Regression Analysis

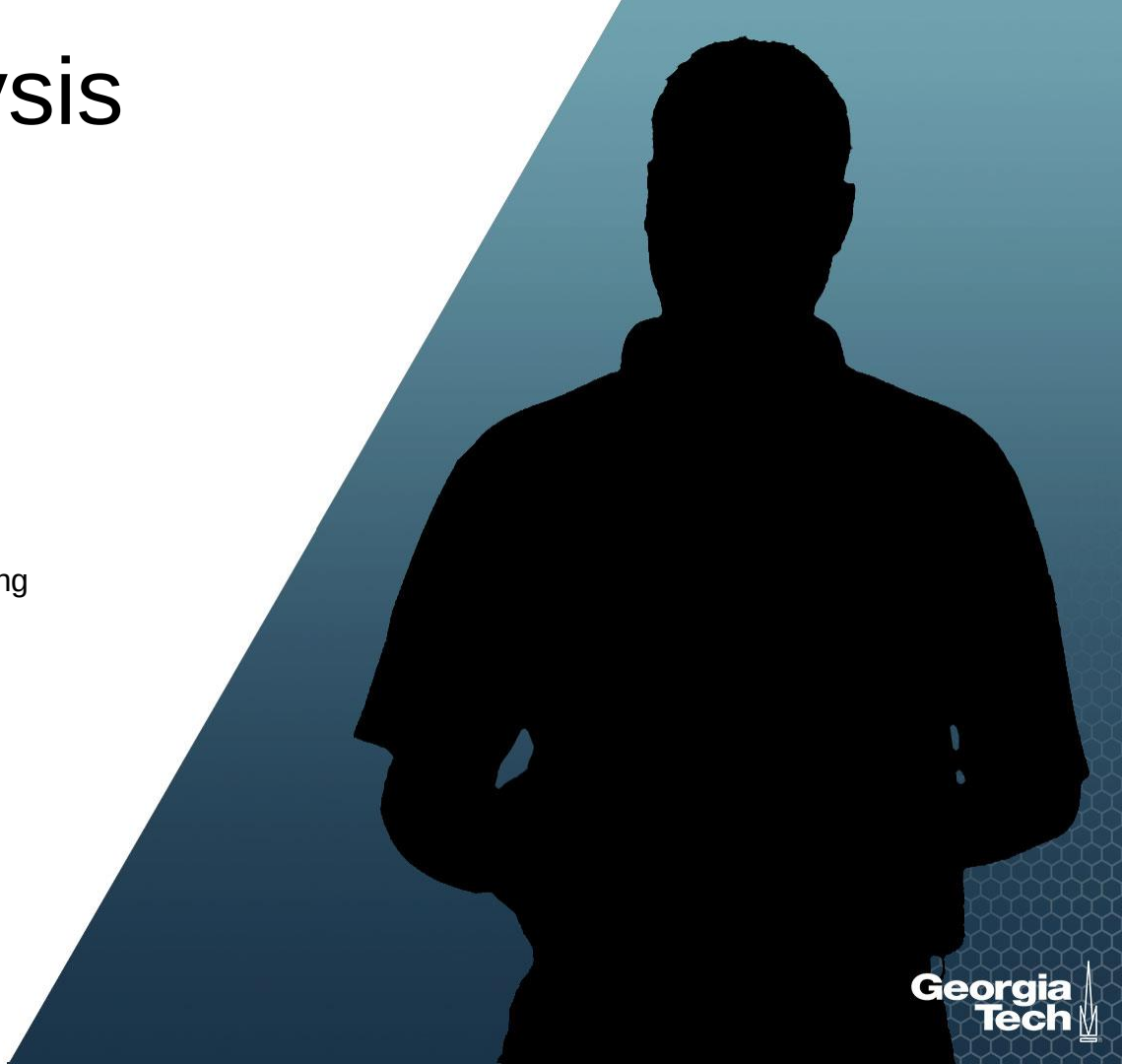
Poisson Regression

Nicoleta Serban, Ph.D.

Professor

Stewart School of Industrial and Systems Engineering

Goodness of Fit Assessment



About This Lesson

Poisson Regression Model

Data: $\{(x_{11},..,x_{1p}),Y_1\},....,\{(x_{n1},..,x_{np}),Y_n\}$ where $Y_1, .., Y_n$ are event count data per observation unit with a Poisson distribution

Assumptions:

- *Linearity Assumption*: $\log(E(Y|x_1, \ldots, x_p)) = \beta_0 + \beta_1 x_1 + \ldots + \beta_p x_p$
- *Independence Assumption*: $Y_1, .., Y_n$ are independent random variables
- *Variance Assumption*: $E(Y|x_1, \ldots, x_p) = V(Y|x_1, \ldots, x_p)$

There is no error term! How to check the assumptions?

Georgia Tech

Residuals in Poisson Regression

Poisson Regression:

$$Y_i|(x_{i1},..,x_{ip}) \sim \text{Poisson}(\lambda(x_{i1},\dots,x_{ip}))$$

- Estimated rates are:

$$\hat{\lambda}_i = \hat{\lambda}(x_{i1,..,}x_{ip}) = e^{\hat{\beta}_0+\hat{\beta}_1 x_{i1}+\dots+\hat{\beta}_p x_{ip}}$$

- Pearson Residuals: $r_i = \frac{Y_i-\hat{\lambda}_i}{\sqrt{\hat{\lambda}_i}}$

- Deviance Residuals:

$$d_i = sgn(Y_i-\hat{\lambda}_i)\sqrt{2\left\{Y_i \log\left(\frac{Y_i}{\hat{\lambda}_i}\right)-(Y_i-\hat{\lambda}_i)\right\}}$$

- Pearson's residuals follow directly a normal approximation to a binomial. Hence approximately N(0,1)
- The deviance residuals are the signed square root of the log-likelihood evaluated at the saturated model vs. the fitted model. Thus approximately N(0,1) if the model is a good fit.
- Deviances play the role of sum of squares in a linear model.

Georgia Tech

Goodness of Fit

GOF Visual Analytics:

- Normal Probability plot & Histogram of the Residuals
- Log of the event rate vs predictors

Hypothesis Testing Procedure:

H_0: *the Poisson model fits the* data

H_A: *the Poisson model does not fit the data*

Deviance test statistic: $D = \sum_{i=1}^{n} d_i^2$

Under null hypothesis, $D \sim \chi^2_{df}$ with $df = n\text{-}p\text{-}1$

Reject the null that the model is correct if $p\text{-value} = P(\chi^2_{df} > D)$ small.

Note that for this test, we want large p-values!!!!

Georgia Tech

What if No Goodness of Fit?

- Add predicting variables, consider interaction terms, or/and transform predicting variables to improve linearity;

- Identify unusual observations (outliers, leverage points);

- The Poisson distribution isn’t appropriate:
  - Overdispersion: the variability of the estimated rates is larger than would be implied by a Poisson model
    - Correlation in the observed responses
    - Heterogeneity in the rates that hasn’t been modeled

Overdispersion

Overdispersion: the variability of the response variable is larger than would be implied by the model

Binomial regression model:

- $\mathrm{V}(Y_i|\mathrm{x}_1, \ldots, \mathrm{x}_\mathrm{p}) = n_i p(\mathrm{x}_{\mathrm{i}1}, \ldots, \mathrm{x}_{ip})(1\text{-}p(\mathrm{x}_{\mathrm{i}1}, \ldots, \mathrm{x}_{ip}))$
- Overdispersed Binomial: $\mathrm{V}(Y_i|\mathrm{x}_1, \ldots, \mathrm{x}_\mathrm{p}) = \phi\, n_i p(\mathrm{x}_{\mathrm{i}1}, \ldots, \mathrm{x}_{ip})(1\text{-}p(\mathrm{x}_{\mathrm{i}1}, \ldots, \mathrm{x}_{ip}))$

Poisson regression model:

- $\mathrm{V}(Y_i|\mathrm{x}_1, \ldots, \mathrm{x}_\mathrm{p}) = \lambda(\mathrm{x}_{\mathrm{i}1}, \ldots, \mathrm{x}_{ip})$
- Overdispersed Poisson: $\mathrm{V}(Y_i|\mathrm{x}_1, \ldots, \mathrm{x}_\mathrm{p}) = \phi\lambda(\mathrm{x}_{\mathrm{i}1}, \ldots, \mathrm{x}_{ip})$

Overdispertion Parameter: ϕ

- Estimate: $\hat{\phi} = \frac{D}{n-p-1}$ where D is the sum of the squared deviances
- If $\hat{\phi} > 2$ then overdispersed model

Georgia Tech

Summary

Georgia Tech