A Min-Cost-Max-Flow Based Algorithm for Reconstructing Binary Image from Two Projections Using Similar Images

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Abstract. The aim of this paper is to study the reconstruction of binary images from two projections using a priori images that are similar to the unknown image. Reconstruction of images from a few projections is preferred to reduce radiation hazards. It is well known that the problem of reconstructing images from a few projections is ill-posed. To handle the ill-posedness of the problem, a priori information such as convexity, connectivity and periodicity are used to limit the number of possible solutions. We use a priori images that are similar to the unknown image, to reduce the class of images having the same two projections. The a priori similar images may be obtained in many ways such as by considering images of neighboring slices or images of the same slice, taken in previous time instances. In this paper, we give a polynomial time algorithm to reconstruct binary image from two projections such that the reconstructed image is optimally close to the a priori similar images. We obtain a solution to our problem by reducing our problem to min cost integral max flow problem.

Keywords: binary matrix reconstruction, computed tomography, discrete tomography, min cost integral max flow problem.

1 Introduction

Discrete Tomography (DT) is an emerging reconstruction technique that reconstructs discrete images from a few projections of the images. As the Computed Tomography requires hundreds of projections to reconstruct images of interior of objects, the object is exposed to more X-ray energy, which causes some side effects such as cancer in medical imaging and destruction of atomic structure in crystalline structure reconstruction. As more projections require more X-ray energy to be transmitted into the object, one of the ways to reduce radiation hazards is to reconstruct images from a few projections. The area of *discrete tomography* is concerned about reconstruction of a discrete object or its geometrical properties from its projections or some other information. This has application in fields such as: image processing [12], statistical data security [6], biplane angiography [10], graph theory, crystallography, medical imaging [4], neutron imaging [8] etc. [5] gives the fundamentals related to this topic.

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Here we consider the problem of reconstructing bi-level image from its projections along row and column, and a priori information namely a set of images that are similar to the unknown image.

An important area where binary image reconstruction obtained is medical imaging, in particular, Digital subtraction angiography [4]. In Digital subtraction angiography, the reconstructed image is the difference between images acquired before and after intra-arterial injection of radio-opaque contrast medium and hence if the difference of a few projections of those two images are given, binary image can be reconstructed. Another area where binary image reconstruction obtained is crystallography. Peter Schwander and Larry Shepp proposed a model that identifies each possible atom location with a cell of integer lattice Z^3 and the electron beams with lines parallel to given direction. The value 1 in a cell of Z^3 denotes the presence of atom in the corresponding location of crystal and the value 0 in a cell of Z^3 denotes the absence of atom in the corresponding location of the crystal. The number of atoms that are present in the line passing through the crystal defines the projection of the structure along the line [7].

The problem of reconstructing 3D-binary matrix is reduced to reconstructing 2D-binary matrix. Reconstructing 2D-binary matrix was studied much before the emergence of its practical application. In 1957 Ryser [11] and Gale [2] gave a necessary and sufficient condition for a pair of vectors being the projections of binary matrices along horizontal and vertical directions. The projections in horizontal and vertical directions are equal to row and column sums of the matrix. They have also given necessary and sufficient conditions for existence of unique 2D-binary matrix which has a given pair of row sum and column sum. In general, the class of binary matrices having same row and column sums is very large. Though the reconstructed matrix and the original matrix have same projections, they may be very different. One of the main issues in $Discrete\ Tomography$ is to reconstruct the object which is more close to the original object with few projections only. One approach to reduce the class of possible solutions is to use some a priori information about the objects. For instance, convex binary matrices have been reconstructed uniquely from projections taken in some prescribed set of four directions in [3]. An another approach is given in [9], where the class of binary matrices having same projections is assumed to have some Gibs distribution. By using this information, object which is close to the original unknown object is reconstructed.

In this paper, We consider the first approach, namely, a set of a priori images that are similar to the unknown image, to limit the possible solutions of 2D-Binary images having given projections. In practice, It is possible to obtain images that are similar to unknown image. One such situation is that the images of the same slice taken in previous time instances may be considered as similar images, and the another situation is that the images of adjacent slices may be considered as similar images. As the patients who have undergone diagnosis may need to undergo diagnosis periodically (more in case of CT-Angiography), images taken in the previous time instances can be used to take images at current instance.

In the next section, we give notations and definitions. In section 3, we give the algorithm. In section 4, we give some results obtained by simulation studies. In section 5, we briefly discuss the correctness and complexity of the proposed algorithm. . The paper concludes with a brief remark in section 6.

2 Notations and Definitions

Let $A = (a_{i,j})$ and $B = (b_{i,j})$ be two binary image of order $m \times n$ where $1 \leq i \leq m$ and $1 \leq j \leq n.$. The images A and B are said to be similar if $\sum_{i=1}^{m} \sum_{j=1}^{n} \, |a_{i,j} - b_{i,j}\,|$ is small.

The row and column projections of $A = (a_{i,j})$ are $R = (r_1, ..., r_m)$ and $C = (c_1, ..., c_n)$ respectively, where $r_i = \sum_{j=1}^{n} a_{i,j}$ and $c_j = \sum_{i=1}^{m} a_{i,j}$

Two integral vectors $R = (r_1, ..., r_m)$ and $C = (c_1, ..., c_n)$ are said to be consistent if $\sum_{i=1}^{m} r_i = \sum_{j=1}^{n} c_j$. For binary matrices T and S of size $m \times n$, we define $|T - S| = \sum_{i=1}^{m} \sum_{j=1}^{n} |T(i,j) - S(i,j)|$

For two integral vectors $R = (r_1, ..., r_m)$ and $C = (c_1, ..., c_n)$, $\varGamma_{(R,C)}$ denotes the class of all binary matrices having row sum R and column sum C.

3 Reconstruction Problem

Given row projection $R = (r_i)$, column projection $C = (c_j)$ and a set of images $S = (S_k)$, where $1 \leq k \leq l$, that are similar to unknown bi-level image $A = (a_{i,j})$, the goal is to obtain a bi-level image $B = (b_{i,j})$ such that R and C are row and column projections of $B = (b_{i,j})$ respectively, and

$$B = arg[\min_{T \in \varGamma_{(R,C)}} \sum_{k=1}^{l} (|T - S_k|)]$$

We construct directed network G for the given a priori image-set S and the projections $R = (r_1,\ r_2,\ \ldots,\ r_m)$ and $C = (c_1,\ c_2,\ \ldots,\ c_n)$ as follows:
$G\ =\ (V,\ E,\ C',\ C'')$ be a weighted directed graph where
$V\ =\ U\ \cup\ W \cup \{s,t\}$
$U\ =\ \{\ u_i \mid 1 \leq i \leq m\}$
$W\ =\ \{\ w_j \mid 1 \leq j \leq n\ \}$
$E = \{(u_i, w_j) \mid 1 \leq i \leq m,\ 1 \leq j \leq n\}$
$\quad \cup \{(s, u_i) \mid\ 1 \leq i \leq m\ \} \cup\ \{(w_j, t) \mid\ 1 \leq j \leq n\ \}$

We define the cost associated with each edge as follows: For each $1 \leq i \leq m$ and $1 \leq j \leq n$,
$C'(u_i, w_j) = -\sum_{k=1}^{m} S_k(i,j),\ C'(s, u_i) = -1$ and $C'(w_j, t) = -1$
where s is the source and t is the sink.

We define the capacity associated with each edge as follows: For each $1 \leq i \leq m$ and $1 \leq j \leq n$,
$C''(u_i, w_j) = 1,\ C''(s, u_i) = r_i$ and $C''(w_j, t) = c_j$

For a binary matrix $A = (a_{i,j})$ of size $m \times n$, the set of all locations with pixel value 0 is denoted by $A_0 = \{(i,j) | a_{i,j} = 0, 1 \leq i \leq m,\ 1 \leq j \leq n\}$, and the set of all locations with pixel value 1 is denoted by
$A_1 = \{(i,j) | a_{i,j} = 1, 1 \leq i \leq m,\ 1 \leq j \leq n\}$.

Lemma 1. *Let B and B' be two binary matrices, having same row projection R and column projection C. Let $S = (S_k)$, where $1 \leq k \leq l$, be the class of images that are* *similar to the unknown image whose row and column projections are also R and C. Then*

$$\sum_{(i,j)\in B_0'}\sum_{k=1}^{l} S_k(i,j) - \sum_{(i,j)\in B_0}\sum_{k=1}^{l} S_k(i,j) = \sum_{(i,j)\in B_1}\sum_{k=1}^{l} S_k(i,j) - \sum_{(i,j)\in B_1'}\sum_{k=1}^{l} S_k(i,j)$$

Proof

From the definition of B_0, B_1, B_0' and B_1', we get

$$\sum_{(i,j)\in B_0}\sum_{k=1}^{l} S_k(i,j) + \sum_{(i,j)\in B_1}\sum_{k=1}^{l} S_k(i,j) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} S_k(i,j) \tag{1}$$

$$\sum_{(i,j)\in B_0'}\sum_{k=1}^{l} S_k(i,j) + \sum_{(i,j)\in B_1'}\sum_{k=1}^{l} S_k(i,j) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} S_k(i,j) \tag{2}$$

From (1) and (2),

$$\sum_{(i,j)\in B_0'}\sum_{k=1}^{l} S_k(i,j) - \sum_{(i,j)\in B_0}\sum_{k=1}^{l} S_k(i,j) = \sum_{(i,j)\in B_1}\sum_{k=1}^{l} S_k(i,j) - \sum_{(i,j)\in B_1'}\sum_{k=1}^{l} S_k(i,j)$$

Hence the Lemma.

Lemma 2. *Let R and C be two integral vectors, $S = (S_k)$, where $1 \leq k \leq l$, be the class of images. Let G be the network associated with R, C and S. Maximum-Flow f of G has minimum cost* **iff** $\sum_{i=1}^{m}\sum_{j=1}^{n} f(u_i, w_j)C(u_i, w_j)$ *is minimum.*

Proof

Cost of flow is

$$\sum_{e\in E} f(e)C(e) = \sum_{i=1}^{m} f(s,u_i)C(s,u_i) + \sum_{i=1}^{m}\sum_{j=1}^{n} f(u_i,w_j)C(u_i,w_j) + \sum_{j=1}^{n} f(w_j,t)C(w_j,t)$$

As the index set for minimization is the set of all binary matrices having R and C as projections, the first and second terms of right hand side of above equation are constants. Hence

$$\sum_{e\in E} f(e)C(e) \text{ is min iff } \sum_{i=1}^{m}\sum_{j=1}^{n} f(u_i,w_j)C(u_i,w_j) \text{ is minimum}$$

Hence the proof.

Theorem 1. *: Let $S = (S_k)$, where $1 \leq k \leq l$, be a set of binary images, and $R = (r_1,\ r_2,\ \ldots,\ r_m)$ and $C = (c_1,\ c_2,\ \ldots,\ c_n)$ be two integral vectors. There* *exists a binary image $B = (b_{i,j})$ such that row and column projections of B are R and C respectively, and*

$$B = arg[\min_{T \in \Gamma_{(R,C)}} \sum_{k=1}^{l} (|T - S_k|)]$$

iff *R and C are consistent and max flow value for the network G corresponds to R, C and S is $|f| = \sum_{i=1}^{m} r_i$, and cost of the flow is minimum.*

Proof

$\Longrightarrow$**:**

Let $R = (r_i)$ and $C = (c_i)$ be two integral vectors, $S = (S_k)$, where $1 \leq k \leq l$, be a set of binary images, and G be the network associated with R, C *and* S. Let us assume that there exists a binary matrix $B = (b_{i,j})$ such that row and column projections of B are R and C respectively, and

$$B = arg[\min_{T \in \Gamma_{(R,C)}} \sum_{k=1}^{l} (|T - S_k|)]$$

let us first prove that R and C are consistent and max flow value for the network G corresponds to R, C and S is $|f| = \sum_{i=1}^{m} r_i$.
Consider the following flow for the network G
For each $1 \leq i \leq m$ and $1 \leq j \leq n$,
$f(s, u_i) = r_i, \ f(w_j, t) = c_j$,
$f(u_i, w_j) = 1$ if $b_{i,j} = 1$,
$f(u_i, w_j) = 0$ if $b_{i,j} = 0$,
$f(u_i, s) = -f(s, u_i), \ f(t, w_j) = -f(w_j, t)$,
$f(w_j, u_i) = -f(u_i, w_j)$
and $f(e) = 0$ for all $e \in V \times V$ such that $f(e)$ is not defined above. The flow f has the following properties:

Capacity constraint: $f(e) \leq C''(e)$ for all $e \in V \times V$. From our definition of f, capacity constraint is evident.

Skew symmetry: $f(u, v) = -f(v, u)$ for all $(u, v) \in V \times V$. Skew symmetry is also evident from our definition of flow f.

Flow conservation: For all $u \in V - \{s, t\}$ $\sum_{v \in V} f(u, v) = 0$. Since for each $1 \leq i \leq m$, row i has r_i $1's$, the number of outgoing edges with capacity 1 from u_i is r_i. Hence total amount outgoing flow from u_i is r_i. The only incoming flow to vertex u_i is from the source, which is also r_i. Since f is skew symmetric and the incoming flow is same as the outgoing flow at node u_i, $\sum_{v \in V} f(u_i, v) = 0$ where $1 \leq i \leq m.$ Since for each $1 \leq j \leq n$, column j has c_j $1's$, the number of incoming edges with capacity 1 to w_j is c_j. Hence total amount of the incoming flow from w_j is c_j. The only outgoing flow from vertex w_j is to the sink, which is also c_j. Since f is skew symmetric and the incoming flow is same as the outgoing flow at node w_j, $\sum_{v \in V} f(w_j, v) = 0$ where $1 \leq j \leq n.$ Hence the flow conservation follows.

The value of flow f is $|f| = \sum_{v \in V} f(s,v) = \sum_{i=1}^{m} f(s,u_i)$. Since $|f| \leq \sum_{v \in V} C''(s,v)$ for any f and for our f, $|f| = \sum_{v \in V} C(s,v)$, $|f| = \sum_{i=1}^{m} r_i$ is the maximum flow. Since $|f| = \sum_{i=1}^{m} f(s,u_i)$ and $|f| = \sum_{i=1}^{n} f(w_j,t)$, $\sum_{i=1}^{m} r_i = \sum_{j=1}^{n} c_j$. Hence R and C are consistent.

Let us now prove that the cost of the flow f is minimum.

Suppose f is not minimum, then there exists an another flow f' such that cost of f' is less than f.ie $\sum_{i=1}^{m} \sum_{j=1}^{n} f'(u_i,w_j)C'(u_i,w_j) < \sum_{i=1}^{m} \sum_{j=1}^{n} f(u_i,w_j)C'(u_i,w_j)$.

Let us construct a binary matrix B' as follows:

$B'(i,j) = 1$ if $f'(u_i,w_j) = 1$.
$B'(i,j) = 0$ otherwise

Since f' is a flow of network G associated with R and C, the row and column projections of B' are R and C respectively.

Claim: $\displaystyle\sum_{k=1}^{l}(|B' - S_k|) < \sum_{k=1}^{l}(|B - S_k|)$

As cost of f' min ,

$$\sum_{(i,j)\in B'_1} \sum_{k=1}^{l} S_k(i,j) > \sum_{(i,j)\in B_1} \sum_{k=1}^{l} S_k(i,j)$$

$$\implies \sum_{(i,j)\in B_1} \sum_{k=1}^{l} S_k(i,j) - \sum_{(i,j)\in B'_1} \sum_{k=1}^{l} S_k(i,j) < 0$$

$$\implies \Big(\sum_{(i,j)\in B'_0} \sum_{k=1}^{l} S_k(i,j) - \sum_{(i,j)\in B_0} \sum_{k=1}^{l} S_k(i,j) \Big)$$

$$+ \Big(\sum_{(i,j)\in B_1} \sum_{k=1}^{l} S_k(i,j) - \sum_{(i,j)\in B'_1} \sum_{k=1}^{l} S_k(i,j) \Big) < 0 \quad \text{(By Lemma 1)}$$

$$\implies \sum_{(i,j)\in B'_0} \sum_{k=1}^{l} S_k(i,j) + (l \sum_{i=1}^{m} r_i) - \sum_{(i,j)\in B'_1} \sum_{k=1}^{l} S_k(i,j)$$

$$< \sum_{(i,j)\in B_0} \sum_{k=1}^{l} S_k(i,j) + (l \sum_{i=1}^{m} r_i) - \sum_{(i,j)\in B_1} \sum_{k=1}^{l} S_k(i,j)$$

$$\implies \sum_{(i,j)\in B'_0} \sum_{k=1}^{l} |B'(i,j) - S_k(i,j)| + \sum_{(i,j)\in B'_1} \sum_{k=1}^{l} |B'(i,j) - S_k(i,j)|$$

$$< \sum_{(i,j)\in B_0} \sum_{k=1}^{l} |B(i,j) - S_k(i,j)| + \sum_{(i,j)\in B_1} \sum_{k=1}^{l} |B(i,j) - S_k(i,j)|$$

Hence the claim.

Since the claim contradicts

$$B = arg[\min_{T \in \Gamma_{(R,C)}} \sum_{k=1}^{l} (|T - S_k|)],$$

the cost of the flow f is minimum.

$\Longleftarrow$**:**

Let us assume that R and C are consistent and max flow value for the network G corresponds to R, C and S is $|f| = \sum_{i=1}^{m} r_i$, and cost of the flow is minimum.

Let us prove that there exists a binary matrix $B = (b_{i,j})$ such that row and column projections of B are R and C respectively, and

$$B = arg[\min_{T \in \Gamma_{(R,C)}} \sum_{k=1}^{l} (|T - S_k|)]$$

Let us construct binary matrix B from flow as follows:
$B(i,j) = 1$ if $f(u_i, w_j) = 1$, $B(i,j) = 0$ otherwise
As the cost of flow is minimum,

$$\sum_{i=1}^{m} \sum_{j=1}^{n} f(u_i, w_j) C'(u_i, w_j) \tag{3}$$

is min (By Lemma 2)
Since f is max-flow for G which is associated with R and C, and R and C are consistent, the row and column projections of B are R and C.

Claim :$B = arg[\min_{T \in \Gamma_{(R,C)}} \sum_{k=1}^{l} (|T - S_k|)]$

Suppose not, there exists B' such that

$$\sum_{k=1}^{l} |B' - S_k| < \sum_{k=1}^{l} |B - S_k|$$

$$\Longrightarrow \sum_{(i,j) \in B'_0} \sum_{k=1}^{l} |B'(i,j) - S_k(i,j)| + \sum_{(i,j) \in B'_1} \sum_{k=1}^{l} |B'(i,j) - S_k(i,j)|$$

$$< \sum_{(i,j) \in B_0} \sum_{k=1}^{l} |B(i,j) - S_k(i,j)| + \sum_{(i,j) \in B_1} \sum_{k=1}^{l} |B(i,j) - S_k(i,j)|$$

$$\Longrightarrow \sum_{(i,j) \in B'_0} \sum_{k=1}^{l} S_k(i,j) + l \sum_{i=1}^{m} r_i - \sum_{(i,j) \in B'_1} \sum_{k=1}^{l} S_k(i,j)$$

$$< \sum_{(i,j) \in B_0} \sum_{k=1}^{l} S_k(i,j) + l \sum_{i=1}^{m} r_i - \sum_{(i,j) \in B_1} \sum_{k=1}^{l} S_k(i,j)$$

$$\Longrightarrow\Big(\sum_{(i,j)\in B_0'}\sum_{k=1}^{l}S_k(i,j)-\sum_{(i,j)\in B_0}\sum_{k=1}^{l}S_k(i,j)\Big)$$
$$+\Big(\sum_{(i,j)\in B_1}\sum_{k=1}^{l}S_k(i,j)-\sum_{(i,j)\in B_1'}\sum_{k=1}^{l}S_k(i,j)\Big)<0$$
$$\Longrightarrow\Big(\sum_{(i,j)\in B_1}\sum_{k=1}^{l}S_k(i,j)-\sum_{(i,j)\in B_1'}\sum_{k=1}^{l}S_k(i,j)\Big)<0 \text{ (By Lemma 1)}$$

Let us construct flow f' as follows
$f'(u_i,w_j)=1$ if $B'(i,j)=1$
$f'(u_i,w_j)=0$ otherwise
It is easy to verify that f' is a max-flow for the network.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}f'(u_i,w_j)C'(u_i,w_j)=\sum_{(i,j)\in B_1'}\sum_{k=1}^{l}-S_k(i,j) \tag{4}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}f(u_i,w_j)C(u_i,w_j)=\sum_{(i,j)\in B_1}\sum_{k=1}^{l}-S_k(i,j) \tag{5}$$

From (3), (4) and (5), $\sum_{i=1}^{m}\sum_{j=1}^{n}f'(u_i,w_j)C'(u_i,w_j)<\sum_{i=1}^{m}\sum_{j=1}^{n}f(u_i,w_j)C'(u_i,w_j)$

which contradicts that the cost of f is minimum. Hence the claim.

Algorithm: $Binary\ image\ reconstruction$
Input: A set of images $S=(S_k)$ and row and column projections $R=(r_1,\ r_2,\ \ldots,\ r_m)$ and $C=(c_1,\ c_2,\ \ldots,\ c_n)$ of A
Output: Bi-level image $B=(b_{i,j})$ such that R and C are the row and column projections of $B=(b_{i,j})$ respectively, and

$$B=arg[\min_{T\in\Gamma_{(R,C)}}\sum_{k=1}^{l}(|T-S_k|)]$$

Initialization: $m :=$ the number of components in R, $n :=$ the number of components in C. For each $1\le i\le m$ and $1\le j\le n,\ b_{i,j}:=0$

Step 1: Compute cost matrix $C'=(c'_{i,j})$ where $c'_{i,j}=-\sum_{k=1}^{l}S_k(i,j)$
and $1\le i\le m$ and $1\le j\le n$,

Step 2: Construct Net work using projections and cost matrix as given below

$G\ =\ (V,\ E,\ C',\ C'')$ be a weighted directed graph where
$V\ =\ U\ \cup\ W\cup\{s,t\}$

$U \ = \ \{\, u_i \mid 1 \leq i \leq m\}$
$W \ = \ \{\, w_j \mid 1 \leq j \leq n\,\}$
$E = \{(u_i, w_j) \mid \ 1 \leq i \leq m, \ 1 \leq j \leq n\}$
$\quad \cup \{(s, u_i) \mid \ 1 \leq i \leq m\,\} \cup \ \{(w_j, t) \mid \ 1 \leq j \leq n\,\}$

We define the cost associated with each edge as follows: For each $1 \leq i \leq m$ and $1 \leq j \leq n$,
$C'(u_i, w_j) = -c_{i,j}, \ C'(s, u_i) = -1$ and $C'(w_j, t) = -1$

We define the capacity associated with each edge as follows: For each $1 \leq i \leq m$ and $1 \leq j \leq n$,
$C''(u_i, w_j) = 1, \ C''(s, u_i) = r_i$ and $C''(w_j, t) = c_j$ where s is the source and t is the sink.

Step 3. Compute Min-cost Max-flow for the network constructed in step 2.

Step 4. Construct image from flow obtained in step 3 as follows For each $1 \leq i \leq m$ and $1 \leq j \leq n$, $b_{i,j} = 1$ if flow from u_i and w_j is 1

4 Simulation Studies

We have taken a real image of a vascular system of size 64×64 (Fig 1.) and created artificial blocks in the blood vessels at random locations. A set of ten images is synthesized from the real image by creating artificial blocks, and those images are considered as similar images (Fig 2(a) through Fig 2(j)). Another set of five images is synthesized from the real image by creating artificial blocks at random locations, and they are considered as test images (Fig 3(a). through Fig 3(e)). We considered each test image as unknown image and computed row and column projections of each test image. We reconstructed each test image without any error from its row and column projections using those similar images we have considered. But, the reconstructed images are not same as the unknown images(test images) when we considered some subsets of Fig 2(a) through Fig 2(j) as shown in the following table. We define the reconstruction **error** as $|A - B|$ where A is the unknown image (test image) B is the reconstructed image. It may be noted that A and B have same row and column projections. We have shown some experimental results for the reconstruction of a test image namely Fig 3(a) using various subsets of Fig 2(a through j). From the experimental results shown in the table, we can infer that more the a priori images or closer the a priori image to the unknown image lesser will be the reconstruction error. Note that when a priori images Fig 2. b,f,h,j are considered separately, the reconstructed errors are 68, 76,50,208 pixels

Fig. 1. Real vascular image

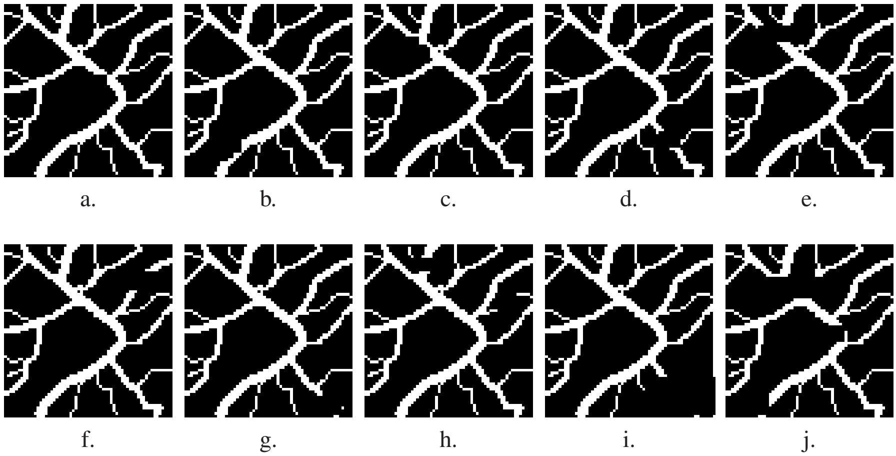

Fig. 2. The a priori images that are similar to the unknown image

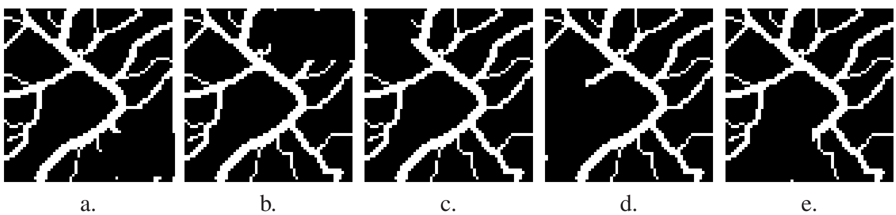

Fig. 3. The unknown images

respectively, but no error while all of them considered together. The error is 32 pixels When Fig 2. f and j are considered together, and 8 pixels are erroneous while Fig 2. b,f,j are considered together. It may also be noted that there is no reconstruction error when Fig 2(d) alone is considered as Fig 2(d) is very close to the unknown image.

Experimental Results

Table 1. Reconstruction error for various set of a priori images

S	error	S	error	S	error
{ a }	4	{ i }	4	{a, b, c }	0
{ b }	68	{ j }	208	{d, e, f }	0
{ c }	14	{ a, b }	4	{g, h, i }	0
{ d }	0	{ c, d }	0	{b, f, j }	8
{ e }	22	{ e, f }	0	{b, f, h, j }	0
{ f }	76	{ g, h }	0	{a, b, c, d, e }	0
{ g }	12	{ i, j }	0	{f, g, h, i, j }	0
{ h }	50	{ f, j }	32		

5 Correctness and Complexity

By Theorem 1, correctness is evident. The time complexity of Step 1 is $O(mnl)$ where m is number of rows , n is number of columns and l is number of similar images. As Min-cost Max-flow can be solved in $O(V^3 log^2 V)$ where V is the number of vertices in the network, Step2 and Step 3 take $O(V^3 log^2 V)$ where $V = m + n + 2$. Step 4 takes $O(mn)$. The dominant part of our algorithm is computing Min-cost Max-flow when l is considered as constant. Hence the time complexity of our algorithm is $O(V^3 log^2 V)$ where $V = m + n + 2$.

6 Conclusion

In this paper we have reconstructed a 2D-bi-level image from its two orthogonal projections using a priori images that are similar to the unknown image. 2D-bi-level image reconstruction from two orthogonal projections has polynomial time algorithm [11], 2D-tri-level image reconstruction from two orthogonal projections is still open, 2D-four-level image reconstruction from two orthogonal projection is NP-Complete [1]. The problem that we have solved is more complex than 2D-bi-level image reconstruction from two orthogonal projections and less complex than 2D-tri-level image reconstruction from two orthogonal projections. We implemented our algorithm and compared the quality of reconstructed image by our algorithm with the quality of reconstructed image by algorithm, which does not consider a priori similar images, given in [11]. The reconstruction of 3D-bi-level image from two orthogonal projections with a priori similar images can be done by slice-by-slice reconstruction using the proposed algorithm. One of the possible areas in which our algorithm can be used is medical imaging. Another area of application is crystallography. Our algorithm can be used to reconstruct crystalline structure from two projections without damaging the crystal. Though our algorithm always constructs an image whose orthogonal projections are the same as the orthogonal projections of the unknown image, the reconstructed image may be distorted from the unknown image if the similar images are not close to the unknown image or the number of a priori images is very small.

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