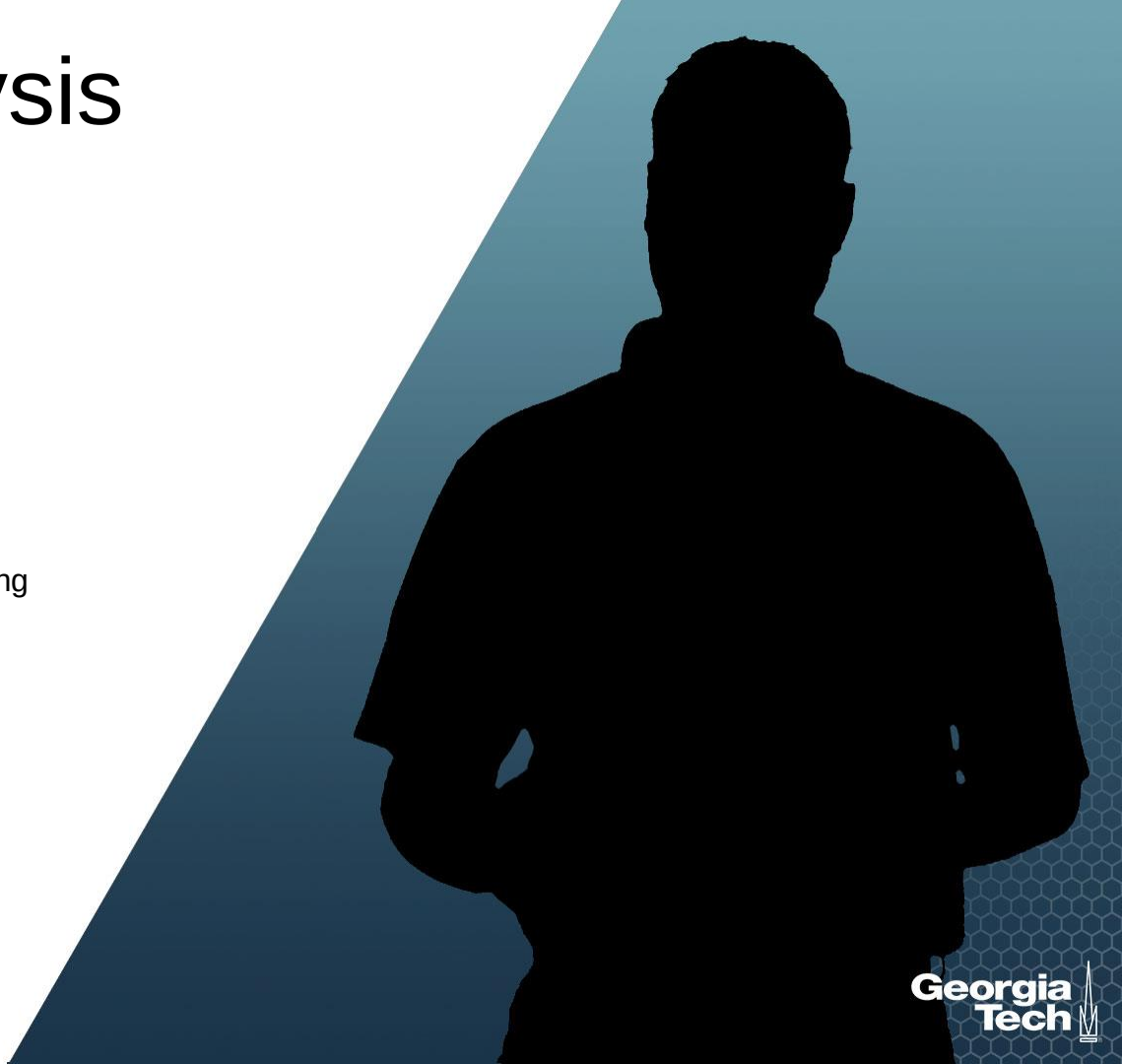
Regression Analysis

Poisson Regression

Nicoleta Serban, Ph.D.

Professor

Stewart School of Industrial and Systems Engineering



Statistical Inference: Data Example

About This Lesson

Data Example 1: High School Awards

Objective: To model and predict the number of awards earned by students at one high school for multiple high schools.

Response Variable: The number of awards earned by students at a high school per *year*

Predicting Variables:

- The type of program in which the student was enrolled, with three levels: 1 = "General", 2 = "Academic" and 3 ="Vocational"; and
- The score on the final exam in math.

Acknowledgement: This data example was acquired from the Institute for Digital Research and Education at University of California, Los Angeles.

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Data Example 1: Statistical Inference

m1 = glm(num_awards ~ prog + math, family="poisson", data=awardsdata)
summary(m1)

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-5.24712	0.65845	-7.969	1.60e-15 ***
progAcademic	1.08386	0.35825	3.025	0.00248 **
progVocational	0.36981	0.44107	0.838	0.40179
math	0.07015	0.01060	6.619	3.63e-11 ***

Null deviance: 287.67 on 199 degrees of freedom
Residual deviance: 189.45 on 196 degrees of freedom

1-pchisq((287.67-189.45),(199-196))
[1] 0

Test for significance β_{math}: p-value≈0 thus statistically significant
Test for overall regression: p-value ≈0 thus at least one predicting variables significantly explains the variability in the number of awards

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Data Example 2: Insurance Claims

Objective: To explain factors that are associated to car insurance claims due to accidents or other events leading to car damage.

Response Variable: The number of car insurance claims per *policyholder*:

- Holders: numbers of policyholders; and
- Claims: numbers of claims

Predicting Variables:

- District of residence of policyholder (1 to 4): 4 is major cities.
- Classification of cars with levels <1 litre, 1–1.5 litre, 1.5–2 litre, >2 litre.
- Age group of the policyholder: <25, 25–29, 30–35, >35.

Data Example 2: Statistical Inference

m.ins = glm(Claims ~ District + Group + Age + offset(log(Holders)), data = Insurance, family = poisson)
summary(m.ins)
Coefficients:

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-1.810508	0.032972	-54.910	< 2e-16 ***
…….				
Age.L	-0.394432	0.049404	-7.984	1.42e-15 ***
Age.Q	-0.000355	0.048918	-0.007	0.994210
Age.C	-0.016737	0.048478	-0.345	0.729910

What are these Age variables?

Null deviance: 236.26 on 63 degrees of freedom
Residual deviance: 51.42 on 54 degrees of freedom

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Data Example 2: Statistical Inference

```
library(MASS)
summary(Insurance)

Ins.dat = within(Insurance, {
  Age = factor(Age, ordered = F)
  Group = factor(Group, ordered = F)
  District = factor(District, ordered = F)})

m.ins = glm(Claims ~ District + Group + Age + offset(log(Holders)), data = Ins.dat, family = poisson)
summary(m.ins)
```

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Data Example 2: Statistical Inference

m.ins = glm(Claims ~ District + Group + Age + offset(log(Holders)), data = Ins.dat, family = poisson)
summary(m.ins)

Coefficients:

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-1.82174	0.07679	-23.724	< 2e-16 ***
…….				
Age25-29	-0.19101	0.08286	-2.305	0.021149 *
Age30-35	-0.34495	0.08137	-4.239	2.24e-05 ***
Age>35	-0.53667	0.06996	-7.672	1.70e-04 ***

Null deviance: 236.26 on 63 degrees of freedom
Residual deviance: 51.42 on 54 degrees of freedom

Test for significance

$\boldsymbol{\beta}_{age25-29}$ = -0.191 or
exp($\boldsymbol{\beta}_{age25-29}$) = 0.826

$\boldsymbol{\beta}_{age25-29}$, $\boldsymbol{\beta}_{age30-35}$ & $\boldsymbol{\beta}_{age>35}$: p-value$<0.05$ thus statistically significant.

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Data Example 2: Statistical Inference (cont'd)

```
m.ins = glm(Claims ~ District + Group + Age + offset(log(Holders)),
data = Ins.dat, family = poisson)
summary(m.ins)

# test for overall regression
1-pchisq((236.26-51.42),(63-54)
```

Test for overall regression: $\text{p-value} \approx 0$ thus at least one predicting variables significantly explains the variability in the number of claims

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Data Example 2: Statistical Inference (cont'd)

 # Is the district of residence of policyholder a statistically significant variable given all other predicting variables in the model?

Full model: District + Group + Age
Reduced model: Group + Age

```
library(aod)
wald.test(b=coef(m.ins), Sigma=vcov(m.ins), Terms=2:4)
```

Wald test:

Chi-squared test:
X2 = 14.6, df = 3, P(> X2) = 0.0022

Test for subsets of coefficients: $\text{p-value} = 0.002$ reject the null hypothesis and conclude that the District variable does have significant explanatory power

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Summary

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