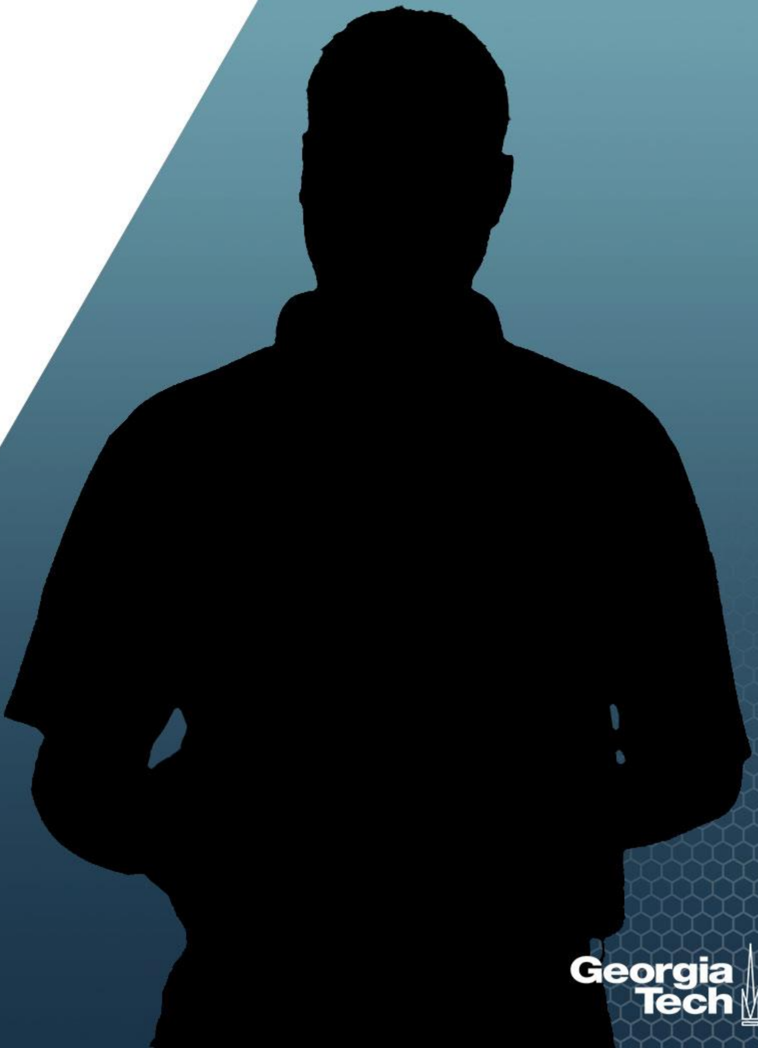
Regression Analysis

Multiple Linear Regression

Nicoleta Serban, Ph.D.

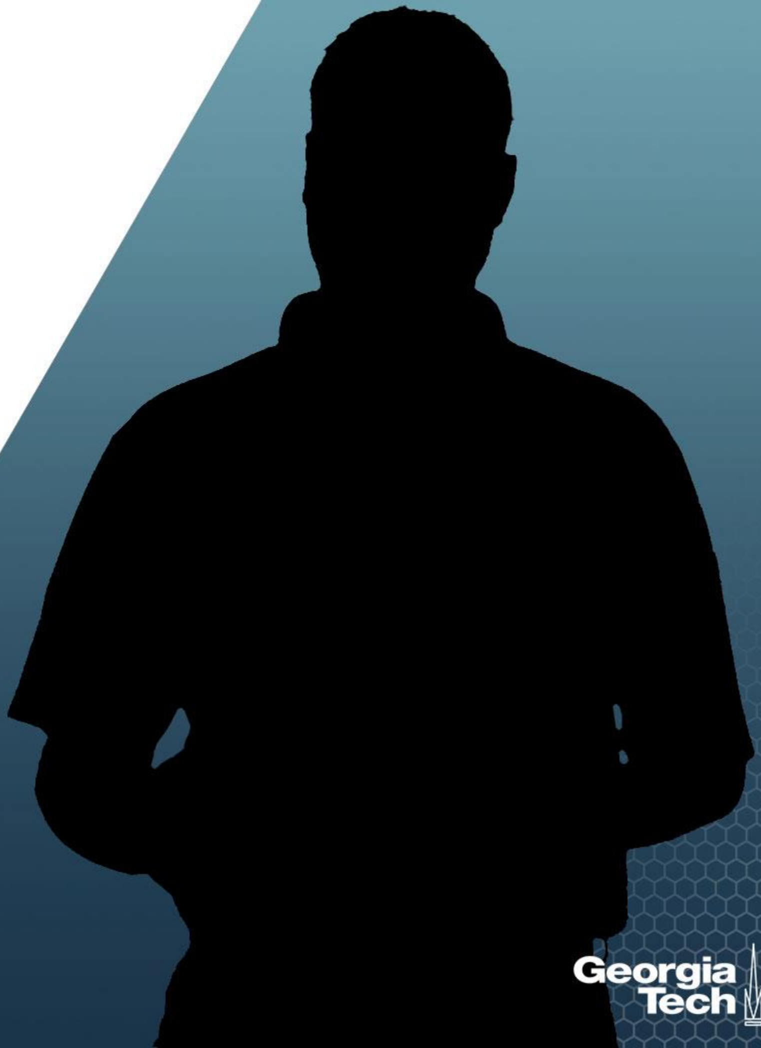
Professor

School of Industrial and Systems Engineering



Estimating Regression Line & Predicting a New Response: Data Example

About This Lesson



Linear Regression: Example 1

Quantitative Predicting Variables:

X_1 = amount (in hundreds of dollars) spent on advertising in 1999

X_2 = total amount of bonuses paid in 1999

X_3 = market share in each territory

X_4 = largest competitor's sales (thousands)

Qualitative Predicting Variable:

X_5 = indicates region where office is located (1 = south, 2 = west, 3 = midwest)

Response Variable:

Y = yearly sales (in thousands of dollars)

Example 1: Mean Response & Prediction

a. For all offices with the characteristics such as those of the first office:
   - What is the average estimated sales?
   - What is the standard deviation?
   - What is the 95% confidence interval for this mean response?

b. If the first office's largest competitor's sales increase to $303,000 (assuming everything else fixed):
   - What sales would you predict for the first office?
   - What is its standard deviation?
   - What is the 95% prediction interval for this prediction?

Example 1: Mean Response Estimation

```
s2 = summary(model)$sigma^2 # Variance estimate
X = model.matrix(model) # Design Matrix
xstar = X[1,] # First office data for formula
resp.var = s2*(xstar%*%solve(t(X)%*%X)%*%xstar) # Variance formula
sqrt(resp.var)

         [,1]
[1,] 33.19118

newdata = meddcor[1,-1] # First office data for confidence interval
predict(model, newdata, interval="confidence") # Confidence Interval

        fit      lwr      upr
1 934.7767 865.0446 1004.509
```

Example 1: Mean Response Estimation

```
s2 = summary(model)$sigma^2 # Variance estimate
X = model.matrix(model) # Design Matrix
xstar = X[1,] # First office data for formula
resp.var = s2*(xstar%*%solve(t(X)%*%X)%*%xstar) # Variance formula
sqrt(resp.var)

        [,1]
[1,] 33.19118

newdata = meddcor[1,-1] # First office data for confidence interval
predict(model, newdata, interval="confidence") # Confidence Interval

       fit      lwr      upr
1 934.7767 865.0446 1004.509
```

a. **Average estimated sales (mean response for sales):**

$$\hat{y} = \mathbf{934.777}$$

Estimated standard deviation:

$$\mathrm{se}(\hat{y}) = \mathbf{33.191}$$

95% Confidence Interval:

$$\boxed{\mathbf{(865.045,\ 1004.509)}}$$

Interpretation: For offices with the same characteristics as the first, the average estimated sales are $934,777, with a lower bound of $865,045 and an upper bound of $1,004,509.

Georgia Tech

Example 1: Mean Response Estimation

```
## Change the competitor's sales for prediction of future observation
xstar.new = xstar
xstar.new[5] = 303

# Variance formula
pred.var = s2*(1+xstar.new%*%solve(t(X)%*%X)%*%xstar.new)
sqrt(pred.var)

         [,1]
[1,] 64.31099

# Prediction Interval
predict(model, xstar.new[-1], interval="prediction")

       fit      lwr      upr
1 911.0569 775.9446 1046.169
```

Example 1: Mean Response Estimation

Change the competitor's sales for prediction of future observation

xstar.new = xstar
xstar.new[5] = 303

 # Variance formula

pred.var = s2(1+xstar.new%*%solve(t(X)%*%X)%*%xstar.new)*
sqrt(pred.var)

```
      [,1]
[1,] 64.31099
```

 # Prediction Interval

predict(model, xstar.new[-1], interval="prediction")

```
       fit       lwr      upr
1 911.0569  775.9446  1046.169
```

b. Predicted sales of the first office given the higher competitor's sales:

$\hat{y} =$ **911.057**

Estimated standard deviation:

$\mathrm{se}(\hat{y})=$ **64.311**

95% Confidence Interval:

(775.945, 1046.169)

Interpretation: If the competitor's sales increase to $303,000 (from $202,220), the predicted sales reduce by $23,720 (from $934,777 to $911,057). Since this is prediction, the standard deviation increases.

Summary