18th IEEE International Conference on Control Applications
Part of 2009 IEEE Multi-conference on Systems and Control
Saint Petersburg, Russia, July 8-10, 2009

Controller Design of an Autonomous Bicycle with Both Steering and Balancer Controls

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Abstract— In this paper, we propose a control algorithm for trajectory tracking control with self balancing of unmanned bicycle by using nonlinear control based on the output-zeroing controller. The simplified model of the bicycle with the balancer is derived from Lagrangian and nonholonomic constraints with respect to translation and rotation relative to the ground plane. We derived a controller using a steering handlebar and a balancer torque to stabilize the bicycle and we performed a local stability analysis of a closed-loop system by using the characteristic polynomial. The trajectory tracking control is derived by an input-output linearization approach to track the path in the ground plane. The proposed control algorithm is guaranteed to maintain bicycle stability even when the linear velocity is zero without requiring a secondary controller. Numerical simulation results are shown to verify the effectiveness of the proposed control strategy.

Index Terms— Balancing Control, Trajectory Tracking, Output Zeroing, Autonomous Bicycle

I. INTRODUCTION

Research on the stabilization of bicycles has gained momentum over the last decade in a number of robotic laboratories around the world. Modeling and control of bicycles became a popular topic for researchers in the latter half of the last century. The bicycle literature is comprehensively reviewed from a control theory perspective in [1], which also describes interesting bicycle-related experiments. But almost all of those papers are focusing on modeling and stabilizing the bicycle with the steering handlebar and the rear wheel. Getz [2] studied feedback control law for nonlinear, nonholonomic, nonminimum phase model of a two-wheeled bicycle with non-zero rear-wheel velocity. Yi [3] presented a trajectory tracking and balancing control for an autonomous motorcycle using only steering handlebar. Dynamic models of the motorcycle were developed from an existing modeling approach [2], modified by adding the bicycle caster angle, and the model can capture the steering effect on the vehicle tracking and balancing. In [4], the simplified dynamic model of a bicycle with a balancer was modeled by using Lagrange dynamic equations. Simulation study has been carried out to show the effectiveness of the proposed model. Yamakita [7], [8] utilized an input-output linearization method to design a trajectory tracking controller for an automatic bicycle. The control methods are designed independently for trajectory tracking and balancing. The proposed algorithm for that problem used an output function which is defined by an angular momentum and the new state function is controlled to zero. In [6], an autonomous bicycle designed and balanced based on output-zeroing controller by using only a balancer to stabilize the bicycle. The performance of this controller was not so good when the mass of the bicycle is increased. Recently, Hwang [5] proposed a controller namely variable structure under-actuated control to balance an electrical bicycle. The balancing control was designed based on a steering handlebar and a balancer and it cannot track a given trajectory.

In this paper, we used both a steering handlebar and a balancer to stabilize the bicycle and track any given complex trajectories. For the balancing control algorithm, we used output-zeroing controller and cooperate between a steering handlebar and a balancer for stabilization of the bicycle. This paper is composed of six sections. In section II, we present a simplified dynamic model of the bicycle with the balancer. In section III, we discuss control system design for balancing stabilization and trajectory tracking. Local stability analysis of closed-loop system is performed in section IV. Numerical simulation results are presented in section V. The conclusions are summarized in section VI.

II. BICYCLE DYNAMICS

In this paper, we use the simplified bicycle models that was developed in [4] and [6]. A detailed model of a bicycle is complex since the system has many degrees of freedom and the motions have constraints. The coordinate system used to analyze the bicycle is defined in Fig. 1. The details of the bicycle assumptions are presented in [4]. The bicycle and the balancer parameters were identified from an experimental setup and it is shown in Table I. The key parameters are:

- β Balancer angle
- ψ Steering shaft angle
- α Roll angle
- θ Yaw angle
- ϕ Front wheel direction angle
- v_r Rear wheel longitudinal velocity
- $v_\perp$ Lateral velocity

We will parameterize the steering angle by

$$\sigma = \frac{\tan\phi}{b}.$$

We consider the generalized velocities $\dot{x}$, $\dot{y}$, $\dot{\theta}$, $\dot{\alpha}$, $\dot{\beta}$, and $\dot{\sigma}$ such that it will be convenient when we introduce the constraints. The generalized velocities of the bicycle with the balancer are partitioned as $\dot{r} = [\dot{\alpha}, \quad v_r, \quad \dot{\beta}, \quad \dot{\sigma}]^T$ and $\dot{s} = [\dot{\theta}, \quad v_\perp]^T$. In these velocity coordinates, the nonholonomic constraints associated with the front and the

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TABLE I

THE BICYCLE AND CONTROL PARAMETERS

Parameter descriptions	Parameters	Value
Bicycle mass	m	45 Kg
Height of the bicycle center of mass	h	0.45 m
Distance between ground and balancer	l_1	0.81 m
Bicycle wheelbase	b	1.06 m
Distance between rear wheel and COG	c	0.5 m
Distance front wheel and reference point	l	0.2 m
Bicycle head angle	η	65°
Bicycle trail	Δ	0.2 m
Moment inertia of steering mechanism	J_s	0.35 Kgm^2
Balancer mass	m_b	13.2 Kg
Height of the balancer center of mass	h_b	0.29 m
Moment inertia of balancer	I_b	0.22 Kgm^2

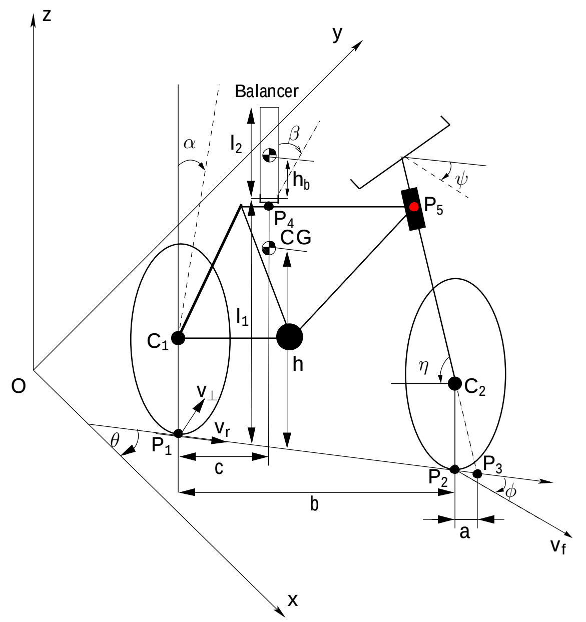

Fig. 1. Coordinate system of the bicycle with the balancer.

rear wheels which are assumed to roll without slipping, are expressed simply by $\dot{s} + A(r,s)\dot{r} = 0$ or

$$\begin{bmatrix} \dot{\theta} \\ v_{\perp} \end{bmatrix} + \begin{bmatrix} 0 & -\sigma & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\alpha} & v_r & \dot{\beta} & \dot{\sigma} \end{bmatrix}^T = 0. \quad (1)$$

From [4], we can obtain the bicycle dynamics with nonholonomic constraints as

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 \\ M_{21} & M_{22} & 0 & 0 \\ M_{31} & 0 & M_{33} & 0 \\ 0 & 0 & 0 & M_{44} \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \dot{v}_r \\ \ddot{\beta} \\ \ddot{\sigma} \end{bmatrix} = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_r \\ \tau_b \\ \tau_\sigma \end{bmatrix}, \quad (2)$$

where F_r is the reaction force at the rear wheel, τ_b is the balancer torque input, τ_σ is the steering torque input, and

$$\begin{aligned}
M_{11} &= I_b + mh^2 + m_b(h_b^2 + l_1^2) + 2h_b l_1 m_b \cos(\beta), \\
M_{12} &= M_{21} = -chm\sigma \cos\alpha, \\
M_{13} &= M_{31} = I_b + h_b m_b (h_b + l_1 \cos\beta), \\
M_{22} &= m(c^2\sigma^2 + (1 + h\sigma \sin\alpha)^2) + \\
&\quad m_b(1 + l_1\sigma \sin\alpha + h_b\sigma \sin(\alpha+\beta))^2, \\
M_{33} &= I_b + h_b^2 m_b, \\
M_{44} &= \frac{J_s b^2}{\sin^2\eta(1 + b^2\sigma^2)^2}, \\
K_1 &= cmg\Delta\sigma \sin\eta \cos\alpha + hm(g \sin\alpha + v_r(\sigma v_r(1 + \\
&\quad h\sigma \sin\alpha) + c\dot{\sigma}) \cos\alpha) + m_b(g(l_1 \sin\alpha + \\
&\quad h_b \sin(\alpha+\beta)) + \sigma v_r^2(l_1(1 + l_1\sigma \sin\alpha) \cos\alpha + \\
&\quad h_b(1 + h_b\sigma \sin(\alpha+\beta)) \cos(\alpha+\beta) + \\
&\quad h_b l_1 \sigma \sin(2\alpha+\beta)) + h_b l_1 \dot{\beta}(2\dot{\alpha} + \dot{\beta}) \sin\beta), \\
K_2 &= -m(h(c\sigma\dot{\alpha}^2 + v_r\dot{\sigma}(1 + h\sigma \sin\alpha)) \sin\alpha + \\
&\quad \sigma v_r(2h\dot{\alpha}(1 + h\sigma \sin\alpha) \cos\alpha + c^2\dot{\sigma}) \\
&\quad -2ch\dot{\alpha}\dot{\sigma} \cos\alpha) - m_b(2h_b\sigma v_r((\dot{\beta} + \dot{\alpha})(1 \\
&\quad +h_b\sigma \sin(\alpha+\beta)) + l_1\sigma\dot{\beta} \sin\alpha) \cos(\alpha+\beta) \\
&\quad +\sigma v_r\dot{\sigma}((l_1 \sin\alpha + h_b \sin(\alpha+\beta))^2) \\
&\quad +h_b v_r\dot{\sigma} \sin(\alpha+\beta) + l_1 v_r\dot{\sigma} \sin\alpha + 2l_1\sigma v_r\dot{\alpha}((1 \\
&\quad +l_1\sigma \sin\alpha) \cos\alpha + h_b\sigma \sin(\alpha+\beta))), \\
K_3 &= h_b m_b(-l_1\dot{\alpha}^2 \sin\beta + g \sin(\alpha+\beta) + \\
&\quad v_r^2\sigma \cos(\alpha+\beta)(1 + \sigma(l_1 \sin\alpha + h_b \sin(\alpha+\beta))), \\
K_4 &= c\Delta gm \sin\eta \sin\alpha - 2chmv_r\dot{\alpha} \cos\alpha \\
&\quad + \frac{2b^4 J_s\sigma\dot{\sigma}^3}{\sin^2\eta(1 + b^2\sigma^2)^3}.
\end{aligned}$$

This gives us a simplified model that we can use to design the controllers for the bicycle stabilization and trajectory tracking.

III. CONTROL ALGORITHM

The control algorithm has to satisfy two requirements. First, the bicycle has to balance autonomously without falling down even when the speed is zero. Secondly, the bicycle must track any smooth desired trajectory.

A. Balancing Control

1) *Model of two-link system:* By projecting the motion of the balancer on $X-Z$ plane, the system can be considered as a two-link system. In the two-link model, the bicycle body and steering handlebar consist of the first link and the balancer is considered as the second link. The control torque for the system is only applied to the second joint of the balancer. We can find the angular momentum L from the first row of equation (2) as

$$\begin{aligned} L &= M_{11}\dot{\alpha} + M_{13}\dot{\beta} \\ &= (d_1 + d_3 + 2d_2 \cos\beta)\,\dot{\alpha} + (d_3 + d_2 \cos\beta)\,\dot{\beta}, \end{aligned} \quad (3)$$

where

$$\begin{aligned} d_1 &= mh^2 + m_b l_1^2, \quad d_2 = m_b l_1 h_b, \\ d_3 &= m_b h_b^2 + I_b. \end{aligned}$$

and it can be easily shown that the time derivative of L just contains a gravity term and it is calculated as

$$\dot{L} = e_1\sigma\cos\alpha + e_2\sin\alpha + e_3\sin(\alpha+\beta), \tag{4}$$

where

$$\begin{aligned} e_1 &= cmg\Delta\sin\eta, \quad e_2 = g(mh + m_b l_1), \\ e_3 &= gm_b h_b. \end{aligned}$$

In (4), we can see that it contains a term σ which means that the steering actions affect to the momentum. Using the angular momentum expressed in (3), a new function p is defined to satisfy the following:

$$L = (d_1 + d_3 + 2d_2\cos\beta)\,\dot{p}. \tag{5}$$

From the equation above, p can be determined as

$$\begin{aligned} p &= \alpha + \int_{\beta_0}^{\beta} \frac{d_3 + d_2\cos\beta}{d_1 + d_3 + 2d_2\cos\beta} d\beta - C \\ &= \alpha + w(\beta), \end{aligned} \tag{6}$$

where C is an integral constant and is determined as $p = 0$ when the system is at the upright position. Using L and p, a new coordinate function $q = (p, L, \beta, \dot{\beta}, \sigma, \dot{\sigma})$ can be represented as

$$\begin{bmatrix} \dot{p} \\ \dot{L} \\ \dot{\beta} \\ \ddot{\beta} \\ \dot{\sigma} \\ \ddot{\sigma} \end{bmatrix} = \begin{bmatrix} L/H(\beta) \\ G(p,\beta,\sigma) \\ \dot{\beta} \\ 0 \\ \dot{\sigma} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_b \\ u_\sigma \end{bmatrix}, \tag{7}$$

where

$$\begin{aligned} H(\beta) &:= d_1 + d_3 + 2d_2\cos\beta, \\ G(p,\beta,\sigma) &:= e_1\sigma\cos(p - w(\beta)) + e_2\sin(p - w(\beta)) \\ &\quad + e_3\sin(p - w(\beta) - \beta). \end{aligned}$$

and u_b, u_σ are new inputs defined as $u_b := \ddot{\beta}$, $u_\sigma := \ddot{\sigma}$ respectively.

2) *Output-zeroing controller:* For the system (7), an output function y is defined as

$$y = L + a_1 p, \tag{8}$$

where $a_1 > 0$ is a design parameter and p is a new coordinate function satisfying

$$\dot{p} = L/H(\beta).$$

Since L and p have relative degree 3 to the control input, we can easily determine a control input which attains the dynamics of the output function. By taking a derivative of L, we have

$$y^{(3)} = L^{(3)} + a_1 p^{(3)}, \tag{9}$$

$$\begin{aligned} \ddot{L} &= \frac{dG}{dt} = \frac{\partial G}{\partial p}\frac{L}{H} + \frac{\partial G}{\partial \beta}\dot{\beta} + \frac{\partial G}{\partial \sigma}\dot{\sigma}, && (10) \\ L^{(3)} &= \frac{d}{dt}\left(\frac{\partial G}{\partial p}\frac{L}{H}\right) + \frac{d}{dt}\left(\frac{\partial G}{\partial \beta}\right)\dot{\beta} + \frac{\partial G}{\partial \beta}u_b && (11) \\ &\quad + \frac{d}{dt}\left(\frac{\partial G}{\partial \sigma}\right)\dot{\sigma} + \frac{\partial G}{\partial \sigma}u_\sigma, \\ \ddot{p} &= \frac{G}{H} - \frac{(\partial H/\partial\beta)\,L}{H^2}\dot{\beta}, && (12) \\ p^{(3)} &= \frac{d}{dt}\left(\frac{G}{H}\right) - \frac{d}{dt}\left(\frac{(\partial H/\partial\beta)\,L}{H^2}\right)\dot{\beta} && (13) \\ &\quad - \frac{(\partial H/\partial\beta)\,L}{H^2}u_b. \end{aligned}$$

We can determine a control input which attains the dynamics of the output function as

$$y^{(3)} + a_2\ddot{y} + a_3\dot{y} + a_4 y = 0, \tag{14}$$

and y converges to zero asymptotically if $a_2 > 0$, $a_3 > 0$, $a_4 > 0$ are determined appropriately.

Let us assume that

$$\frac{u_\sigma}{u_b} = \gamma, \tag{15}$$

where $\gamma < 0$. By rearranging (8) to (15) then the control input u_b is given by

$$\begin{aligned} u_b = -&\left(\frac{\partial G}{\partial \beta} + \gamma\frac{\partial G}{\partial \sigma} - a_1\frac{(\partial H/\partial\beta)\,L}{H^2}\right)^{-1} \\ &\left(\frac{d}{dt}\left(\frac{\partial G}{\partial p}\frac{L}{H}\right) + \frac{d}{dt}\left(\frac{\partial G}{\partial \beta}\right)\dot{\beta} + \frac{d}{dt}\left(\frac{\partial G}{\partial \sigma}\right)\dot{\sigma}\right. \\ &- a_1\frac{d}{dt}\left(\frac{(\partial H/\partial\beta)\,L}{H^2}\right)\dot{\beta} + a_1\frac{d}{dt}\left(\frac{G}{H}\right) \\ &\left. + a_2\ddot{y} + a_3\dot{y} + a_4 y\right), \end{aligned} \tag{16}$$

and the control input u_σ is given by

$$u_\sigma = \gamma u_b. \tag{17}$$

The zero dynamics become stable and all states $[y, \dot{y}, \ddot{y}, p]^T$ converge to zero.

B. Trajectory Control

In this section, we will take (x_r, y_r) as our reference position and cause this position to track desired counterparts (x_d, y_d). From [6] and Figure 2, the trajectory control

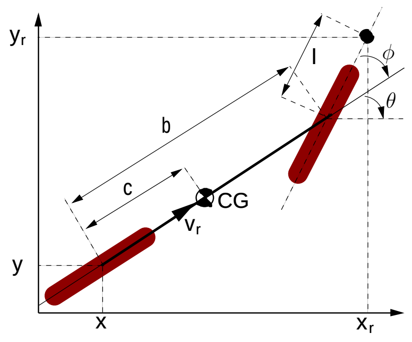

Fig. 2. Top-view of bicycle model.

equations can be written as

$$\begin{bmatrix} \ddot{x}_r \\ \ddot{y}_r \end{bmatrix} = \begin{bmatrix} (E+F)\begin{bmatrix} 1 \\ 0 \end{bmatrix}(1+\sigma) & F\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{b}{1+b^2\sigma^2} \end{bmatrix} \cdot \begin{bmatrix} \dot{v}_r \\ \ddot{\sigma} \end{bmatrix} + E\begin{bmatrix} 0 \\ 1 \end{bmatrix}\sigma,$$

$$\begin{bmatrix} \ddot{x}_r \\ \ddot{y}_r \end{bmatrix} = X\begin{bmatrix} \dot{v}_r \\ \ddot{\sigma} \end{bmatrix} + Y, \tag{18}$$

where

$$E = \begin{bmatrix} -b\sin\theta - l\sin(\theta+\phi) & -l\sin(\theta+\phi) \\ b\cos\theta + l\cos(\theta+\phi) & l\cos(\theta+\phi) \end{bmatrix},$$

$$F = \begin{bmatrix} \cos\theta & -v_r^2\sin\theta \\ \sin\theta & v_r^2\cos\theta \end{bmatrix}.$$

We define a new input for the rear wheel as $u_r = \dot{v}_r$ and u_σ is the same as the previous section. Finally, we can design an external system controller to track the desire trajectory (x_d, y_d) asymptotically gives

$$\begin{bmatrix} u_r \\ u_\sigma \end{bmatrix} = X^{-1}\left(\begin{bmatrix} u_x \\ u_y \end{bmatrix} - Y\right), \tag{19}$$

where

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} \ddot{x}_d \\ \ddot{y}_d \end{bmatrix} - b_i \sum_{i=1}^{2} \begin{bmatrix} x_r^{(i-1)} - x_d^{(i-1)} \\ y_r^{(i-1)} - y_d^{(i-1)} \end{bmatrix}. \tag{20}$$

The constants b_i, $i = 1, 2$, are chosen such that the polynomial equation $s^2 + b_2 s + b_1 = 0$ is Hurwitz.

IV. Local Stability Analysis of Closed-Loop System

The local stability of the closed-loop system for the balancing control is shown by an approximated linearized model of the bicycle when the bicycle at zero speed. When we assume $v_r = 0$, we could not find Lyapunov function for the nonlinear system.

A. Bicycle Linear Model

From [4], The bicycle linear model around upright position can be written as:

$$\dot{x} = Ax + Bu, \tag{21}$$

where $x = [\alpha, \beta, \sigma, \dot{\alpha}, \dot{\beta}, \dot{\sigma}]^T$ is a state variable.
By using the nominal parameter value of the bicycle and balancer (in Table I), we can find the elements of the state equation as

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 44.21 & -15.4 & 1.11 & 0 & 0 & 0 \\ -175.5 & 79.612 & -3.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2.1 \\ 0 & 0 & 0 & -0.316 & 1.8 & 0 \end{bmatrix}^T.$$

The eigenvalues of the system are 10.81, -10.81, 2.64, -2.64, 0, 0. There are some positive eigenvalues and we can conclude that the open-loop linear system is unstable.

B. Output-Zeroing Linearization

We can linearize the output-zeroing controller by assuming that $\cos\alpha \approx 1$, $\sin\alpha \approx \alpha$, $\cos\beta \approx 1$, $\sin\beta \approx \beta$, $\dot{\alpha}^2 \approx 0$, $\dot{\beta}^2 \approx 0$ and $\dot{\alpha}\dot{\beta} \approx 0$ and substituting the parameters in Table II into the control input equation. Output-zeroing controller

TABLE II

The Control Parameters for Balancing

Parameters	Values	Parameters	Values	Parameters	Values
a_1	150	a_2	30	b_1	15
a_3	80	a_4	150	b_2	80
γ	-0.8				

is a state feedback controller. In order to simplify the control input, we combine actual control (input transformation from $\ddot{x}$ to u) [6] and linear output-zeroing controller. Then we will get a new control input as

$$u = kx, \tag{22}$$

where

$$k = \begin{bmatrix} 590.5 & -70.9 & 48.6 & 120.3 & 6.1 & 6.2 \\ -340.8 & 18.5 & -32.2 & -83.1 & -4.2 & -4.3 \end{bmatrix}.$$

The block diagram shows the closed-loop system.

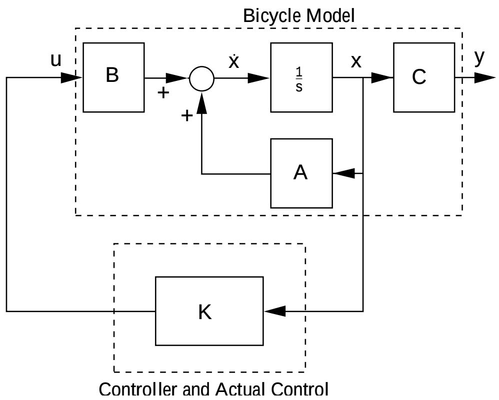

Fig. 3. Bicycle closed-loop control.

C. Closed-Loop System

From Fig. 3, we can find the closed-loop system as

$$A_c = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -142 & 6.98 & -14 & -38 & -1.9 & -2 \\ 889.8 & -48 & 84 & 217 & 11 & 11.1 \\ -712 & 38.7 & -67 & -174 & -8.8 & -9 \end{bmatrix},$$

$$B_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2.1 \\ 0 & 0 & 0 & -0.316 & 1.8 & 0 \end{bmatrix}^T,$$

and the eigenvalues of the closed-loop system are -26.95, -6.69, -2.03, -0.26, $-6.8 \times 10^{-16} + i9.98 \times 10^{-8}$, $-6.8 \times 10^{-16} - i9.98 \times 10^{-8}$. We can see that all eigenvalues are negative and we can conclude that the balancing control system is stable.

V. Numerical Simulation

The simulation is conducted on an Intel Core 2 Duo, 2.2GHz, 2GB RAM computer, and all simulations were performed in MATLAB using an adaptive step-size Runge-Kutta integrator, ode45. In order to explain the effectiveness of the proposed method, several numerical simulations are shown where the parameters of bicycle and control parameters are shown in Table I and Table II. The parameters of the bicycle were identified from an experimental setup. Figure 4 shows the bicycle closed-loop control for balancing and trajectory tracking. w_1 and w_2 are weighting gains of steering input for balancing control and trajectory control. In this case, we chose weighting gains $w_1 = w_2 = 0.5$ for the simulation.

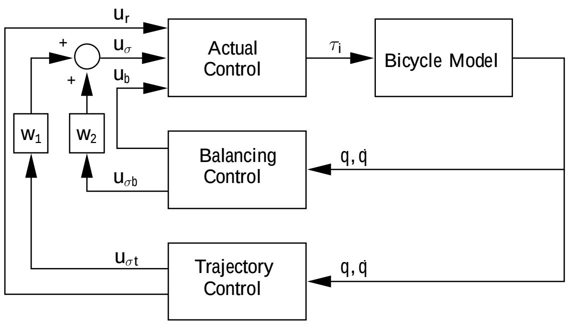

Fig. 4. Bicycle closed-loop control.

A. Balancing at Zero Velocity

To show the validity of the proposed control for bicycle stabilization at zero velocity, we perform two simulations with the same parameters to compare the response of the bicycle stability.

1) Bicycle stabilization with only balancer (Fig. 5)
2) Bicycle stabilization with a steering handlebar and a balancer (Fig.6)

For the bicycle with only the balancer, it can stabilize the bicycle under maximum initial conditions roll angle $\alpha_0 = 7°$. However, for the bicycle with a steering handlebar and a balancer, it can stabilize the bicycle with maximum initial roll angle until $\alpha_0 = 13°$. In Fig. 5 and 6, it can be seen that the state converges to the upright position. From this simulation results, we clearly see that the new proposed control has better performance than the previous one [6] and the designed output-zeroing controller can stabilize the bicycle even when the forward velocity is zero.

B. Path Tracking with Balance

We show the results of two simulations of path tracking using two different reference trajectories: a straight line and a circle path.

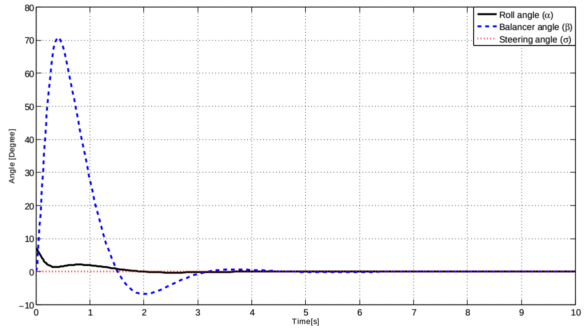

Fig. 5. Roll angle, Balancer angle and Steering angle.

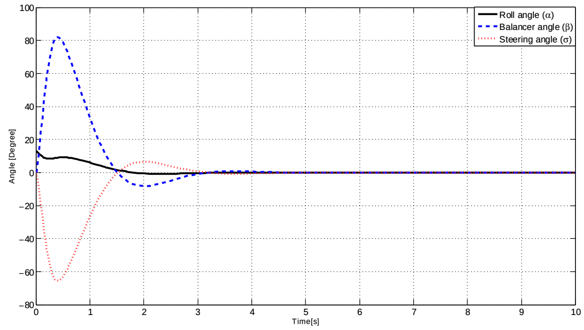

Fig. 6. Roll angle, Balancer angle and Steering angle.

1) Straight Path with variable Speed: For the first simulation the desired path trajectory was along a straight line with variable speed.

$$x_{rd} = \begin{cases} 2t, & t \leq 5s; \\ 10, & 5s < t < 8s; \\ 0.5t^2 - 8t + 42, & t \geq 8s. \end{cases} \tag{23}$$

$$y_{rd} = 0. \tag{24}$$

The initial positions for this simulation is $x_r(0) = 0[m]$ and $y_r(0) = 2[m]$. Fig. 7(a) shows the roll angle, balancer angle and steering angle versus t. Fig. 7(b) shows the angular velocity of system. Fig. 7(c) shows the rear wheel linear velocity $v_{rd} = \sqrt{\dot{x}^2 + \dot{y}^2}$. It can be seen that the bicycle has maintained stability at all times even when we change the desired velocity. The tracking of the desired trajectory is presented in Fig. 8.

2) Circle Path: For the second simulation, the desired path trajectory was a circle at constant speed 2[m/s] and

$$x_{rd}(t) = 10\sin(2t/10), \tag{25}$$

$$y_{rd}(t) = 10\cos(2t/10). \tag{26}$$

The initial positions for this simulation are $x_r(0) = 0[m]$ and $y_r(0) = 9[m]$. Fig. 9(a) shows the roll angle, balancer angle and steering angle versus t. Fig. 9(b) shows the angular velocity versus t. Fig. 9(c) shows the control input of the balancer torque and the steering torque versus t. Note that

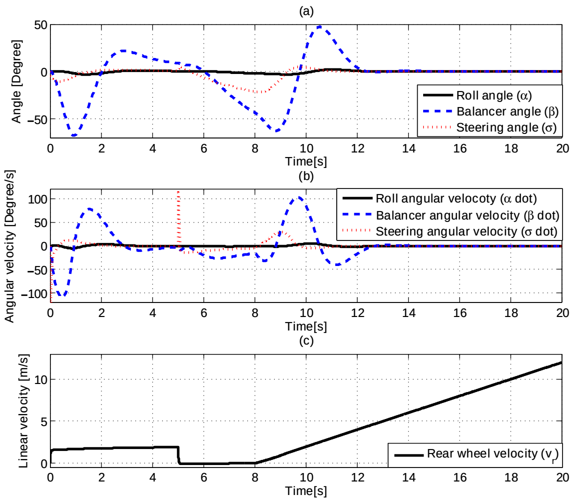

Fig. 7. (a) Roll angle (α), Balancer angle (β) and Steering angle (σ), (b) Angular velocity ($\dot{\alpha}, \dot{\beta}, \dot{\sigma}$), and (c) Rear wheel linear velocity (v_r).

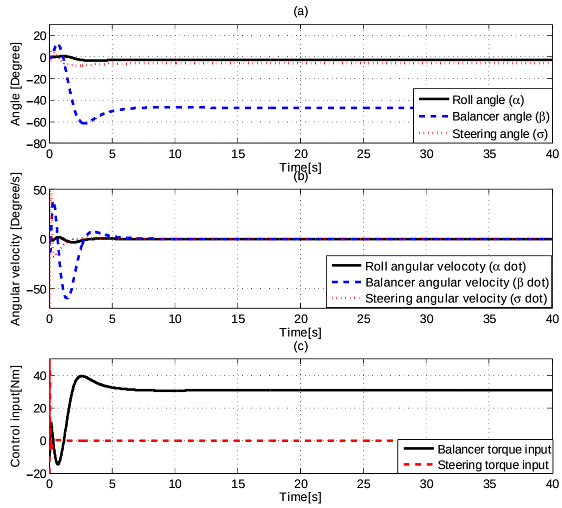

Fig. 9. (a) Roll angle (α), Balancer angle (β) and Steering angle (σ), (b) Angular velocity ($\dot{\alpha}, \dot{\beta}, \dot{\sigma}$), and (c) Control input (F_r, τ_b, τ_σ).

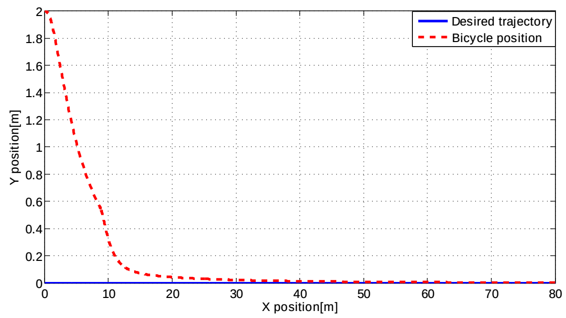

Fig. 8. Tracking a straight line in the plane.

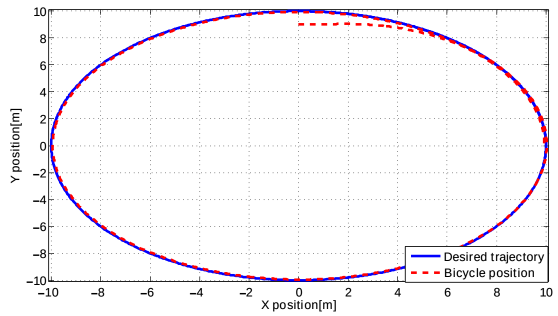

Fig. 10. Tracking a sinusoidal path in the plane.

the balancer guarantees stability of the bicycle, but not the roll angle, as the bike must lean to counteract centrifugal forces. Figure 10 shows the tracking of the circle path. The bicycle start from initial position $(0, 9m)$ then it will track to the desired path.

VI. Conclusions

In this paper, we presented a new proposed control for a bicycle with an inverted pendulum balancer and also presented a closed-loop stability analysis of the bicycle using a linear model. From simulation results of balancing bicycle with a steering handlebar and a balancer, it is shown that a novel proposed control has better performance than balancing control of a bicycle with only a balancer. The simulation results showed that the trajectory tracking and balancing control systems can work very well, even when the forward velocity is zero.

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