CS 374 : Computational and Numerical Methods

Set 3

The Bisection Method

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The Bisection Method

In mathematics, the bisection method is a root-finding method that applies to any continuous functions for which one knows two values with opposite signs. The method consists of repeatedly bisecting the interval defined by these values and then selecting the sub interval in which the function changes sign, and therefore must contain a root.It is a very simple and robust method, but it is also relatively slow.

1 $f(x) = x^6 - x - 1$

We observe that $f(1) = 1^6 - 1 - 1 = -1$ and at 2, $f(2) = 2^6 - 2 - 1 = 61$, it blows up. Hence the positive root lies between 1 and 2.
We again see that at 0, $f(0) = -1$ and at -1, $f(-1) = 1$. Hence the negative root lies between -1 and 0.

1.1 Plots

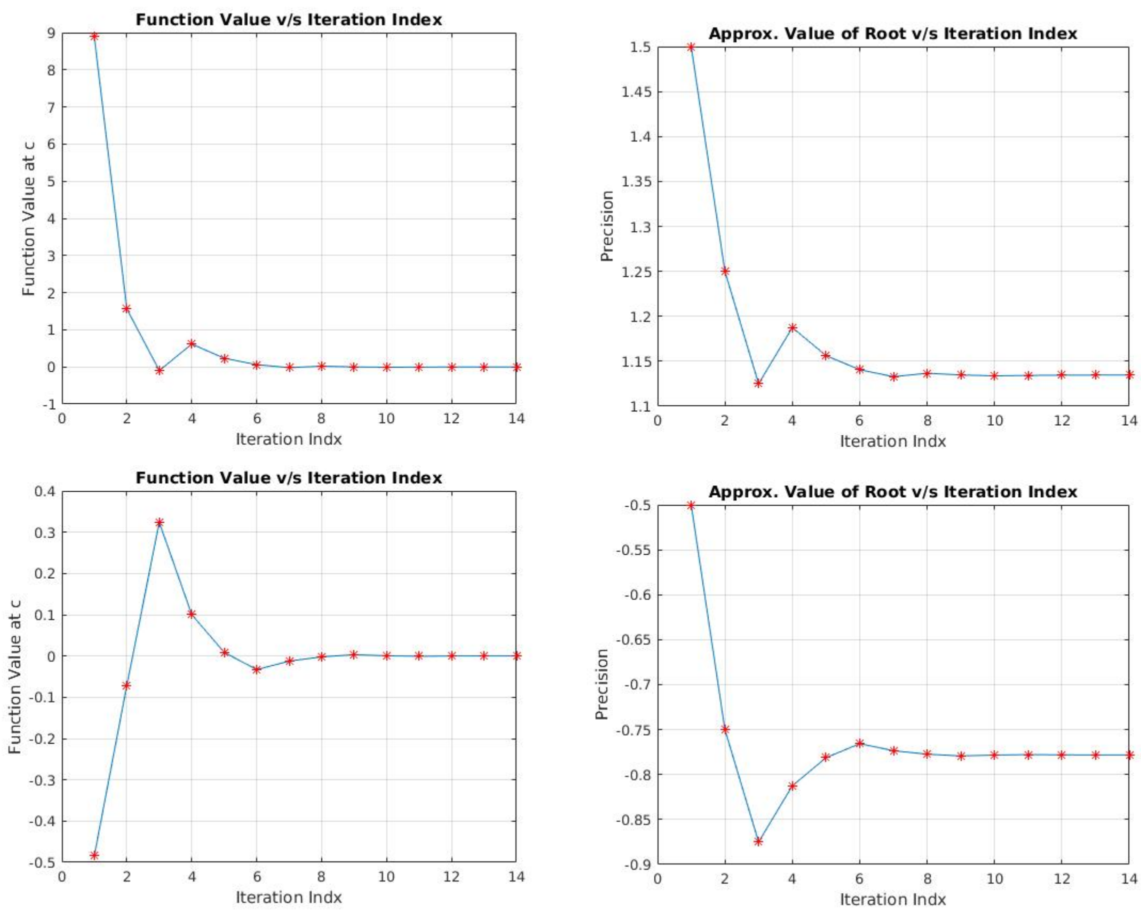


Figure 1: Fig 1.1,1.2 : Positive root of $x^6 - x - 1 = 0$ | | Fig 1.3,1.4 : Negative root of $x^6 - x - 1 = 0$

1.2 Tables

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	1	2	1.5	8.8906	<0	0.5	Set b=c
2	1	1.5	1.25	1.5647	<0	0.25	Set b=c
3	1	1.25	1.125	-0.0977	>0	0.125	Set a=c
4	1.125	1.25	1.1875	0.6167	<0	0.0625	Set b=c
5	1.125	1.1875	1.1563	0.2333	<0	0.0313	Set b=c
6	1.125	1.1563	1.1406	0.0616	<0	0.0156	Set b=c
7	1.125	1.1406	1.1328	-0.0196	>0	0.0078	Set a=c
8	1.1328	1.1406	1.1367	0.0206	<0	0.0039	Set b=c
9	1.1328	1.1367	1.1348	0.0004	<0	0.002	Set b=c
10	1.1328	1.1348	1.1338	-0.0096	>0	0.001	Set a=c
11	1.1338	1.1348	1.1343	-0.0046	>0	0.0005	Set a=c
12	1.1343	1.1348	1.1345	-0.0021	>0	0.0002	Set a=c
13	1.1345	1.1348	1.1346	-0.0008	>0	0.0001	Set a=c
14	1.1346	1.1348	1.1347	-0.0002	>0	0.0001	Set a=c

Table 1: Positive Root of $x^6 - x - 1 = 0$

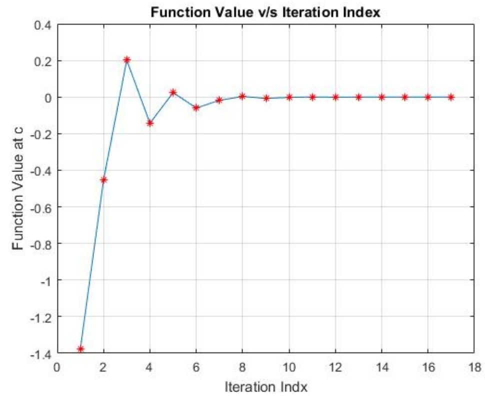
ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	-1	0	-0.5	-0.4844	<0	0.5	Set b=c
2	-1	-0.5	-0.75	-0.072	<0	0.25	Set b=c
3	-1	-0.75	-0.875	0.3238	>0	0.125	Set a=c
4	-0.875	-0.75	-0.8125	0.1002	>0	0.0625	Set a=c
5	-0.8125	-0.75	-0.7813	0.0086	>0	0.0313	Set a=c
6	-0.7813	-0.75	-0.7656	-0.033	<0	0.0156	Set b=c
7	-0.7813	-0.7656	-0.7734	-0.0125	<0	0.0078	Set b=c
8	-0.7813	-0.7734	-0.7773	-0.002	<0	0.0039	Set b=c
9	-0.7813	-0.7773	-0.7793	0.0033	>0	0.002	Set a=c
10	-0.7793	-0.7773	-0.7783	0.0006	>0	0.001	Set a=c
11	-0.7783	-0.7773	-0.7778	-0.0007	<0	0.0005	Set b=c
12	-0.7783	-0.7778	-0.7781	0	<0	0.0002	Set b=c
13	-0.7783	-0.7781	-0.7782	0.0003	>0	0.0001	Set a=c
14	-0.7782	-0.7781	-0.7781	0.0001	>0	0.0001	Set a=c

Table 2: Negative Root of $x^6 - x - 1 = 0$

2 $f(x) = x^3 - x^2 - x - 1$

We have $f'(x) = 3x^2 - 2x - 1$. We find that the local minima occurs at $x = 1$, since the function raises itself to ∞ after that. We also note that at $x = 2$, $f(x) = 1$. So we get an estimate that the root lies between 1 and 2. At the local maxima, i.e $x = -\frac{1}{3}$, the function is negative. Hence we are sure that the function has only one real root.

2.1 Plots



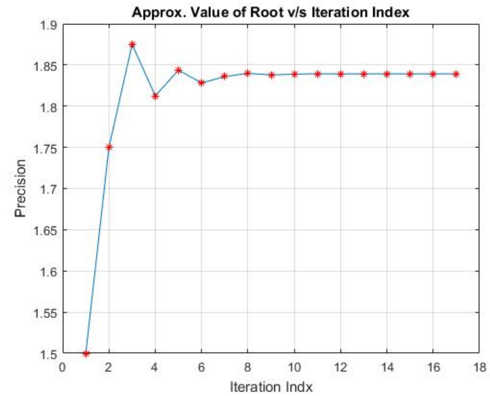


Figure 2: Fig 1.1,1.2 : Positive root of $x^3 - x^2 - x - 1 = 0$

2.2 Tables

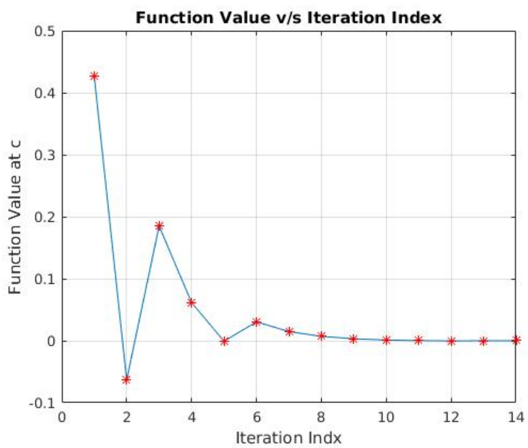
ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	1	2	1.5	-1.375	>0	0.5	Set a=c
2	1.5	2	1.75	-0.4531	>0	0.25	Set a=c
3	1.75	2	1.875	0.2012	<0	0.125	Set b=c
4	1.75	1.875	1.8125	-0.1433	>0	0.0625	Set a=c
5	1.8125	1.875	1.8438	0.0245	<0	0.0313	Set b=c
6	1.8125	1.8438	1.8281	-0.0605	>0	0.0156	Set a=c
7	1.8281	1.8438	1.8359	-0.0183	>0	0.0078	Set a=c
8	1.8359	1.8438	1.8398	0.003	<0	0.0039	Set b=c
9	1.8359	1.8398	1.8379	-0.0076	>0	0.002	Set a=c
10	1.8379	1.8398	1.8389	-0.0023	>0	0.001	Set a=c
11	1.8389	1.8398	1.8394	0.0004	<0	0.0005	Set b=c
12	1.8389	1.8394	1.8391	-0.001	>0	0.0002	Set a=c
13	1.8391	1.8394	1.8392	-0.0003	>0	0.0001	Set a=c
14	1.8392	1.8394	1.8393	0	<0	0.0001	Set b=c

Table 3: Positive Root of $x^3 - x^2 - x - 1 = 0$

3 $f(x) = 1 + 0.3cos(x) - x$

From a rough plot of the function, we can see that there can exist only one intersection between the curves, $f_1(x) = 0.3\cos x + 1$ and $f_2(x) = x$. At $x = 0$, $f(x) = 1.3$ and at $x = \frac{\pi}{2}$, $f(x) = 1 - \frac{\pi}{2} < 0$.

3.1 Plots



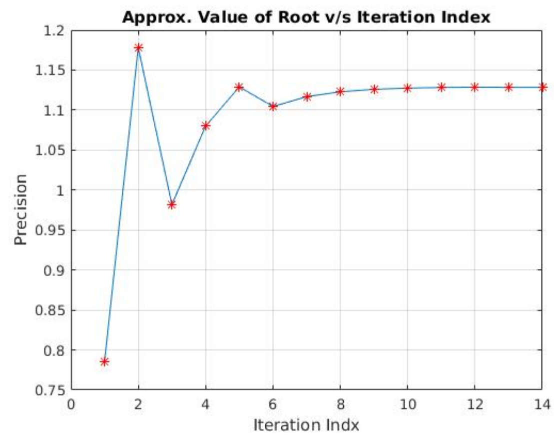


Figure 3: Fig 1.1,1.2 : Positive root of $f(x) = 1 + 0.3cos(x) - x$

3.2 Tables

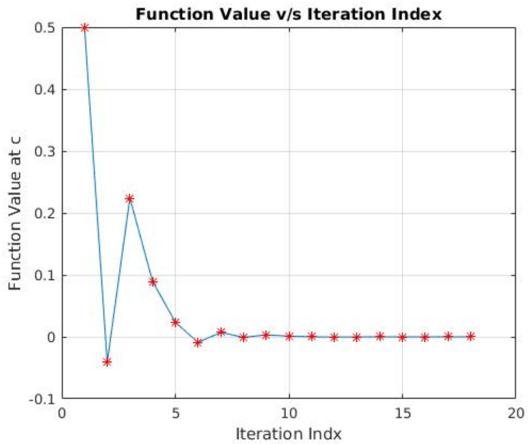
ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	0	1.5708	0.7854	0.42673	>0	0.7854	Set a=c
2	0.7854	1.5708	1.1781	-0.06329	<0	0.3927	Set b=c
3	0.7854	1.1781	0.98175	0.18492	>0	0.19635	Set a=c
4	0.98175	1.1781	1.0799	0.0615	>0	0.09817	Set a=c
5	1.0799	1.1781	1.129	-0.00074	<0	0.04909	Set b=c
6	1.0799	1.129	1.1045	0.03042	>0	0.02454	Set a=c
7	1.1045	1.129	1.1167	0.01485	>0	0.01227	Set a=c
8	1.1167	1.129	1.1229	0.00705	>0	0.00614	Set a=c
9	1.1229	1.129	1.1259	0.00316	>0	0.00307	Set a=c
10	1.1259	1.129	1.1275	0.00121	>0	0.00153	Set a=c
11	1.1275	1.129	1.1282	0.00023	>0	0.00077	Set a=c
12	1.1282	1.129	1.1286	-0.00026	<0	0.00038	Set b=c
13	1.1282	1.1286	1.1284	-1e-05	<0	0.00019	Set b=c
14	1.1282	1.1284	1.1283	0.00011	>0	0.0001	Set a=c

Table 4: Positive Root of $f(x) = 1 + 0.3cos(x) - x$

4 $f(x) = 0.5 + \sin x - \cos x$

We see that at $x = 0$, $f_1(x) = 0.5 + \sin x = 0.5$ and $f_2(x) = \cos x = 1$. And at $x = \frac{\pi}{2}$, $f_1 \frac{\pi}{2} = 1.5$ and $f_2(x) = 0$. Note that the functions $f_1(x)$ and $f_2(x)$ are monotonic between $x = 0$ and $x = \frac{\pi}{2}$ so there can't be any other intersection between the two.

4.1 Plots



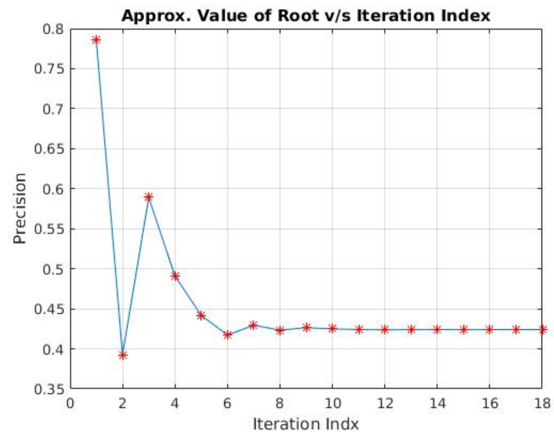


Figure 4: Fig 1.1,1.2 : Positive root of $f(x) = 0.5 + \sin x - \cos x$

4.2 Tables

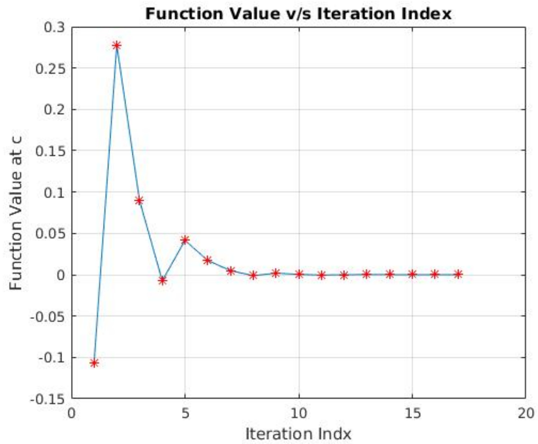
ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	0	1.5708	0.7854	0.5	<0	0.7854	Set b=c
2	0	0.7854	0.3927	-0.0412	>0	0.3927	Set a=c
3	0.3927	0.7854	0.589	0.2241	<0	0.1963	Set b=c
4	0.3927	0.589	0.4909	0.0895	<0	0.0982	Set b=c
5	0.3927	0.4909	0.4418	0.0236	<0	0.0491	Set b=c
6	0.3927	0.4418	0.4172	-0.009	>0	0.0245	Set a=c
7	0.4172	0.4418	0.4295	0.0073	<0	0.0123	Set b=c
8	0.4172	0.4295	0.4234	-0.0009	>0	0.0061	Set a=c
9	0.4234	0.4295	0.4264	0.0032	<0	0.0031	Set b=c
10	0.4234	0.4264	0.4249	0.0012	<0	0.0015	Set b=c
11	0.4234	0.4249	0.4241	0.0002	<0	0.0008	Set b=c
12	0.4234	0.4241	0.4238	-0.0004	>0	0.0004	Set a=c
13	0.4238	0.4241	0.424	-0.0001	>0	0.0002	Set a=c
14	0.424	0.4241	0.424	0	<0	0.0001	Set b=c

Table 5: The smallest Positive Root of $f(x) = 0.5 + \sin x - \cos x$

5 $f(x) = x - e^{-x}$

Note that the function $f_1(x) = e^{-x}$ is monotonically decreasing and $f_2(x) = x$ is monotonically increasing. Hence there exist only one root. At $x = 0$, $f(x) = -1$ and at $x = 1$, $f(x) = 0$. Hence the root lies between these two values.

5.1 Plots



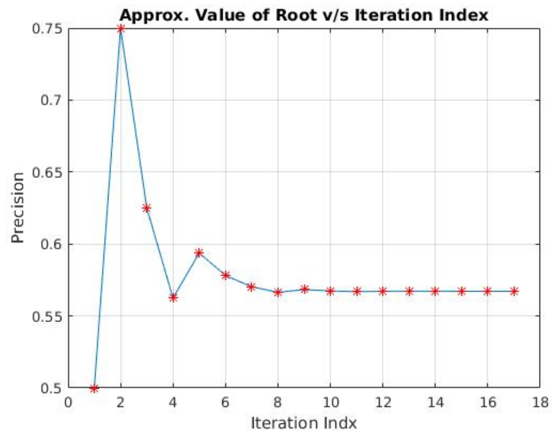


Figure 5: Fig 1.1,1.2 : Positive root of $f(x) = x - -e^{-x}$

5.2 Tables

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	0	1	0.5	-0.1065	>0	0.5	Set a=c
2	0.5	1	0.75	0.2776	<0	0.25	Set b=c
3	0.5	0.75	0.625	0.0897	<0	0.125	Set b=c
4	0.5	0.625	0.5625	-0.0073	>0	0.0625	Set a=c
5	0.5625	0.625	0.5938	0.0415	<0	0.0313	Set b=c
6	0.5625	0.5938	0.5781	0.0172	<0	0.0156	Set b=c
7	0.5625	0.5781	0.5703	0.005	<0	0.0078	Set b=c
8	0.5625	0.5703	0.5664	-0.0012	>0	0.0039	Set a=c
9	0.5664	0.5703	0.5684	0.0019	<0	0.002	Set b=c
10	0.5664	0.5684	0.5674	0.0004	<0	0.001	Set b=c
11	0.5664	0.5674	0.5669	-0.0004	>0	0.0005	Set a=c
12	0.5669	0.5674	0.5671	0	>0	0.0002	Set a=c
13	0.5671	0.5674	0.5673	0.0002	<0	0.0001	Set b=c
14	0.5671	0.5673	0.5672	0.0001	<0	0.0001	Set b=c

Table 6: The smallest Positive Root of $f(x) = x - e^{-x}$

6 $f(x) = \sin x - e^{-x}$

At $x = 0$, the values of $f_2(x) = e^{-x}$ enter the range of values of $f_1(x) = \sin x$ with $f(x) = -1$. At $x = \frac{\pi}{2}$, $f(x) > 0$. Between 0 and $\frac{\pi}{2}$, both the functions are monotonic. Hence there can exist only one root between the two values.

The second root occurs when $f_2(x) = e^{-x}$ cuts $f_1(x) = \sin x$ in the first lobe for the second time. The first time being between $0 << \frac{pi}{2}$ and the second time being between $\frac{pi}{2} < x < \pi$.

6.1 Plots

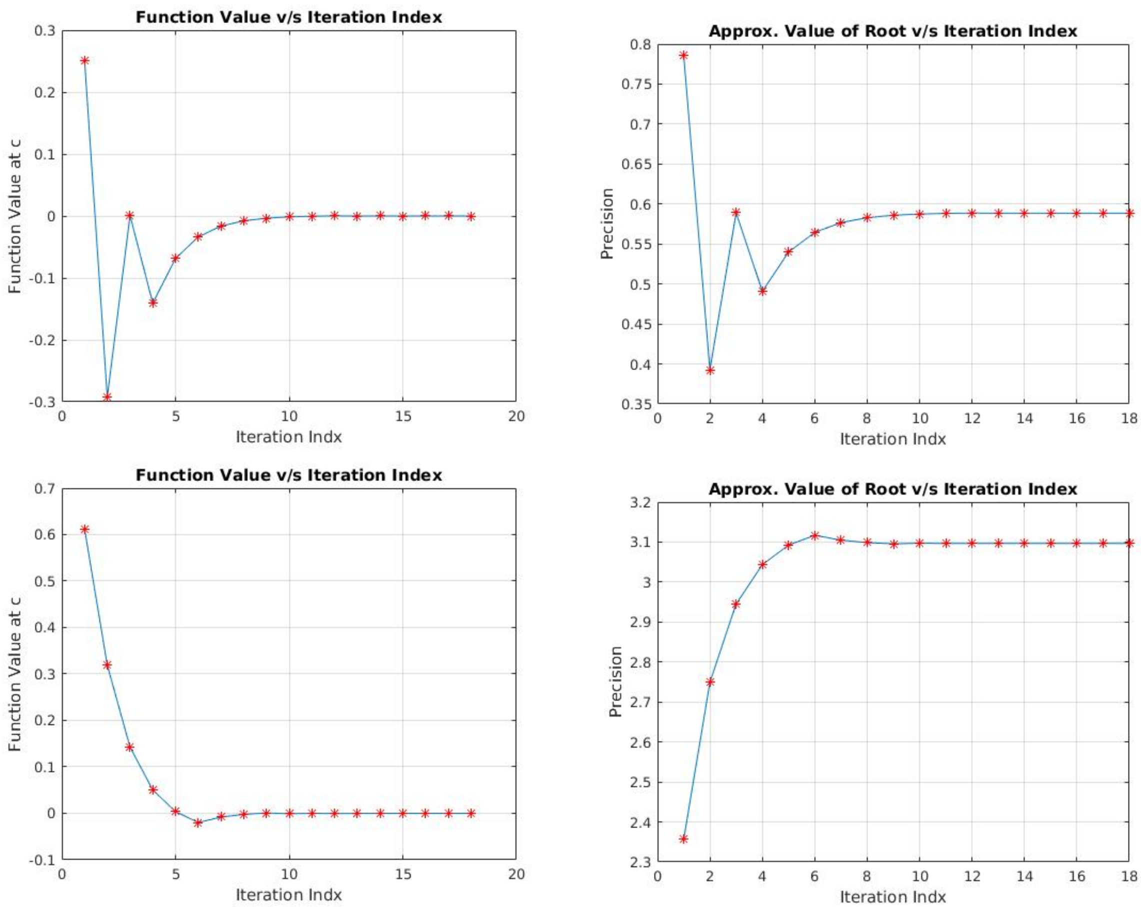


Figure 6: Fig 1.1,1.2 : First smallest Positive root of $f(x) = \sin x - e^{-x}$ || Fig 1.3,1.4 : Second Smallest Positive root of $f(x) = \sin x - e^{-x}$

6.2 Tables

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	0	1.5708	0.7854	0.2512	<0	0.7854	Set b=c
2	0	0.7854	0.3927	-0.2925	>0	0.3927	Set a=c
3	0.3927	0.7854	0.589	0.0007	<0	0.1963	Set b=c
4	0.3927	0.589	0.4909	-0.1407	>0	0.0982	Set a=c
5	0.4909	0.589	0.54	-0.0687	>0	0.0491	Set a=c
6	0.54	0.589	0.5645	-0.0336	>0	0.0245	Set a=c
7	0.5645	0.589	0.5768	-0.0164	>0	0.0123	Set a=c
8	0.5768	0.589	0.5829	-0.0078	>0	0.0061	Set a=c
9	0.5829	0.589	0.586	-0.0035	>0	0.0031	Set a=c
10	0.586	0.589	0.5875	-0.0014	>0	0.0015	Set a=c
11	0.5875	0.589	0.5883	-0.0003	>0	0.0008	Set a=c
12	0.5883	0.589	0.5887	0.0002	<0	0.0004	Set b=c
13	0.5883	0.5887	0.5885	-0.0001	>0	0.0002	Set a=c
14	0.5885	0.5887	0.5886	0.0001	<0	0.0001	Set b=c

Table 7: The first smallest Positive Root of $f(x) = \sin x - e^{-x}$

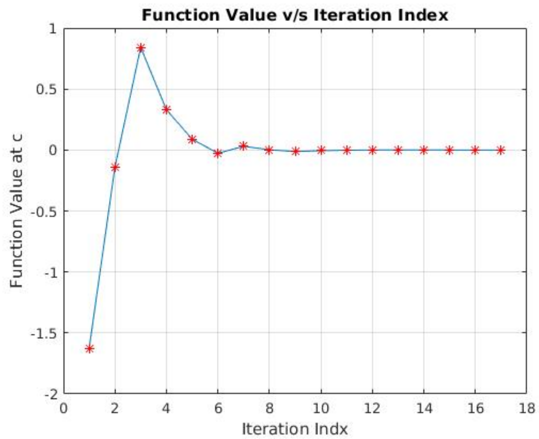
ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	1.5708	3.1416	2.3562	0.6123	>0	0.7854	Set a=c
2	2.3562	3.1416	2.7489	0.3187	>0	0.3927	Set a=c
3	2.7489	3.1416	2.9452	0.1425	>0	0.1963	Set a=c
4	2.9452	3.1416	3.0434	0.0503	>0	0.0982	Set a=c
5	3.0434	3.1416	3.0925	0.0037	>0	0.0491	Set a=c
6	3.0925	3.1416	3.117	-0.0197	<0	0.0245	Set b=c
7	3.0925	3.117	3.1048	-0.008	<0	0.0123	Set b=c
8	3.0925	3.1048	3.0986	-0.0022	<0	0.0061	Set b=c
9	3.0925	3.0986	3.0956	0.0008	>0	0.0031	Set a=c
10	3.0956	3.0986	3.0971	-0.0007	<0	0.0015	Set b=c
11	3.0956	3.0971	3.0963	0	>0	0.0008	Set a=c
12	3.0963	3.0971	3.0967	-0.0003	<0	0.0004	Set b=c
13	3.0963	3.0967	3.0965	-0.0002	<0	0.0002	Set b=c
14	3.0963	3.0965	3.0964	-0.0001	<0	0.0001	Set b=c

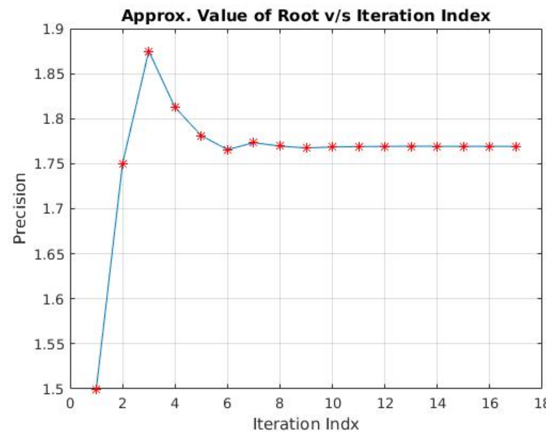
Table 8: The Second smallest Positive Root of $f(x) = \sin x - e^{-x}$

7 $f(x) = x^3 - 2x - 2$

We have $f'(x) = 3x^2 - 2$. Hence the local maxima lies at $x = -\sqrt{\frac{2}{3}}$ and at this point $f(-\sqrt{\frac{2}{3}}) < 0$, so there is only root of this equation. At $x = 1$, $f(x) = -3$ and at $x = 2$, $f(x) = 2$. Hence the only real root of $f(x)$ lies between 1 and 2.

7.1 Plots





7.2 Tables

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	1	2	1.5	-1.625	>0	0.5	Set a=c
2	1.5	2	1.75	-0.1406	>0	0.25	Set a=c
3	1.75	2	1.875	0.8418	<0	0.125	Set b=c
4	1.75	1.875	1.8125	0.3293	<0	0.0625	Set b=c
5	1.75	1.8125	1.7813	0.0891	<0	0.0313	Set b=c
6	1.75	1.7813	1.7656	-0.027	>0	0.0156	Set a=c
7	1.7656	1.7813	1.7734	0.0307	<0	0.0078	Set b=c
8	1.7656	1.7734	1.7695	0.0018	<0	0.0039	Set b=c
9	1.7656	1.7695	1.7676	-0.0127	>0	0.002	Set a=c
10	1.7676	1.7695	1.7686	-0.0054	>0	0.001	Set a=c
11	1.7686	1.7695	1.769	-0.0018	>0	0.0005	Set a=c
12	1.769	1.7695	1.7693	0	>0	0.0002	Set a=c
13	1.7693	1.7695	1.7694	0.0009	<0	0.0001	Set b=c
14	1.7693	1.7694	1.7693	0.0004	<0	0.0001	Set b=c

Table 9: The Positive Root of $f(x) = x^3 - 2x - 2$

8 $f(x) = x^4 - x - 1$

Graphically, we can see that there are only 2 roots of this equation since $f_1(x) = x^4$ and $f_2(x) = x + 1$ cut in only 2 points, once in positive x-axis and once in negative x-axis.

At $x = 1$, $f(x) = -1$ and at $x = 2$, $f(x) = 13$. Hence the positive root lies between 1 and 2. Again t $x = 0$, $f(x) = -1$ and at $x = -1$, $f(x) = 1$. Hence the negative root lies between -1 and 0.

8.1 Plots

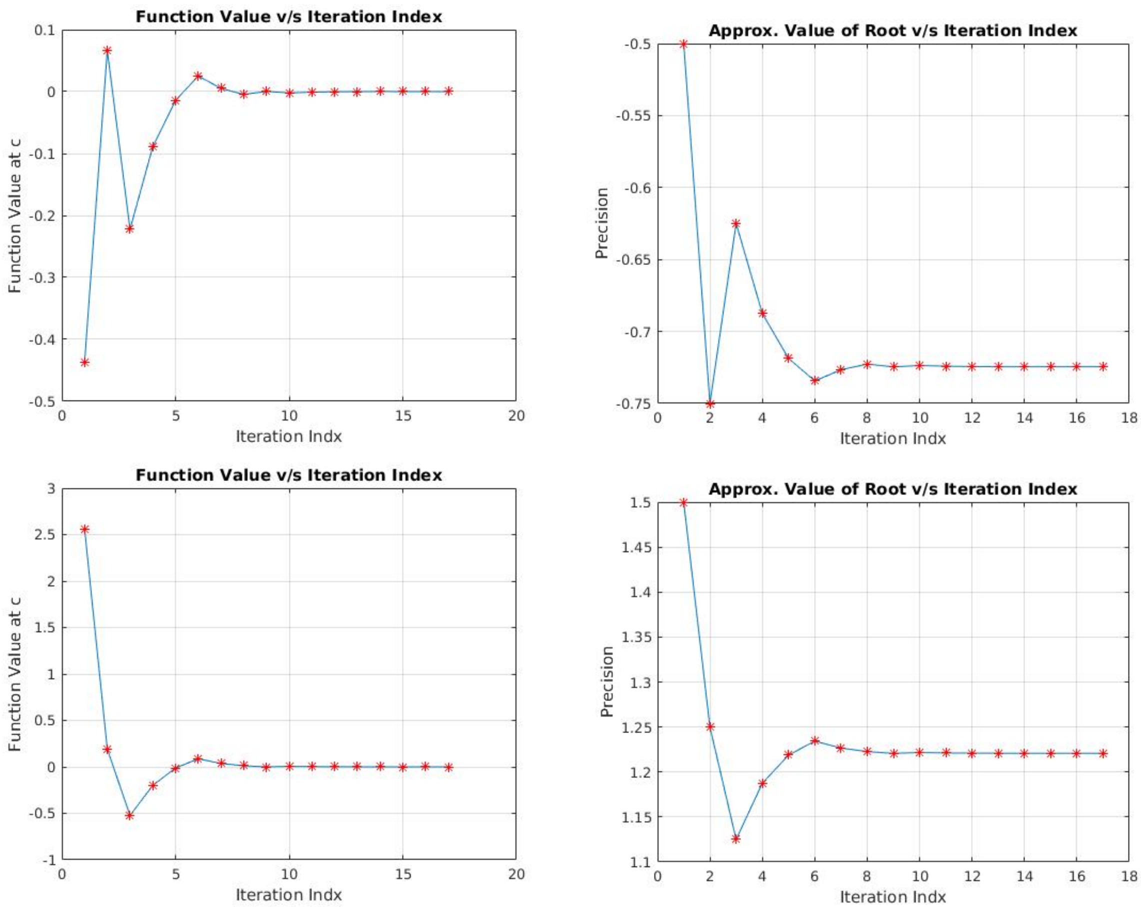


Figure 7: Fig 1.1,1.2 : Negative root of $f(x) = x^4 - x - 1$ | | Fig 1.3,1.4 : Positive root of $f(x) = x^4 - x - 1$

8.2 Tables

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	1	2	1.5	2.5625	<0	0.5	Set b=c
2	1	1.5	1.25	0.1914	<0	0.25	Set b=c
3	1	1.25	1.125	-0.5232	>0	0.125	Set a=c
4	1.125	1.25	1.1875	-0.199	>0	0.0625	Set a=c
5	1.1875	1.25	1.2188	-0.0125	>0	0.0313	Set a=c
6	1.2188	1.25	1.2344	0.0872	<0	0.0156	Set b=c
7	1.2188	1.2344	1.2266	0.0368	<0	0.0078	Set b=c
8	1.2188	1.2266	1.2227	0.012	<0	0.0039	Set b=c
9	1.2188	1.2227	1.2207	-0.0003	>0	0.002	Set a=c
10	1.2207	1.2227	1.2217	0.0059	<0	0.001	Set b=c
11	1.2207	1.2217	1.2212	0.0028	<0	0.0005	Set b=c
12	1.2207	1.2212	1.2209	0.0013	<0	0.0002	Set b=c
13	1.2207	1.2209	1.2208	0.0005	<0	0.0001	Set b=c
14	1.2207	1.2208	1.2208	0.0001	<0	0.0001	Set b=c

Table 10: The Positive Root of $f(x) = x^4 - x - 1$

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	-1	0	-0.5	-0.4375	<0	0.5	Set b=c
2	-1	-0.5	-0.75	0.0664	>0	0.25	Set a=c
3	-0.75	-0.5	-0.625	-0.2224	<0	0.125	Set b=c
4	-0.75	-0.625	-0.6875	-0.0891	<0	0.0625	Set b=c
5	-0.75	-0.6875	-0.7188	-0.0144	<0	0.0313	Set b=c
6	-0.75	-0.7188	-0.7344	0.0252	>0	0.0156	Set a=c
7	-0.7344	-0.7188	-0.7266	0.0052	>0	0.0078	Set a=c
8	-0.7266	-0.7188	-0.7227	-0.0046	<0	0.0039	Set b=c
9	-0.7266	-0.7227	-0.7246	0.0003	>0	0.002	Set a=c
10	-0.7246	-0.7227	-0.7236	-0.0022	<0	0.001	Set b=c
11	-0.7246	-0.7236	-0.7241	-0.0009	<0	0.0005	Set b=c
12	-0.7246	-0.7241	-0.7244	-0.0003	<0	0.0002	Set b=c
13	-0.7246	-0.7244	-0.7245	0	<0	0.0001	Set b=c
14	-0.7246	-0.7245	-0.7245	0.0001	>0	0.0001	Set a=c

Table 11: The Negative Root of $f(x) = x^4 - x - 1$

9 $f(x) = e^x - x - 2$

The functions $f_1(x) = e^x$ and $f_2(x) = x + 2$ intersect at only one point. At $x = 1$ at which $f(1) = e - 3 < 0$ and at $x = 2$ at which $f(2) = e^2 > 0$. Hence the only real root of this equation lies between 1 and 2.

9.1 Plots

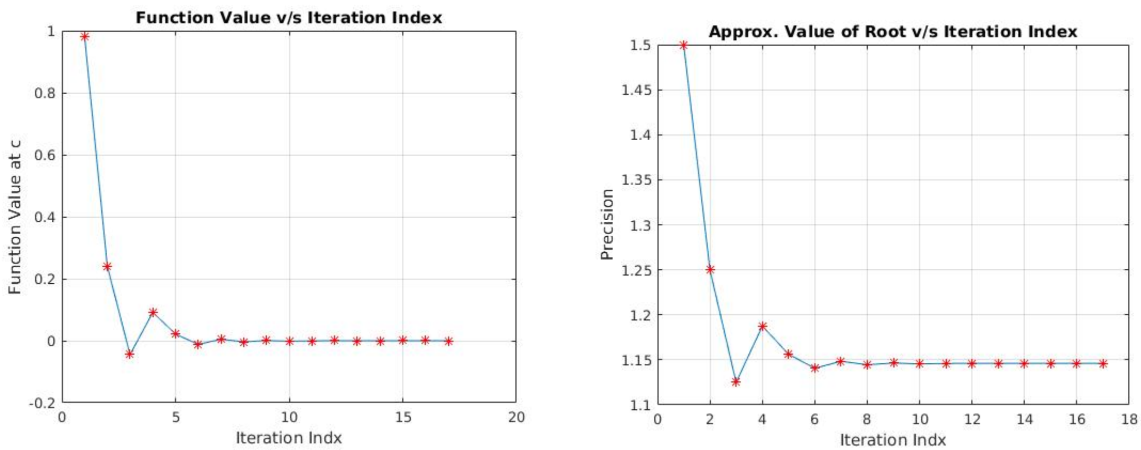


Figure 8: Fig 1.1,1.2 :Positive root of $f(x) = e^x - x - 2$

9.2 Tables

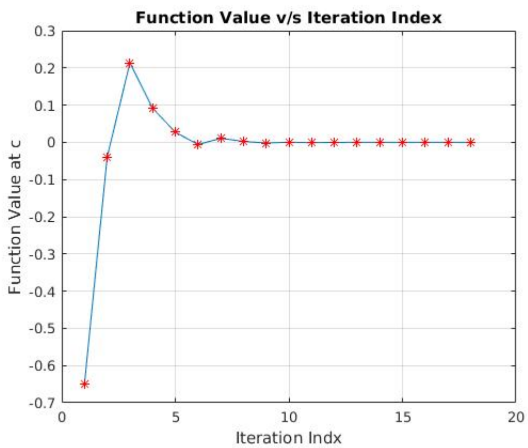
ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	1	2	1.5	0.9817	<0	0.5	Set b=c
2	1	1.5	1.25	0.2403	<0	0.25	Set b=c
3	1	1.25	1.125	-0.0448	>0	0.125	Set a=c
4	1.125	1.25	1.1875	0.0914	<0	0.0625	Set b=c
5	1.125	1.1875	1.1563	0.0217	<0	0.0313	Set b=c
6	1.125	1.1563	1.1406	-0.0119	>0	0.0156	Set a=c
7	1.1406	1.1563	1.1484	0.0048	<0	0.0078	Set b=c
8	1.1406	1.1484	1.1445	-0.0036	>0	0.0039	Set a=c
9	1.1445	1.1484	1.1465	0.0006	<0	0.002	Set b=c
10	1.1445	1.1465	1.1455	-0.0015	>0	0.001	Set a=c
11	1.1455	1.1465	1.146	-0.0004	>0	0.0005	Set a=c
12	1.146	1.1465	1.1462	0.0001	<0	0.0002	Set b=c
13	1.146	1.1462	1.1461	-0.0002	>0	0.0001	Set a=c
14	1.1461	1.1462	1.1462	0	>0	0.0001	Set a=c

Table 12: The Positive Root of $f(x) = e^x - x - 2$

10 $f(x) = 1 - x + \sin x$

Graphically it is easy to see that $f_1(x) = x - 1$ and $f_2(x) = \sin x$ would intersect in only one point. At $x = \frac{\pi}{2}$, $f(x) = 1 - \frac{\pi}{2} < 0$ and at $x = pi$, $f(x) = 2 - \pi > 0$. Hence the only root lies between $\frac{\pi}{2}$ and π.

10.1 Plots



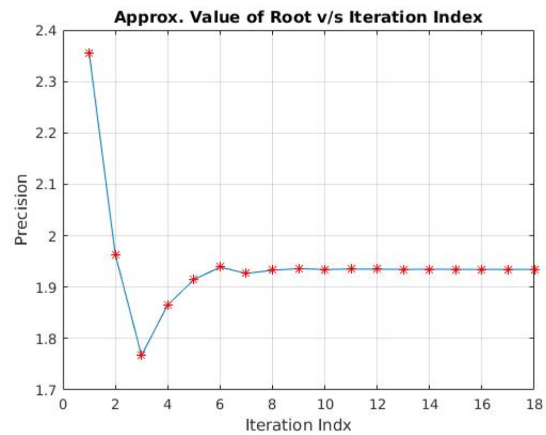


Figure 9: Fig 1.1,1.2 :Smallest Positive root of $f(x) = 1 - x + \sin x$

10.2 Tables

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	1.5708	3.1416	2.3562	-0.6491	<0	0.7854	Set b=c
2	1.5708	2.3562	1.9635	-0.0396	<0	0.3927	Set b=c
3	1.5708	1.9635	1.7671	0.2136	>0	0.1963	Set a=c
4	1.7671	1.9635	1.8653	0.0916	>0	0.0982	Set a=c
5	1.8653	1.9635	1.9144	0.0271	>0	0.0491	Set a=c
6	1.9144	1.9635	1.939	-0.006	<0	0.0245	Set b=c
7	1.9144	1.939	1.9267	0.0107	>0	0.0123	Set a=c
8	1.9267	1.939	1.9328	0.0024	>0	0.0061	Set a=c
9	1.9328	1.939	1.9359	-0.0018	<0	0.0031	Set b=c
10	1.9328	1.9359	1.9343	0.0003	>0	0.0015	Set a=c
11	1.9343	1.9359	1.9351	-0.0008	<0	0.0008	Set b=c
12	1.9343	1.9351	1.9347	-0.0002	<0	0.0004	Set b=c
13	1.9343	1.9347	1.9345	0	>0	0.0002	Set a=c
14	1.9345	1.9347	1.9346	-0.0001	<0	0.0001	Set b=c

Table 13: The smallest Positive Root of $f(x) = 1 - x + \sin x$

11 $f(x) = -x + \tan x$

The functions $f_1(x) = x$ and $f_2(x) = \tan x$ cut many times however only once in each period of $f_2(x) = \tan x$ on account of $\tan x$ being monotonic in each period. x cuts $\tan x$ in period of $\frac{\pi}{2} < x < \frac{3\pi}{2}$. At $x = 0$, $\tan x < x$ and at $x = \frac{3\pi}{2}$ $\tan x > x$ so the root lies between these 2 values.
However, $\tan x$ is undefined at $x = \frac{3\pi}{2}$. So we take a value $x = \frac{3\pi}{2} - \epsilon$ where ideally $\epsilon \to 0$ but computationally can be taken as $\epsilon = 0.00001$ since we want our answer accurate upto the 4th decimal place only.
As discussed earlier, the roots lie in intervals of $k\pi$ to $\frac{3}{2}k\pi$. So first we find the interval of $x = 100$. We get $k = 31$ since $k = \lfloor \frac{100}{\pi} \rfloor$.

11.1 Plots

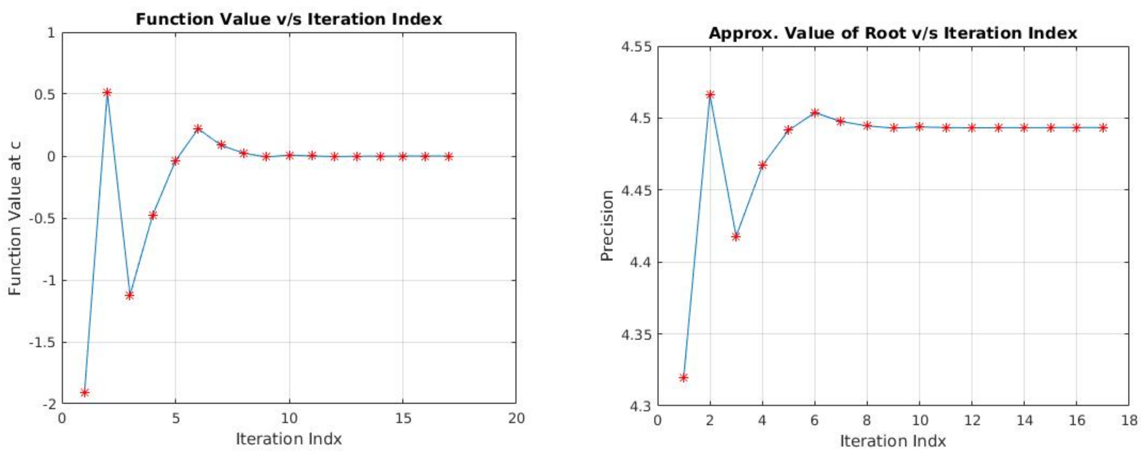


Figure 10: Fig Smallest Positive root of $f(x) = -x + \tan x$

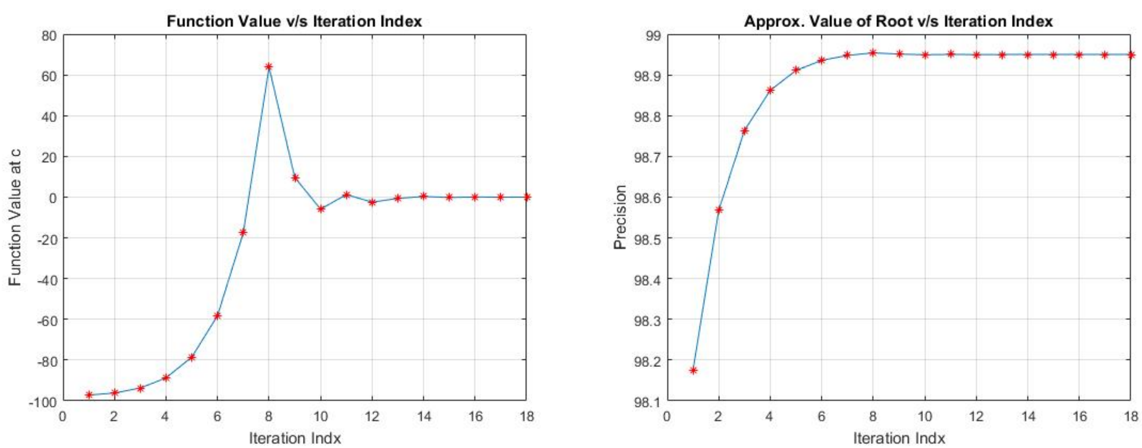


Figure 11: Fig Root of $f(x) = -x + \tan x$ near $x = 100$

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	3.1416	4.7124	3.927	-2.927	>0	0.7854	Set a=c
2	3.927	4.7124	4.3197	-1.9055	>0	0.3927	Set a=c
3	4.3197	4.7124	4.516	0.5113	<0	0.1963	Set b=c
4	4.3197	4.516	4.4179	-1.1213	>0	0.0982	Set a=c
5	4.4179	4.516	4.467	-0.4747	>0	0.0491	Set a=c
6	4.467	4.516	4.4915	-0.0383	>0	0.0245	Set a=c
7	4.4915	4.516	4.5038	0.2199	<0	0.0123	Set b=c
8	4.4915	4.5038	4.4976	0.087	<0	0.0061	Set b=c
9	4.4915	4.4976	4.4946	0.0234	<0	0.0031	Set b=c
10	4.4915	4.4946	4.493	-0.0077	>0	0.0015	Set a=c
11	4.493	4.4946	4.4938	0.0078	<0	0.0008	Set b=c
12	4.493	4.4938	4.4934	0.0001	<0	0.0004	Set b=c
13	4.493	4.4934	4.4932	-0.0038	>0	0.0002	Set a=c
14	4.4932	4.4934	4.4933	-0.0019	>0	0.0001	Set a=c

Table 14: The smallest Positive Root of $f(x) = -x + \tan x$

ItrNo	a	b	c	f(c)	f(a)*f(c)	a-c=b-c	Assign
1	97.389	98.96	98.175	-97.175	>0	0.7854	Set a=c
2	98.175	98.96	98.567	-96.153	>0	0.3927	Set a=c
3	98.567	98.96	98.764	-93.737	>0	0.1963	Set a=c
4	98.764	98.96	98.862	-88.71	>0	0.0982	Set a=c
5	98.862	98.96	98.911	-78.56	>0	0.0491	Set a=c
6	98.911	98.96	98.936	-58.217	>0	0.0245	Set a=c
7	98.936	98.96	98.948	-17.531	>0	0.0123	Set a=c
8	98.948	98.96	98.954	63.754	<0	0.0061	Set b=c
9	98.948	98.954	98.951	9.5785	<0	0.0031	Set b=c
10	98.948	98.951	98.949	-5.9107	>0	0.0015	Set a=c
11	98.949	98.951	98.95	1.2387	<0	0.0008	Set b=c
12	98.949	98.95	98.95	-2.4683	>0	0.0004	Set a=c
13	98.95	98.95	98.95	-0.6497	>0	0.0002	Set a=c
14	98.95	98.95	98.95	0.2855	<0	0.0001	Set b=c
15	98.95	98.95	98.95	-0.1843	>0	0	Set a=c

Table 15: The Positive Root of $f(x) = -x + \tan x$ around 100