Branch & Bound Algorithm for Aircraft Ground Movement Optimization

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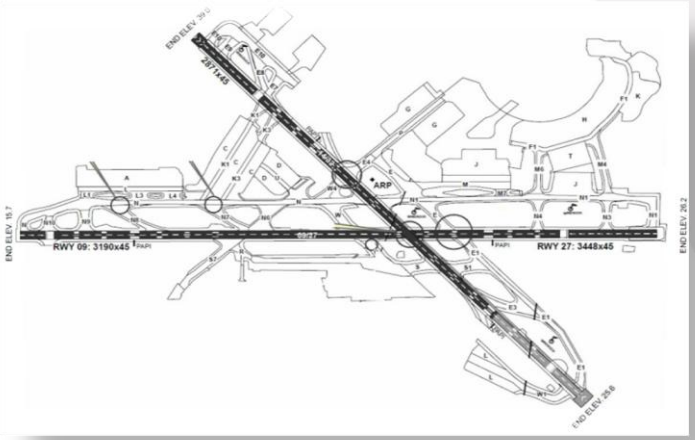
Air Traffic Ground Control

- Given aircraft data with:
  - tentative Entry-time
  - Entry & Exit points on the airport
- Route aircraft through network of runways & taxiways
- Resolve overlap conflicts: Minimize overall delays



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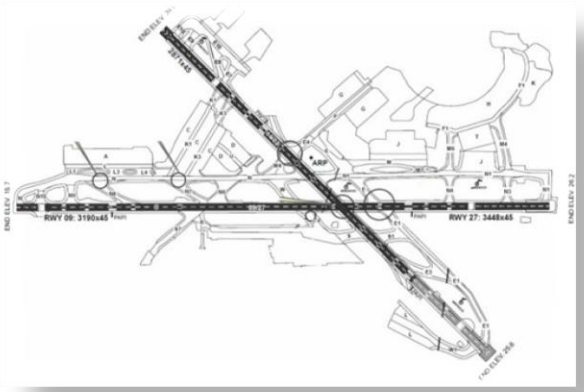
Motivation

- Heavy congestion: 12.5% of flight delays in USA in 2012 estimated to have occurred in vicinity of airports (FAA)
- Over-scheduling by airlines leads to unrestrained push-backs
- Prominently manual FCFS scheduling by ATCs: Myopic schedules
- Existing methods don't guarantee solution optimality while maintaining minimal run-times

Modeling

Modeling

- Airport map:



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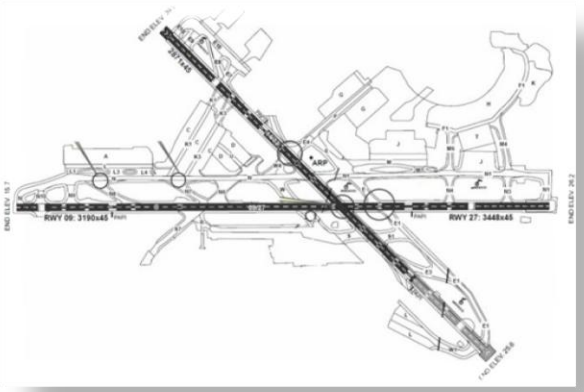
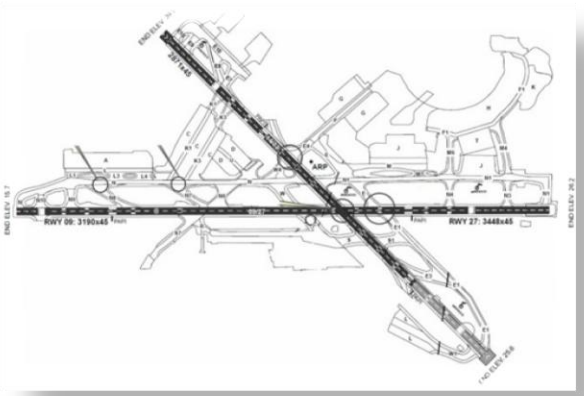
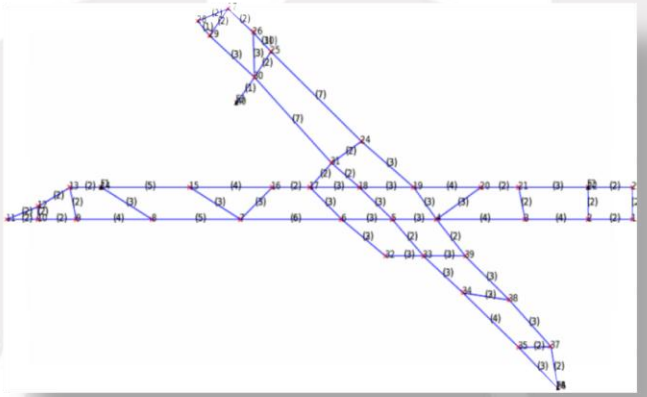


- Aircraft:

Modeling

- Airport map: Graph with nodes and arcs





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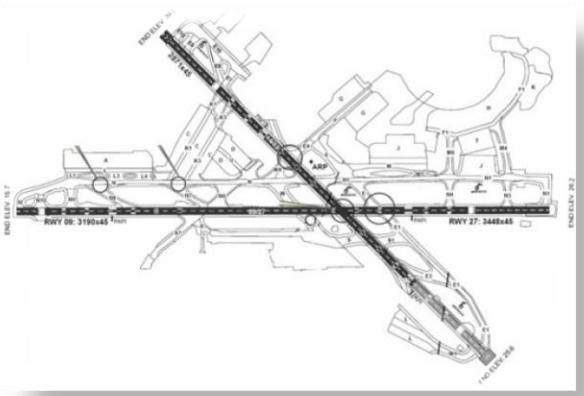
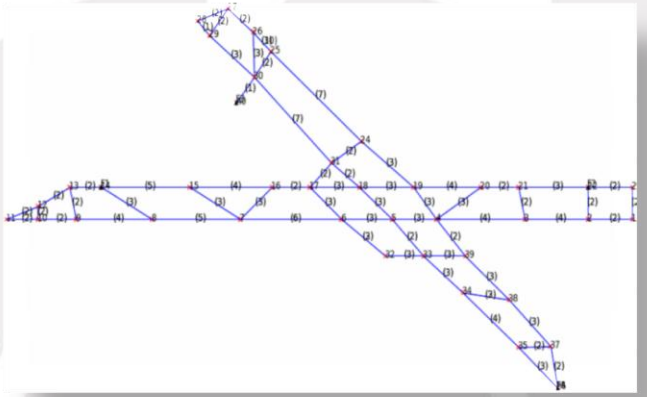
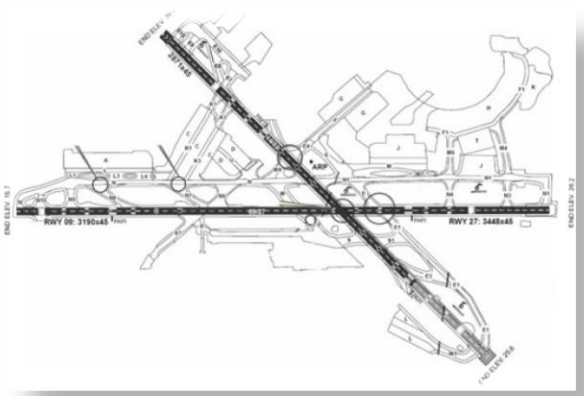


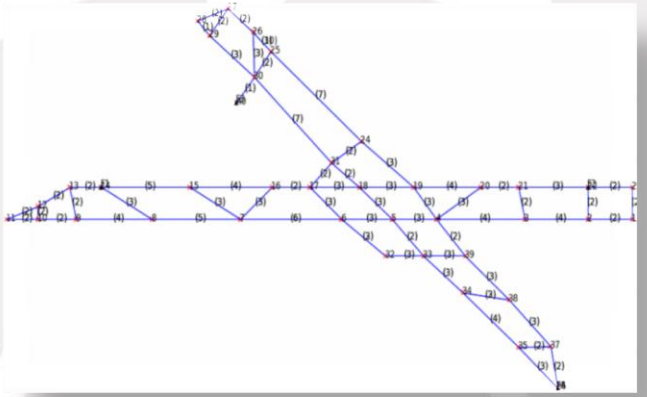


- Aircraft: Point object with constant speed

Modeling

- Airport map: Graph with nodes and arcs

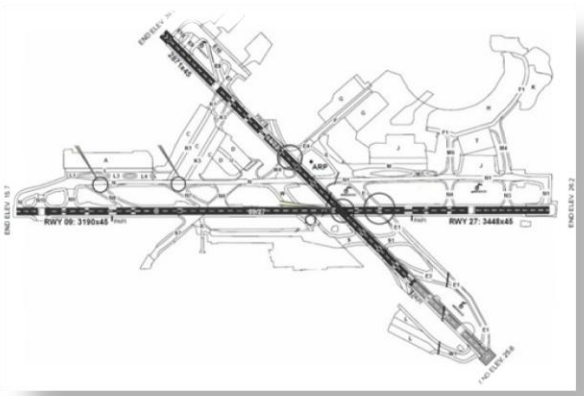


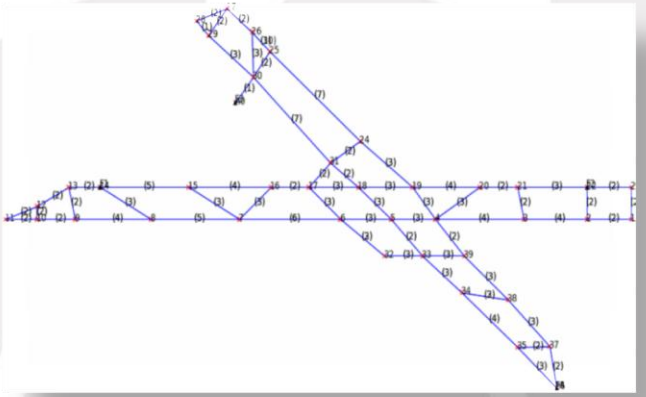


- Aircraft: Point object with constant speed
  - Entry-time

Modeling

- Airport map: Graph with nodes and arcs

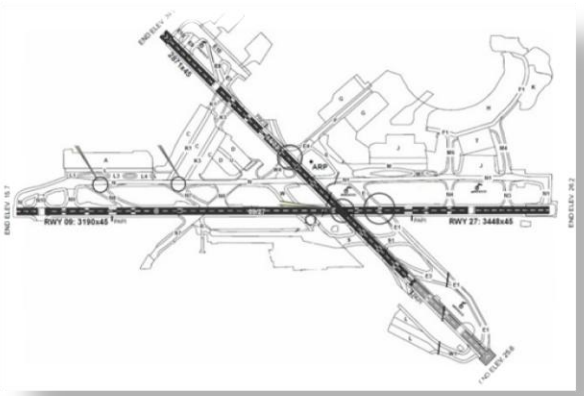


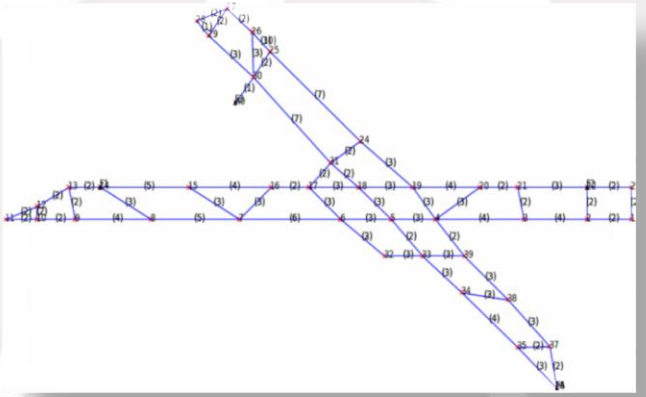


- Aircraft: Point object with constant speed
  - Entry-time
  - Origin

Modeling

- Airport map: Graph with nodes and arcs





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  - Origin
  - Destination

Modeling

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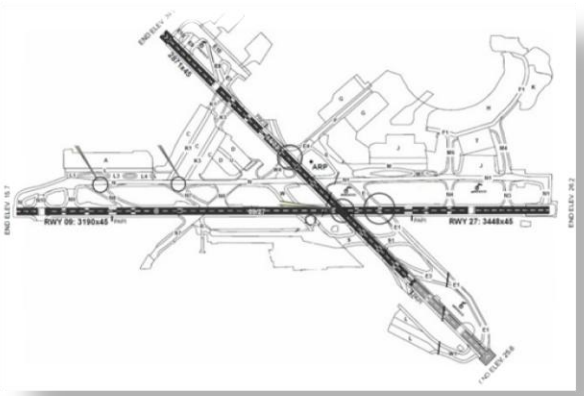
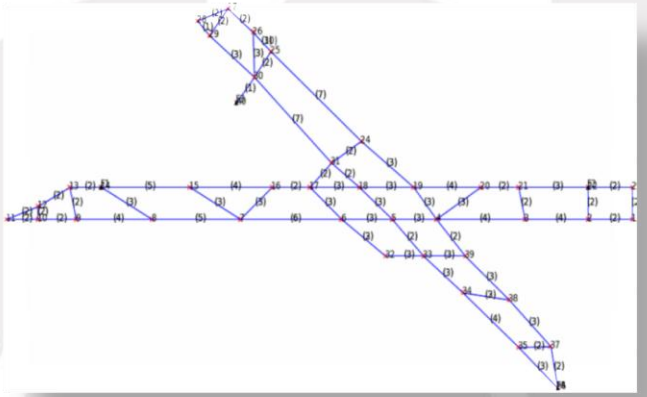
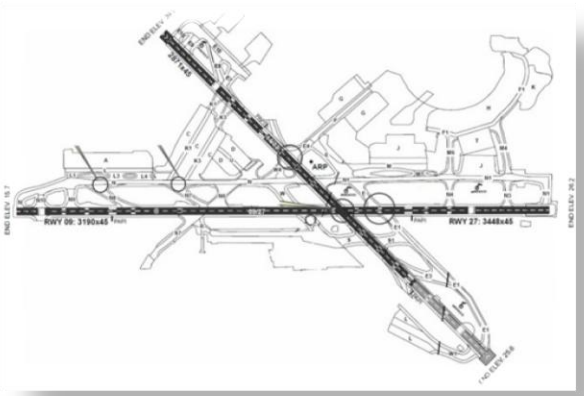


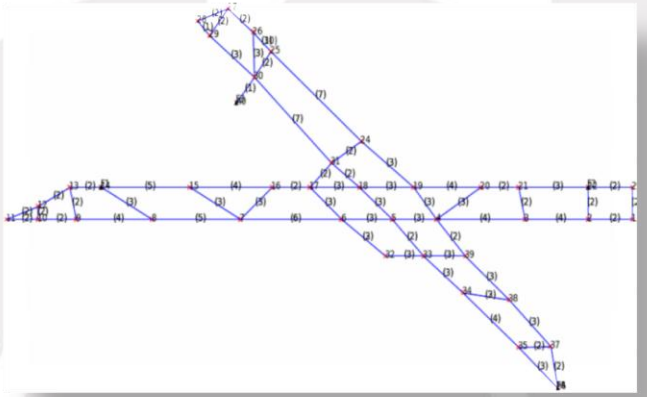


- Aircraft: Point object with constant speed
  - Entry-time
  - Origin
  - Destination
  - Speed (constant)

Modeling

- Airport map: Graph with nodes and arcs





- Aircraft: Point object with constant speed
  - Entry-time
  - Origin
  - Destination
  - Speed (constant)
  - Trailing separation (turbulence mitigation)

Modeling

- Airport map: Graph with nodes and arcs

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  - Take-off/Landing distance

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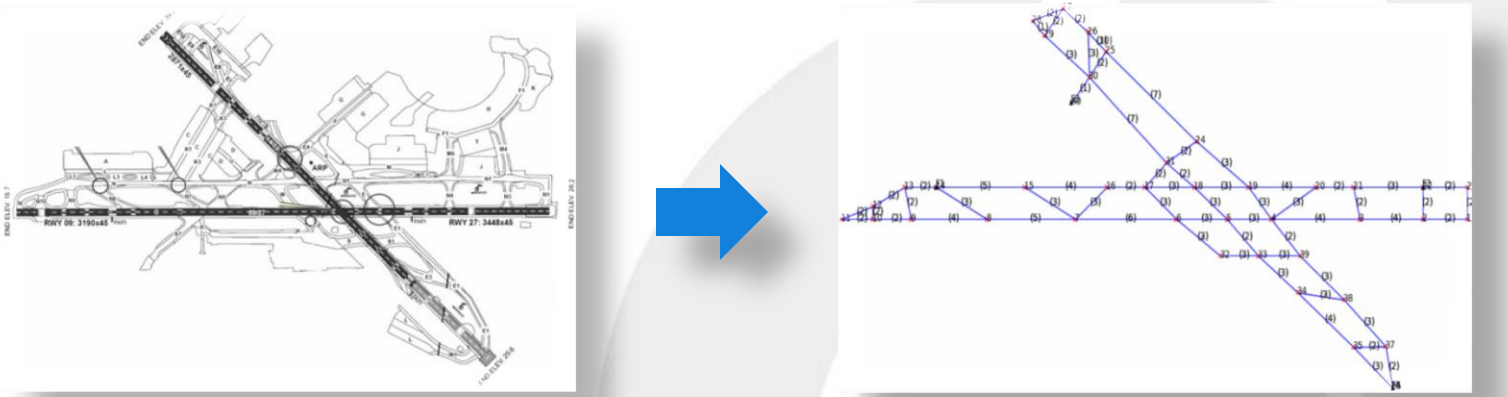


- Aircraft: Point object with constant speed
  - Entry-time
  - Origin
  - Destination
  - Speed (constant)
  - Trailing separation (turbulence mitigation)
  - Take-off/Landing distance
  - Priority

Constraints

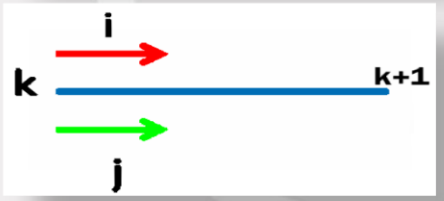
Conjunctive *(Path traversal)*

- Travel-time

Disjunctive *(Precedence)*

- Head-on

- Trailing



- Runway

Objective

- Route and schedule all aircraft to:
  - Eliminate overlap conflicts
  - Minimize overall taxi and waiting time weighted by priority

$$Minimize: \sum_{i=0}^{N} P_i t_{in}$$

output : values of t_{ik} for $i \in (0, N)$ & $k \in (0, n)$

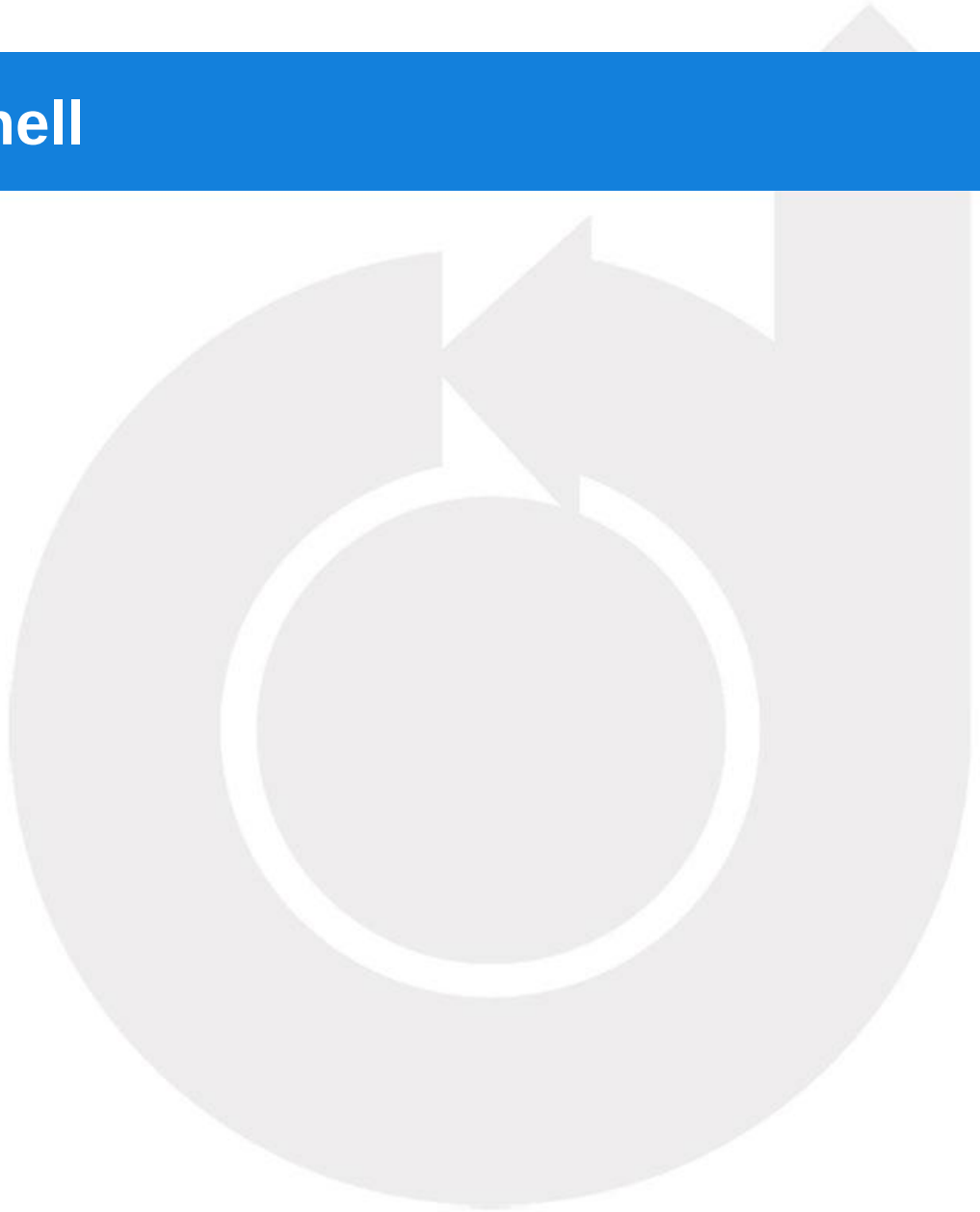
t_{ik} : Time when aircraft i reaches node k

P_i : Priority level of aircraft i.

N : Total no. of flights

n : Last / Destination node in the path of flight i

Algorithm in a Nutshell



Algorithm in a Nutshell

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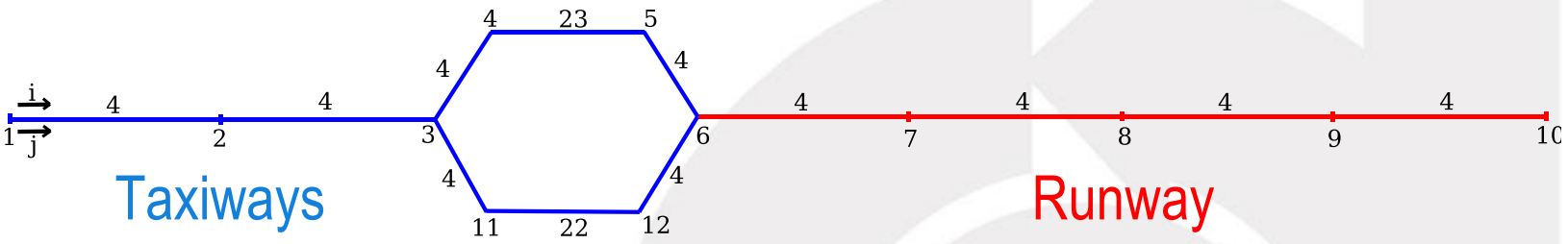
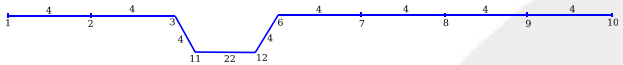
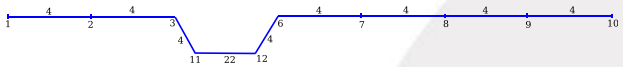
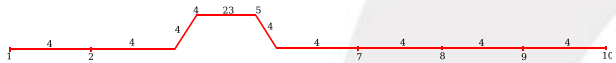
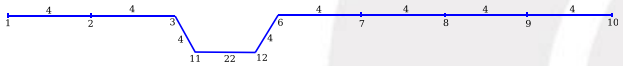
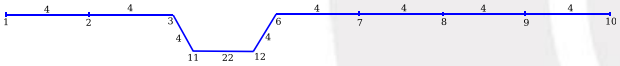
- Overlap constraints resolved at every level
- Upper bound updated for every better solution
- Solution space spanned and global optimum identified

Problem Illustration



Flight	Start-time	Origin	Destination	Speed	Trailing Separation	Take-off/Landing Distance	Priority
i	0	1	10	1	20	16	1
j	4	1	10	2	20	16	1

Flight-Path Generation

			Length
1.	*i* :		54
	j :		54
2.	*i* :		55
	j :		54
3.	*i* :		54
	j :		55
4.	*i* :		55
	j :		55

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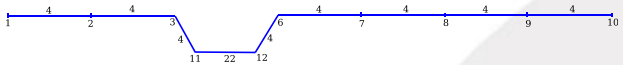
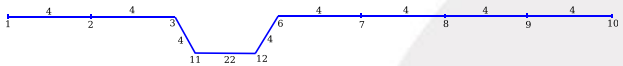
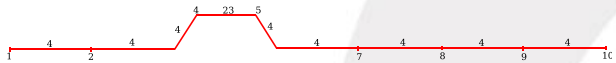
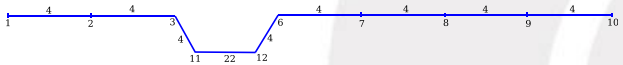
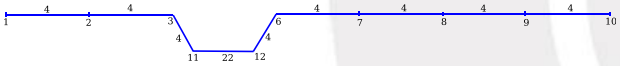
1.	*i* :		54
	j :		54
2.	*i* :		55
	j :		54
3.	*i* :		54
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4.	*i* :		55
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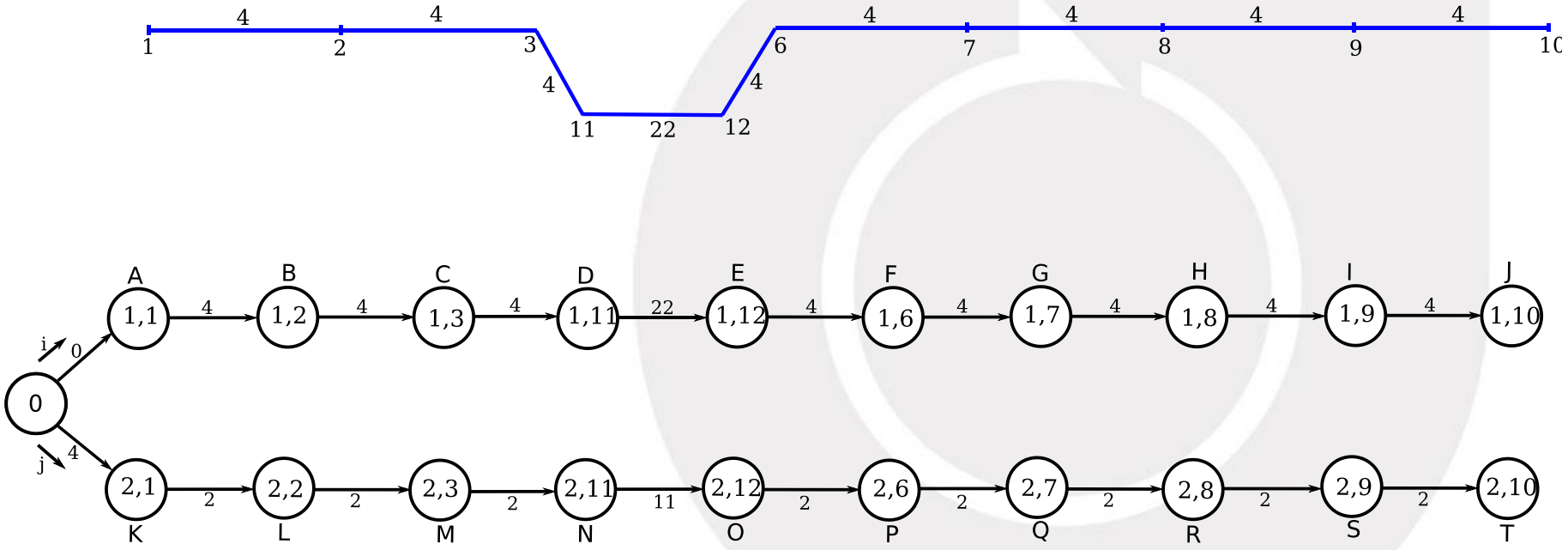
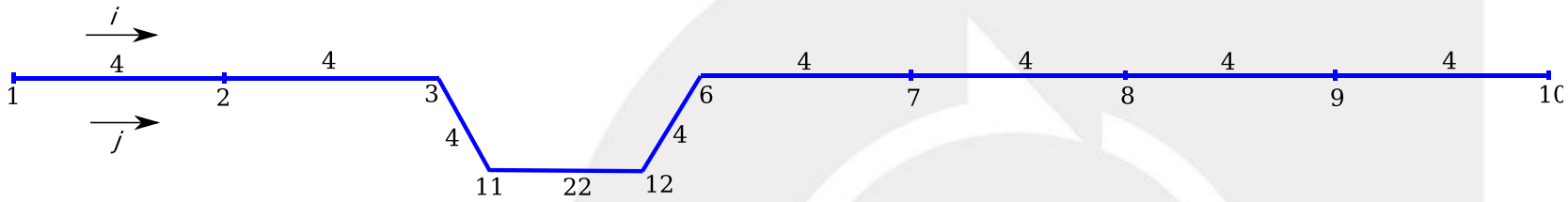
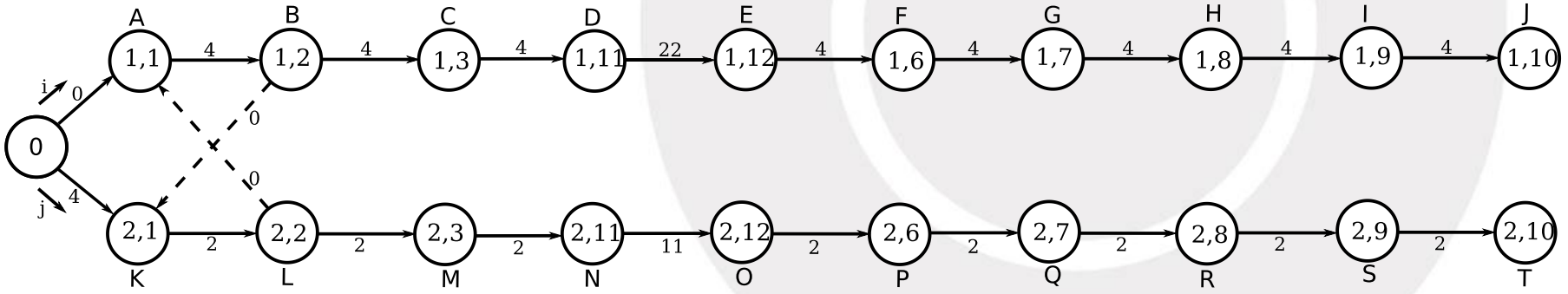
Constraint Generation



Conjunctive constraints

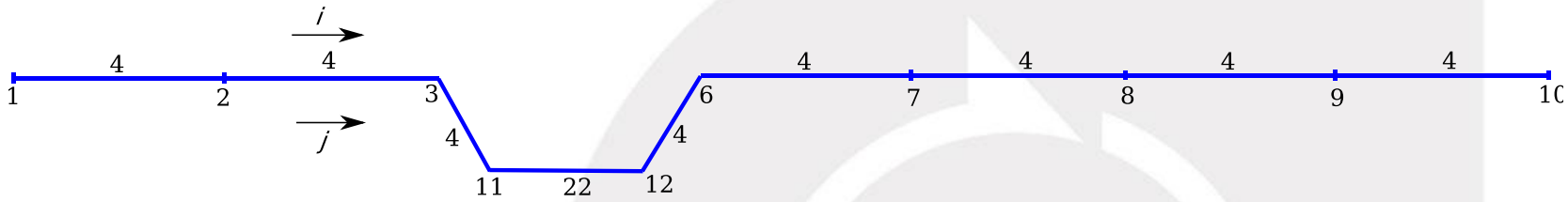
Constraint Generation

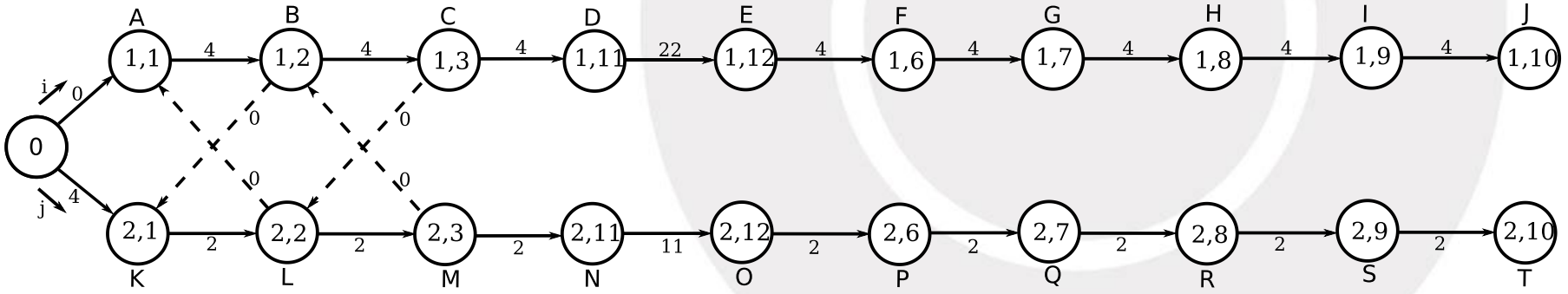




Disjunctive constraints

Constraint Generation





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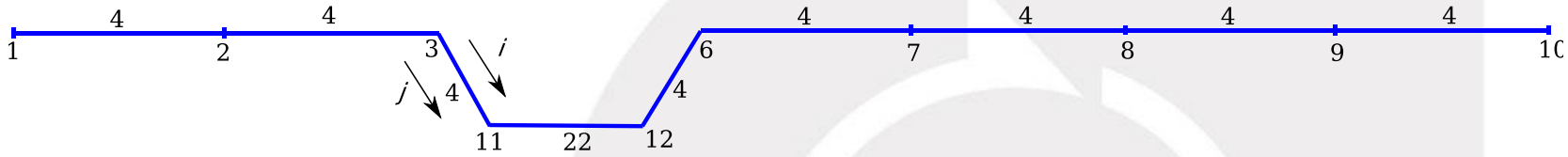

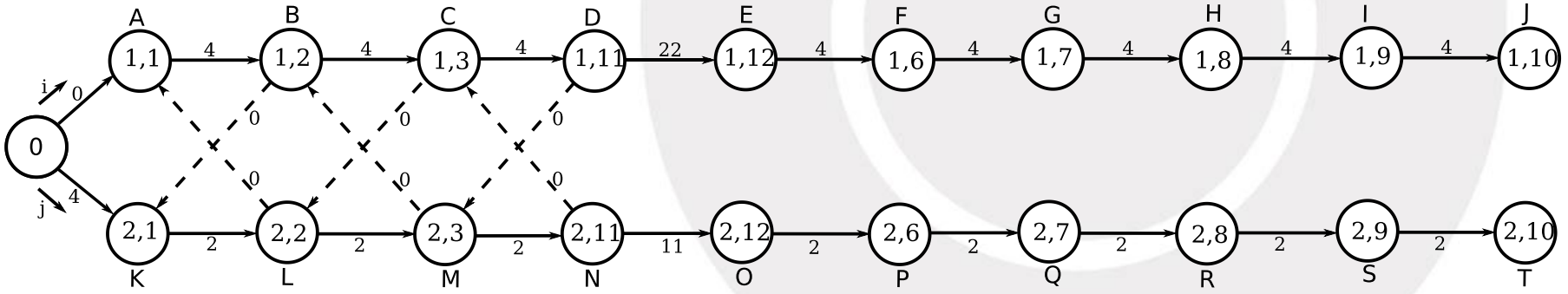

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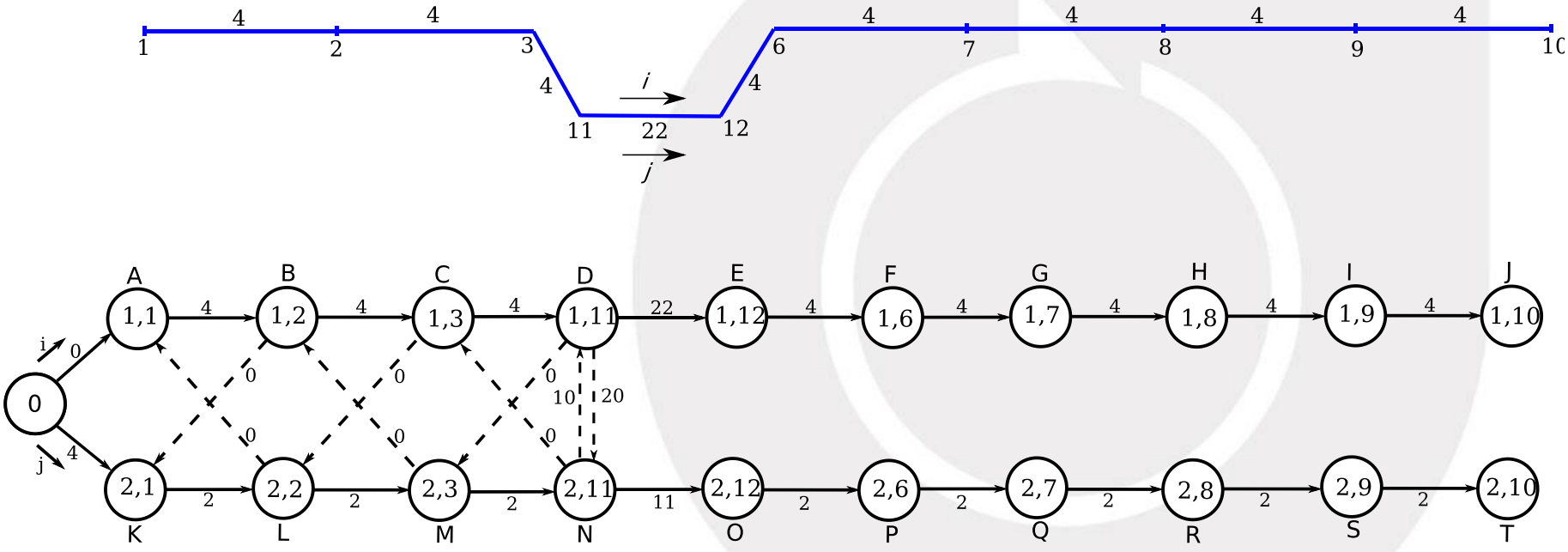

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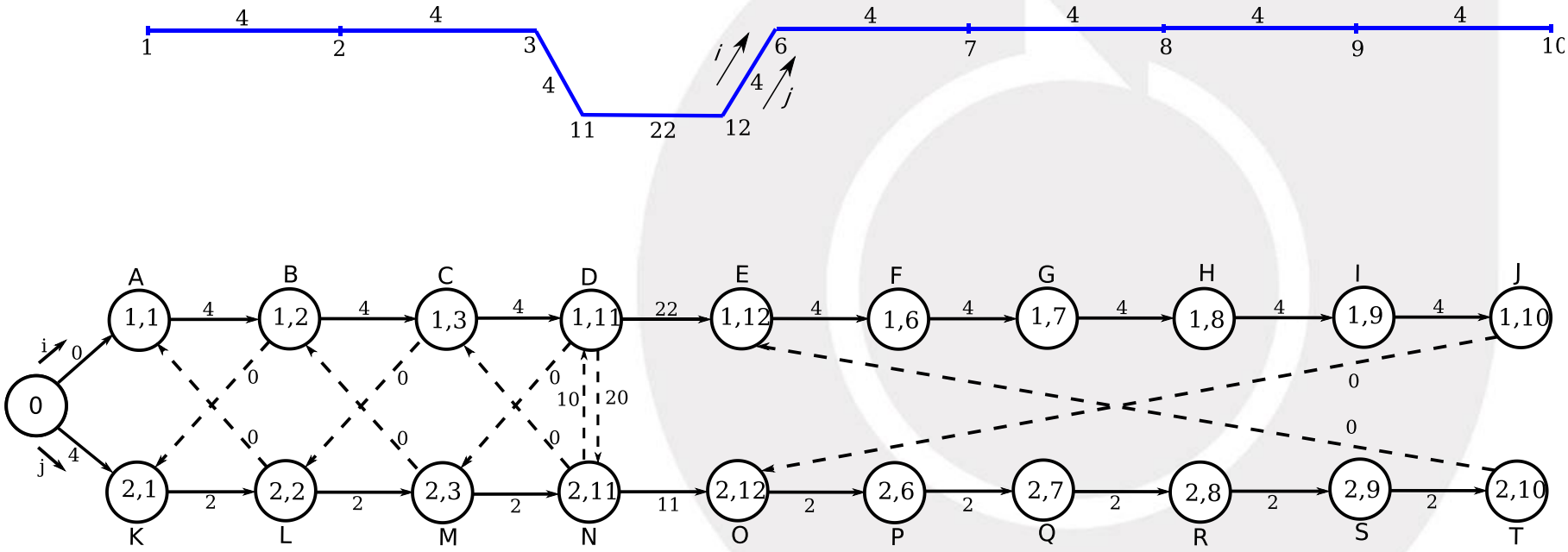
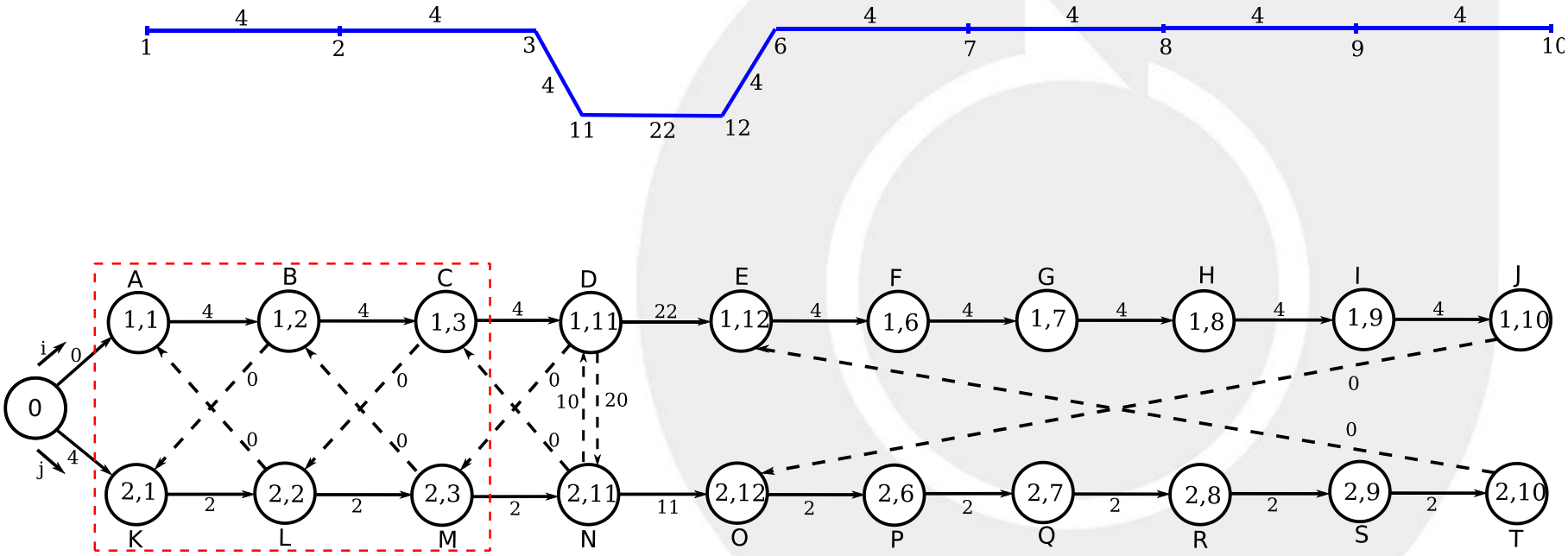


Disjunctive constraints

Constraint Feasibility



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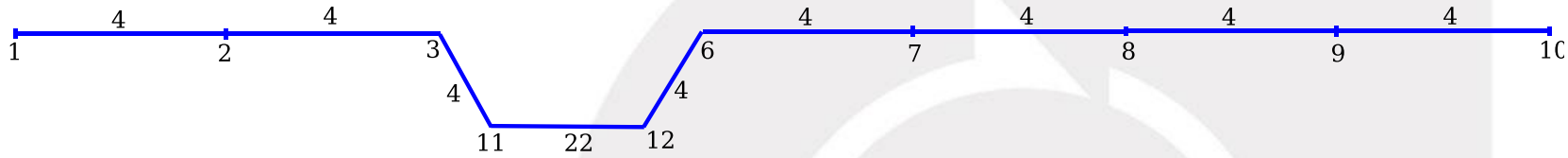
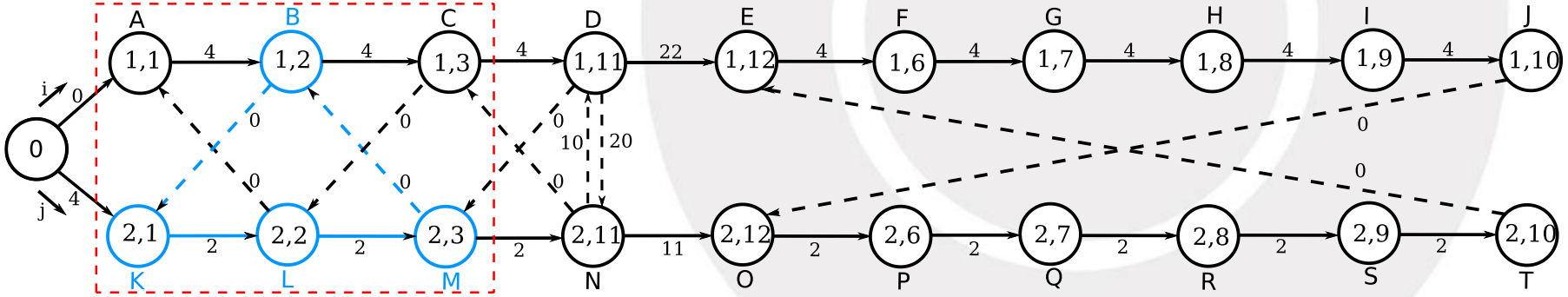
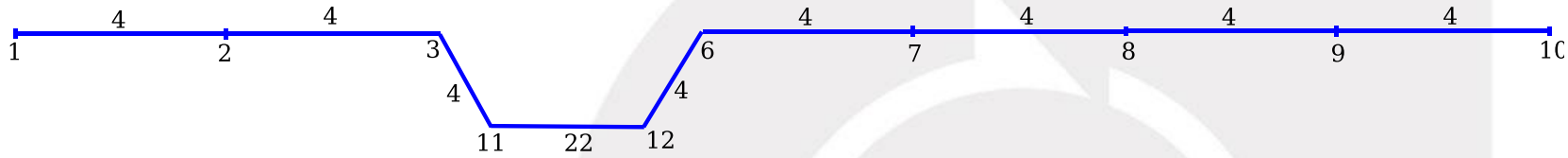
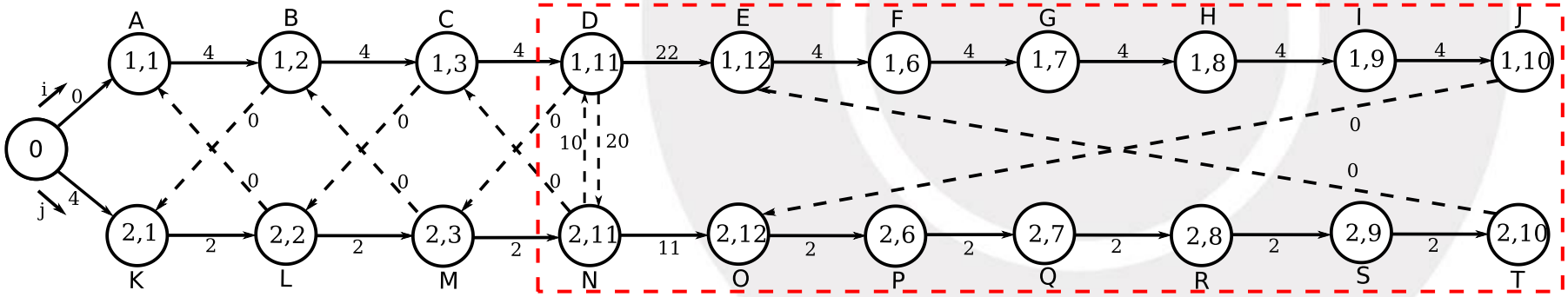




Cycle

Constraint Feasibility





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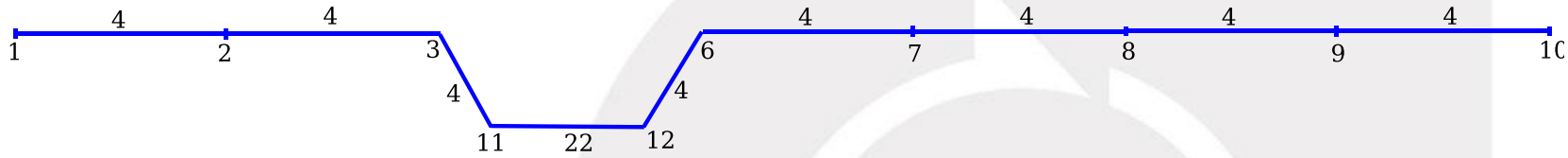
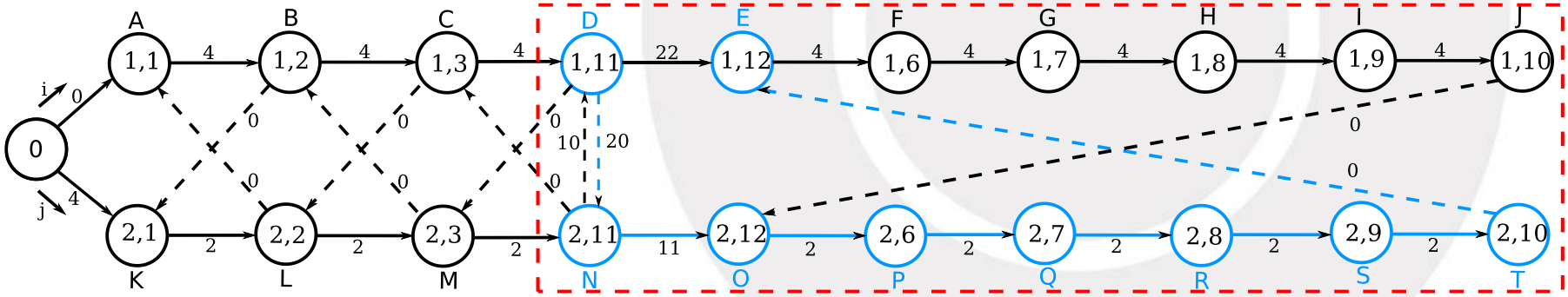
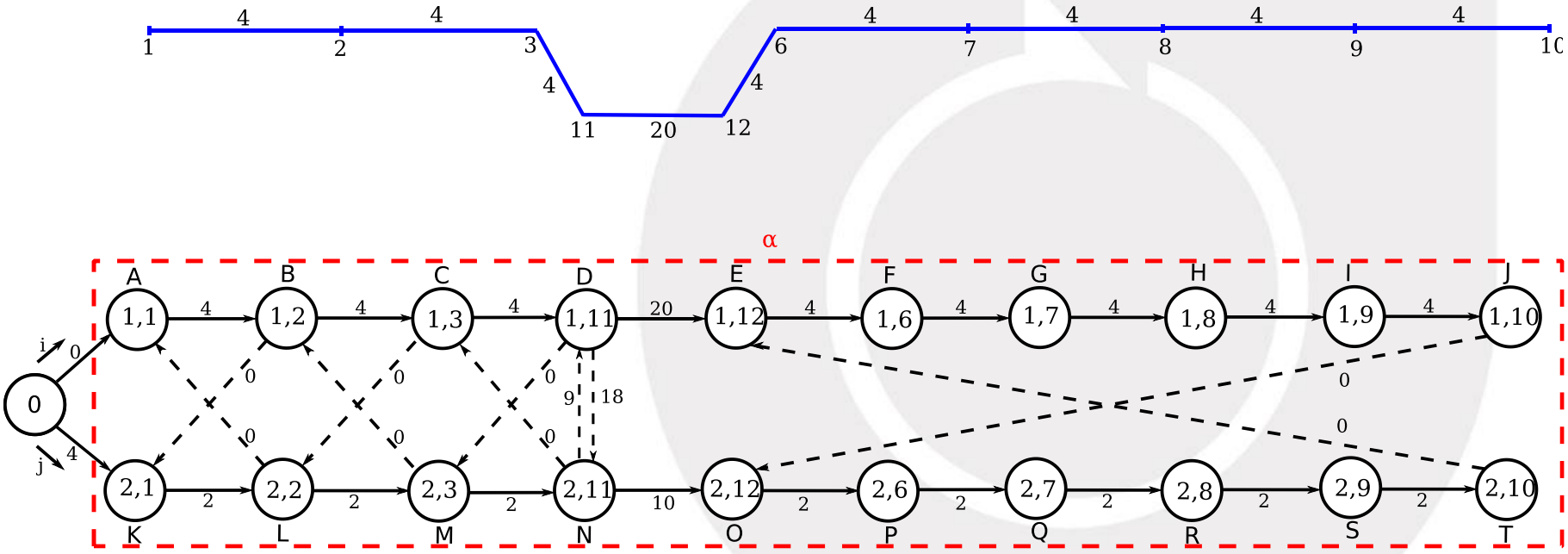




No cycle

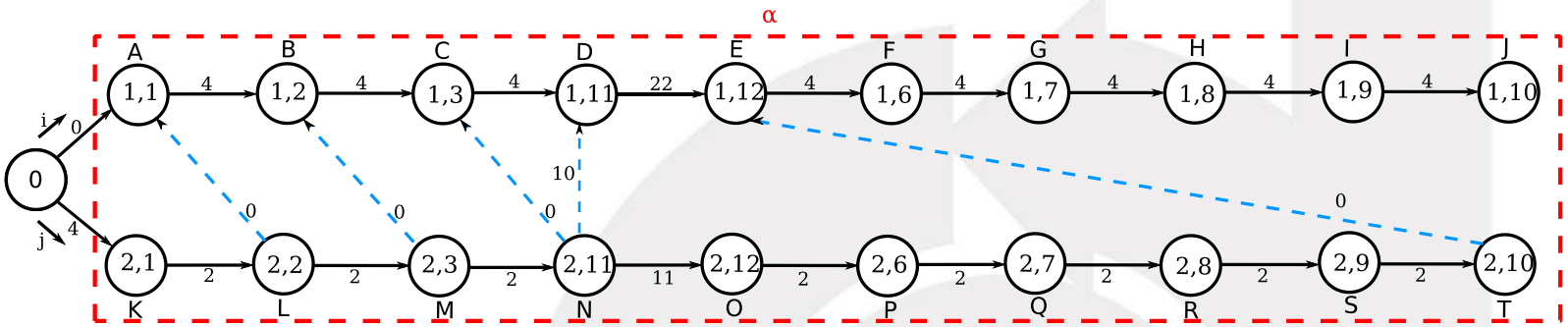
Constraint Feasibility



Grouping

Constraint Grouping

j leads



OR

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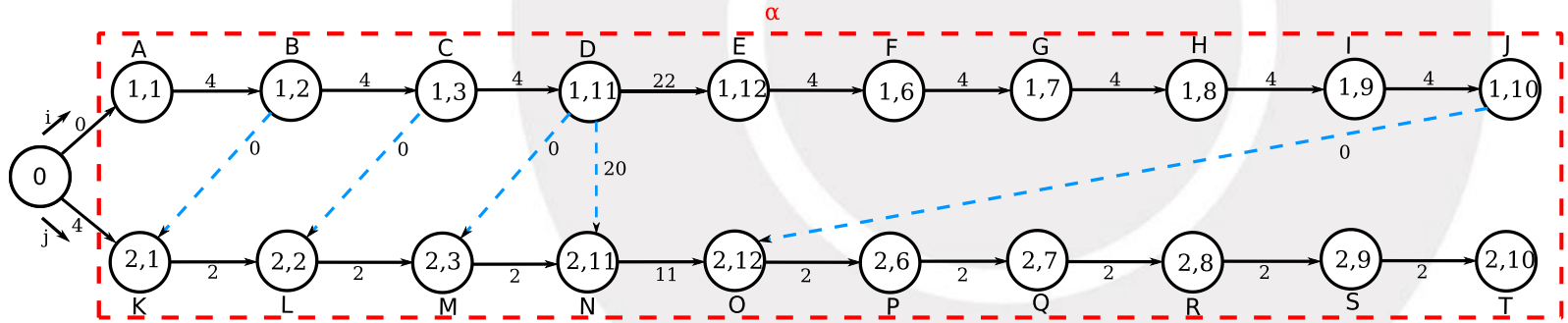

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Branch & Bound

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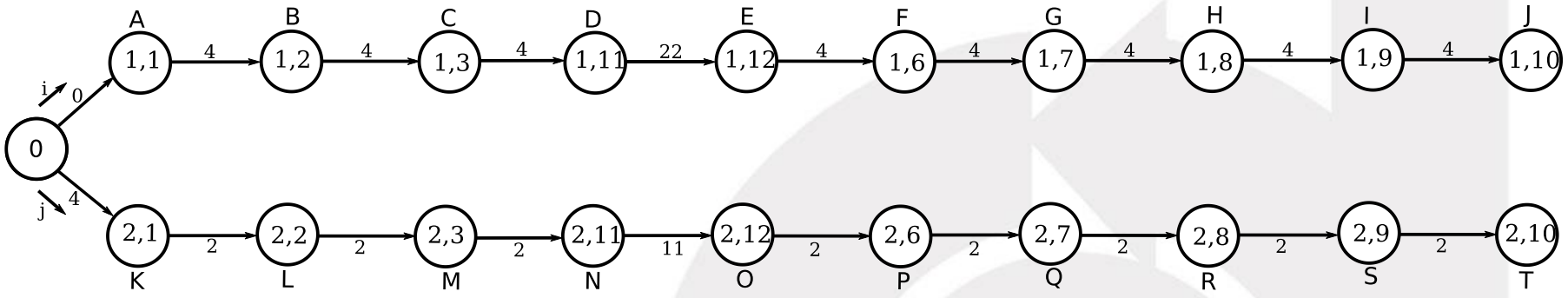
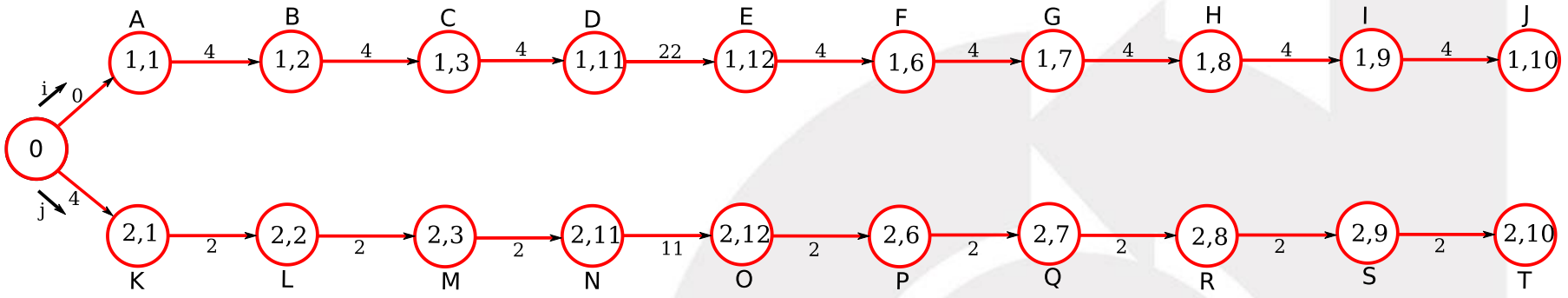


Branch & Bound

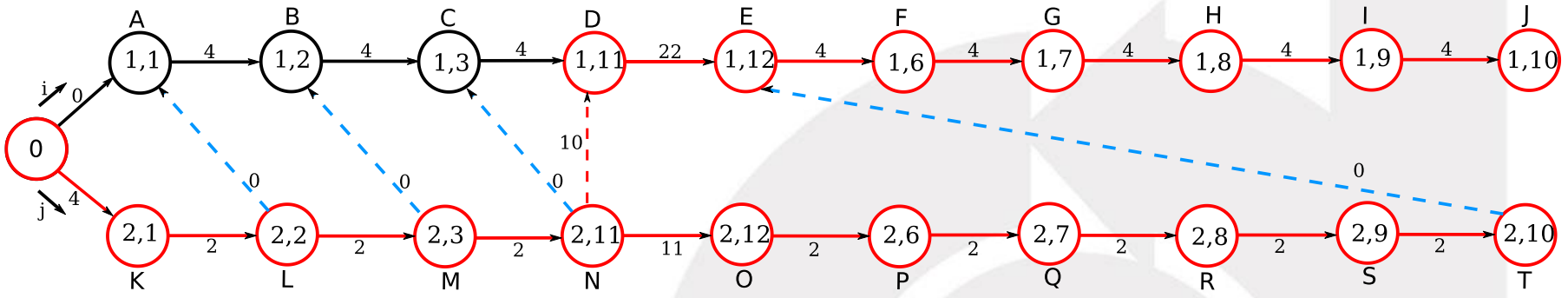


Longest path from origin to destination (Cost) : 54 + 31 = 85

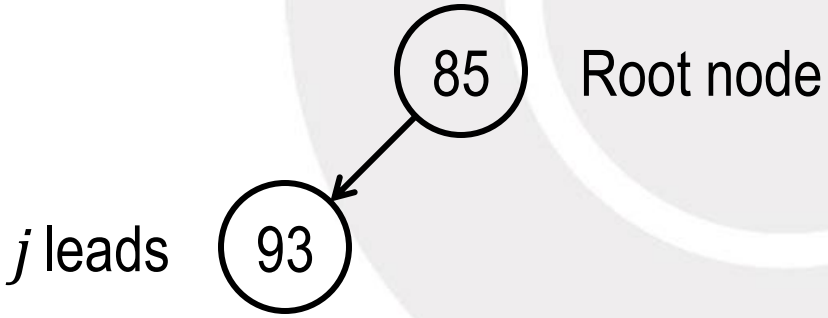
85 Root node

Upper bound = ∞

Branch & Bound

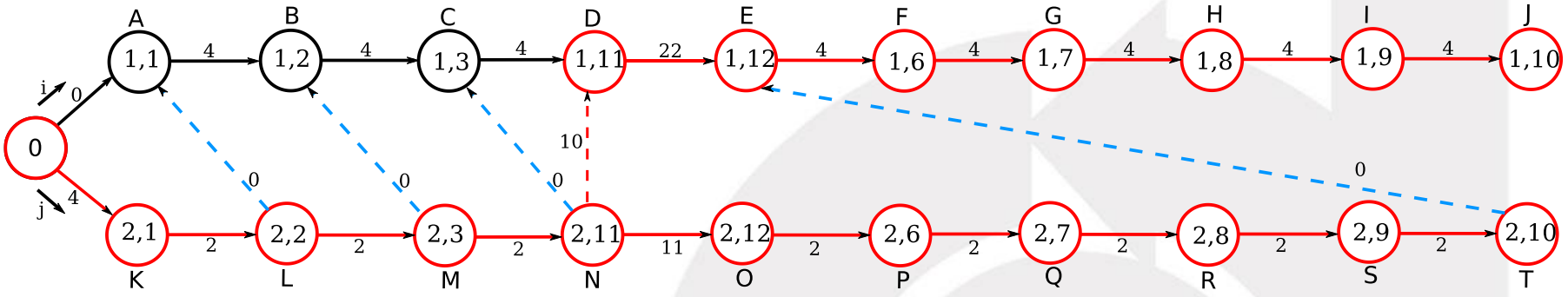


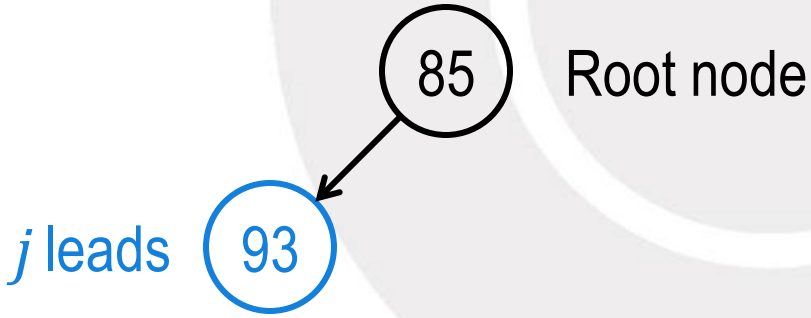
Longest path from origin to destination (Cost) : 62 + 31 = 93



Upper bound = ∞

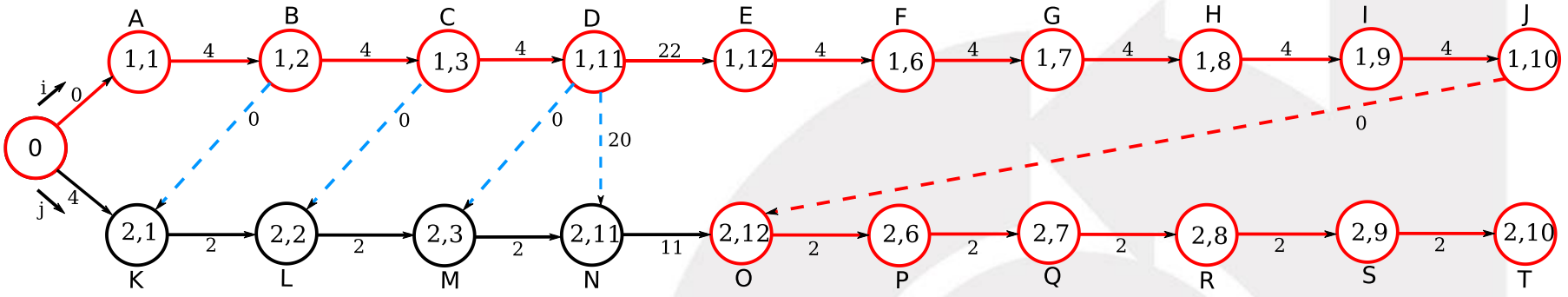
Branch & Bound



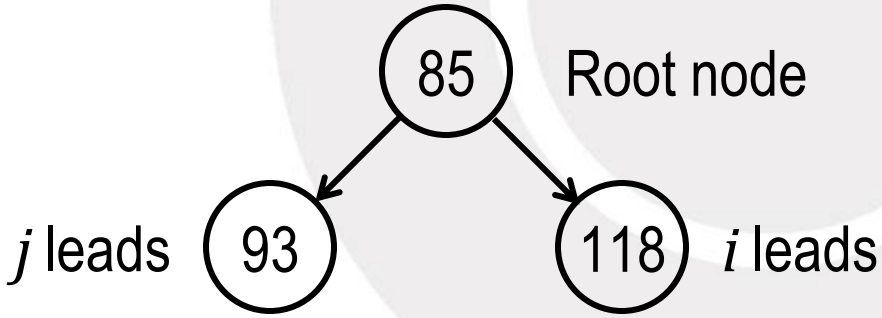


Upper bound = 93

Branch & Bound

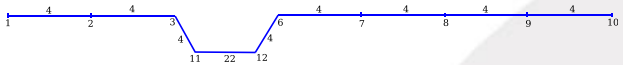
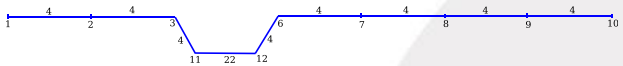
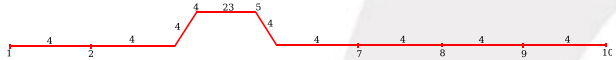
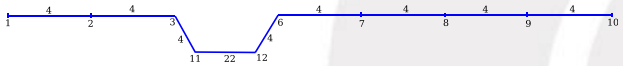
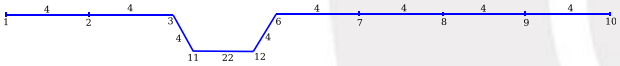


Longest path from origin to destination (Cost) : 54 + 64 = 118

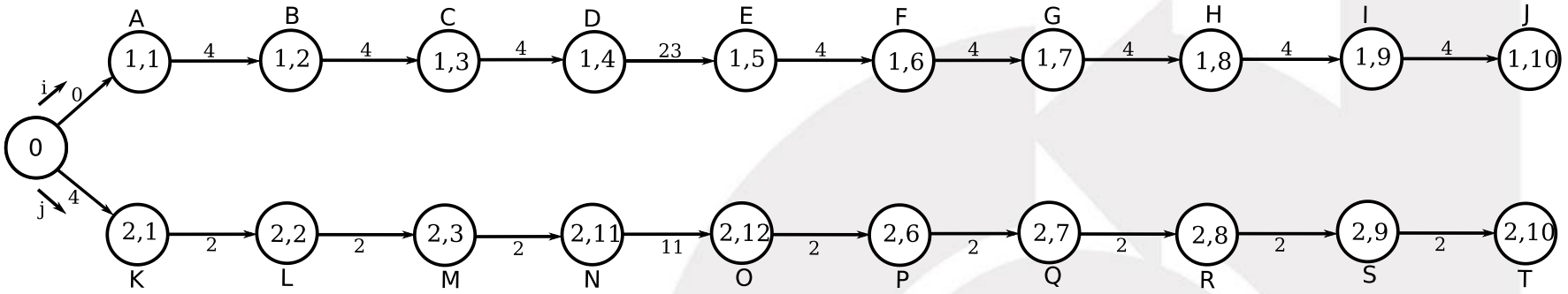


Upper bound = 93

Flight-Path Generation

			Length
1.	i :		54
	j :		54
2.	i :		55
	j :		54
3.	i :		54
	j :		55
4.	i :		55
	j :		55

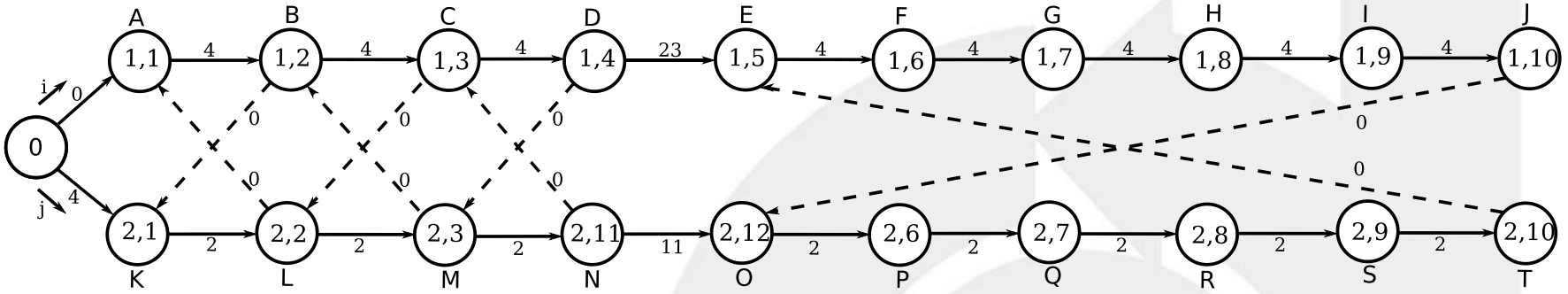
Conjunctive graph



86 Root node

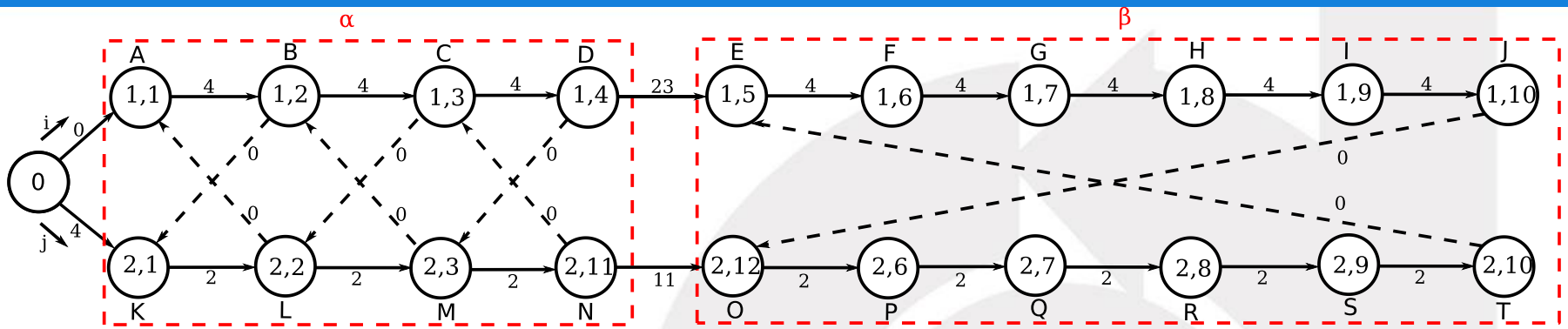
Upper bound = 93

Constraint generation



Upper bound = 93

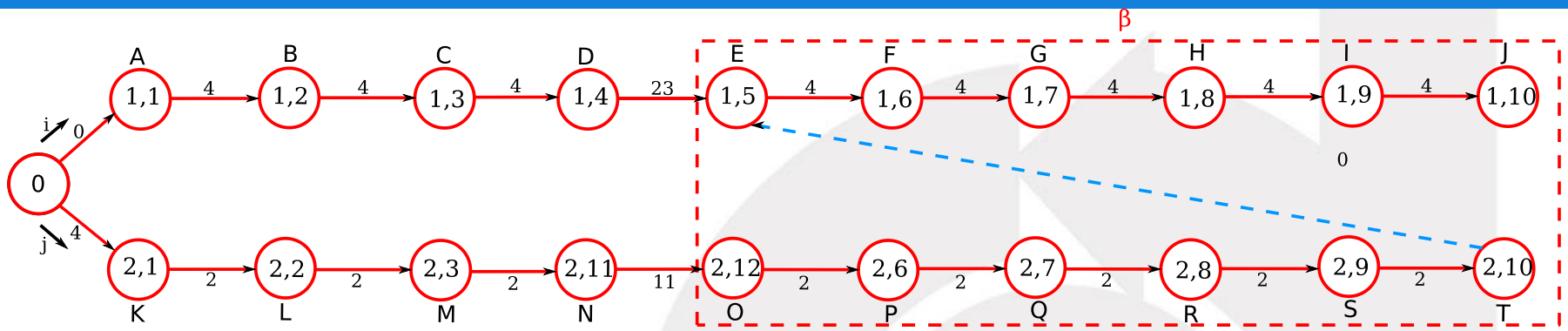
Constraint grouping



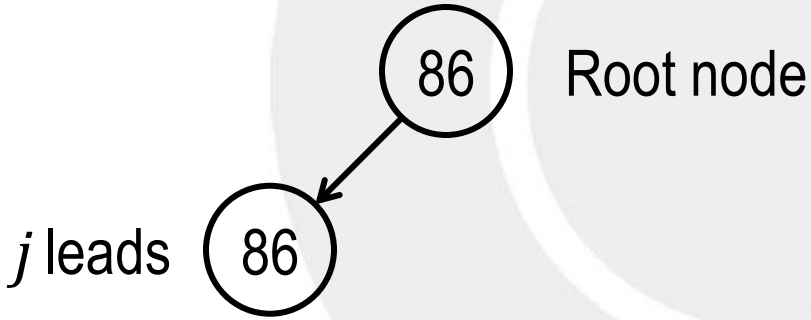
86 Root node

Upper bound = 93

Constraint resolution (Bottle-neck)

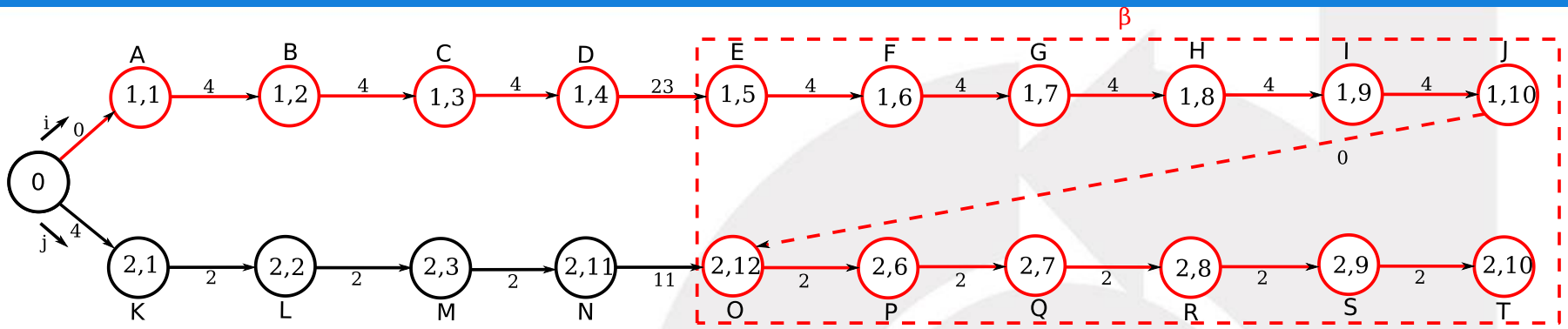


Longest path from origin to destination (Cost) : 55 + 31 = 86

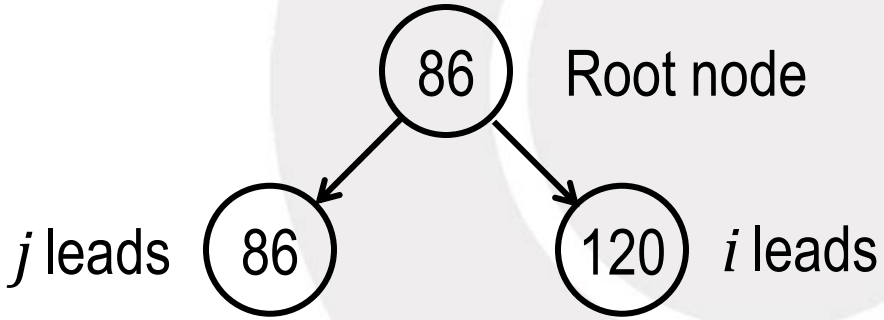


Upper bound = 93

Constraint resolution (Bottle-neck)

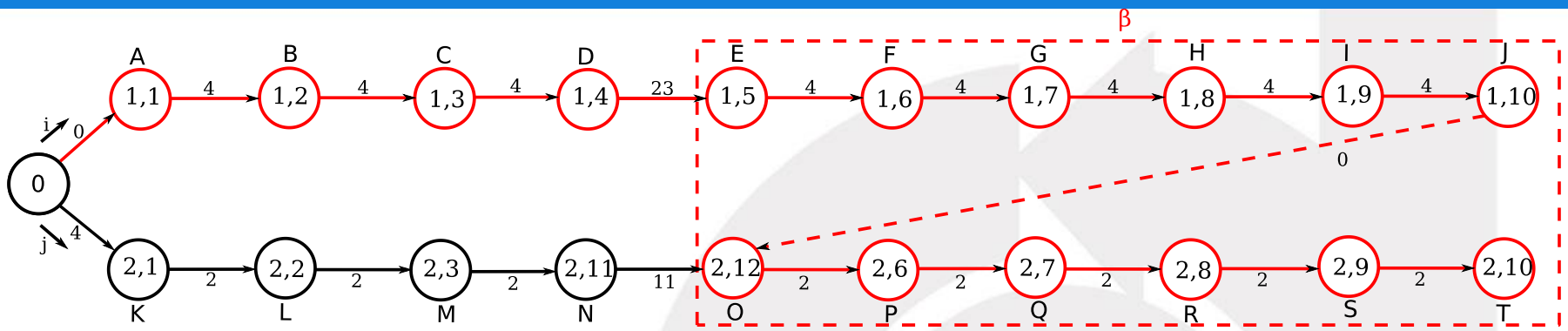


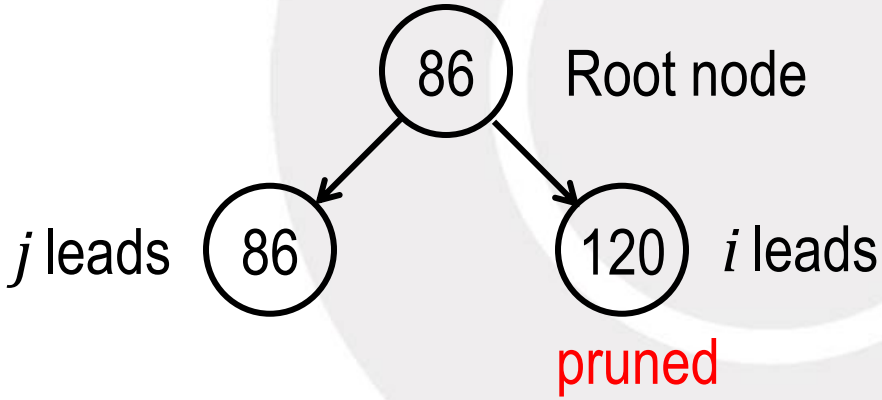
Longest path from origin to destination (Cost) : 55 + 65 = 120



Upper bound = 93

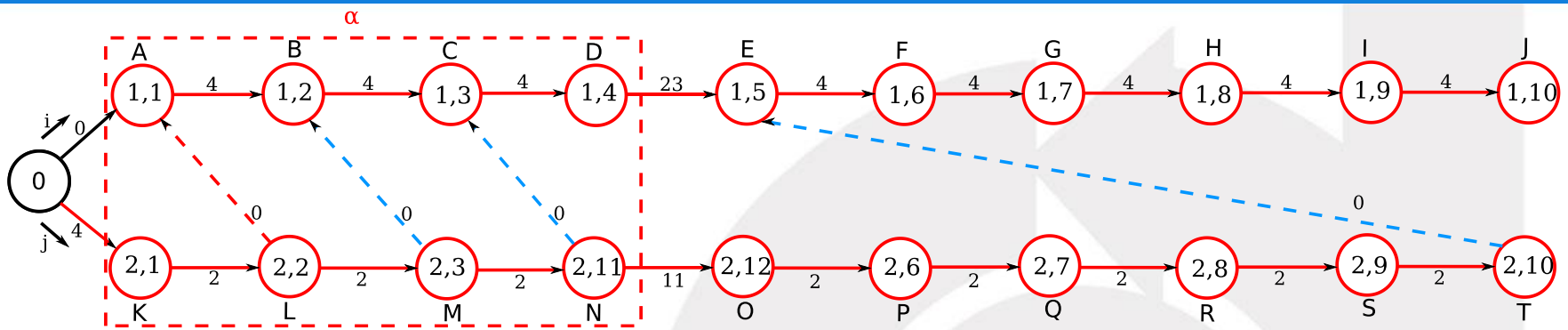
Constraint resolution



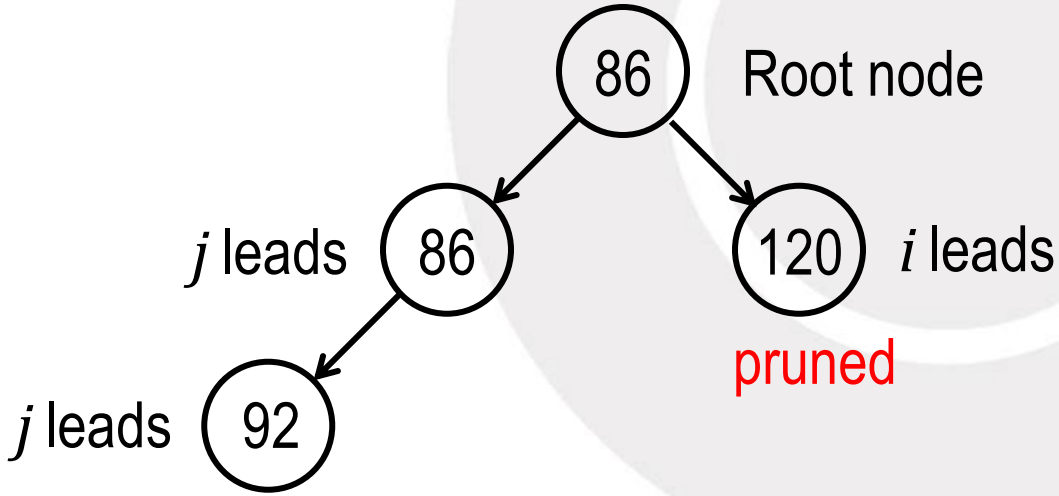


Upper bound = 93

Constraint resolution

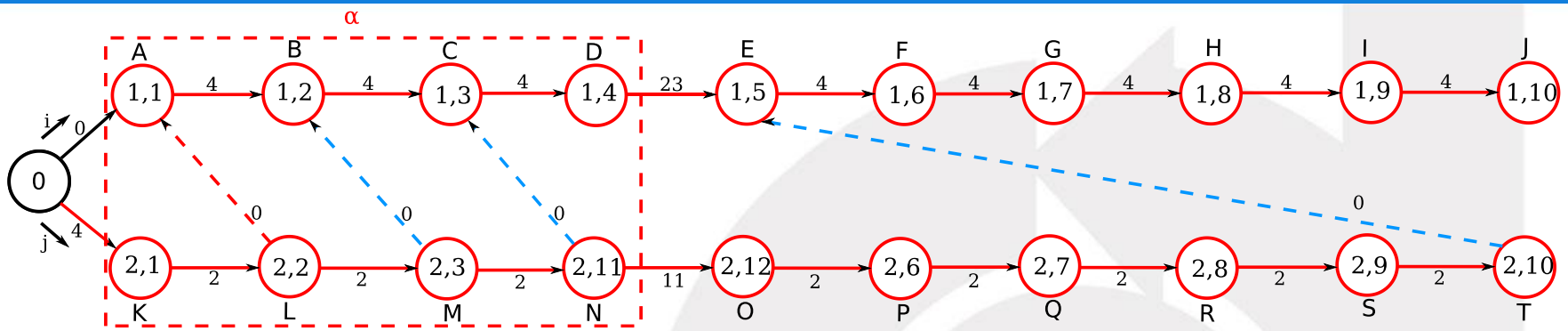


Longest path from origin to destination (Cost) : 61 + 31 = 92

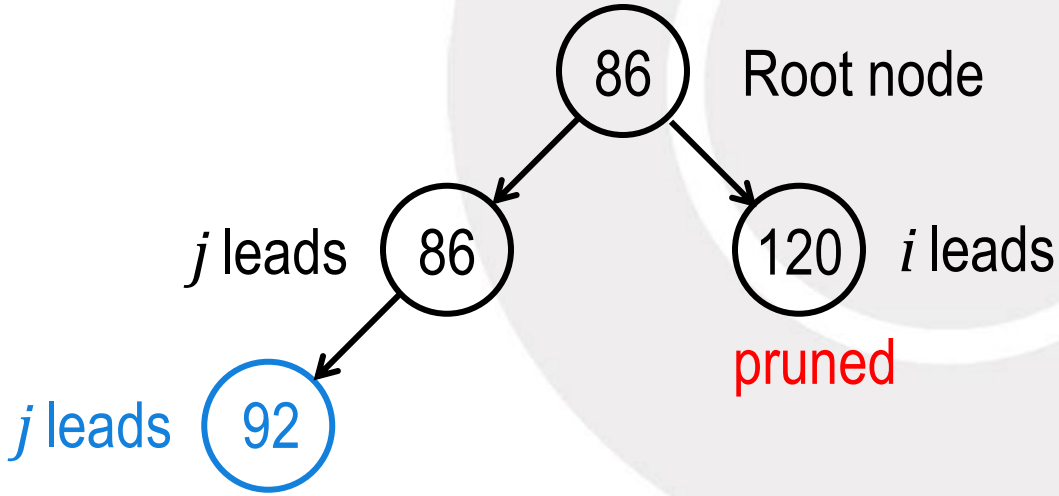


Upper bound = 93

Constraint resolution

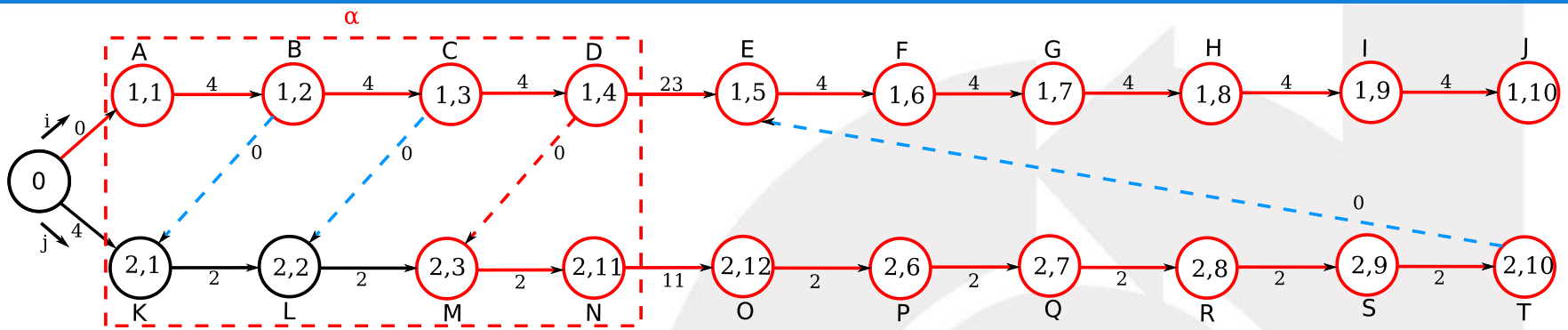


Longest path from origin to destination (Cost) : 61 + 31 = 92

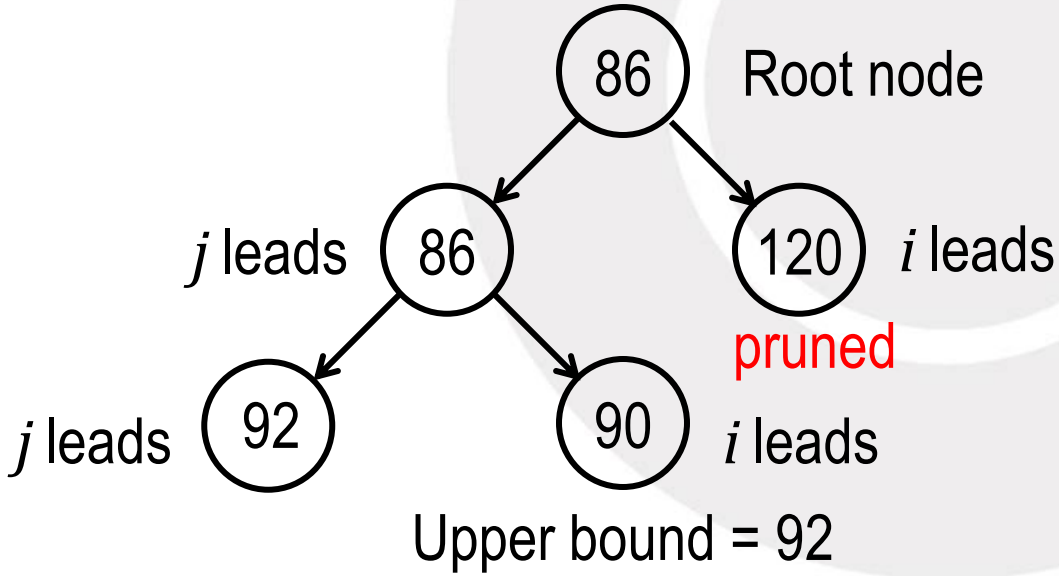


Upper bound = 92

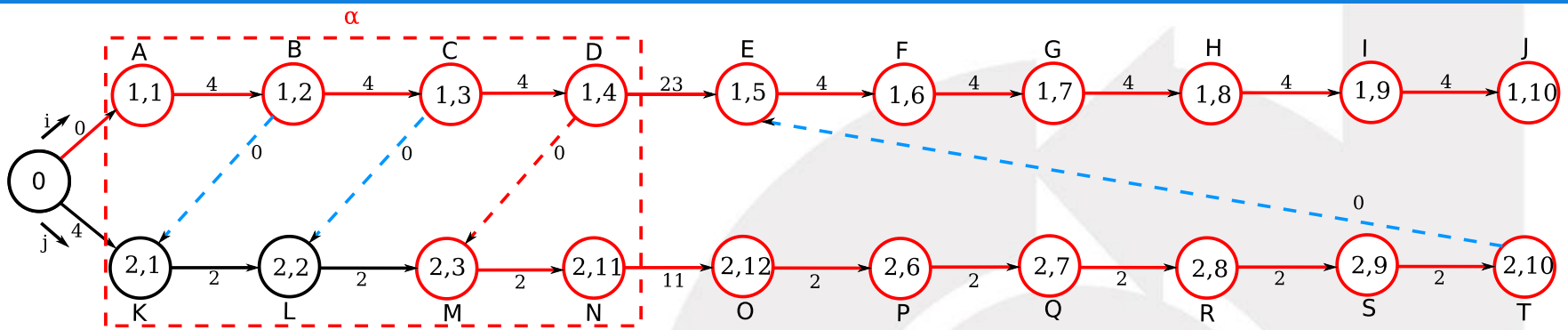
Constraint resolution



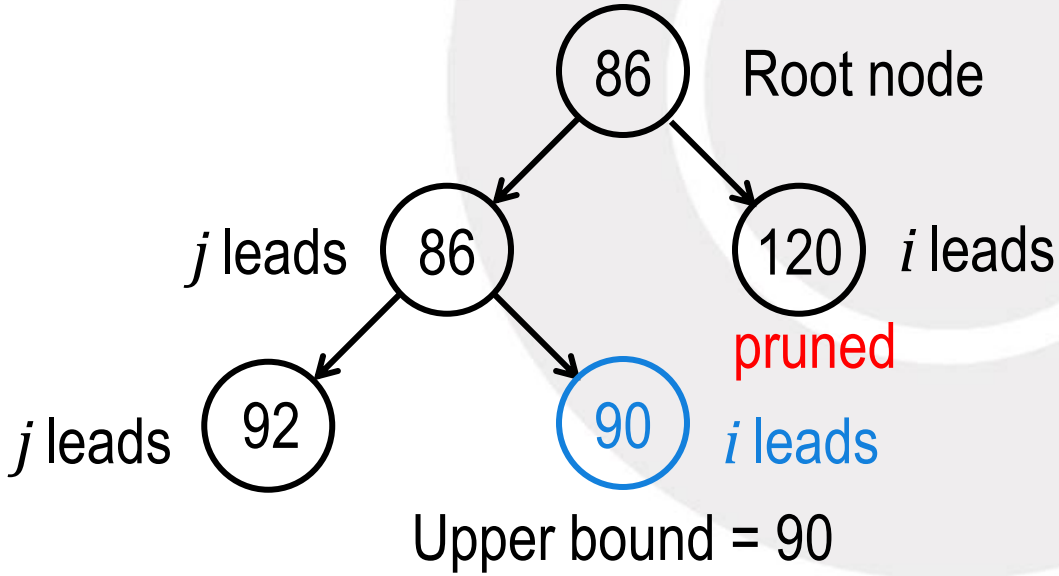
Longest path from origin to destination (Cost) : 55 + 35 = 90



Constraint resolution



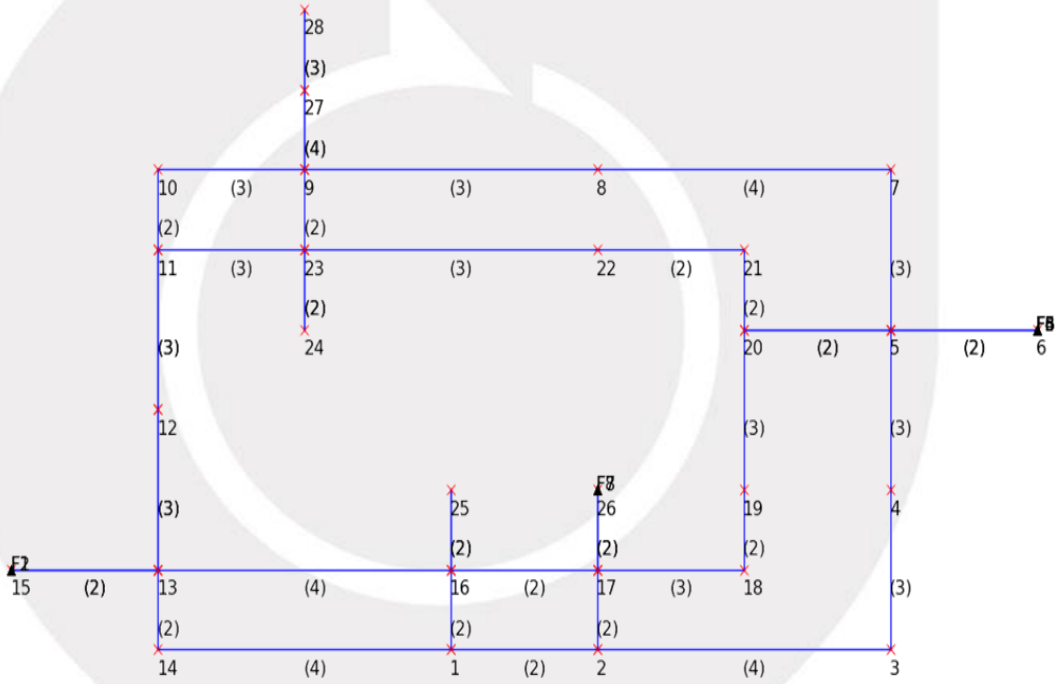
Longest path from origin to destination (Cost) : 55 + 35 = 90



Results

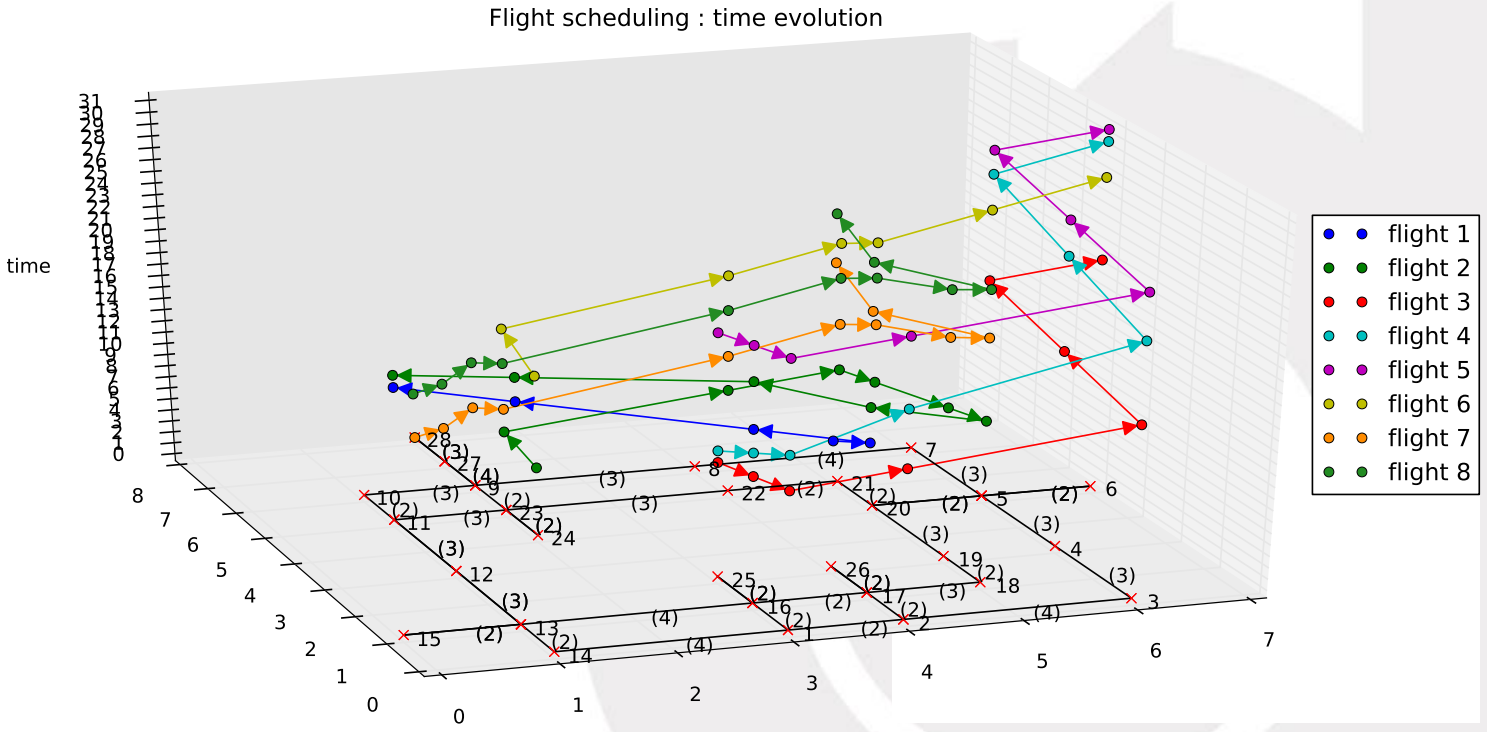
"Optimal airport surface traffic planning using mixed-integer linear programming"(Roling et al.)

Fno.	O	D	t_0	Sp	T_{sep}	P
1	26	15	7	1	4	1
2	24	15	6	2	4	1
3	25	6	10	2	4	1
4	25	6	8	1	4	1
5	25	6	16	2	4	1
6	24	6	14	1	4	1
7	28	26	0	1	4	1
8	28	26	3	1	4	1



Runway constraint relaxed

Results

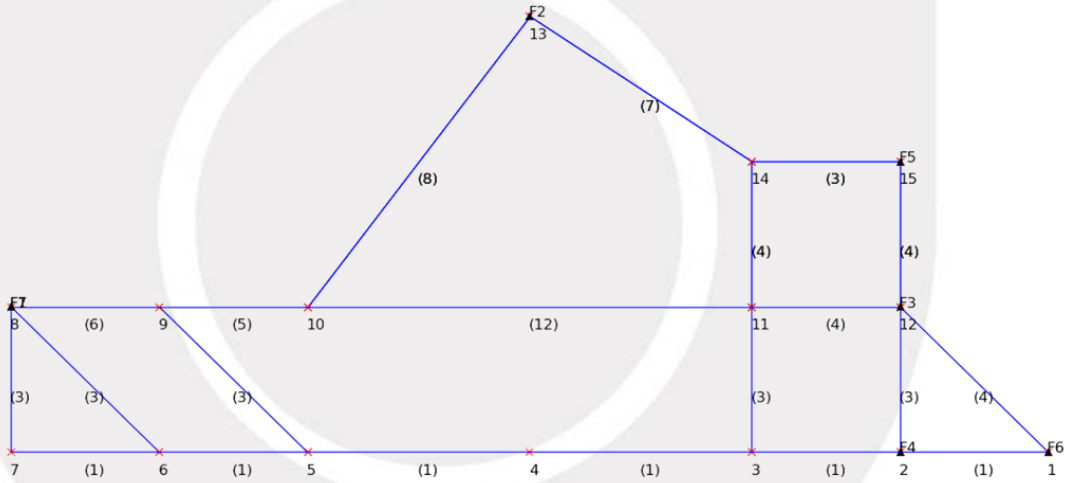


Roling et al. solution		B&B solution (Global optimum)			Flight-path combinations	Cost improvement
Cost	Runtime	Cost	Time to find optimum	Total runtime		
362	–	204	1 sec	7 sec	7776	75%

Results

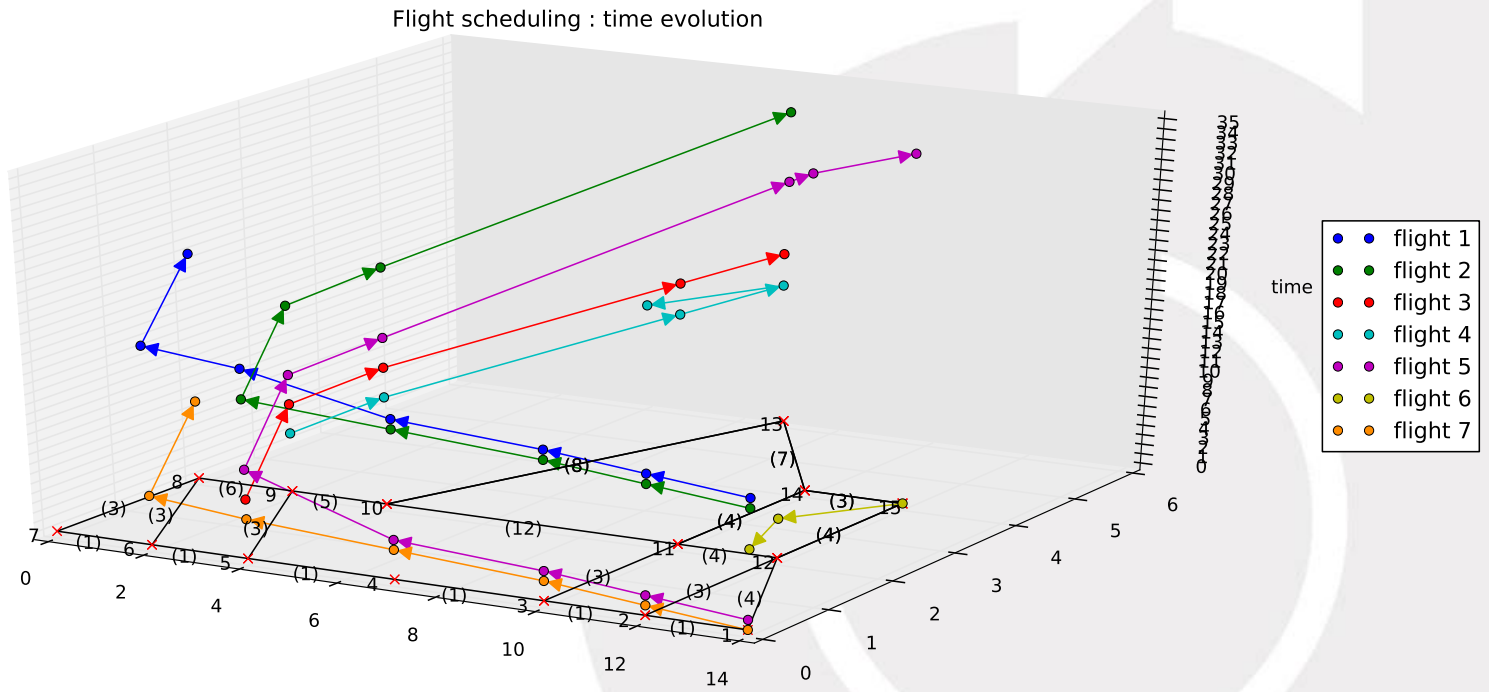
"Airport ground movement optimization using bacterial foraging algorithm"(Baijal et al.)

Fno.	O	D	t_0	Sp	T_{sep}	P
1	1	8	13	1	3	1
2	1	13	12	1	3	1
3	5	12	6	1	3	1
4	9	2	6	1	3	1
5	1	15	0	1	3	1
6	15	1	0	1	3	1
7	1	8	0	1	3	1



Runway constraint relaxed

Results

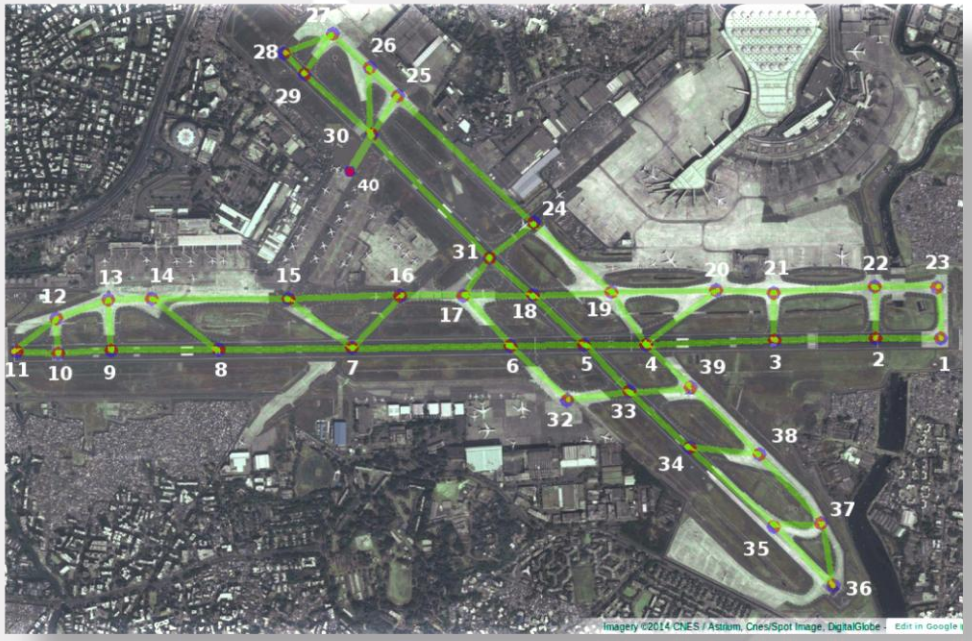


Roling et al. solution		B&B solution (Global optimum)			Flight-path combinations	Cost improvement
Cost	Runtime	Cost	Time to find optimum	Total runtime		
186	–	166	2 sec	18 sec	124416	12%

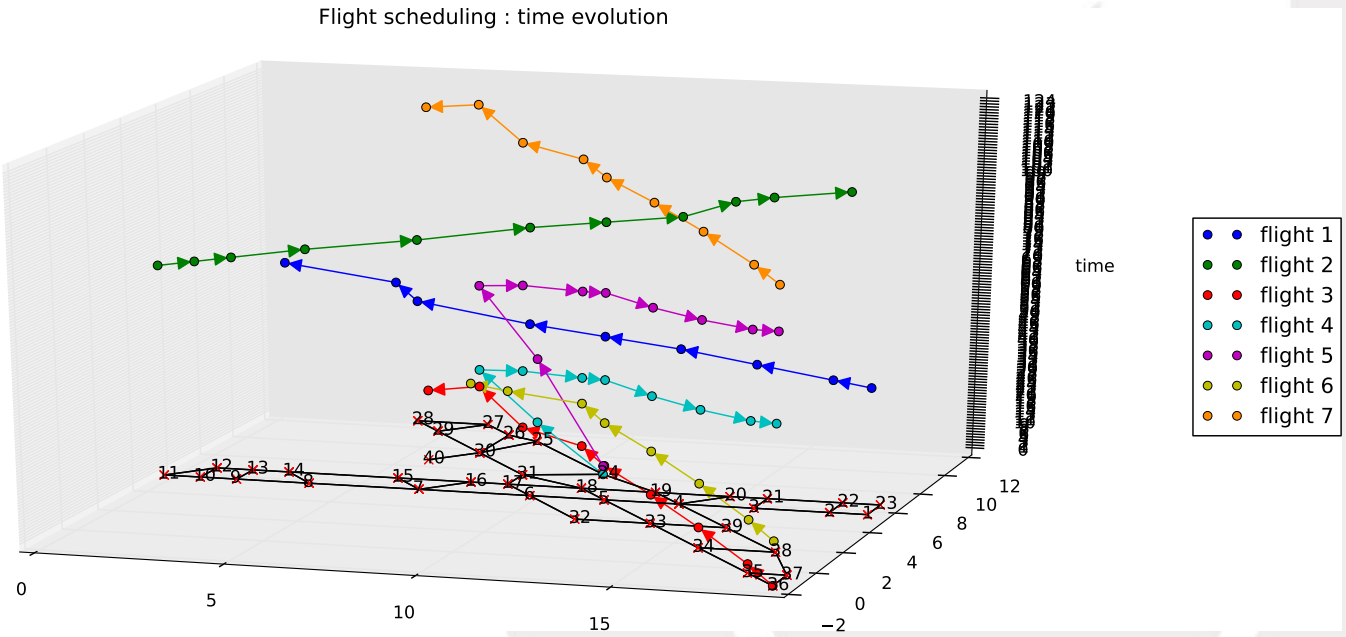
Results

Mumbai International & Domestic airport (Cross-runways)

Fno.	O	D	t_0	Sp	T_{sep}	R_{dist}	P
1	1	14	0	1	3	20	1
2	11	22	10	1	3	20	1
3	36	40	20	1	3	20	1
4	24	36	30	1	3	20	1
5	24	36	40	1	3	20	1
6	36	16	50	1	3	15	1
7	36	40	60	1	3	20	1



Results



No. of flights	Optimal cost	Flight - path combinations	Time to find optimum	Runtime
4	227	120	0 sec	1 sec
5	335	480	0 sec	5 sec
6	445	1440	0 sec	1 min 14 sec
7	575	1440	2 sec	6 min 45 sec

Results

Tolerance

- Global optimum generally identified in first 1% of flight-path combinations
- Closely spaced solutions cause heavy branching
- Relax upper bound within tolerance to improve pruning
- No better solution than the relaxed upper bound exists

Tolerance / Flight	Optimal cost	Relaxed UB	Runtime
0	575	575	6 min 45 sec
2	575	561	4 min 35 sec
4	575	547	2 min 54 sec
6	575	533	1 min 53 sec
8	575	519	1 min 21 sec
10	575	505	58 sec

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Conclusions & Future work

- Aircraft Ground Movement Optimization modeled as a job-shop scheduling problem
- Key feature: Global optimum identification
- Small runtimes: Suitable for use as a real-time decision support tool
- Need to incorporate aircraft speed-profiling
- Rolling horizons for continuous scheduling like at the ATC

Thank you!