

Numerical Methods for Conservation Laws

Assignment 1 (Scalar Laws)

Use the fluctuation-signal/splitting algorithm to numerically solve for the following scalar hyperbolic conservation laws:

1. $u_t + (u^2/2)_x = 0$, in the domain $[-2,2]$, with initial conditions:

$$u(x, 0) = 1, \quad |x| < 1/3 \quad (1)$$

$$u(x, 0) = 0, \quad |x| > 1/3 \quad (2)$$

Plot u versus x obtained numerically at $t = 2/3$ and $t = 4/3$ and compare with analytical solution.

2. Consider the traffic equation $\rho_t + (\rho u_{max}[1 - \rho])_x = 0$, $0 \leq \rho \leq 1$, $u_{max} = 1.0$
 - (a) Solve with initial conditions $\rho(x, 0) = 0.25 + 0.75 \exp(-0.25x^2)$, domain $[-30, 30]$. Plot $\rho(x, 25)$.
 - (b) Solve with initial discontinuous data $\rho_l = 0.25, \rho_r = 1.0$, jump at $x = 0.0$, domain $[-40, 10]$. Plot $\rho(x, 36)$.
 - (c) Solve with initial discontinuous data $\rho_l = 1.0, \rho_r = 0.0$, jump at $x = 0.0$, domain $[-30, 20]$. Plot $\rho(x, 18)$.

Use 100 points in the domain and time step based on a CFL number of 0.8 for solving the above problems..