Regression Analysis
Model Selection

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Introduction

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About This Lesson

Objectives

- **High Dimensionality:** When we have a very large number of predicting variables to consider, it can be difficult to interpret and work with the fitted model.

- **Multicollinearity:** When the predicting variables are correlated, it is important to select variables in such a way that the impact of multicollinearity is minimized.

- **Prediction vs Explanatory Objective:** The variables selected for the two objectives will most often be different.

➔ **Variable Selection** addresses all these concerns.

Implications and Words of Caution

- **Controlling vs. Explanatory Variables**
  - Consider research hypothesis as well as potential controlling variables
- **Targeted Predicting Variables**
  - Include target variable in model if specified by research hypothesis
- **Over-Interpretation**
  - Selected variables are not necessarily special!
    - Highly influenced by correlations between variables
    - Interpretation of regression coefficients
    - Causality vs. Association

No Magic Bullet

- Variable selection for large number of predicting variables is an "unsolved" problem in statistics
- In some sense, model selection is "data mining"
- Data miners / machine learners often work with many predictors
- There are no magic procedures to get you the "best model"

"All models are wrong, but some are useful." —George Box

Notation

Given

$S \subset \{1, \ldots, p\}$ a subset of indices

and

(x_j for $j \in S$) the subset of predicting variables with indices in S:

- $\widehat{\boldsymbol{\beta}}(S)$ is the vector of estimated regression coefficients for the submodel with $\boldsymbol{X}_S = (x_j \text{ for } j \in S)$ predicting variables

- $\widehat{\boldsymbol{Y}}(S)$ is the vector of fitted values for the submodel with $\boldsymbol{X}_S = (x_j \text{ for } j \in S)$ predicting variables
  - E.g., for regression assuming normality, $\widehat{\boldsymbol{Y}}(S) = \boldsymbol{X}_S \widehat{\boldsymbol{\beta}}(S)$

➔ I will refer to this model as the **S submodel**.

Summary