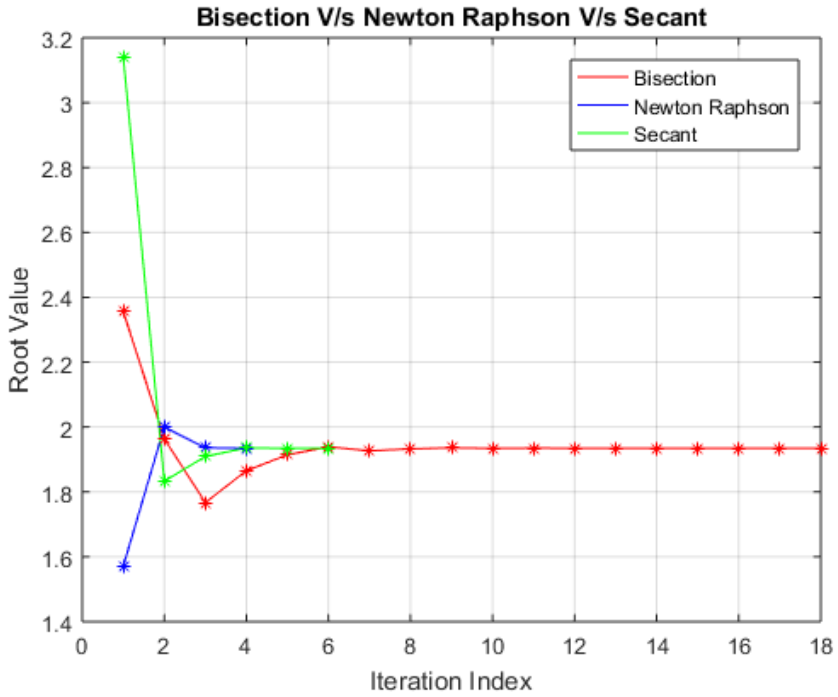


Figure 10: Smallest Positive root of $f(x) = 1 - x + \sin x$

10.2 Tables

ItrNo	x_n	$f(x_n)$	$f(x_n) - f(x_{n-1})$	$x_n - x_{n-1}$	x_{n+1}	$x_{n+1} - x_n$
1	1.5708	0.4292	0	0	3.1416	0
2	3.1416	-2.1416	1.5708	-2.5708	1.8331	-1.3086
3	1.8331	0.13276	-1.3086	2.2744	1.9094	0.07638
4	1.9094	0.03378	0.07638	-0.09898	1.9355	0.02607
5	1.9355	-0.00127	0.02607	-0.03504	1.9346	-0.00094
6	1.9346	1e-05	-0.00094	0.00128	1.9346	1e-05
7	1.9346	0	1e-05	-1e-05	1.9346	0

Table 13: The smallest Positive Root of $f(x) = 1 - x + \sin x$

11 $f(x) = -x + \tan x$

11.1 Plots

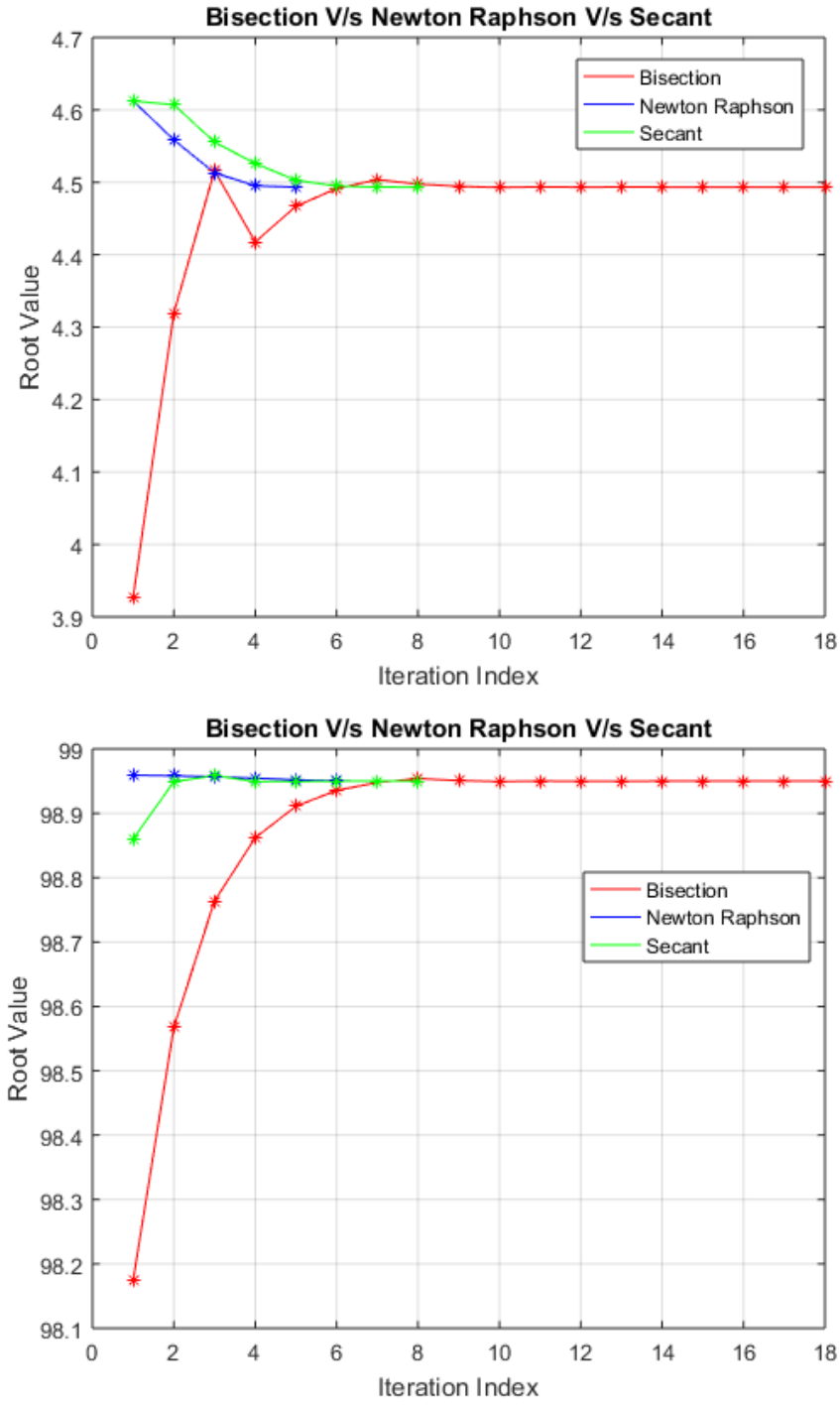


Figure 11: 1.1 : First Root of $f(x) = -x + \tan x$ | 1.2 : Root of $f(x) = -x + \tan x$ near $x = 100$

ItrNo	x_n	$f(x_n)$	$f(x_n)-f(x_{n-1})$	x_n-x_{n-1}	x_{n+1}	$x_{n+1}-x_n$
1	4.7024	95.294	0	0	4.6124	0
2	4.6124	5.3543	-0.09	-89.94	4.607	-0.00536
3	4.607	4.8493	-0.00536	-0.50497	4.5556	-0.05145
4	4.5556	1.7692	-0.05145	-3.0801	4.526	-0.02955
5	4.526	0.77754	-0.02955	-0.99167	4.5028	-0.02317
6	4.5028	0.19952	-0.02317	-0.57803	4.4948	-0.008
7	4.4948	0.02935	-0.008	-0.17017	4.4935	-0.00138
8	4.4935	0.0013	-0.00138	-0.02805	4.4934	-6e-05
9	4.4934	1e-05	-6e-05	-0.00129	4.4934	0

Table 14: The smallest Positive Root of $f(x) = -x + \tan x$

ItrNo	x_n	$f(x_n)$	$f(x_n)-f(x_{n-1})$	x_n-x_{n-1}	x_{n+1}	$x_{n+1}-x_n$
1	98.95	1.0465	0	0	98.86	0
2	98.86	-88.894	-0.09	-89.94	98.949	0.08895
3	98.949	-8.4321	0.08895	80.461	98.958	0.00932
4	98.958	480.67	0.00932	489.11	98.949	-0.00916
5	98.949	-7.0959	-0.00916	-487.77	98.949	0.00013
6	98.949	-5.9575	0.00013	1.1384	98.95	0.0007
7	98.95	0.49221	0.0007	6.4497	98.95	-5e-05
8	98.95	-0.03137	-5e-05	-0.52358	98.95	0
9	98.95	-0.00016	0	0.03122	98.95	0

Table 15: The Positive Root of $f(x) = -x + \tan x$ around 100