# Regression Analysis

## Multiple Linear Regression

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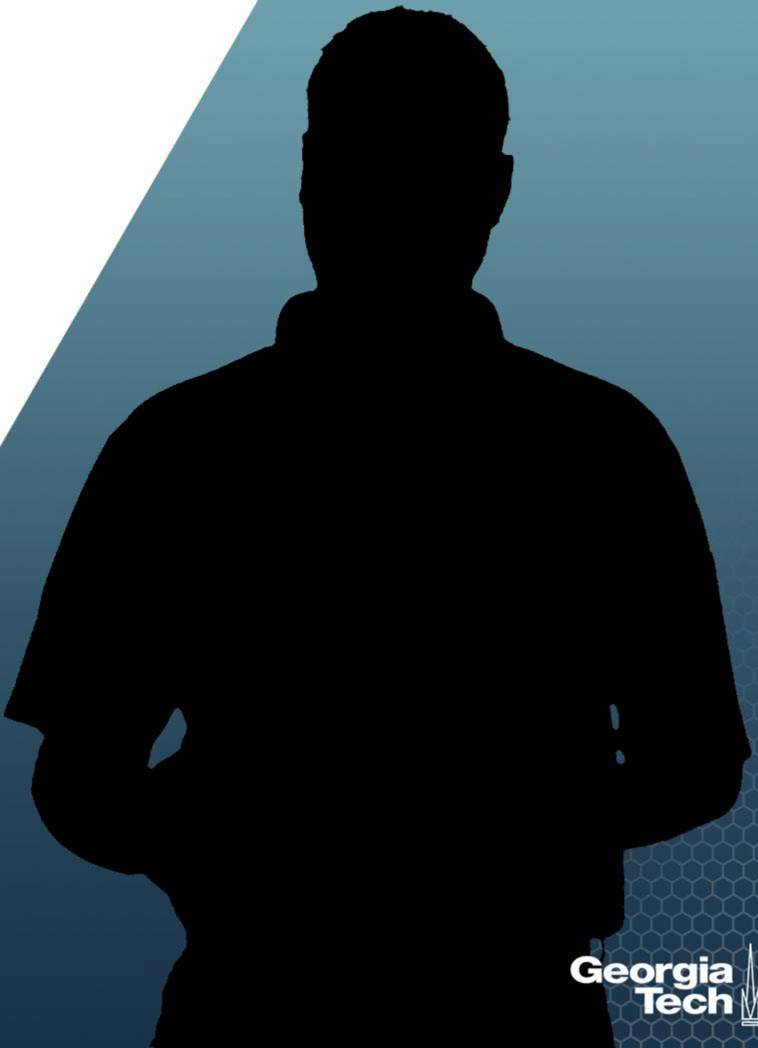
Predicting Demand for Rental Bikes: Regression Analysis



# About This Lesson

# Linear Regression Analysis in R

```
# Applying multiple linear regression model
model1 = lm(cnt ~ .,data=train)
summary(model1)
```

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | -79.4356 | 7.4390 | -10.678 | < 2e-16 *** |
| season2 | 34.9268 | 5.4110 | 6.455 | 1.12e-10 *** |
| season3 | 27.0055 | 6.4438 | 4.191 | 2.80e-05 *** |
| season4 | 65.3435 | 5.4690 | 11.948 | < 2e-16 *** |
| yr1 | 85.3415 | 1.7487 | 48.804 | < 2e-16 *** |
| mnth2 | 4.1666 | 4.3853 | 0.950 | 0.342060 |
| mnth3 | 16.4733 | 4.9267 | 3.344 | 0.000829*** |
| mnth4 | 12.5834 | 7.3038 | 1.723 | 0.084936 . |
| mnth5 | 26.4616 | 7.8357 | 3.377 | 0.000735 *** |
| mnth6 | 11.5056 | 8.0535 | 1.429 | 0.153131 |
| ⋮ | | | | |

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 101.8 on 13851 degrees of freedom
Multiple R-squared:  0.6852,    Adjusted R-squared:  0.684
F-statistic: 591 on 51 and 13851 DF,  p-value: < 2.2e-16
```

In the full output there are 51 predictor rows in addition to the intercept.

$\hat{\sigma} = \boxed{101.8}$

$df = n-p-1 = 13{,}903 - 51 - 1 = \boxed{13{,}851}$

$R^2 \approx \boxed{0.6852} \approx 68.5\%$ variability explained

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# Coding Dummy Variables in R

```r
## Create Dummy Variables
weathersit  = data$weathersit
weathersit.1 = rep(0,length(weathersit))
weathersit.1[weathersit==1] = 1
weathersit.2 = rep(0,length(weathersit))
weathersit.2[weathersit==2] = 1
weathersit.3 = rep(0,length(weathersit))
weathersit.3[weathersit==3] = 1

## Include all dummy vars without intercept
fit.1 = lm(cnt ~ weathersit.1 + weathersit.2 +weathersit.3 - 1)

## Include 3 dummy variables with intercept
fit.2 = lm(cnt ~ weathersit.1 + weathersit.2)

## Use categorical variable
weathersit  = as.factor(data$weathersit)
fit.3 = lm(cnt ~ weathersit)
```

**summary**(fit.1)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| weathersit.1 | 204.869 | 1.680 | 121.97 | <2e-16 *** |
| weathersit.2 | 175.165 | 2.662 | 65.80 | <2e-16 *** |
| weathersit.3 | 111.501 | 4.758 | 23.43 | <2e-16 *** |

**summary**(fit.2)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | 111.501 | 4.758 | 23.43 | <2e-16 *** |
| weathersit.1 | 93.369 | 5.046 | 18.50 | <2e-16 *** |
| weathersit.2 | 63.665 | 5.452 | 11.68 | <2e-16 *** |

**summary**(fit.3)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | 204.869 | 1.680 | 121.972 | <2e-16 *** |
| weathersit2 | -29.704 | 3.148 | -9.437 | <2e-16 *** |
| weathersit3 | -93.369 | 5.046 | -18.503 | <2e-16 *** |

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**Codding Dummy Variables**

R Sets the “first” class as being the baseline

- If a different class is the baseline, either use dummy variables or specify with ‘contr.treatment’

Be careful when using a model without intercept in R!

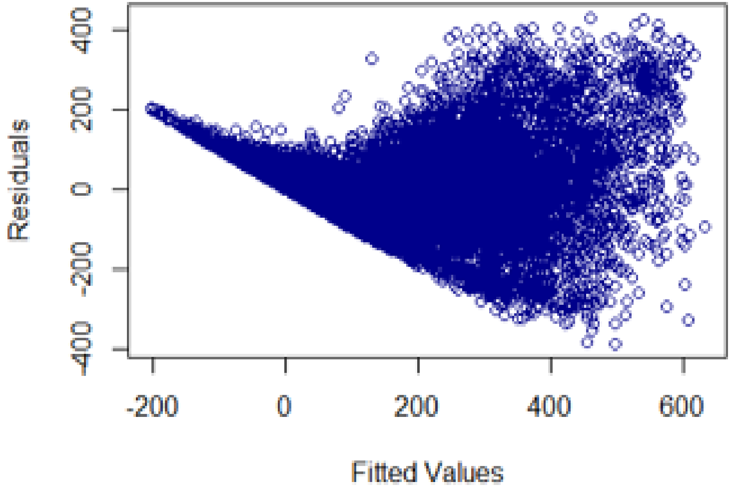
- No baseline comparison

# Goodness of Fit: Constant Variance Assumption

## Fitting the model

# Creating scatterplot of residuals vs fitted values

```
resids = rstandard(model1)
fits = model1$fitted
plot(fits,
     resids,
     xlab="Fitted Values",
     ylab="Residuals",
     main="Scatterplot",
     col="darkblue")
```




- The constant variance assumption does not hold -- the variance increases when moving from lower to higher fitted values.
- The residuals, at low y values, seem to follow a straight-line pattern. The linear pattern in the beginning suggests that the response variable stays constant for a range of predictor values.

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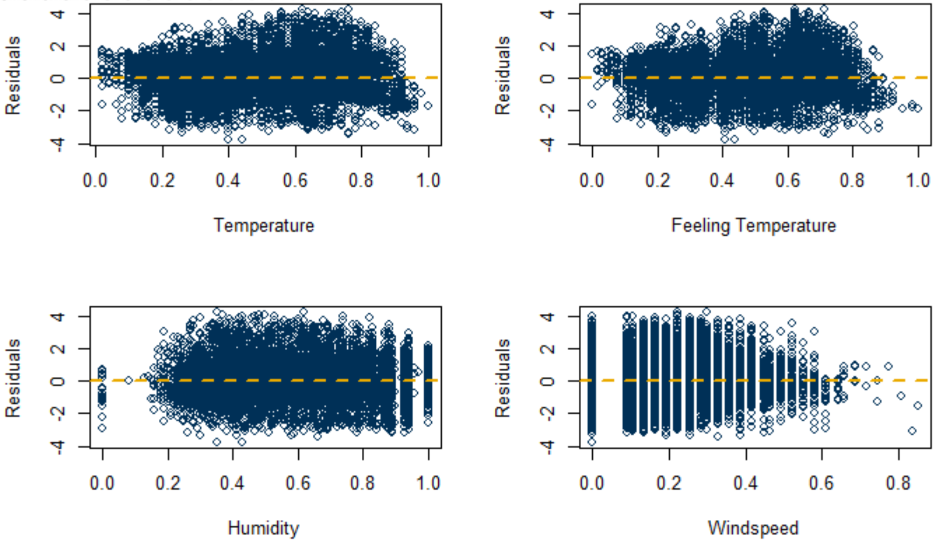
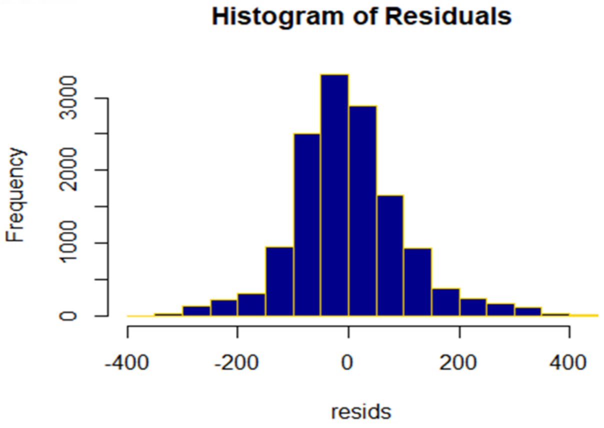
# Goodness of Fit: Linearity Assumption



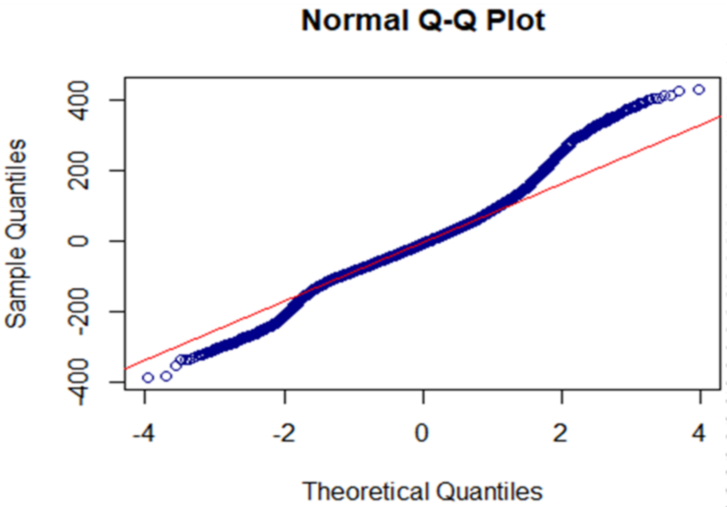

The residuals do not vary with the any of the numeric predicting variables. No transformation of the predicting variable is needed.

# Goodness of Fit: Normality Assumption

```
## Checking normality
# histogram
hist(resids,
     nclass=20,
     col="darkblue",
     border="gold",
     main="Histogram of residuals")
```




```
# q-q plot
qqnorm(resids,
     col="darkblue")
qqline(resids,
     col="red")
```

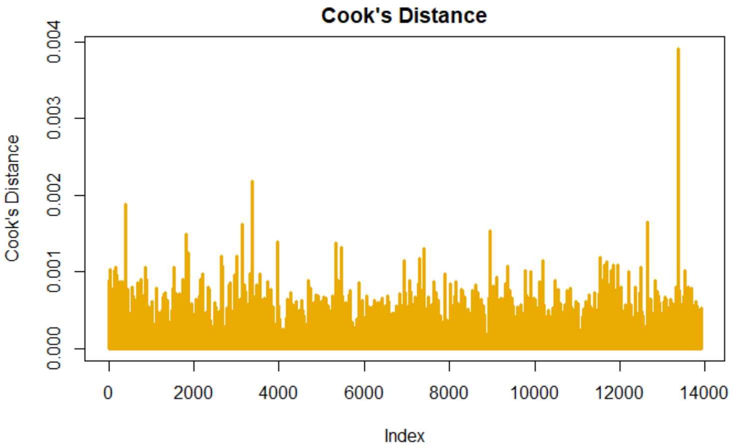


# Goodness of Fit: Outliers

**# Cook's Distance**

```
cook = cooks.distance(model1)

plot(cook,
      type="h",
      lwd=3,
      col="darkred",
      ylab = "Cook's Distance",
      main="Cook's Distance")
```

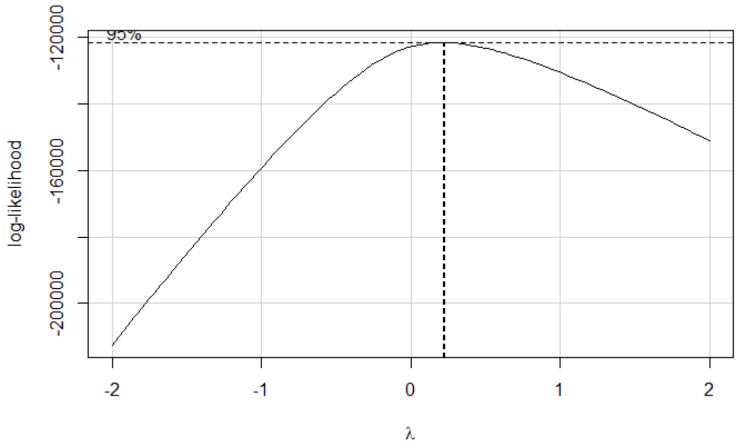



There is one observation with a Cook's Distance noticeably higher than the other observations. However, its Cook's distance is close to 0.004, suggesting that there are likely no outliers.

# Transformation of the Response Variable

```
## Box Cox transformation
bc <- boxcox(model1)
lambda <- bc$x[which(bc$y==max(bc$y))]

## Fitting the model with square root transformation
model2<-lm(sqrt(cnt)~.,data=train)
summary(model2)
```




- The optimal value of lambda or the power provided by the Box Cox transformation is 0.22.
- Generally, when the response data consist of count data, a theoretically recommended transformation is the square root, corresponding to a 0.5 power transformation.

# Regression Analysis after Transformation

## Fitting the model with square root transformation

```
model2<-lm(sqrt(cnt)~.,data=train)
summary(model2)
```

## Find Insignificant Values

```
which(summary(model2)$coeff[,4]>0.05)
```

| mnth2 | mnth4 | mnth6 | mnth7 | mnth8 | mnth10 | mnth11 | weekday1 |
|---|---|---|---|---|---|---|---|
| 6 | 8 | 10 | 11 | 12 | 14 | 15 | 41 |

## Multicollinearity

```
vif(model2)
```

| | GVIF | Df | GVIF^(1/(2*Df)) |
|---|---|---|---|
| season | 165.308 | 3 | 2.343 |
| yr | 1.025 | 1 | 1.012 |
| mnth | 323.778 | 11 | 1.300 |
| hr | 1.771 | 23 | 1.012 |
| holiday | 1.121 | 1 | 1.059 |
| weekday | 1.137 | 6 | 1.011 |
| weathersit | 1.386 | 2 | 1.085 |
| temp | 51.283 | 1 | 7.161 |
| atemp | 43.748 | 1 | 6.614 |
| hum | 1.921 | 1 | 1.386 |
| windspeed | 1.251 | 1 | 1.118 |

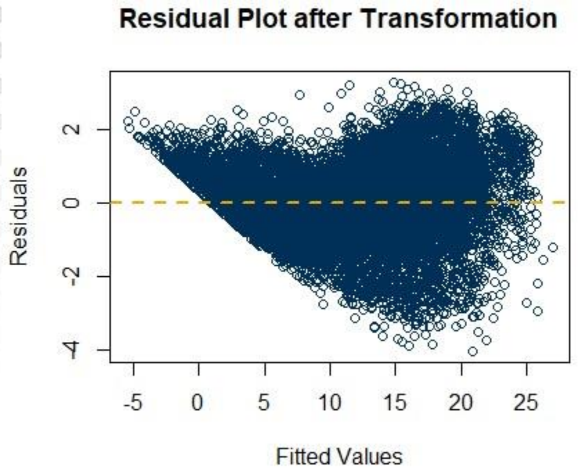
## Model Performance

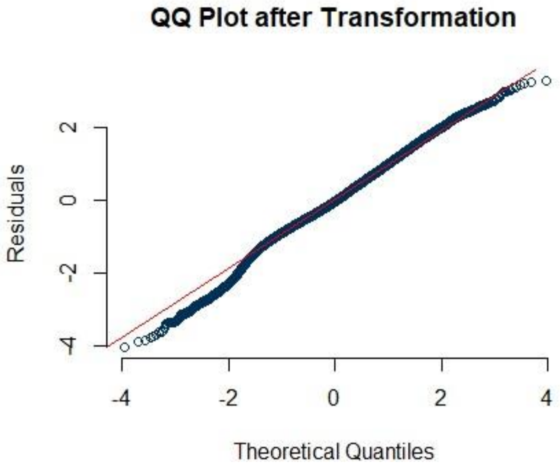
```
summary(model2)$r.squared
```

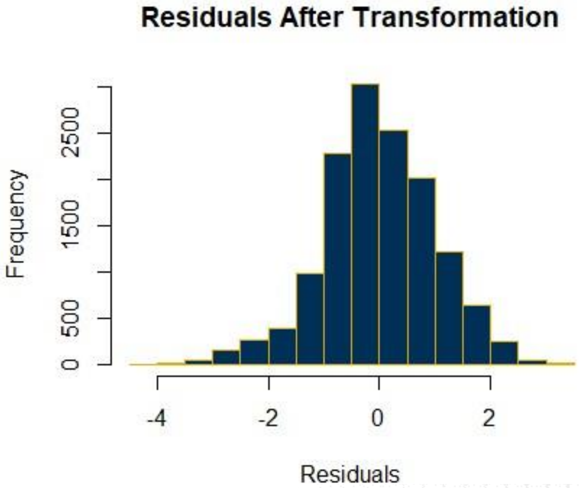
## [1] 0.786535

As VIFs of the season, mnth, temp, atemp factors are greater than max(10, $1/(1-R^2)$), it indicates there is a problem of multicollinearity in the linear model. So, we should not use all the predictors in the model.

# Goodness of Fit after Transformation




The constant variance assumption is still violated. The transformation has not improved the goodness of fit even though the model performance is better with respect to the coefficient of determination.

# Removing Low Demand Data

```
## Remove data for hours 0-6
hrs <- as.numeric(data$hr)
data_red <- data[which(hrs>=7),]
## Test/Train Data
set.seed(9) # for uniformity
sample_size  <- floor(0.8*nrow(data_red))
picked  <- sample(seq_len(nrow(data_red)),size = sample_size)
train_red  <- data_red[picked, -c(1,2,9,15,16)]
test_red  <- data_red[-picked, -c(1,2,9,15,16)]

## Fitting the model with square root transformation
model3<-lm(sqrt(cnt)~.,data=train_red)
summary(model3)$r.squared
[1] 0.6579021
df<-which(summary(model3)$coeff[,4]>0.05)

 mnth7 mnth11 mnth12   hr14   hr15   hr20 
    11     15     16     23     24     29 
```

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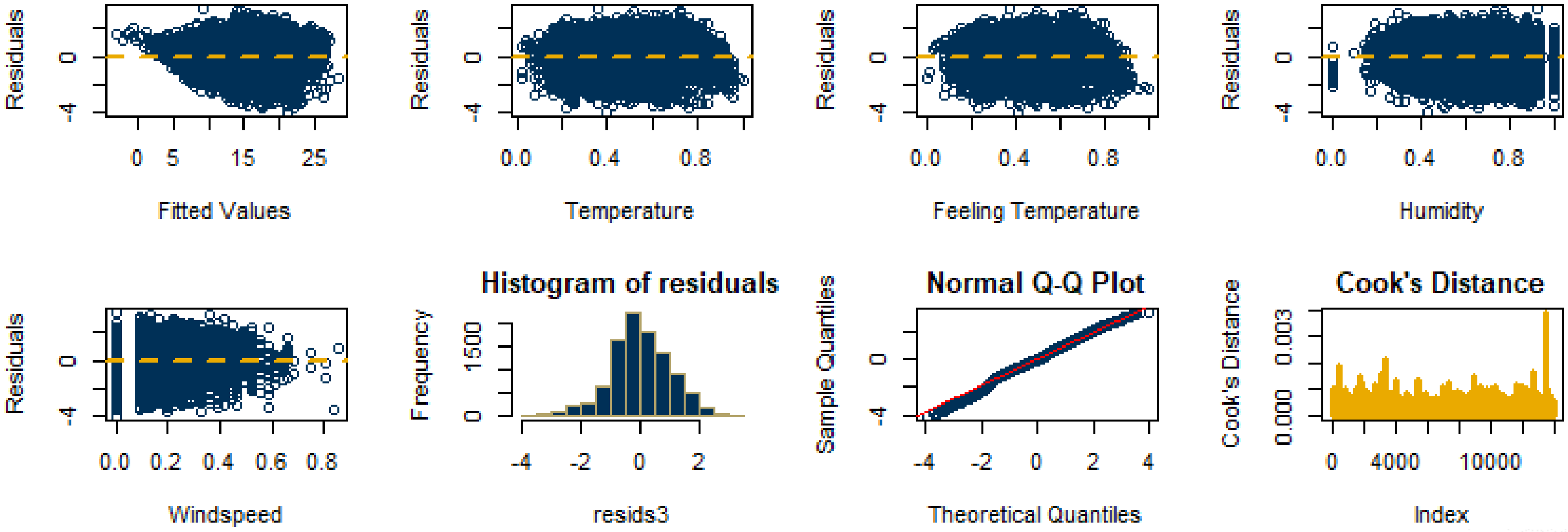
# Goodness of Fit without Low Demand Data




- The constant variance assumption is still violated even for the model without the low demand data and with the transformed response.
- The implication of the constant variation assumption violation is that the uncertainty in predicting bike demand when in high demand will be higher than estimated using the multiple regression models in this lesson.

# Summary