Regression Analysis

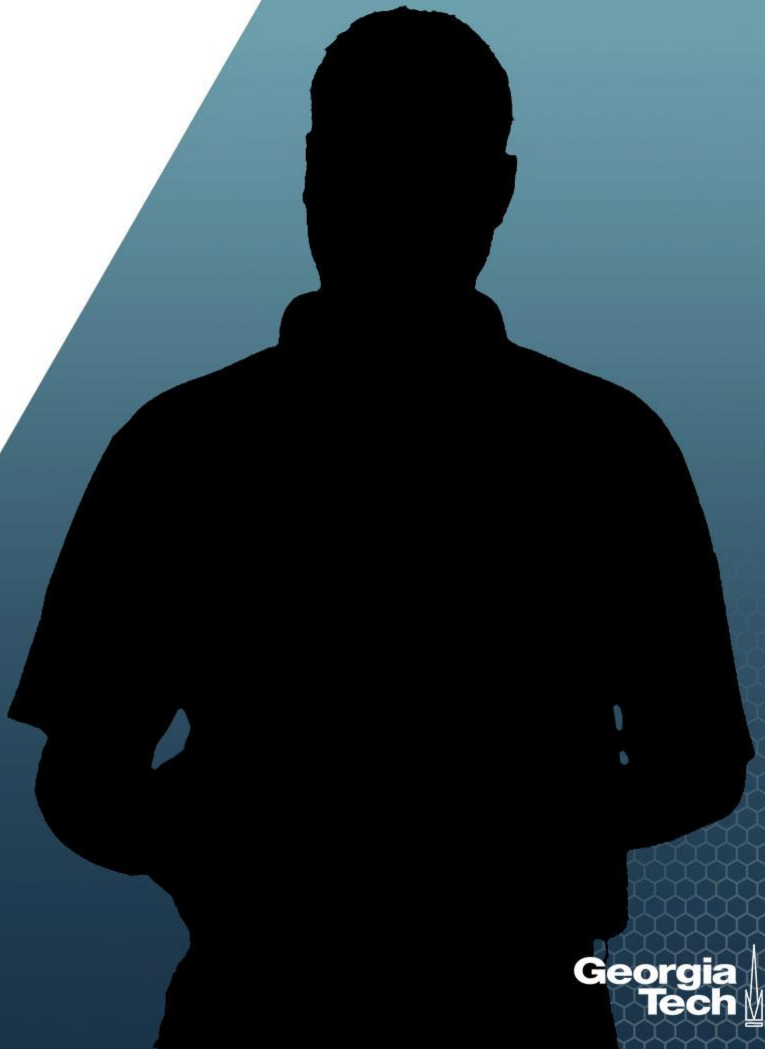
Simple Linear Regression

Nicoleta Serban, Ph.D.

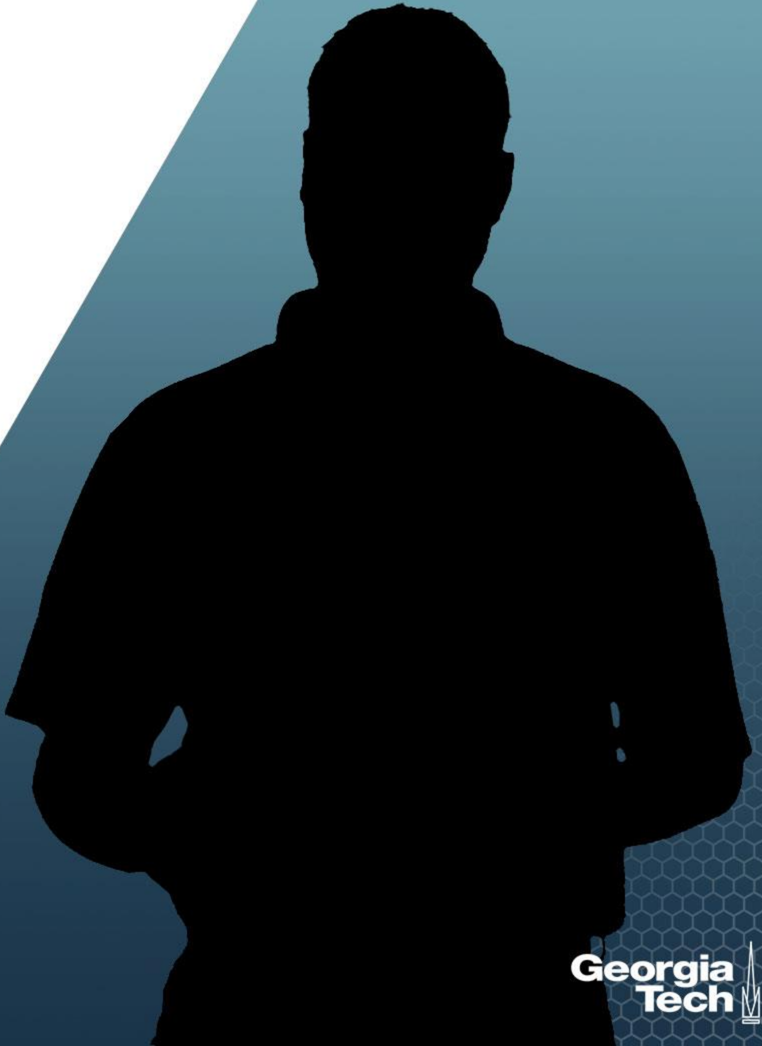
Professor

School of Industrial and Systems Engineering

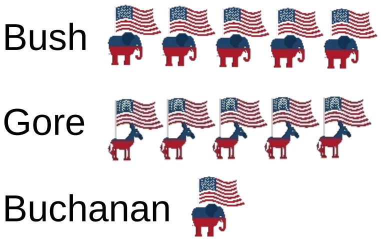
Example 2: 2000 Presidential Elections in Florida



About This Lesson



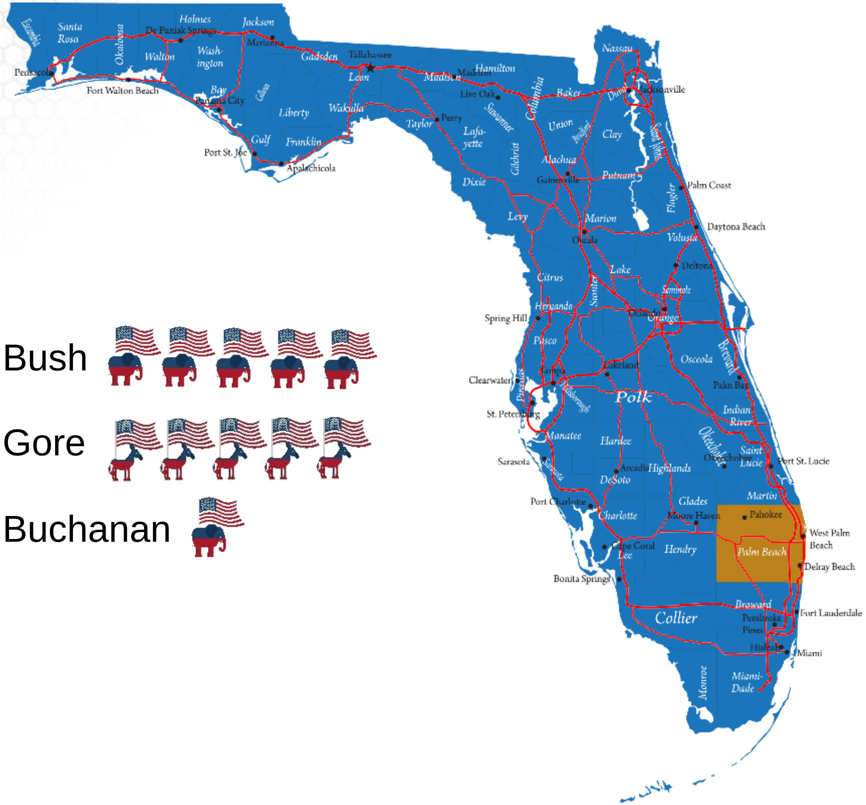
Elections in 2000: Florida



Bush

Gore

Buchanan



Data Example in R

```
## Read data with read.table R command which is used for reading ASCII files
     elections = read.table("elections.txt",header=TRUE)
```

```
## Check the data content elections[1:4,]
  co  lat   lon   npop  whit blac hisp  o65 hsed coll  inco  bush  gore brow
1  1 29.7  82.4 198326  74.4 21.8  4.7  9.4 82.7 34.6 19412 34124 47365  658
2  2 30.3  82.3  20761  82.4 16.8  1.5  7.7 64.1  5.7 14859  5610  2392   17
3  3 30.2  85.6 146223  84.2 12.4  2.4 11.9 74.7 15.7 17838 38637 18850  171
4  4 29.9  82.2  24646  76.1 22.9  2.6 11.8 65.0  8.1 13681  5414  3075   28
  nade harr hage buch mcre phil moor
1 3226    6   42  263    4   20   21
2   53    0    3   73    0    3    3
3  828    5   18  248    3   18   27
4   84    0    2   65    0    2    3
```

The data file includes many other variables characterizing the counties. We will focus only on the number of votes in this analysis.

Exploratory Data Analysis in R

### Extract number of votes for each candidates

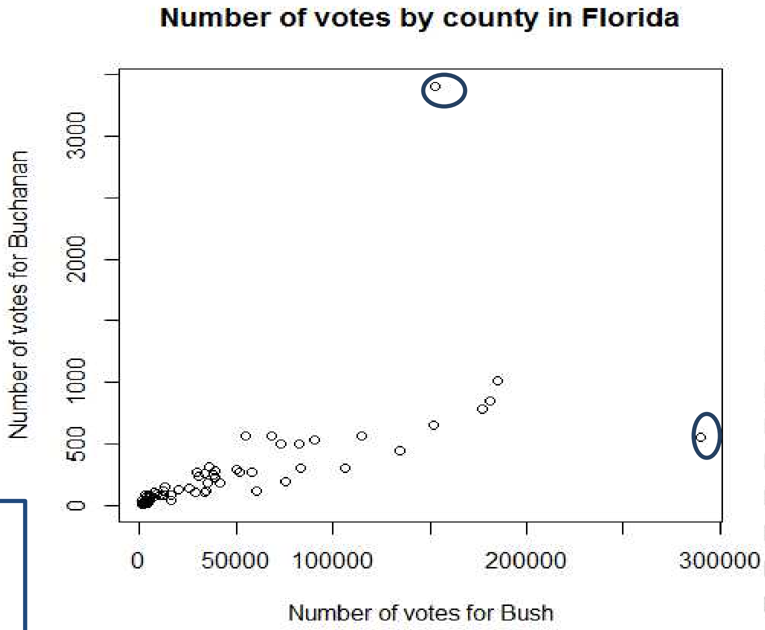
```
buch = elections$buch
bush = elections$bush
```

### Visualize the relationship between number of votes between Buchanan and Bush

```
plot(bush,buch,xlab="Number of votes for Bush",ylab="Number of votes for Buchanan", main="Number of votes by county in Florida")
cor(buch,bush)
```

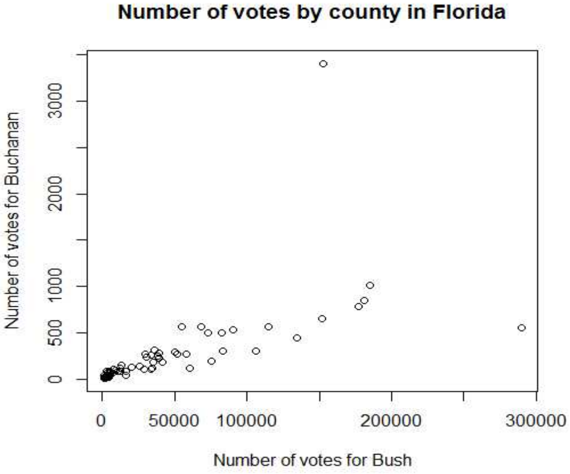
Linearity Assumption:

- The scatterplot shows a strong positive relationship between the number of votes for the two candidates except for two outliers, one corresponding to the Palm Beach county. The correlation is high also (0.625).
- Curvature in the relationship – consider transformations

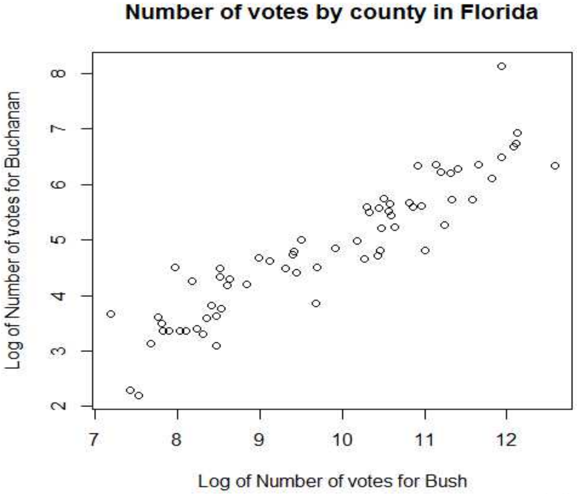


Linearity using Transformation

```
### Transform both variables using the log-transformation
    plot(log(bush),log(buch),xlab="Log of Number of votes for
    Bush",ylab="Log of Number of votes for Buchanan",
    main="Number of votes by county in Florida")
    cor(log(bush),log(buch))
```



Number of votes by county in Florida



Number of votes by county in Florida

Linearity using Transformation

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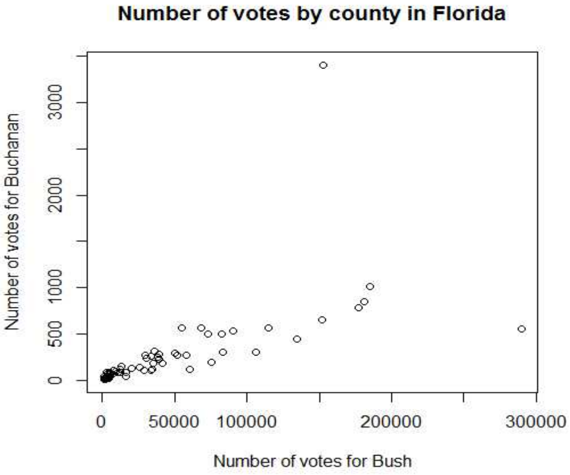
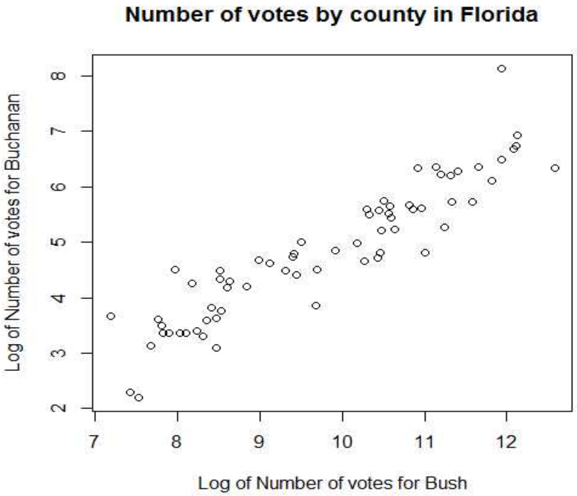
        Bush",ylab="Log of Number of votes for Buchanan",
        main="Number of votes by county in Florida")
        cor(log(bush),log(buch))
```

Linearity Assumption:

- **The linear relationship has improved with the transformations**
- **The correlation has increased from 0.625 to 0.922**
- **We will perform the regression analysis using the transformed data**





Linear Regression Analysis

model = lm(log(buch)~log(bush))

```
Coefficients:
              Estimate  Std. Error  t value   Pr(>|t|)
(Intercept) -2.55079    0.38903   -6.557   1.04e-08 ***
log(bush)   0.75620     0.03934  19.222   < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

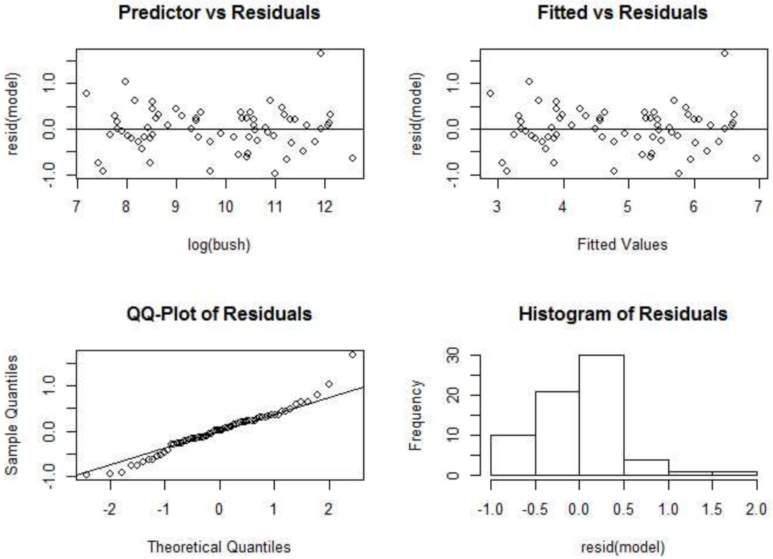
Residual standard error: 0.4672 on 65 degrees of freedom
Multiple R-squared: 0.8504,   Adjusted R-squared: 0.8481
F-statistic: 369.5 on 1 and 65 DF,  p-value: < 2.2e-16
```

$\hat{\beta}_0 = -2.55$, se($\hat{\beta}_0$) = 0.389
$\hat{\beta}_1 = 0.756$, se($\hat{\beta}_1$) = 0.039
Test for statistical significance:
$\hat{\beta}_0$: t-value= -6.557, p-value ≈ 0
$\hat{\beta}_1$: t-value= 19.22, p-value ≈ 0

$\hat{\sigma}$= 0.4672, n-2 = 65
$R^2 \sim 85\%$ variability explained

Residual Analysis

```
## Perform Residual Analysis
        par(mfrow=c(2,2))
        plot(log(bush),resid(model), main="Predictor vs
        Residuals”)
        abline(0,0)
        plot(fitted(model),resid(model),main="Fitted vs
        Residuals",
            xlab="Fitted Values")
        abline(0,0)
        qqnorm(resid(model),main="QQ-Plot of Residuals")
        qqline(resid(model))
        hist(resid(model),main="Histogram of Residuals")
```



Predictor vs Residuals

Fitted vs Residuals

QQ-Plot of Residuals

Histogram of Residuals

Model Interpretation

```
## Estimated Regression Coefficients
        betas = coef(model)
        Betas
          (Intercept)   log(bush)
         -2.5507857   0.7561963
## Confidence intervals for the coefficients
        confint(model)
                                2.5 %            97.5 %
        (Intercept) -3.3277351   -1.7738363
        log(bush)    0.6776289    0.8347638
```

Interpretation:

- As number of log-votes for Bush increase by 1% the expected % increase of log-votes for Buchanan is 0.756.
- The minimum % increase is 0.677 and the maximum % increase is 0.834

Is Palm Beach an Outlier?

## Omit Palm Beach

```
model.red = lm(log(buch[-50])~log(bush[-50]))
summary(model.red)
Coefficients:
                         Estimate    Std. Error  t value  Pr(>|t|)
(Intercept)       -2.31657    0.35470    -6.531   1.23e-08 ***
log(bush[-50])  0.72960    0.03599   20.271  < 2e-16 ***
```

## Obtain the predicted vote count for Palm Beach given the fitted model without

```
new = data.frame(bush = bush[50])
## The difference between predicted on the original scale and the observed vote count
buch[50 ]-exp(predict(model.red,new))
[1] 2809
```

## Prediction Confidence Interval for log(vote count)

```
predict(model.red,new,interval='prediction',level=.95)
```

## Prediction Confidence Interval on the original scale

```
exp(predict(model.red,new,interval='prediction',level=.95))
  fit              lwr           upr
597.5019 252.738 1412.564
```

## Is the observed vote count in the prediction interval?

```
buch[50]
[1] 3407
```

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## Omit Palm Beach
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## Prediction Confidence Interval on the original scale
        exp(predict(model.red,new,interval='prediction',level=.95))
          fit            lwr          upr
        597.5019 252.738 1412.564
## Is the observed vote count in the prediction interval?
        buch[50]
        [1] 3407
```

Interpretation:

- The difference between predicted and observed vote count for Bush in the Palm Beach county is 2809.
- The upper bound of the prediction confidence interval for the vote count is 1412 which is much lower than the observed vote count, 3407.
- While a difference of 2809 votes is not large given the total U.S. votes, this was particularly decisive for the 2000 elections.
- Recall that George W. Bush won Florida by a margin of 537 votes.

Summary