Regression Analysis

Analysis of Variance

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Mean Pairwise Comparison

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About This Lesson

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Pairwise Comparison of Means

One primary goal of ANOVA might be to determine which treatment means are bigger or smaller. One way to do this is to compare all *k(k-1)/2* pairs of treatments. For a (1 - α) confidence interval for the mean difference $\mu_i - \mu_j$:

$$\left(\hat{\mu}_i - \hat{\mu}_j\right) \pm q_{\alpha,k,N-k}\sqrt{\frac{MSE}{2}\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}$$

- Estimate of difference in means
- α percentile of "studentized range" distribution
- Standard deviation/error of estimator

Difference Between t_α and q_α

Correct for simultaneous inference:

- $q > t$ (at any fixed α and df)
- Intervals are wider to compensate for the fact that we are making simultaneous comparisons (multiplicity correction)

Why?

95% CIs for two populations $\Rightarrow$ (.95)(.95) ≈ .90 $\Rightarrow$ The simultaneous or joint confidence level for the two parameters is roughly **90%**.

95% CIs for three populations $\Rightarrow$ (.95)(.95)(.95) ≈ .86 $\Rightarrow$ The simultaneous or joint confidence level for the three parameters is roughly **86%**.

Example1: Global Suicide by Region

Which country regions have different suicide rates?

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Pairwise Comparison

TukeyHSD(aov(suicidesper100k ~ region, data=suicide_data))

Tukey multiple comparisons of means

95% family-wise confidence level

$region

	diff	lwr	upr	p adj
EASTERN EUROPE-ASIA	7.1256986	-0.8654681	15.1168654	0.1218931
GLOBAL WEST-ASIA	1.3948384	-6.3253621	9.1150390	0.9998655
LATIN AMER. & CARIB-ASIA	-2.4242761	-9.7079484	4.8593961	0.9848625
MIDDLE EAST-ASIA	-7.8183246	-17.4646356	1.8279865	0.2171605
NORTHERN AMERICA-ASIA	1.8826591	-18.6470201	22.4123382	0.9999996
OCEANIA-ASIA	-0.6423728	-13.5277421	12.2429965	1.0000000
SUB-SAHARAN AFRICA-ASIA	-4.2457218	-17.1310911	8.6396474	0.9858800
WESTERN ASIA-ASIA	-9.6996143	-30.2292935	10.8300649	0.8717761
WESTERN EUROPE-ASIA	2.4643324	-10.4210369	15.3497016	0.9997844
GLOBAL WEST-EASTERN EUROPE	-5.7308602	-12.5740866	1.1123662	0.1809537
LATIN AMER. & CARIB-EASTERN EUROPE	-9.5499748	-15.8966379	-3.2033117	0.0002123
MIDDLE EAST-EASTERN EUROPE	-14.9440232	-23.9039098	-5.9841367	0.000026

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Pairwise Comparison

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- 10 different categories, total of 45 different pairwise comparisons
- Two groups with only one observation and three groups with three observations – not sufficient data for comparison
- Only three pairs have an adjusted p-value smaller than 0.05: Latin America vs Eastern Europe, Middle East vs Eastern Europe and Middle East vs Global West

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ANOVA Example 2: Keyboard Layout

Three different keyboard layouts are being compared in terms of typing speed.

Which mean typing times for the three keyboard layouts are different?

Layout 1	Layout 2	Layout 3
23.8	30.2	27.0
25.6	29.9	25.4
24.0	29.1	25.6
25.1	28.8	24.2
25.5	29.1	24.8
26.1	28.6	24.0
23.8	28.3	25.5
25.7	28.7	23.9
24.3	27.9	22.6
26.0	30.5	26.0
24.6	*	23.4
27.0	*	*

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Pairwise Comparison

TukeyHSD(aov(speed ~ layout))
Tukey multiple comparisons of means
95% family-wise confidence level

Fit: aov(formula = speed ~ layout)

$layout

	diff	lwr	upr	p adj
2-1	3.9850000	2.854395	5.1156053	0.0000000
3-1	-0.3613636	-1.463581	0.7408538	0.7008915
3-2	-4.3463636	-5.500092	-3.1926352	0.0000000

- Keyboard layout 2 has a statistically significantly higher typing time than keyboard layouts 1 and 3, on average.
- It is plausible that keyboard layouts 1 and 3 have similar typing time, on average.

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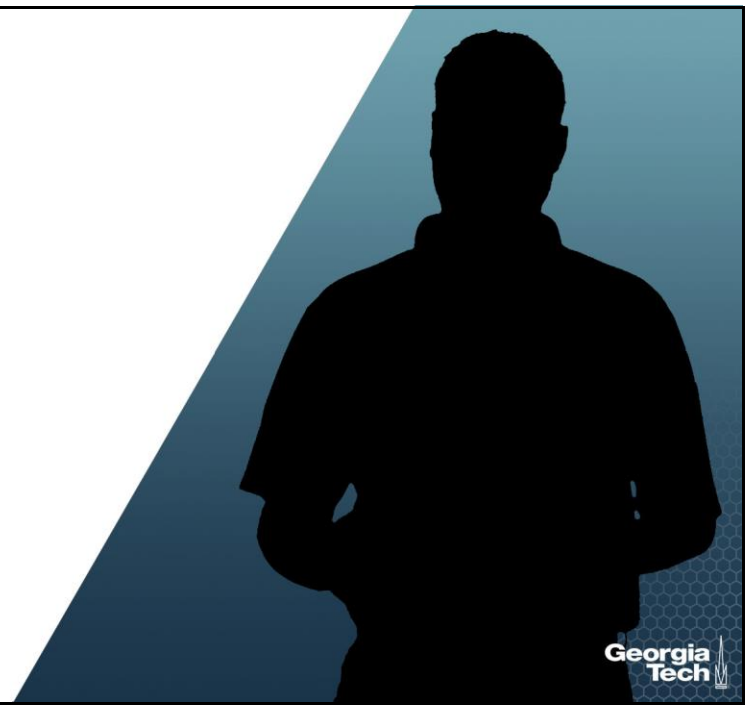
Summary



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