CS 374 : COMPUTATIONAL AND NUMERICAL METHODS

ASSIGNMENTS

COMPUTATIONAL AND NUMERICAL METHOD

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Contents

1 Assignment 1

1.1 The Entropy Function

We have the entropy function as,

$$\langle I \rangle = -k \sum p_i \log p_i$$

1.2 Plots

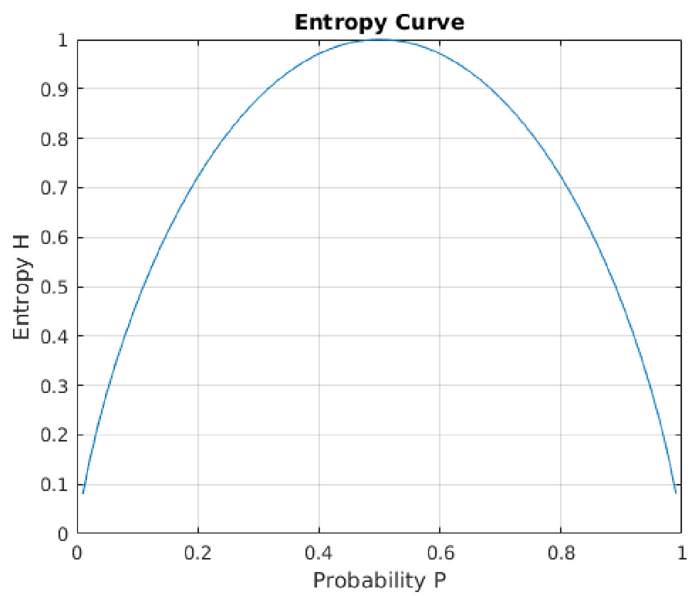

Figure 1: Entropy Function for k = 1

1.2.1 Observation

- We can see that the function peaks at $p = \frac{1}{2}$
- We can interpret this as maximum information available at $p = \frac{1}{2}$ so the function peaks at that point. At $p = 0$ or $p = 1$, we already know what event is going to take place so the information we obtain from that event is 0.

1.2.2 Equations

$$\langle I \rangle = -k[p \log_2 p + (1-p)\log_2(1-p)]$$

$$\frac{\mathrm{d}}{\mathrm{d}p}\langle I \rangle = -k[\log_2 p + 1 + -\log_2(1-p) - 1]/ln(2) = 0$$

So we have,

$$\begin{aligned}
-k[\log_2 p - \log_2(1-p)]/ln(2) &= 0\\
\log_2 p &= \log_2(1-p)\\
p &= 1-p\\
p &= \frac{1}{2}
\end{aligned}$$

1.3 Approximation

Apply a very small perturbation as $p = \frac{1}{2} + \epsilon$, in which $\epsilon << \frac{1}{2}$. Show that in this perturbation approach $\langle I \rangle \approx a - b\epsilon^2$, with $a = k$ and $b = \frac{4k}{\ln 2}$.
We will be using the approximation $\ln(1+x) \approx x$

We have

$$\begin{aligned}
\langle I \rangle &= -k[p\log_2 p + (1-p)\log_2(1-p)]\\
&= -k[(\frac{1}{2}+\epsilon)\log_2(\frac{1}{2}+\epsilon) + (\frac{1}{2}-\epsilon)\log_2(\frac{1}{2}-\epsilon)]\\
&= -k[\frac{1}{2}(\log_2(\frac{1}{2}+\epsilon) + \log_2(\frac{1}{2}-\epsilon)) + \epsilon(\log_2(\frac{1}{2}+\epsilon) - \log_2(\frac{1}{2}-\epsilon))]\\
&= -\frac{k}{\ln 2}[\frac{1}{2}(\ln(\frac{1}{2}+\epsilon) + \ln(\frac{1}{2}-\epsilon)) + \epsilon(\ln(\frac{1}{2}+\epsilon) - \ln(\frac{1}{2}-\epsilon))]\\
&= -\frac{k}{\ln 2}[\frac{1}{2}(\ln(\frac{1+2\epsilon}{2}) + \ln(\frac{1-2\epsilon}{2})) + \epsilon(\ln(\frac{1+2\epsilon}{2}) - \ln(\frac{1-2\epsilon}{2}))]\\
&= -\frac{k}{\ln 2}[\frac{1}{2}(\ln(1+2\epsilon) + \ln(1-2\epsilon) - 2\ln 2) + \epsilon(\ln(1+2\epsilon) - \ln(1-2\epsilon) + \ln 2 - \ln 2)]\\
&\approx -\frac{k}{\ln 2}[\frac{1}{2}(2\epsilon - 2\epsilon - 2\ln 2) + \epsilon(2\epsilon - (-2\epsilon))]\\
&\approx -\frac{k}{\ln 2}[\frac{1}{2}(-2\ln 2) + \epsilon(4\epsilon)]\\
&\approx -\frac{k}{\ln 2}[-\ln 2 + 4\epsilon^2]\\
&\approx k - (\frac{4k}{\ln 2})\epsilon^2
\end{aligned}$$

Thus we have $\langle I \rangle \approx k - (\frac{4k}{\ln 2})\epsilon^2$ with $a = k$ and $b = \frac{4k}{\ln 2}$ by comparison.

1.4 Plots

$$\langle I \rangle = -k[p \log_2 p + (1-p) \log_2 (1-p)]$$

$$\langle I \rangle \approx k - (\frac{4k}{\ln 2})\epsilon^2$$

$$\text{where } p = \tfrac{1}{2} + \epsilon$$

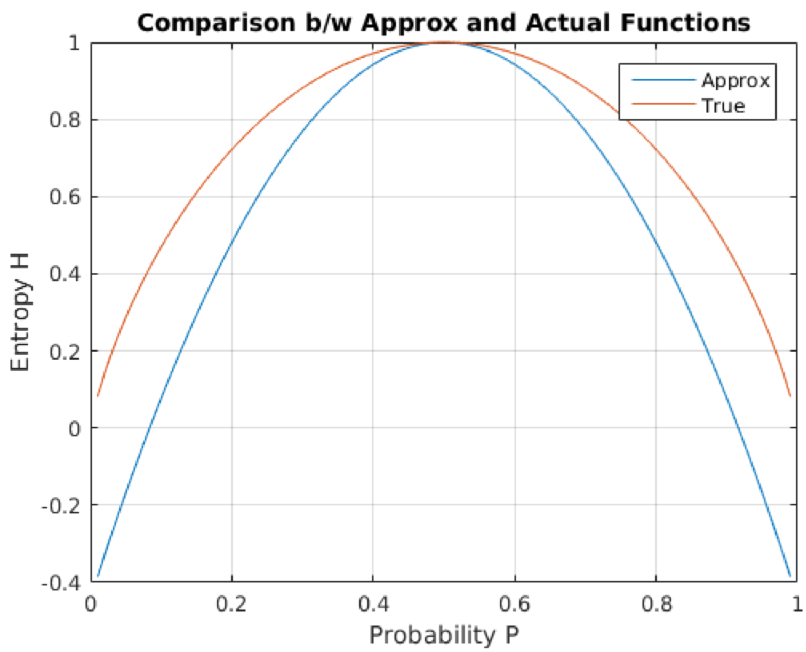

Figure 2: Actual Function V/s Approximation

2 Assignment 2

2.1 Plots

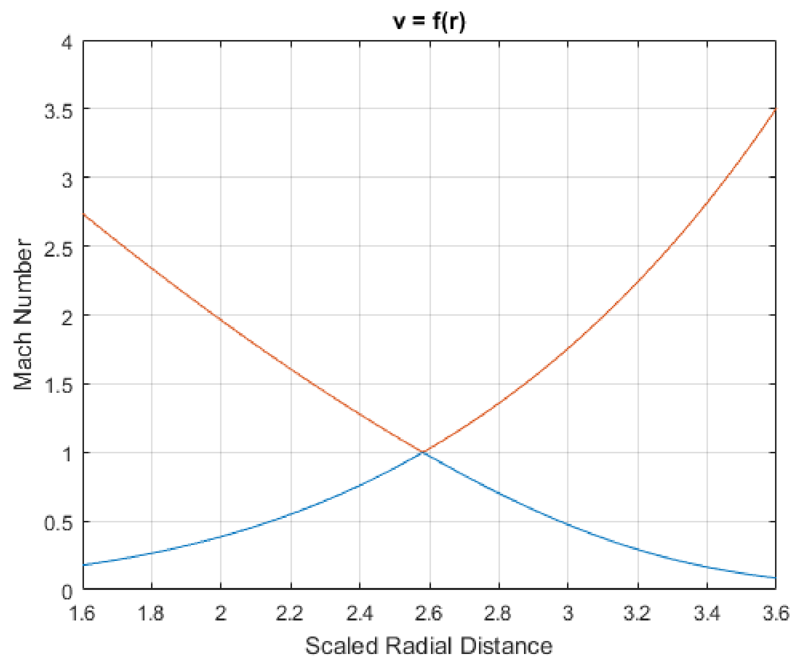

Figure 3: Two plots corresponding to $v = f(r)$

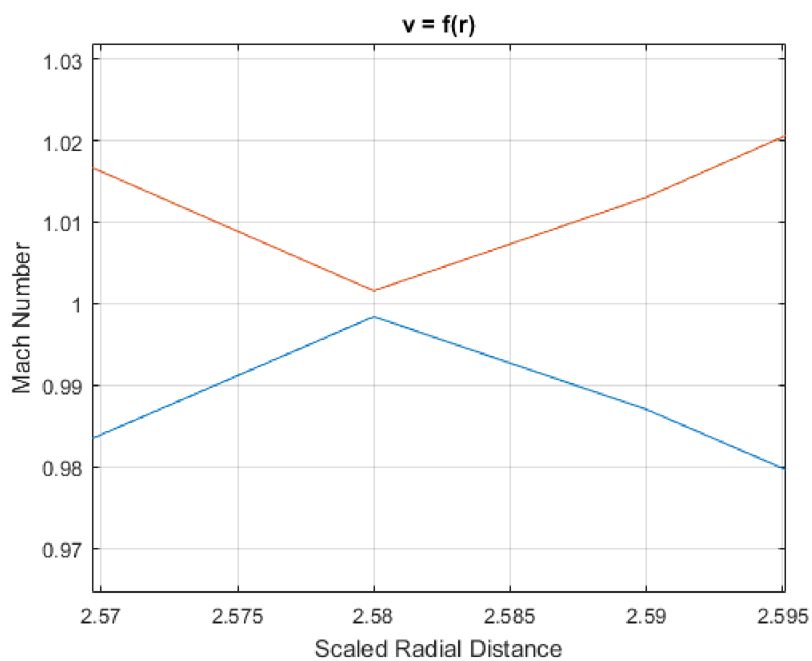

Figure 4: The Plots don't meet but they do come close

2.2 Code Run-time

Function Name	Calls	**Total Time**	Self Time*	Total Time Plot (dark band = self time)
assign2	1	15.163 s	4.325 s	
accretion	4050150	10.601 s	10.601 s	
xlabel	1	0.137 s	0.137 s	
bisection	402	0.121 s	0.041 s	

Figure 5: $\triangle V = 10$

Function Name	Calls	**Total Time**	Self Time*	Total Time Plot (dark band = self time)
assign2	1	139.903 s	39.618 s	
accretion	40227738	100.192 s	100.192 s	
bisection	402	0.105 s	0.037 s	

Figure 6: $\triangle V = 1$

Function Name	Calls	**Total Time**	Self Time*	Total Time Plot (dark band = self time)
assign2	1	1353.255 s	380.050 s	
accretion	402024532	973.114 s	973.114 s	
bisection	402	0.091 s	0.032 s	

Figure 7: $\triangle V = 0.1$

3 Assignment 3

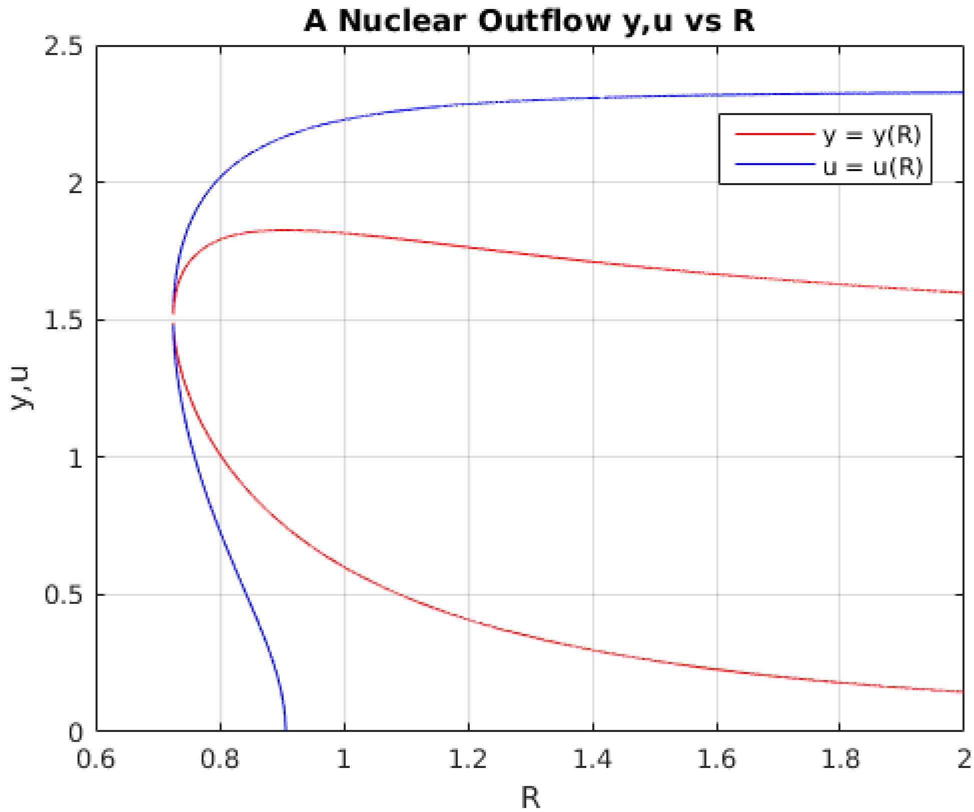

Figure 8: $y = y(R), u = u(R)$

3.1 Analytical Methods

$$xyR^2 = 1$$

$$y^4 - By^2 - \frac{4y}{R^2} + \frac{3}{R^4} = 0$$

$$\frac{dy}{dR} = \frac{2(3-2yR^2)}{R^3(2y^3R^2 - ByR^2 - 2)}$$

The turning point in the velocity profile occurs when the numerator in the RHS becomes zero. Hence we have $y = \frac{3}{2R^2}$, which gives us $x = \frac{2}{3}$.

Solution for the y will be

$$y = -\frac{-R^4 + 4R^2 + \sqrt{(R^4 - 4r^2)^2 + 12R^4B}}{2R^4B}$$

$$y = -\frac{-R^4 + 4R^2 - \sqrt{(R^4 - 4r^2)^2 + 12R^4B}}{2R^4B}$$

4 Assignment 4

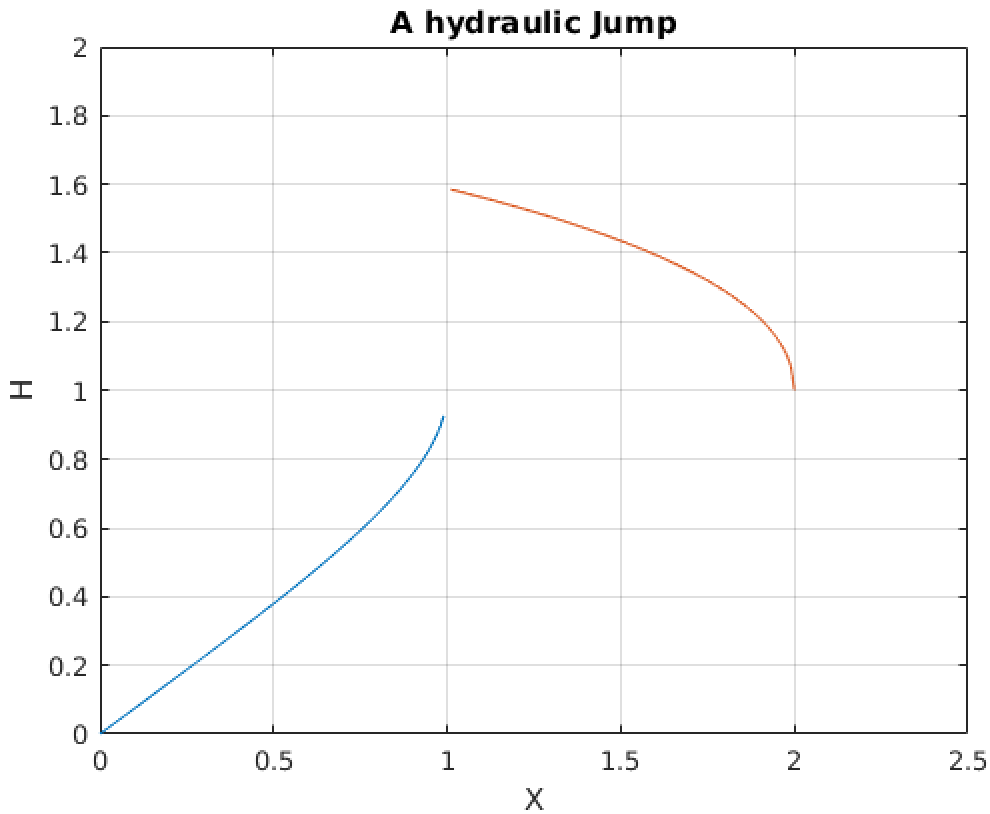

Figure 9: $H = H(X)$

4.1 Analytical Methods

$$4H - H^4 = 3(X - D)$$
$$\frac{dH}{dX} = \frac{3}{4(1-H^3)}$$

By looking at the above equation we can say that at $H = 1$ slope of the function become ∞. Thus at point $H = 1$,function will take hydraulic jump and will be discontinuous.

5 Assignment 5