CS 374 : Computational and Numerical Methods

Assignment 1 - Set 2

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Contents

1 Approximation of functions using Taylor Polynomials

Consider the following functions:

- $y = e^x$
- $y = \ln x$
- $y = \sin x$
- $y = \cos x$

Produce the first, the second and the third-degree Taylor polynomials for each of the foregoing functions, using a=1 as the point of approximation for $\ln x$ and a=0 for the rest.In a suitably chosen neighborhood of a, follow how the accuracy of a Taylor polynomial improves with its increasing degree. For this you will have to estimate the difference between f(x) and its Taylor polynomials in a code. Present your result for each function along with its Taylor polynomials of all 3 degrees.

1.1 Approximation for e^x

1.1.1 Plot

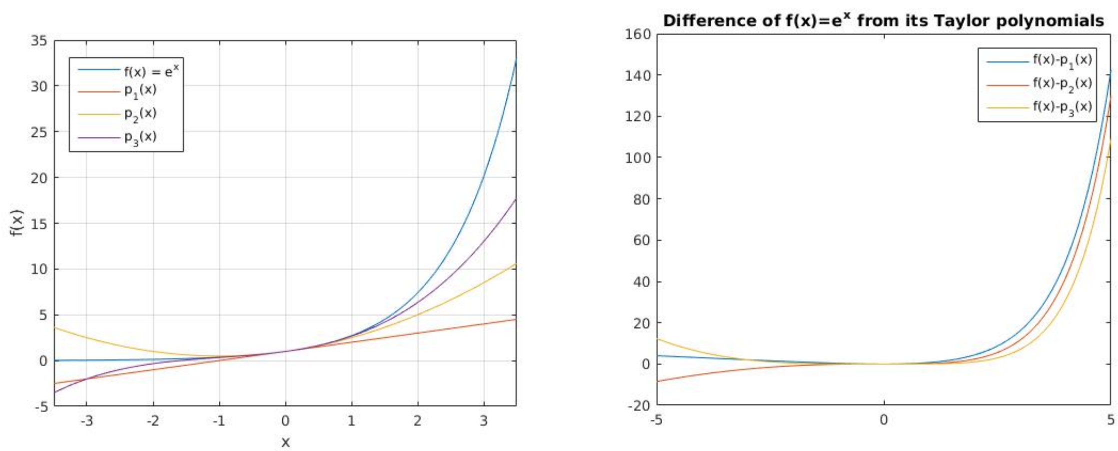


Figure 1: Behaviour of Different Order polynomial

1.1.2 Observations

- Using Taylor Series which is given by

$$f(x) = f(x_0) + \frac{f'(x_0)(x-x_0)}{1!} + \frac{f''(x_0)(x-x_0)^2}{2!} + \dots \tag{1}$$

  We approximate the function till the third order of the series around $a = 0$.

- As we observed that the even order polynomial rises up in the positive direction as $x \to -\infty$ and exactly this kind of graph we got in the figure 1. **As $p_2(x)$ in the graph lies above the actual graph**.

- In the case of odd order polynomial equation which goes to $-\infty$ as $x \to -\infty$ and that is why we got $p_1(x)$ and $p_3(x)$ which goes to $-\infty$ shown in the figure.**These polynomial lies under the actual function curve.**

- **The positive side of the graph, sign will not affect the Taylor polynomial and all the Taylor polynomials lies below the actual function as we have approximated the function.**

1.2 Approximation for $\ln x$

1.2.1 Plot

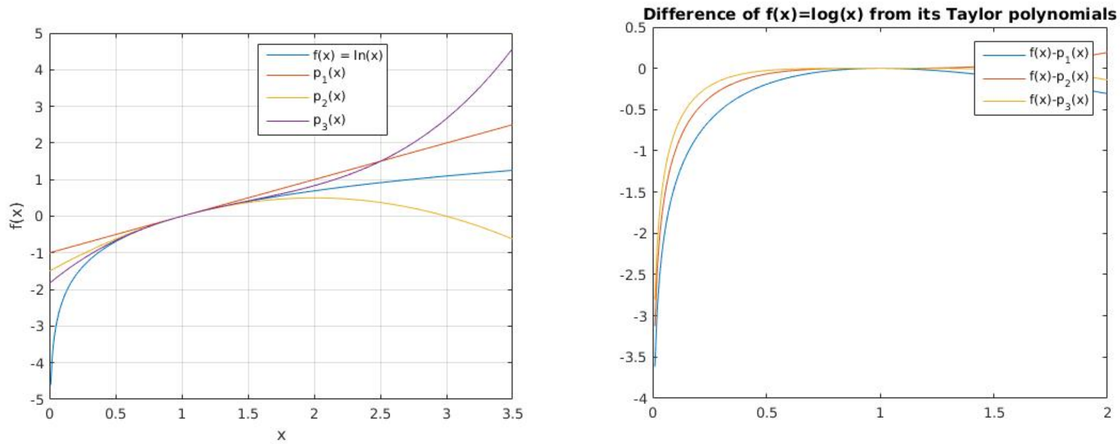


Figure 2: Behaviour of Different Order polynomial

1.2.2 Observations

- Using Taylor Series which is given by equation 1, we approximate the function till the third order of the series around $a = 1$ as it is given in the question.
- As we observed that the value of even order polynomial decreases or goes to $\rightarrow -\infty$ as the value of x increases in the positive direction.Thus for smaller value of x even order polynomial behaves like $\ln x$. But for large value of x, the higher even order with negative sign brigs the function down in the negative direction. **As $p_2(x)$ in the graph lies below the actual graph**.
- In the case of odd order polynomial equation which goes to ∞ as $x \rightarrow \infty$ and that is why we got $p_1(x)$ and $p_3(x)$ which goes to ∞ shown in the figure.**These polynomial lies above the actual function curve.**
- **In between [0-1], value of x will not affect the Taylor polynomial and all the Taylor polynomials lies above the actual function as we have approximated the function.**
- As we consider more and more degree in the Taylor Polynomial Series, we get closer and closer graph compared to the actual function.

1.3 Approximation for $\sin x$

1.3.1 Plot

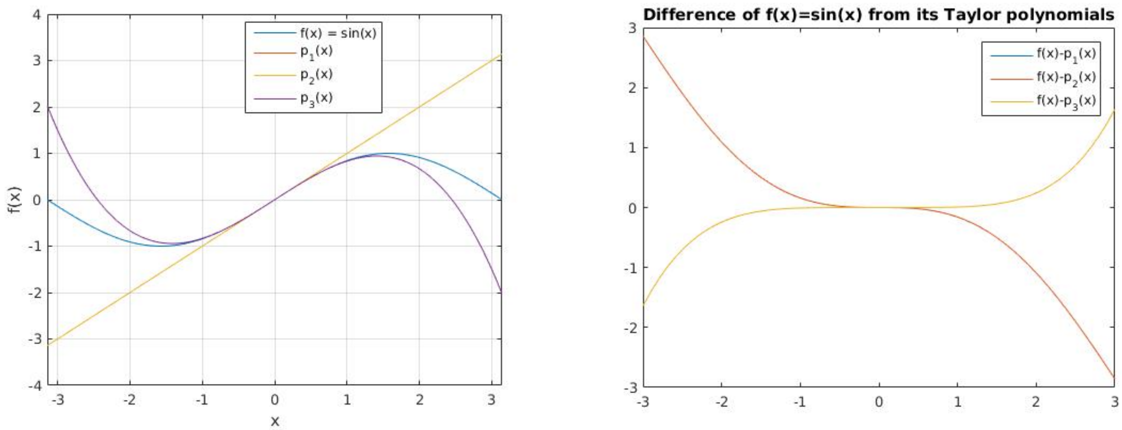


Figure 3: Behaviour of Different Order polynomial

1.3.2 Observations

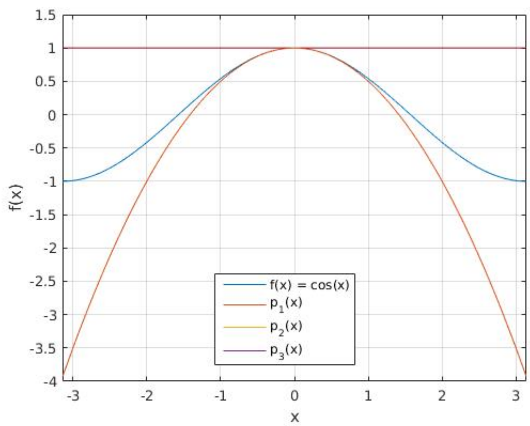
- Using Taylor Series which is given by equation 1, we approximate the function till the third order of the series around $a = 0$ as it is given in the question.
- For any value of x, the first order Taylor polynomial which is nothing but a straight line does not have any turning point and thus form small value of x it shows the correct output but shows incorrect output for large value of x.
- For small value of x eg.[0-1], the third order Taylor polynomial will behave like straight line as we neglect the x^3 part of the polynomial.But as the value for the x increases, the higher order term with the minus sign brings the function down toward $-\infty$. In the negative direction, since higher order of the function is three with minus sign takes the function up in the $+\infty$.

  Since the first derivative of the $p_3(x)$ is $p_3'(x) = 1 - \frac{x^2}{2}$ which has two turning points at $x = \pm\sqrt{2}$.Also the second derivative of $p_3(x)$ is $p_3''(x) = -x$. Thus function has minima at $x = \sqrt{2}$ and maxima at $x = -\sqrt{2}$. And thus more and more degree of the Taylor polynomial takes us more closer to the actual function.

  Since $p_3(x)$ has root at $x = \sqrt{6}$, the decreasing rate of the function is more than the actual function.

1.4 Approximation for $\cos x$

1.4.1 Plot



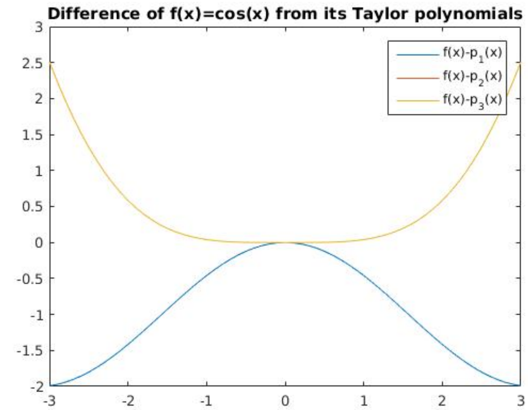


Figure 4: Behaviour of Different Order polynomial

1.4.2 Observations

- Using Taylor Series which is given by equation 1, we approximate the function till the third order of the series around $a = 0$ as it is given in the question.
- For any value of x, the first order Taylor polynomial which is nothing but a constant line does not have any turning point and thus form small value of x it shows the correct output but shows incorrect output for large value of x.
- For small value of x eg.$[-\frac{\pi}{2}$-$\frac{\pi}{2}]$, the second order Taylor polynomial will behave like cosine function as we neglect the x^2 part of the polynomial.But as the value for the x increases, the higher order term with the minus sign brings the function down toward $-\infty$. In the negative direction, since higher order of the function is two with minus sign will not affect the function and takes down in the $-\infty$.

  Since the first derivative of the $p_2(x)$ is $p_2'(x) = -x$ which has only one turning point at $x = 0$.Also the second derivative of $p_2(x)$ is $p_2''(x) = -1$. Thus function has maxima at $x = 0$. And thus for some values eg. $[-\frac{\pi}{2}$-$\frac{\pi}{2}]$ function behaves like cosine function but the function goes to $-\infty$ for increasing values of x in positive as well as negative direction.

  Since $p_3(x)$ has root at $x = \pm\sqrt{2} \approx \pm 1.41$, the decreasing rate of the function is more than the actual function.