

Exam	City	Marks
E1	C1	30
E1	C2	50
E2	C3	10
E2	C1	60
E3	C2	40
E3	C3	50
		$\bar{y} = 40$

Exam	City
$\hookrightarrow E_1 \rightarrow 2 (40)$	$\hookrightarrow C_1 \rightarrow 2 (45)$
$E_2 \rightarrow 2 (35)$	$\hookrightarrow C_2 \rightarrow 2 (45)$
$E_3 \rightarrow 2 (45)$	$\hookrightarrow C_3 \rightarrow 2 (30)$

$$\begin{aligned}
 V(\text{Root}) &= \sum_{y_i} (y_i - \bar{y})^2 \\
 &= \frac{1}{n} \left[(30-40)^2 + (50-40)^2 + (10-40)^2 \right. \\
 &\quad \left. (60-40)^2 + (40-40)^2 + (50-40)^2 \right] \\
 &= \frac{1}{6} [400 + 100 + 900 + 400 + 0 \\
 &\quad + 100] \\
 &= \frac{1900}{6} = 316.66
 \end{aligned}$$

for exam : E₁(2, 40) E₂(2, 35) E₃(2, 45)

$$\begin{aligned}
 V(E_1) &= \frac{1}{2} [(30-40)^2 + (50-40)^2] \\
 &= \frac{1}{2} [100 + 100] \\
 &= \frac{200}{2} = 100
 \end{aligned}$$

$$\begin{aligned}
 V(E_2) &= \frac{1}{2} [(10-35)^2 + (60-35)^2] \\
 &= \frac{1}{2} [(625) + (625)] \\
 &= 625
 \end{aligned}$$

$$\begin{aligned}
 V(E_3) &= \frac{1}{2} [(40-45)^2 + (50-45)^2] \\
 &= \frac{1}{2} [25 + 25] \\
 &= 25
 \end{aligned}$$

$$V(\text{Exam}) = \frac{2}{6} [100] + \frac{2}{6} [625] + \frac{2}{6} [25]$$

$$V(E) = 33 \cdot 33 + 208 \cdot 33 + 8 \cdot 33 \\ = 249 \cdot 99$$

$$VR = 316 \cdot 66 - 249 \cdot 99 \\ = 66 \cdot 67$$

for City: $C_1(2, 45)$ $C_2(2, 45)$ $C_3(2, 30)$

$$V(C_1) = \frac{1}{2} [(30-45)^2 + (60-45)^2] \\ = \frac{1}{2} [225 + 225] \\ = 225$$

$$V(C_2) = \frac{1}{2} [(50-45)^2 + (40-45)^2] \\ = \frac{1}{2} [25 + 25] \\ = 25$$

$$V(C_3) = \frac{1}{2} [(10-30)^2 + (50-30)^2] \\ = \frac{1}{2} [400 + 400] \\ = 400$$

$$\begin{aligned}
 V(C) &= \frac{2}{6} [225] + \frac{2}{6} [25] + \frac{2}{6} [400] \\
 &= 75 + 8.33 + 133.33 \\
 &= 216.66
 \end{aligned}$$

$$\begin{aligned}
 VR &= 316.66 - 216.66 \\
 &= 100
 \end{aligned}$$

$$\begin{array}{ccc}
 66.67 & & 100 \\
 \therefore VR(E) & \swarrow & VR(C) \\
 & & \checkmark
 \end{array}$$