CS 374 : Computational and Numerical Methods

Set 8

Numerical Differentiation and Integration

Purvil Mehta (201701073)
Bhargey Mehta (201701074)

Dhirubhai Ambani Institute of Information and Communication Technology
Gandhinagar

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1 Spline Interpolation

Carry out a cubic spline interpolation of the data provided below. Present your result by plotting the spline functions.

x	0.0	1.0	2.0	3.0	4.0	5.0	6.0
y	2.0000	2.1592	3.1697	5.4332	9.1411	14.406	21.303

1.1 Plots

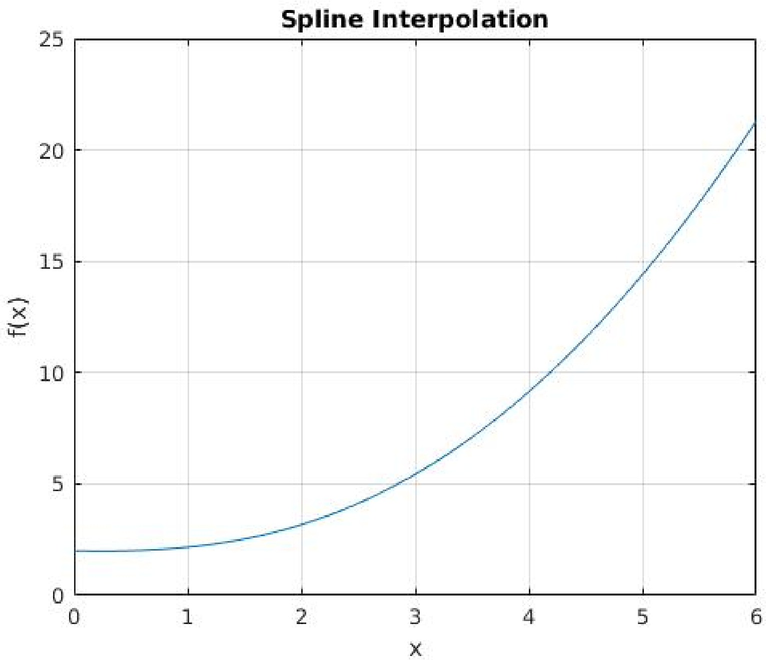

Figure 1: Spline Interpolation

2 Spline and Quadratic Interpolation

Carry out a quadratic spline interpolation of the data provided below. Present your result by plotting the spline functions.

x	-2	-1	0
y	-15	-8	-3

2.1 Plots

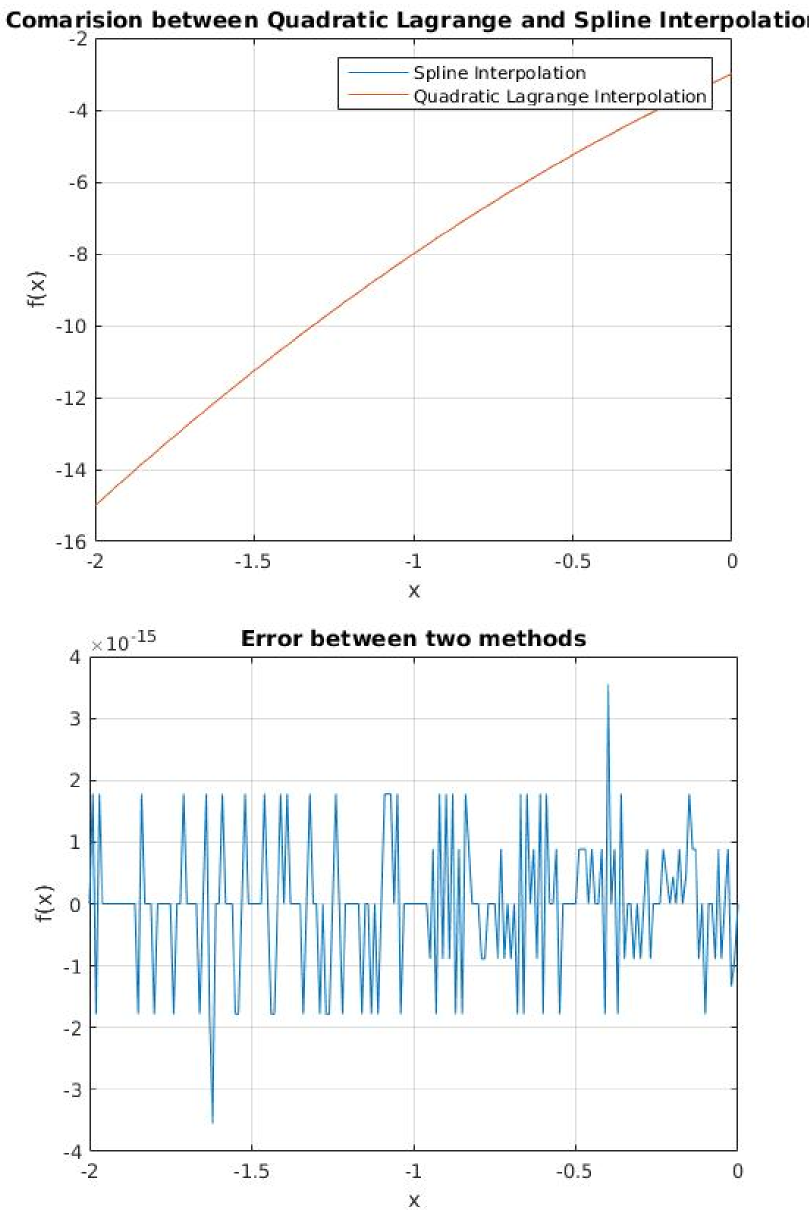

Figure 2: Comparison of Linear Piece-wise and Spline

3 Numerical Integration

3.1 $e^x \cos 4x$

Actual Value of the Integral

$$I = \int_0^\pi e^x \cos 4x dx = \frac{e^\pi - 1}{17}$$

n	T_n	$error(T_n)$	S_n	$error(S_n)$
2	26.516	25.214	22.715	21.413
4	3.2491	1.9467	-4.5067	-5.8091
8	1.6245	0.32213	1.083	-0.21938
16	1.3757	0.073329	1.2928	-0.009605
32	1.3203	0.017918	1.3018	-0.000552
64	1.3068	0.004454	1.3024	-3.4e-05
128	1.3035	0.001112	1.3024	-2e-06
256	1.3027	0.000278	1.3024	0
512	1.3025	6.9e-05	1.3024	0

3.2 $x^{\frac{5}{2}}$

Actual Value of the Integral

$$I = \int_0^1 x^{\frac{5}{2}} dx = \frac{2}{7}$$

n	T_n	$error(T_n)$	S_n	$error(S_n)$
2	0.33839	0.052674	0.28452	-0.001196
4	0.29879	0.013077	0.28559	-0.000122
8	0.28897	0.00326	0.2857	-1.2e-05
16	0.28653	0.000814	0.28571	-1e-06
32	0.28592	0.000203	0.28571	0
64	0.28577	5.1e-05	0.28571	0
128	0.28573	1.3e-05	0.28571	0
256	0.28572	3e-06	0.28571	0
512	0.28572	1e-06	0.28571	0

3.3 $\frac{1}{1+(x-\pi)^2}$

Actual Value of the Integral

$$I = \int_0^1 \frac{1}{1+(x-\pi)^2} dx = \arctan 5 - \pi + \arctan \pi$$

n	T_n	$error(T_n)$	S_n	$error(S_n)$
2	2.1667	-0.17311	2.6251	0.28533
4	2.2687	-0.071099	2.3027	-0.037094
8	2.3323	-0.007496	2.3535	0.013705
16	2.3378	-0.001953	2.3397	-0.000106
32	2.3393	-0.000489	2.3398	-1e-06
64	2.3396	-0.000122	2.3398	0
128	2.3397	-3.1e-05	2.3398	0
256	2.3398	-8e-06	2.3398	0
512	2.3398	-2e-06	2.3398	0

3.4 e^{-x^2}

n	T_n	S_n
2	2.5	1.6667
4	1.2548	0.83977
8	0.88943	0.76763
16	0.88623	0.88516
32	0.88623	0.88623
64	0.88623	0.88623
128	0.88623	0.88623
256	0.88623	0.88623
512	0.88623	0.88623

3.5 $\arctan 1 + x^2$

n	T_n	S_n
2	13.528	14.126
4	14.231	14.466
8	14.374	14.422
16	14.378	14.379
32	14.378	14.378
64	14.378	14.378
128	14.378	14.378
256	14.378	14.378
512	14.378	14.378

4 Numerical Differentiation

4.1 $\arctan x^2 - x + 1$

$$f'(x) = \frac{2x - 1}{1 + (x^2 - x + 1)^2}$$

at $x = 1$,

$$f'(1) = \frac{2 * 1 - 1}{1 + (1^2 - 1 + 1)} = \frac{1}{2}$$

h	D_h	$error(D_h)$	C_h	$error(C_h)$
0.1	0.52086	0.020855	0.49586	-0.0041443
0.05	0.51146	0.01146	0.49896	-0.0010403
0.025	0.50599	0.0059897	0.49974	-0.00026033
0.0125	0.50306	0.0030599	0.49993	-6.5099e-05
0.00625	0.50155	0.0015462	0.49998	-1.6276e-05

4.2 $\arctan 100x^2 - 199x + 100$

$$f'(x) = \frac{200x - 199}{1 + (100x^2 - 199x + 100)^2}$$

at $x = 1$,

$$f'(1) = \frac{200 * 1 - 199}{1 + (100 * 1^2 - 199 + 100)} = \frac{1}{2}$$

h	D_h	$error(D_h)$	C_h	$error(C_h)$
0.1	3.4098	2.9098	0.20029	-0.29971
0.05	2.5941	2.0941	0.39043	-0.10957
0.025	1.6757	1.1757	0.46978	-0.030224
0.0125	1.1093	0.60933	0.49226	-0.0077385
0.00625	0.80839	0.30839	0.49805	-0.0019461

4.3 Observation

- We can clearly see that the error of the derivative calculated by the central difference is substantially less than the derivative calculated by the forward difference method on account of the error being proportional to h^2 in case of central difference and being proportional to h in case of forward difference.