# CS 302 : MODELING AND SIMULATION

BASS MODEL

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# Contents

**1 Abstract** **2**

**2 Introduction** **2**

**3 Model of Innovation Diffusion** **2**
3.1 External Influence . . . . . . . . . . 2
3.2 Internal Influence . . . . . . . . . . 2
3.3 Bass Model (Mixed Influence) . . . . . . . . . . 3
3.4 Result on Parameter Perturbation . . . . . . . . . . 3
3.5 Conclusion . . . . . . . . . . 4

**4 Population Growth Modeling** **4**
4.1 Linear Model . . . . . . . . . . 4
4.2 Logistic Model . . . . . . . . . . 4
4.3 Results and Conclusion . . . . . . . . . . 4

# 1 Abstract

This report aims at analysing the effects of advertisement and word of mouth on product acceptance. We analyse how sensitive the rate of acceptance is to the parameters. We also try to use the same model to model the population growth by fitting past census data to a linear and logistic curve using least squares method.

# 2 Introduction

Whenever a new product come to the market, there are specific set of actions that take place. Firstly the new product is bought by innovators probably very small chunk of the total adopters. Then early adopters are the one who buy the product and influence others to buy the same. And from then the number of adopters increases followed by early and late majority people. The graph then saturates and consists of the laggards.

The innovators introduce the product but they do not boost the acceptance. The early adopters pull the acceptance rate up and by the time, the early majority period ends, the rate of acceptance is at it's peak. From here the rate begins to decrease and finally becomes very small or we can say that the number people who accept the product saturate to a certain value.

# 3 Model of Innovation Diffusion

We would be following the convention

- $p$ is a constant and captures the innovators or people who adopt the product on their own without being influenced by others.
- $q$ is a constant and captures the people who adopt the product on being influenced by others.
- $C$ maximum number of potential users
- $N(t)$ total number who have adopted the product till time t
- $\alpha(t)$ is the coefficient of diffusion,

We analysed three type of the model here.One with no influences, second with only influences and third is including both.

## 3.1 External Influence

In this model we assumed that there is no influence from any adopters. In this model p is the constant which captures the innovators or people who adopt the product by their own. It can be mathematically modeled as

$$\frac{dN}{dt} = p(C - N(t))$$

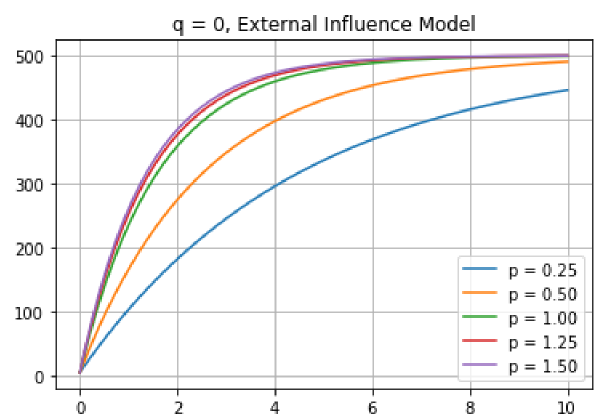
where p is the rate at which product growth happening. It is easy to observe that as the value of p increases, the number of adopters increases more rapidly and the product captures the potential market quickly. Solving the differential equation we get,

$$N(t) = C(1 - e^{-pt})$$

Define a saturation fraction s such that $N(T_{sat}) = sC$. Putting it back into the equation we can easily get

$$T_{sat} = \frac{1}{p} ln \frac{1}{1-s}$$

So as the p increases, the time to saturate at $N(t) = C$ also decreases and thus the more p is the faster the product reaches to the total population.

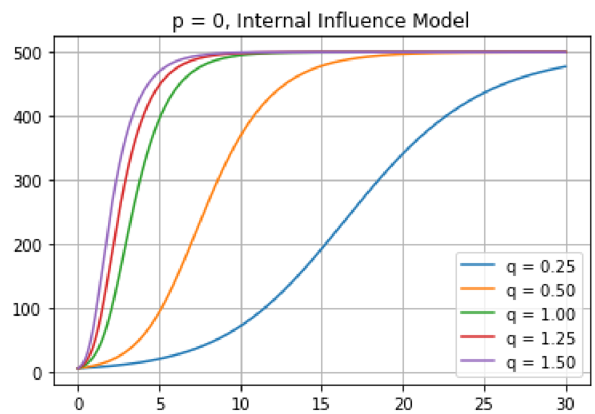



## 3.2 Internal Influence

In this model we assumed that there is only influence from any adopters. In this model q is a constant and captures the innovators or people who adopt the product on being influenced by others. Thus Model can be mathematically modeled as

$$\frac{dN}{dt} = \frac{qN(t)}{C}(C - N(t))$$

Intuitively it's clear that as we decreases the value of the parameter q, the total people influenced by the other decreases per unit time. Thus it will take longer time to adopt the product by everyone.

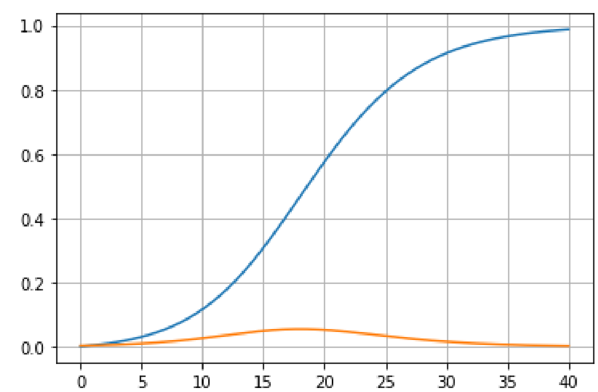



## 3.3 Bass Model (Mixed Influence)

When there are both parameter on which the product growth depends then we observed the S-shape curved meaning lesser adoption in the beginning and sudden increase after certain time. This model is called as Bass Model given by

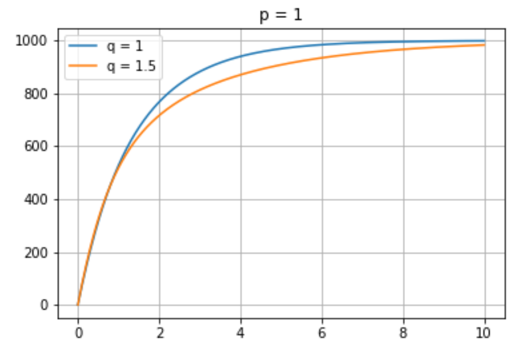
$$\frac{dN}{dt} = (p + \frac{qN(t)}{C})(C - N(t))$$

This phenomena is also observed in our model as shown below. The blue curve is the fraction that adopted the product and the red curve is the change in the fraction. The latter is the derivative for the former.

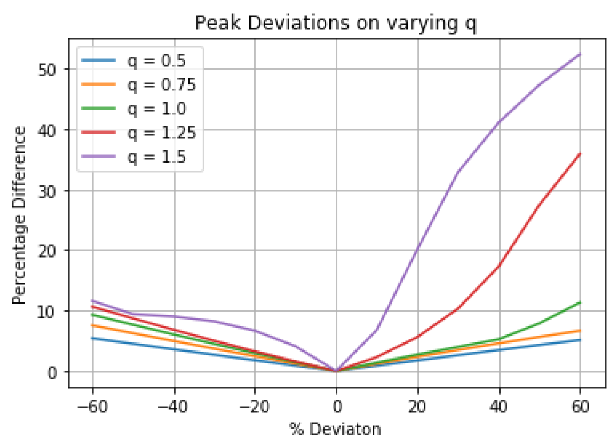


## 3.4 Result on Parameter Perturbation

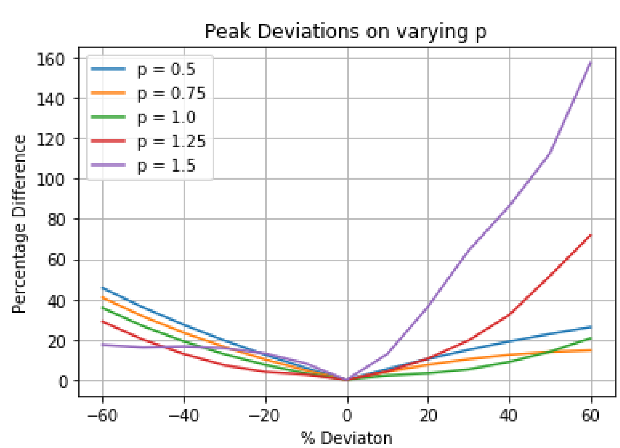
We observe that changing the parameters p and q by equal amounts does not translate to equal change in the behaviour of the system. Moreover, perturbing the parameters in the negative direction are less effective in producing a change in the overall behaviour than doing the same in the positive direction. This translates to a faster saturation on adding some amount of people or influence but not to a slower saturation on removing the same amount of people or influence.




We see that increasing q by 50% leads to a faster saturation. We aim to measure this difference of behaviour by varying p to different amounts i.e. 0-60%. We increase or decrease by q some percentage and observe the peak difference percentage till both the plots saturate.




One observation is that larger the value of q, larger is the effect of perturbation on it. This is trivial since a larger q translates to a larger absolute quantity for a fixed ratio. Next we observe that a negative perturbation always results in a smaller peak deviation than an equal positive perturbation. We plot the same graph for p and observe the following.



### 3.5 Conclusion

We observe similarities between the effect of the two plots. Again negative perturbations don't produce a high peak deviation but. For the same p, q values, a perturbation of x% in p produces a larger peak deviation than in q. For example, here in this case, a perturbation of +0.6 in p produces a peak deviation of 22% but the same in q produces just 8%.

Thus we conclude that the model is somewhat more sensitive to the value of p than q.

## 4 Population Growth Modeling

We try to model the population growth using a logistic model

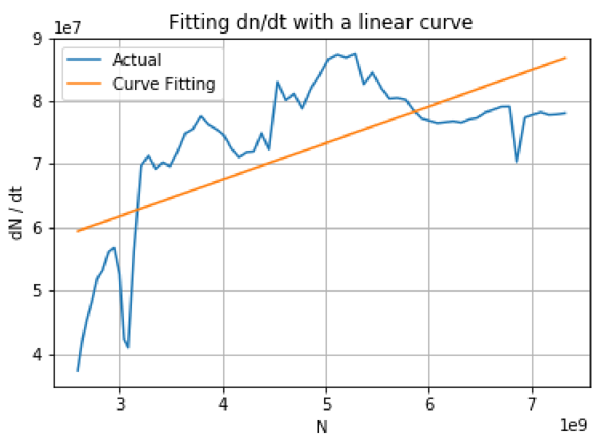
$$\frac{dN}{dt} = (p + \frac{q}{C}N) * (C - N)$$

We assume a translation of constants such that $\frac{dN}{dt} = \theta_0 + \theta_1 N + \theta_2 N^2$ We have

- $\theta_0 = pC$
- $\theta_1 = \frac{q}{C} - p$
- $\theta_2 = -\frac{q}{C}$

### 4.1 Linear Model

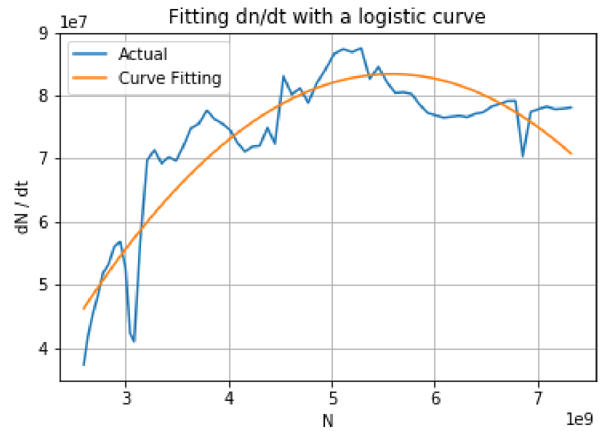
Fitting using a linear model which results in Malthusian explosion. It assumes that the more the number of people, the more the rate of change. However it fails to take into account that the resources needed to sustain that population are not present. However this model works well if the resources are available in large quantities.




We can see that it is not a very good estimate given the past year's data. We then try to fit the data with a logistic curve.

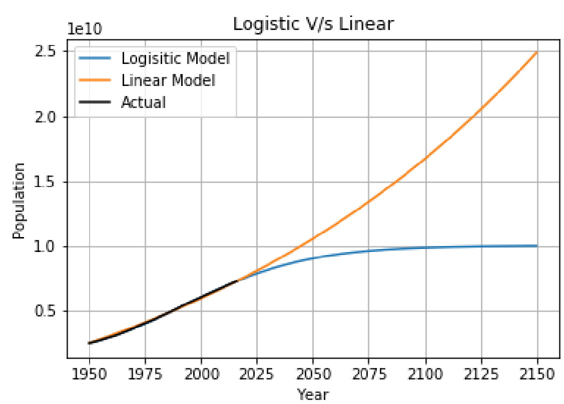
### 4.2 Logistic Model

This model assumes that the resources are not infinite and that the population must saturate to it's carrying capacity where the resources are being utilised to the fullest. This model fits the data better compared to the previous linear model.




The problem with this model is that the curve fitting algorithm doesn't take into account that the constants have a physical significance. For example, both p and C are positive constants and so their product should also be positive but the data forces the curve to fit in such a way that $\theta_0 < 0$. We can solve this problem with some other machine learning algorithms or statistical methods that penalise the algorithm when $\theta_0$ tries to be negative.

### 4.3 Results and Conclusion




However we can still simulate with this curve. We see that both the models fit the actual population data quite well. But after sometime around 2030, the Malthusian explosion occurs and the linear model predicts the population to blow up to 25 billion by 2150 whereas the the logistic model predicts that by 2150 the population will saturate to 10 billion.