CS 302 : MODELING AND SIMULATION

SARS MODEL

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1 Abstract

This report aims to study the spread of SARS in a population with quarantine measures employed. It tries to qualitatively analyse the cost of the control measures and their effectiveness in controlling the spread.

2 Introduction

This problem is motivated by the fact that certain control measures such as quarantine and vaccination can be employed to better control the spread of SARS. The control measures in practice translate to monetary resources. If we can understand the spread better, we can use those resources in an efficient manner and hence save more lives.

We will be studying the steady state behaviour of the 4 fractions, the fraction of susceptible who didn't catch SARS during the epidemic, the fraction of deaths that occur due to SARS, and the fraction of people who caught SARS and recovered from it and the fraction of people who where infected either inside or outside the quarantine. We will be studying the dependence of these fractions on the parameters that model the spread and also on the parameters that model the control measures such as quarantines.

3 Model

A total of 9 different compartments and 8 parameters will be used to capture the different categories that an individual from the population can belong to.

3.1 Qualitative behaviour

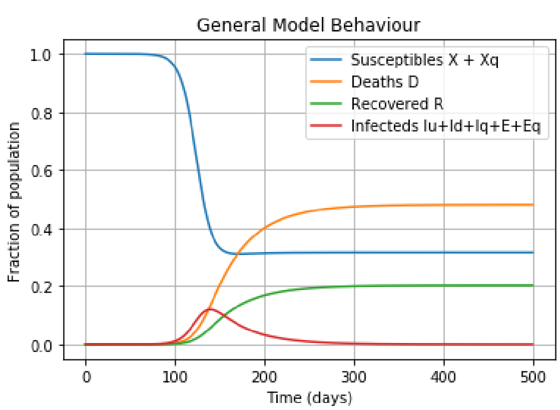

Figure 1: General qualitative behaviour

3.2 Mathematical Model

$$\frac{dX}{dt} = u\,X_q - qk(1-b)\frac{X}{N}I_u - kbI_u\frac{X}{N}$$
$$\frac{dX_q}{dt} = -u\,X_q + qk(1-b)\frac{X}{N}I_u$$
$$\frac{dE}{dt} = -p\,E - bk(1-q)\frac{X}{N}I_u$$
$$\frac{dE_q}{dt} = -p\,E - bkq\frac{X}{N}I_u$$
$$\frac{dI_u}{dt} = p\,E - (m+v+m)I_u$$
$$\frac{dI_q}{dt} = p\,E - (m+v+m)I_q$$
$$\frac{dD}{dt} = m(I_q + I_d + I_u)$$
$$\frac{dR}{dt} = v(I_q + I_d + I_u)$$
$$\frac{dI_d}{dt} = w\,(I_q + I_u) - mI_d - vI_d$$

Figure 2: Mathematical model of SARS

3.3 Compartments

- X - Susceptible (doesn't have SARS but can be infected by Infectious Undetected I_u)
- X_q - Susceptible Quarantined (doesn't have SARS and cannot be infected because they are in quarantine)
- E - Exposed (has SARS, can't infected others)
- E_q - Exposed Quarantined (have SARS and detected, thus move aside to stop infecting others)
- I_u - Infectious Undetected (have undetected SARS, can transmit)
- I_q - Infectious Quarantined (have SARS and detected, thus move aside to stop infecting others)
- I_d - Infectious Isolated (have SARS,can't infect others because isolated from the main population)
- D - Deaths due to SARS
- R - Recovered (recovered from SARS)

3.4 Parameters

Parameters used in the model and their values unless otherwise stated.

- b - Transmission Probability that a contact between I_u and X results in infection ($b = 0.06$)
- k - Number of contacts per day someone in I_u has in X ($k = 10$/day)
- q - Fraction of people in X who were exposed to SARS and go to E_q, rest $1-q$ go to E ($q = 0.2$/day)
- u - Fraction of people in X_q who are allowed to return to X ($u = 0.1$/day)
- p - Fraction of people who become infectious ($E \to I_u, E_q \to I_q$) ($k = 0.2$/day)
- m - Death rate ($m = 0.095$/day)
- v - Recovery rate ($k = 0.04$/day)
- w - Fraction of people who are transferred from $I_q, I_u \to I_d$ ($w = 0.0625$/day)

4 Results

4.1 Quarantine Fraction q

The quarantine fraction q determines the size of the population that can be quarantined. Ideally we want it to be 1, so that the first person to catch SARS recovers from it while not spreading it further. However this is a very idealistic scenario and is not feasible. The reproduction number $R_0 = \frac{kb(1-q)}{v+m+w} \approx 30.38(1-q)$. $R_0 < 1 \implies q > 0.67$. Hence we get the threshold value of $q \approx 0.7$.

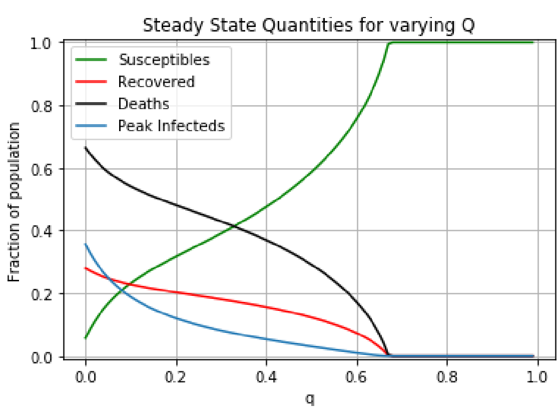

Figure 3: Dependence on quarantine fraction

There is no effect of quarantining more than 70% of the population since any more would not change the long term behaviours of the system but it would mean a 30% decrease in the monetary resources. One other important deduction is that, there is an almost linear change in the fractions for $q \in (0, 0.5)$ but after that, any more increase translates to more than linear decrease in deaths. We conclude that if more than 50% quarantine is affordable, efforts should be made to make it to 70%.

4.2 Number of contacts/day k

This is one of the parameters that decides how fast SARS is likely to spread since every contact is a potential new infection. If an infected person doesn't come in contact with anyone (case $k = 0$) then there are no chances on a new infection and hence the epidemic dies with the recovery of the already infected population. The reproduction number $R_0 = \frac{kb(1-q)}{v+m+w} \approx 0.24k$. $R_0 < 1 \implies k < 4.11$ and so we get the threshold value of $k \approx 0.4$.

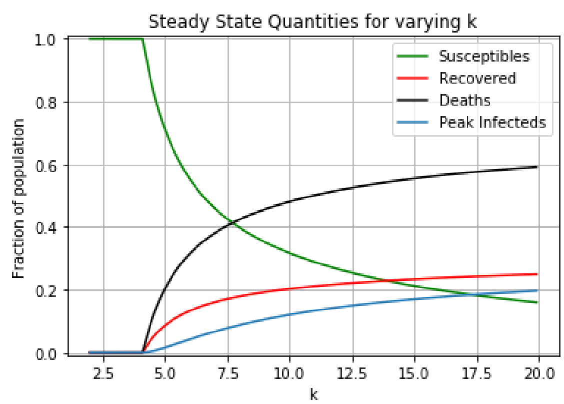

Figure 4: Dependence on number of contacts

We can see that the spread is close to 0 when the number of average contacts each person has is below 4. There is a sharp increase for $k \in (4, 6)$. That means even a slight increase in contacts triggers the spread of SARS. Then there is a portion of linear increase in deaths with k in the region $k \in (6, 8)$. After that the plot begins to approach its saturation value. The results suggest that the efforts to decrease contact amongst the population is worthless if it cannot be brought down to less than a threshold since the end results would not change much.

4.3 Delay T_q and size τ of quarantine

It is very unpractical to assume that, we can begin using control measures as soon as there is even one positive case. Intuitively it seems better to start quarantining people as soon as possible in order to contain the infections but the model suggests that there there is an optimum window in which the control measures should be practised.

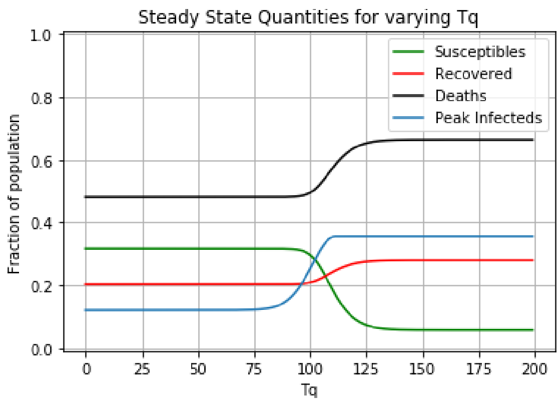

Figure 5: Dependence on quarantine delay

From the results, it is clear that the optimum window is somewhere between day 80 and the day 110. If quarantining is begun before day 80 then there is no difference between the number of deaths and infecteds that occur as opposed to when it was started at day 80 itself. This means that we can save 80 days worth of monetary resources and still have no significant change in the end results.

However the window is very sensitive. Any more delay than 80 days would result in very high spread of the infection. Any effort to quarantine after day 110 is practically useless since the number of the infecteds and the deaths would be the same as if hadn't employed it in the first place.

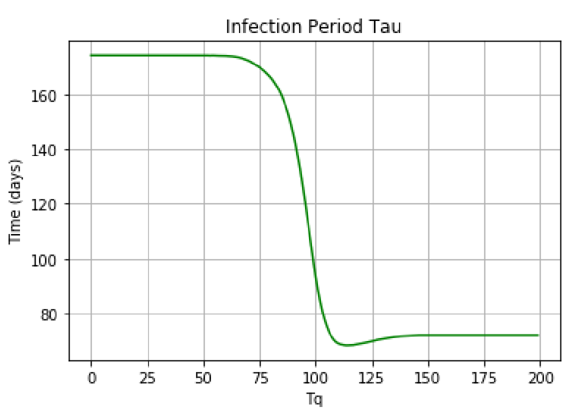

Figure 6: Quarantine delay and infection period

We also note the effect of delay on the infection period τ which is defined as the period when the total infected population is at least 5% of the peak infected population. As per figure 5, a delay of 75 days results in a larger infection period than the same for the case of delay of 100 days. The motivation is that if we are constrained by the number of days we can use the control measures, we want to employ it at the most optimum moment. We also want to see how sensitive the fractions are to the number of days we quarantine, of course assuming that they are employed at the optimum delay.

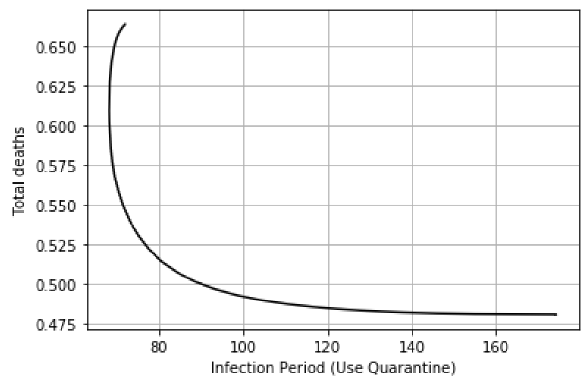

Figure 7: Death fraction and infection period

We find that there is a threshold window size of around 80 days. Any decrease in the quarantine window of size 80 days results in a more than linear increase in the number of deaths but the same increase in the window size results in a less than linear decrease in the same. So efforts should be made to make the window size at least of length 80 days.

4.4 Transmission probability b

As the probability of an encounter resulting in a SARS infection for a susceptible increases, more and more susceptibles will get infected by infectious undetected people. Initially when the transmission probability is very low the lesser number of people will be infected and the total spread will die without an epidemic. In addition, the death rate, infected isolation rate and recovery rate are much more higher than the infection rate (which is contributed to by the transmission probability). Thus there will be very less fraction of people who got infected via SARS and so the deaths and recoverable people are.

We observe a threshold value of the transmission probability at around $b = 0.025$ crossing which SARS becomes very deadly. At $b = 0.025$, we observe insignificant deaths whereas at $b = 0.05$, we observe that 40% of the population would die. The death fraction saturates to around 70% of the population for

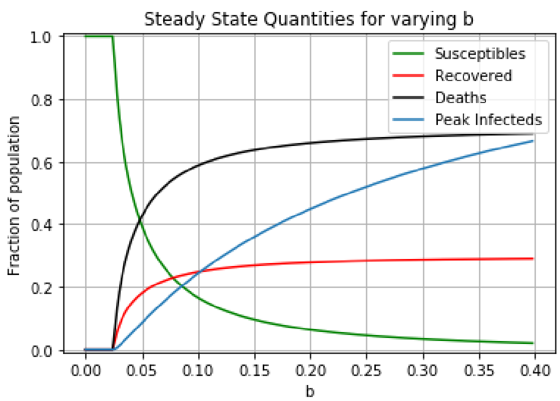

Figure 8: Dependence on transmission probability b

very high values of b. The reproduction number $R_0 \approx 40.51b$, $R_0 < 1 \implies b < 0.024$.

4.5 Termination rate $v + m + w$

It is clear that higher the outflow rate of infected population is, the lower are the deaths and infected people. The rate from which infected people are moving out from the compartment is higher, the lesser will be the deaths. Thus for higher rate we get peak infected very less close to zero.

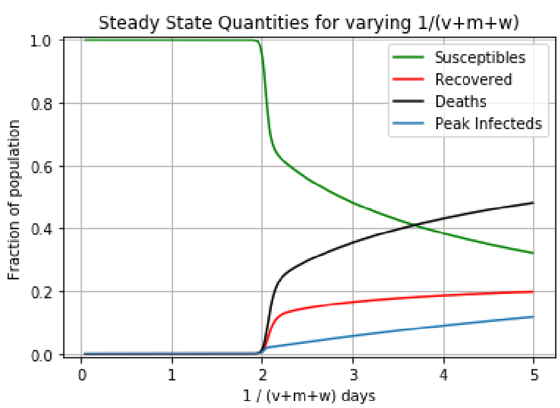

Figure 9: Dependence on termination rates

The inverse of these rates determine how much time on an average an individual stays in this compartment. If that time is high then it is very intuitive that more susceptibles would eventually become infected. As soon as it hits the critical value of rate, death fraction increases since infecteds will remain infected for larger amount of time and infect more susceptible. The reproduction number $R_0 \approx \frac{0.48}{v+m+w}$, $R_0 < 1 \implies \frac{1}{v+m+w} < 2.08$.

5 Conclusions

Note: The SARS model assumes some parameter values as described earlier.

The conclusions presented here will qualitatively remain the same but at different threshold values for different parameter values.

1. The size of the epidemic depends linearly on the quarantine fraction for $q \in (0.1, 0.3)$. For the region $q \in (0.35, 0.7)$, there is more than linear decrease in the size. At around $q = 0.7$, the epidemic size is negligible and hence any more measures taken after this point are fruitless.

2. Like the transmission probability, the number of contacts/day k also has a threshold value beyond which there is a rapid increase in the size of the epidemic. That value is $k = 4$ contacts/day.

3. The delay window is around $\triangle T_q = 110 - 80 = 30$ days.

   (a) Monetary resources can be saved since quarantining at day 75 yields the same results as same for day 0.

   (b) If the delay is even a little bit more 110 days then there is no effect of the quarantining.

4. Death fraction $D_f \simeq 0.475 + \frac{c}{\tau}$, assuming that quarantining is done for the full infection period τ. The optimum value is around $\tau = 80$ days. Any decrease from this value results in a more than linear increase in D_f and vice-versa.

5. There is an outbreak of SARS at a threshold value of $b = 0.025$. There is rapid increase in the size of the epidemic upto $b = 0.1$, after which it saturates.

6. The smaller the termination rates v, m and w are, the more time an infected individual stays infected and more are the susceptibles who become infected. If $\frac{1}{v+m+w} < 2$ days then there is no outbreak.