CS 374 : Computational and Numerical Methods

Set 6

The Bisection Method

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Contents

1 Linear Interpolation of $f(x) = \sqrt{x}$

Carry out the Lagrange linear interpolation between (1,1) and (4,2). Plot the linear interpolation function together with $f(x) = \sqrt{x}$.

1.1 Plots

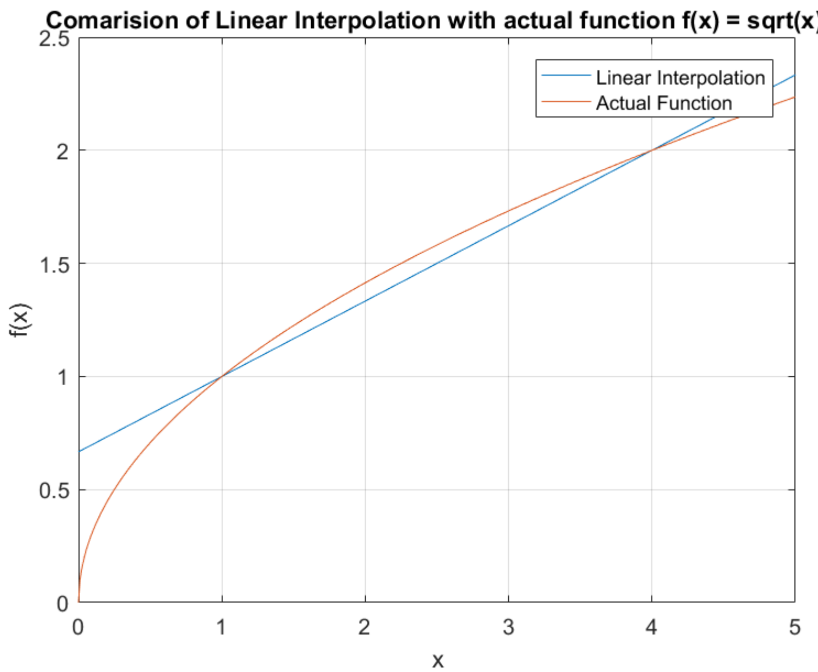

Figure 1: Linear Interpolation of $f(x) = \sqrt{x}$

1.2 Equations

The general formula for the first order Lagrange's Interpolation is defined as,

$$p_1(x) = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1$$

1.3 Observation

By looking at the graph above we say that , the more order in the Lagrange's Equation we use,the less and less error we get in the approximation.

2 Linear Interpolation of $f(x) = e^x$

Carry out a Lagrange linear interpolation for (0.82, 2.270500) and (0.83, 2.293319). Extend your study with a Lagrange quadratic polynomial using (0.84, 2.316367). Compare your polynomials with the function $y = e^x$, plotting all of them on the same graph.

2.1 Plots

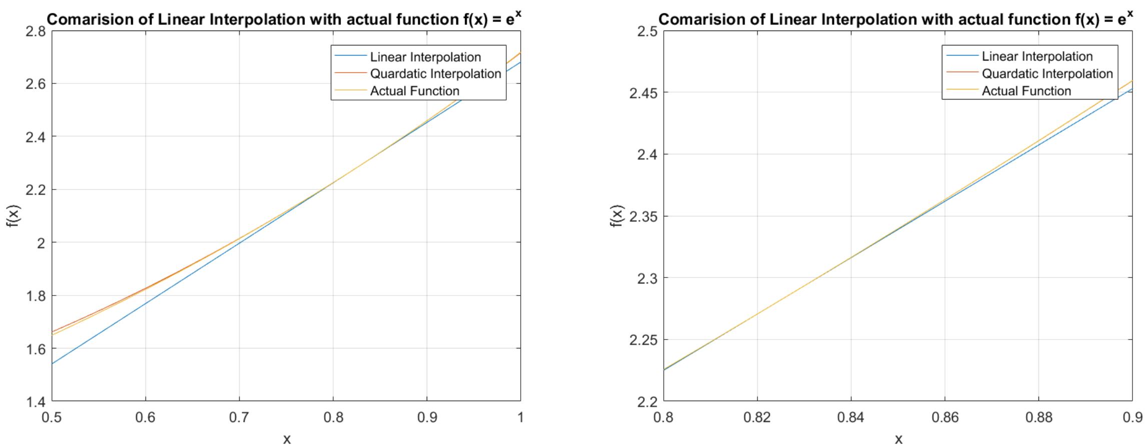

Figure 2: Linear Interpolation of $f(x) = e^x$

2.2 Equations

The general formula for the first order and second order Lagrange's Interpolation is defined as,

$$p_1(x) = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1$$

$$p_2(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_1)(x - x_0)}{(x_2 - x_1)(x_2 - x_0)} y_2$$

By putting the values of $x_0 = 0.82$,$x_1 = 0.83$,$x_2 = 0.84$,$y_0 = 2.270500$,$y_1 = 2.293319$ and $y_2 = 2.316367$ in the above equation we get approximation as ,

$$p_1(x) = -(x - 0.83)(0.22705) + (x - 0.83)(0.2293319)$$

$$p_2(x) = (x - 0.83)(x - 0.84)(0.022705) - (x - 0.82)(x - 0.84)(0.02293319) - (x - 0.83)(x - 0.82)(0.02316367)$$

2.3 Observation

- By looking at the graph above we say that , the more order in the Lagrange's Equation we use,the less and less error we get in the approximation.
- The quadratic and linear interpolation around the given data points are almost the same.

3 Interpolation using Lagrange's Method and Newton's Method

Construct a quadratic Lagrange polynomial using the points (0,-1), (1,-1) and (2, 7). Plot your result. Extend this entire exercise with Newton's divided-difference quadratic polynomial and compare the two methods.

3.1 Plots

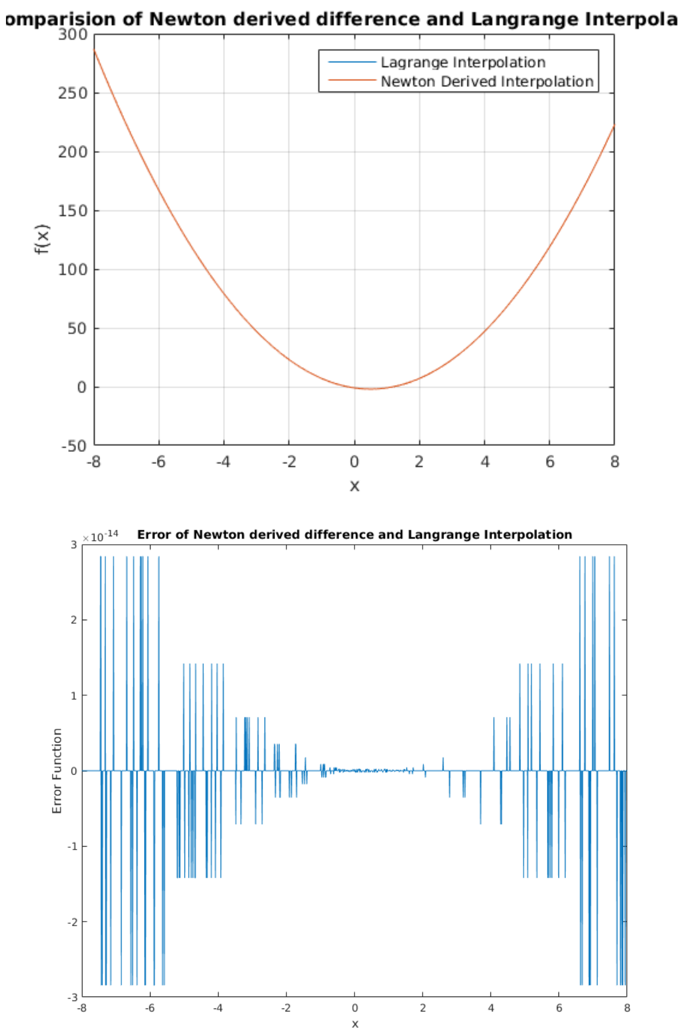

Figure 3: Quadratic Interpolation of Newton's Derived Difference and Lagrange's Method

3.2 Equations

The general formula for the first order and second order Lagrange's Interpolation is defined as,

$$p_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_1)(x-x_0)}{(x_2-x_1)(x_2-x_0)}y_2$$

$$n_1(x) = y_0 + (x-x_0)f[x_0,x_1]$$

$$n_2(x) = n_1(x) + (x-x_0)(x-x_1)f[x_0,x_1,x_2]$$

where,

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$
$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

3.3 Observation

- The error between two method is very less around 10^{-14}.
- By looking at the graph above we say that , the Lagrange's Interpolation and Newton's Derived Difference Interpolation tends to the unique function and hence we got similar looking graph in both cases.

4 Interpolation using Lagrange's Method and Newton's Method

For given data points,

- $f(3.35) = 0.298507$
- $f(3.40) = 0.294118$
- $f(3.50) = 0.285714$
- $f(3.60) = 0.277778$

(a) Produce Lagrange polynomials of the linear, quadratic and cubic orders with increasing values of x. (b) Produce Newton's divided-difference polynomials of all the three foregoing orders. (c) Plot the results of both methods on the same graph and compare them with the function $y = \frac{1}{x}$. Also comment on the respective computational advantages of the two methods above.

4.1 Plots

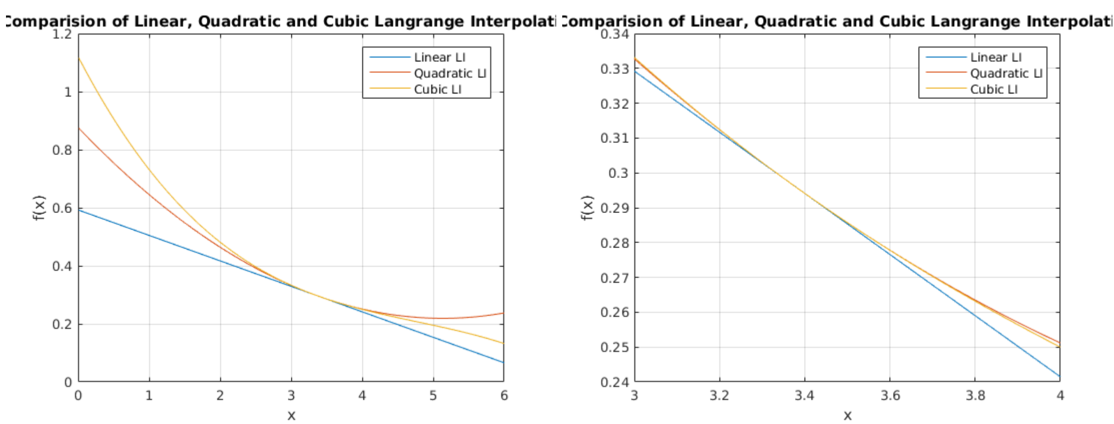

Figure 4: Interpolation using Lagrange's Method

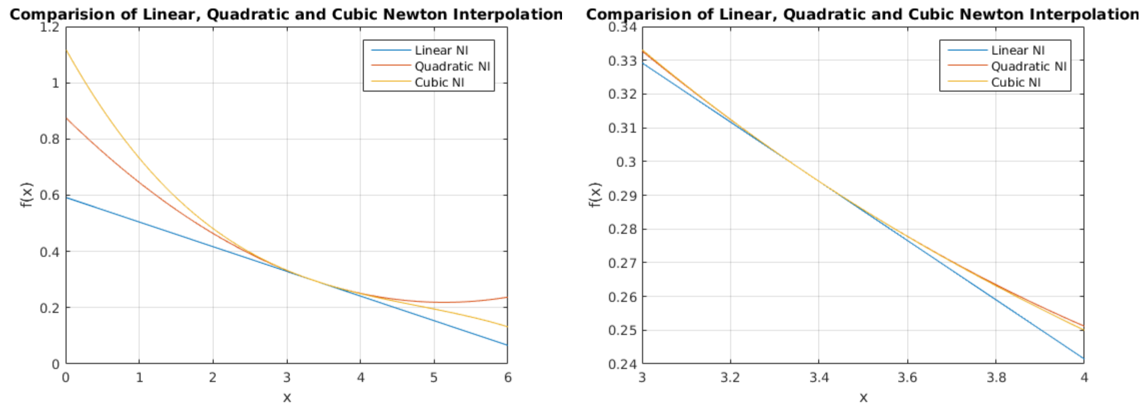

Figure 5: Interpolation using Newton's Derived Difference

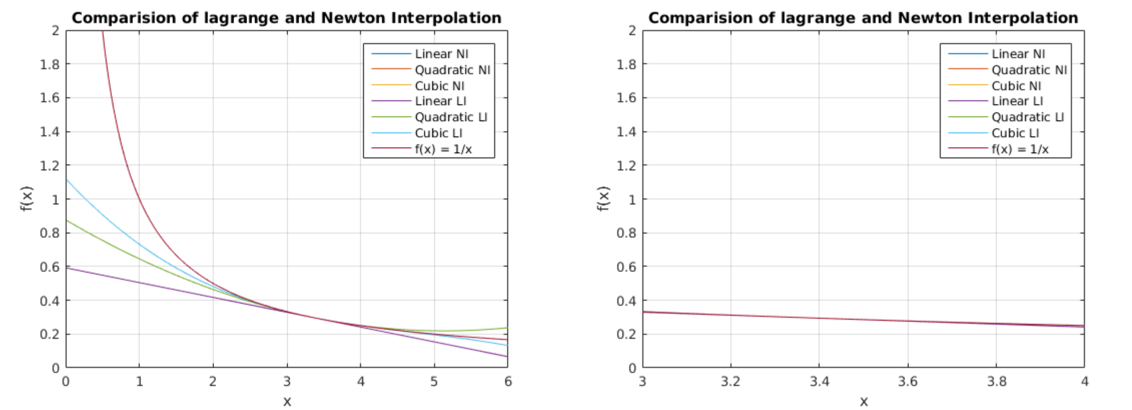

Figure 6: Comparison with Actual Function $f(x) = \exp x$

4.2 Equations

The general formula for the first order and second order Lagrange's Interpolation is defined as,

$$p_3(x) = \frac{(x-3.40)(x-3.50)(x-3.60)}{(-0.05)(-0.15)(-0.25)}(0.298507) + \frac{(x-3.35)(x-3.50)(x-3.60)}{(-0.05)(-0.10)(-0.20)}(0.294118) + \frac{(x-3.35)(x-3.40)(x-3.60)}{(-0.05)(-0.10)(-0.10)}(0.295778) + \frac{(x-3.35)(x-3.50)(x-3.40)}{(-0.25)(-0.20)(-0.10)}(0.298507)$$

$$n_1(x) = y_0 + (x-x_0)f[x_0,x_1]$$

$$n_2(x) = n_1(x) + (x-x_0)(x-x_1)f[x_0,x_1,x_2]$$

$$n_3(x) = n_2(x) + (x-x_0)(x-x_1)(x-x_2)f[x_0,x_1,x_2,x_3]$$

where,

$$f[x_0,x_1] = \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$$

$$f[x_0,x_1,x_2,x_3] = \frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0}$$

4.3 Observation

- By looking at the graph above we say that , the Lagrange's Interpolation and Newton's Derived Difference Interpolation tends to the unique function in each and every order and hence we got similar looking graph in both cases.
- Computationally , the Lagrange's method would be less expensive as compared as the Newton's method due to it's recursive nature.

5 Interpolation using Lagrange's Method and Newton's Method

For given data points,

- $f(0) = 2.5$
- $f(1) = 0.5$
- $f(2) = 0.5$
- $f(2.5) = 1.5$
- $f(3) = 1.5$
- $f(3.5) = 1.125$
- $f(4) = 0$

(a) Interpolate successive points by straight line segments. This is known as piece wise linear interpolation. (b) On each of the three following sub intervals of x [0, 2], [2, 3] and [3, 4] interpolate using both Lagrange's quadratic polynomial and Newton's divided-difference interpolation polynomial. (c) Plot the results of both methods covering all the three sub intervals on the same graph and compare them.

5.1 Plots

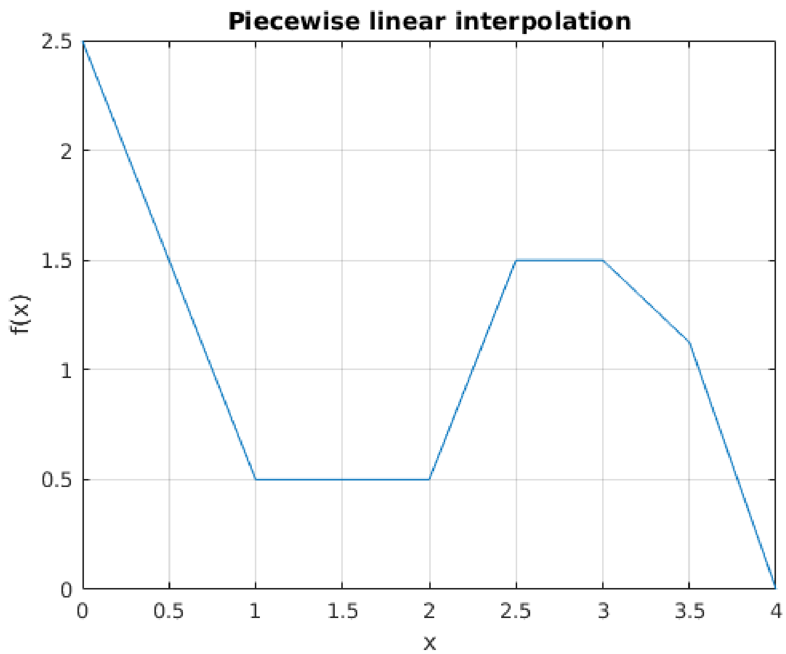

Figure 7: Piece wise Linear Interpolation

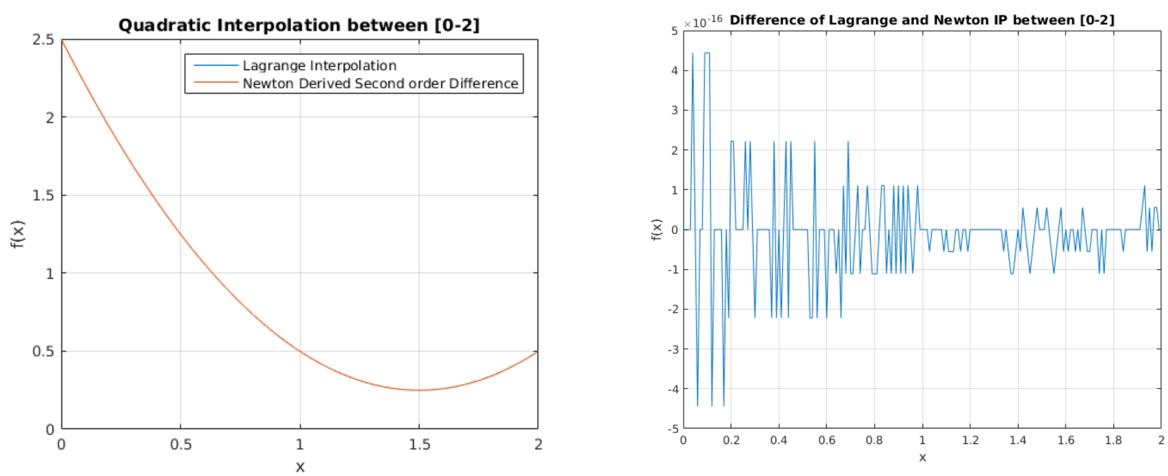

Figure 8: Interpolation between [0-2]

5.2 Observation

- By looking at the graph above we say that , the Lagrange's Interpolation and Newton's Derived Difference Interpolation tends to the unique function in each and every order and hence we got similar looking graph in both cases.

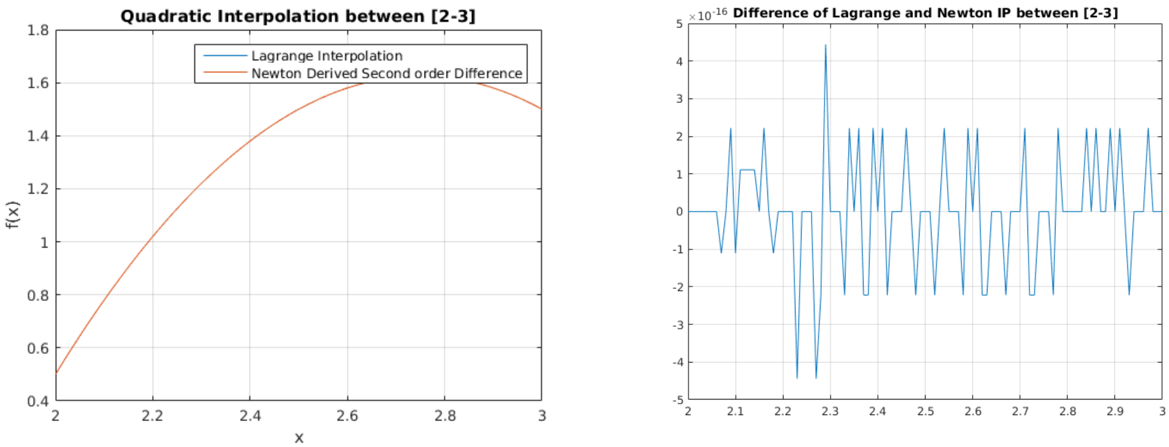

Figure 9: Interpolation between [2-3]

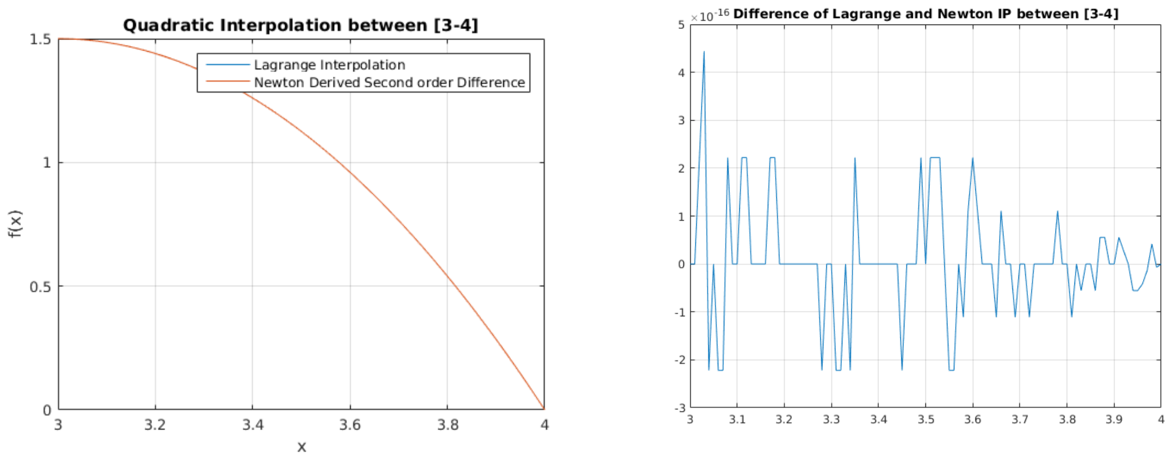

Figure 10: Interpolation between [3-4]

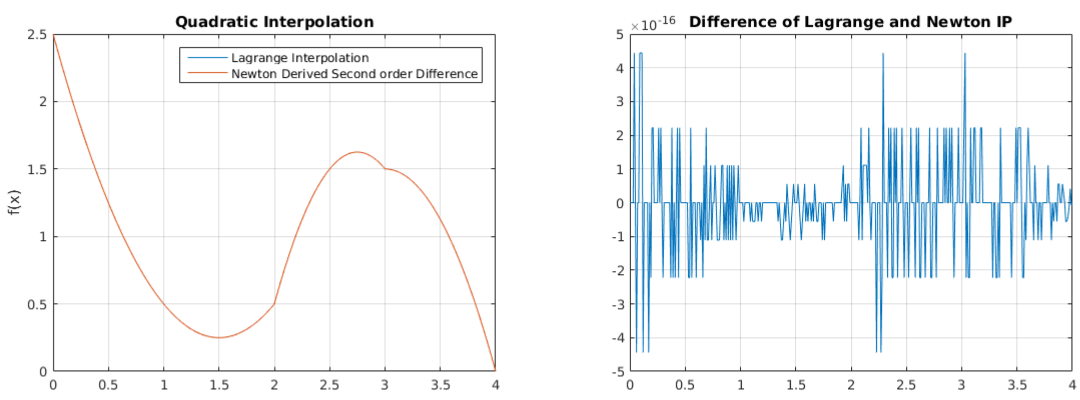

Figure 11: Interpolation between [0-3]