Active Vibration Control: Complete Guide

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Introduction to Euler-Bernoulli Beam Theory

Basic Concepts:

This theory is essential in understanding the behavior of beams under various loading conditions.

It's a simplification of the more general Timoshenko beam theory and is particularly accurate for slender beams where the shear deformation is much smaller than the bending deformation.

The small deflection of a beam as a function of the moment distribution is given by:

$$M(x) = EI\frac{d^2y(x)}{dx^2}$$

- **Shear Force and Distributed Loads**: The relationship between shear force and applied distributed loads is given by $\frac{dV(x)}{dx} = w(x)$.
- **Moment and Shear Force**: The relationship between moment and shear force is expressed as $\frac{dM(x)}{dx} = V(x)$.

- ***Differential Equations***

- The Euler-Bernoulli beam equation is a fourth-order differential equation, necessitating four boundary conditions. These conditions can be Dirichlet (applied deflection), Neumann (applied slope), or higher order (applied shear or moment).

- ***Galerkin Weak Statement***

- The Galerkin Weak Statement (GWS) is a method used to approximate the solutions of differential equations. It involves multiplying the differential equation by a test function and integrating over the domain. This approach is essential in finite element analysis.

- ***Discretization and Numerical Implementation***

- The Euler-Bernoulli equation breaks down into two differential equations connected through the definition of moment. This approach simplifies the problem and allows for numerical implementation, particularly in MATLAB.

- The concept of discretizing the domain and forming a matrix statement (LHS*Q=RHS) is introduced. Here, LHS is a square matrix, Q is the column vector of unknowns, and RHS is the column vector of knowns.

- ***Boundary Conditions***

- The boundary conditions for solving the Euler-Bernoulli beam problem include shear (Neumann) and moment (Dirichlet) on the boundary. These conditions are crucial for accurately modeling the behavior of the beam under various loading and support scenarios.

Derivation of the Euler-Bernoulli Beam Equation

Step 1: Relationship Between Moment and Curvature

The basic assumption of the Euler-Bernoulli beam theory is that the curvature of the beam (κ) is proportional to the bending moment (M) at any point along its length. Mathematically, this is expressed as:

$$M(x) = EI\frac{d^2y(x)}{dx^2}$$

where:

- $M(x)$ is the bending moment at a point x along the beam.
- E is the modulus of elasticity of the material.
- I is the moment of inertia of the beam's cross-section.
- $\frac{d^2y(x)}{dx^2}$ is the second derivative of the deflection curve $y(x)$, representing the curvature.

Step 2: Shear Force and Moment Relationship

The shear force (V) in a beam section is related to the bending moment by the derivative:

$$\frac{dM(x)}{dx} = V(x)$$

Step 3: Relationship Between Shear Force and Load

The shear force is related to the distributed load $(w(x))$ acting per unit length of the beam. This relationship is given by:

$$\frac{dV(x)}{dx} = w(x)$$

Step 4: Combining the Equations

By differentiating the moment equation with respect to x and using the shear force relationship, we get:

$$\frac{d^2M(x)}{dx^2} = \frac{dV(x)}{dx} = w(x)$$

Step 5: Substituting Moment-Curvature Relationship

Substitute the moment-curvature relationship into the above equation:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2y(x)}{dx^2}\right) = w(x)$$

- The Euler-Bernoulli beam equation is a fundamental concept in structural engineering for analyzing beam deflection under various loading conditions.
- It simplifies the complex interactions in a beam to a manageable mathematical model, allowing engineers to predict the behavior of structural elements under different scenarios.

Step 6: Euler-Bernoulli Beam Equation

Rearranging and simplifying, we arrive at the Euler-Bernoulli beam equation:

$$EI\frac{d^4y(x)}{dx^4} = w(x)$$

This is a fourth-order linear differential equation where $\frac{d^4y(x)}{dx^4}$ represents the beam's deflection due to the distributed load $w(x)$.

$$Generalized\ Version:$$

$$L(y) = -\frac{d^2}{dx^2}\left(EI(x)\frac{d^2y(x)}{dx^2}\right) + w(x) = 0 \quad \Omega \in (0, L)$$

Problem: Beam Deflection Analysis

Scenario:

You are tasked with assessing the structural integrity of a beam in a construction project. The beam, with a length of 6 meters, is subjected to multiple types of loads:

1. A uniformly distributed load (UDL) of 5000 N/m over its entire length.
2. A concentrated point load of 10000 N at the midpoint.
3. A triangular load with a maximum intensity of 8000 N/m.

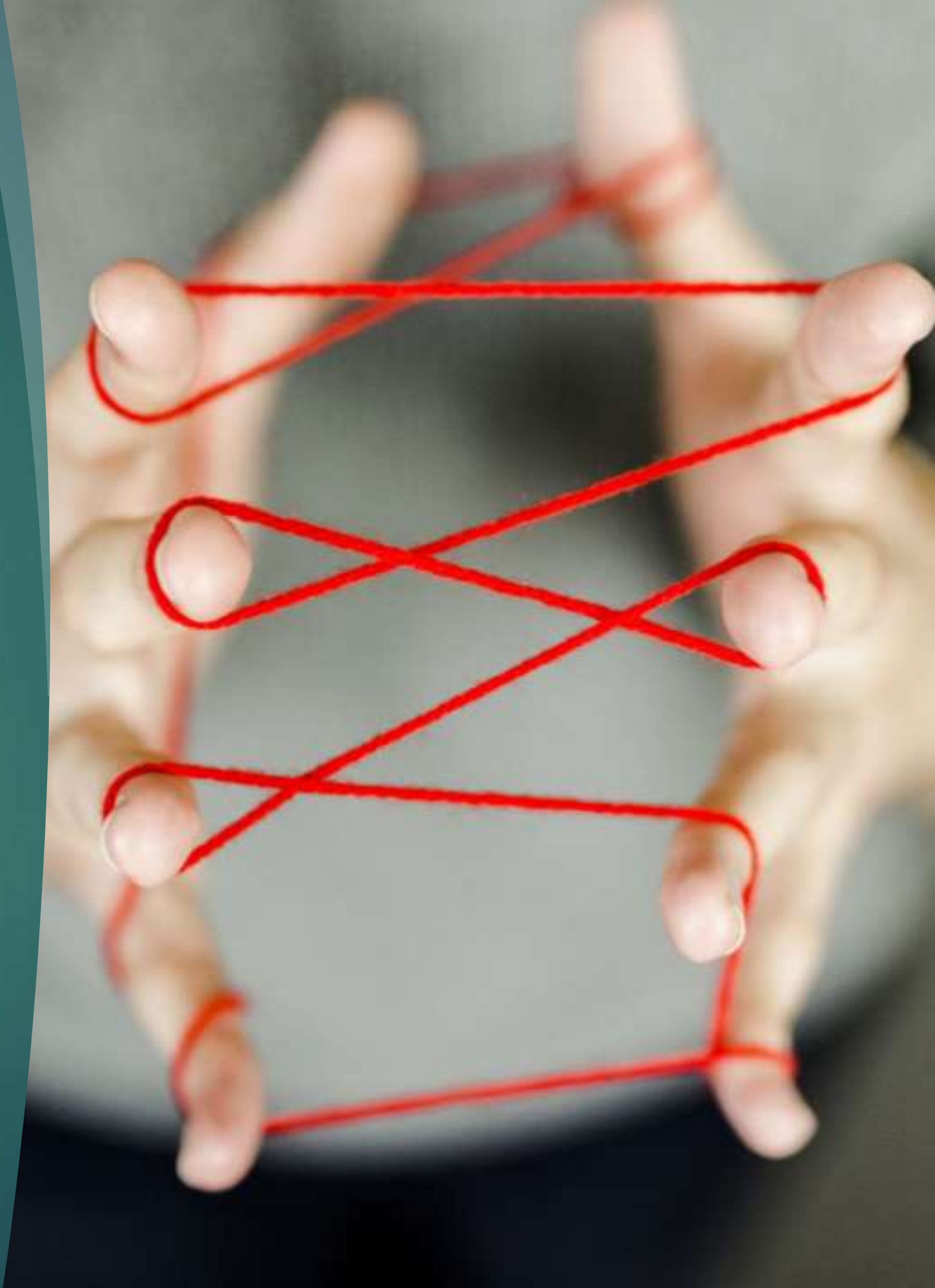
The beam's material has a modulus of elasticity of 200 GPa and a moment of inertia of 8×10^-6 m^4. Using Euler-Bernoulli beam theory, you need to:

- Determine the reactions at the supports.
- Calculate the shear force and bending moment along the beam.
- Analyze the deflection of the beam under these loading conditions.
- Identify the points of maximum deflection and assess whether they are within acceptable limits for safety and functionality.

- **Specific Questions for Analysis:**

1. What are the reactions at the left and right supports?
2. How do the shear force and bending moment vary along the length of the beam?
3. What is the deflection of the beam at its midpoint, at 2 meters from the left support, and at the right support?
4. Where does the maximum deflection occur along the beam, and what is its magnitude?
5. Considering the given loads and material properties, is the beam design safe and functional?

- This problem will require you to integrate your understanding of structural mechanics, particularly focusing on beam loading and deflection characteristics. Your analysis should ensure that the beam's design meets the required safety and performance standards.

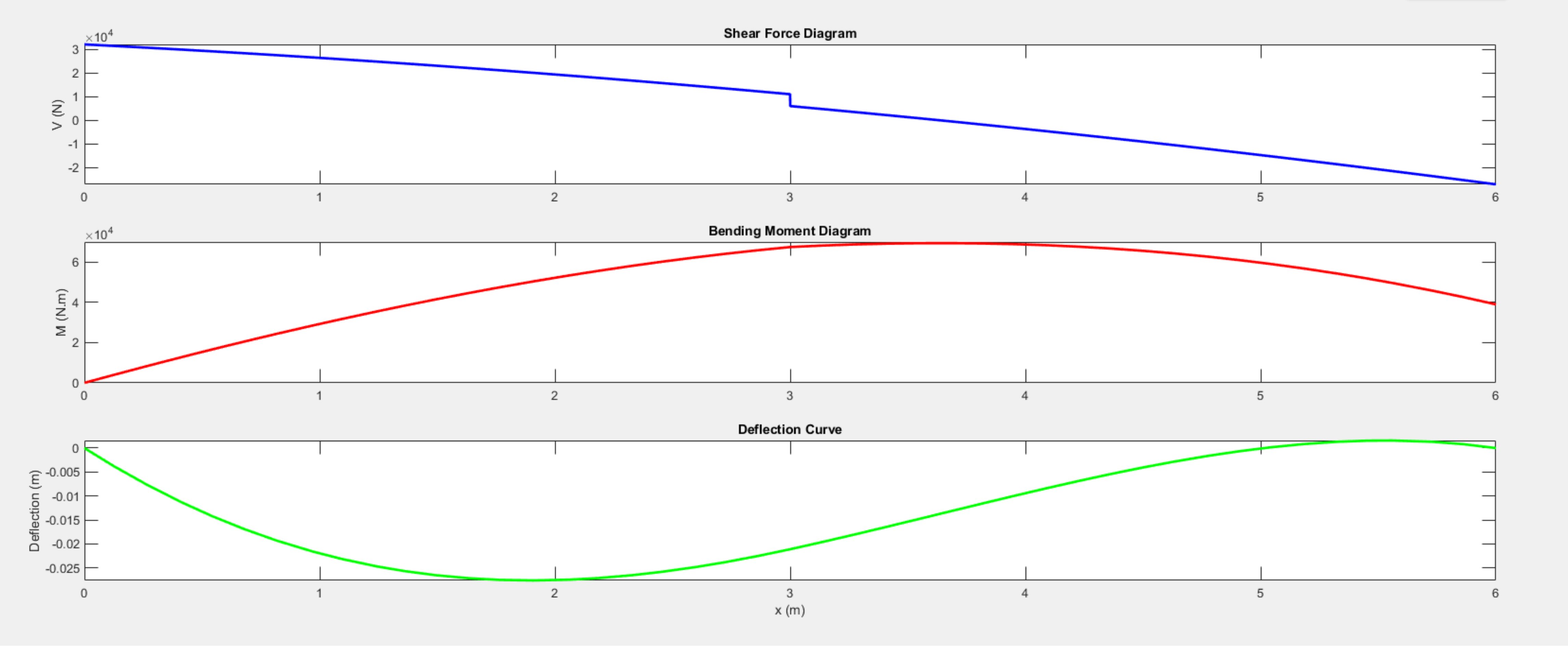


1. **Initialization of Given Data:**
   - `L = 6;` sets the beam's length to 6 meters.
   - `E = 200e9;` specifies the modulus of elasticity (E) of the beam material in Pascals (Pa).
   - `I = 8e-6;` defines the moment of inertia (I) of the beam's cross-section in m^4.
   - `w = 5000;` is the magnitude of the uniformly distributed load (UDL) in Newtons per meter (N/m).
   - `P = 10000;` indicates the point load in Newtons (N) applied at the beam's midpoint.
   - `w_max = 8000;` represents the maximum intensity of the triangular distributed load in N/m.
2. **Calculating Reactions at Supports:**
   - The reactions at the supports (`R1` and `R2`) are calculated considering the beam's symmetric loading. It accounts for the UDL, point load, and triangular load.
3. **Shear Force and Bending Moment Calculations:**
   - MATLAB's symbolic math toolbox is used (`syms x C1 C2;`) to define symbolic variables for the calculation.
   - The shear force `V` and the bending moment `M` are expressed as functions of the position along the beam (`x`).

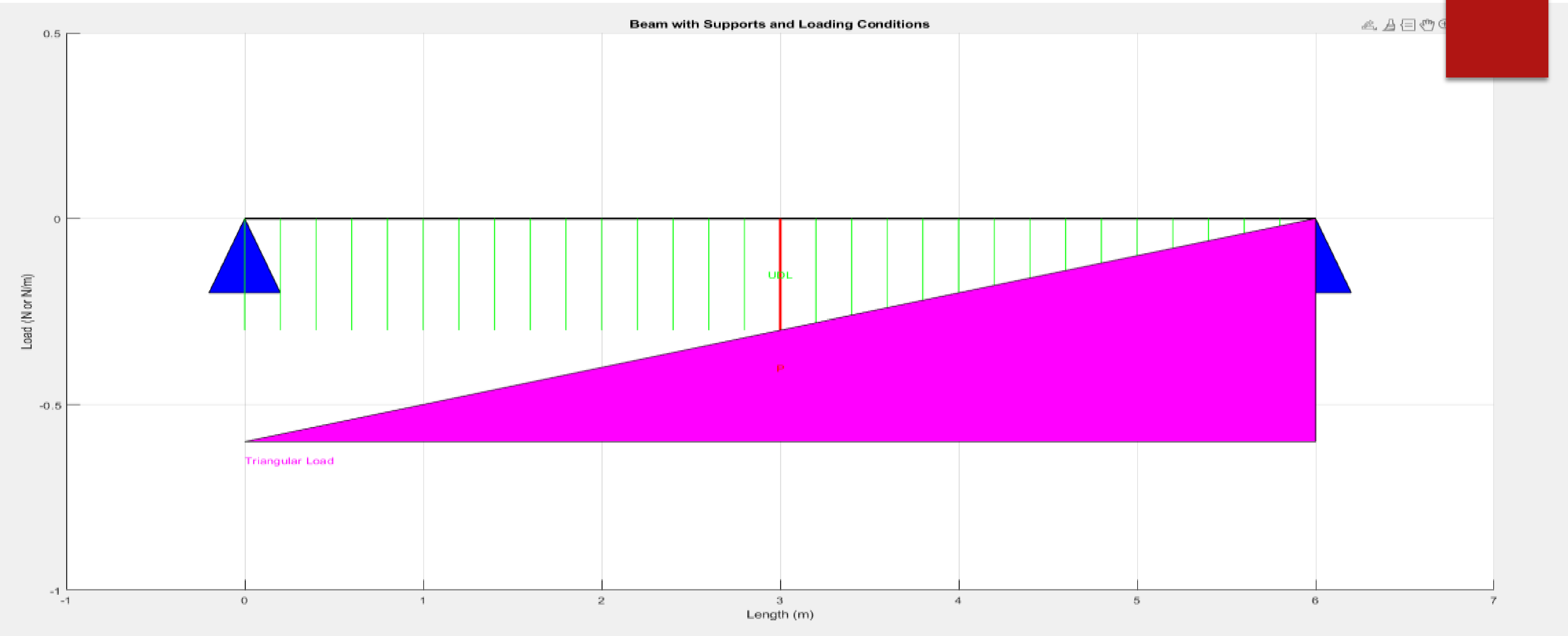
beamdeflection.m

4. **Deflection Analysis:**
   - The second derivative of the bending moment `M` with respect to `x` is divided by the product of modulus of elasticity `E` and moment of inertia `I` to get the differential equation of the deflection curve.
   - This equation is integrated twice to get the deflection `y_int`, and integration constants `C1` and `C2` are added.
5. **Applying Boundary Conditions:**
   - Boundary conditions are applied to solve for the integration constants `C1` and `C2`. For a simply supported beam, these are typically that the deflection is zero at both supports.
6. **Substituting Back the Constants:**
   - The solutions for `C1` and `C2` are substituted back into the deflection equation to get the final form of the deflection curve `y`.
7. **Numerical Evaluation of Deflection:**
   - A function handle `y_func` is created for numerical evaluation of the deflection.
   - The deflection is calculated at specific points: midpoint, 2m from the left support, and at the right support. The maximum deflection is also determined.
8. **Displaying Results:**
   - The script prints out the calculated reactions at supports, deflections at specific points, and the maximum deflection.
9. **Plotting Beam Analysis:**
   - The script generates several plots to visually represent the shear force diagram, bending moment diagram, and deflection curve.
10. **Visual Representation of Beam and Loading Conditions:**
    - A second figure is created to visually represent the beam, supports, and loading conditions. This includes drawing the beam, supports, UDL, point load, and triangular load.

This code provides a comprehensive analysis of the beam under various loading conditions, calculating key structural parameters like shear force, bending moment, and deflection, which are crucial for assessing the structural integrity of the beam.



Shear Force Diagram
×10^4
V (N)
3
2
1
0
-1
-2
0
1
2
3
4
5
6
Bending Moment Diagram
×10^4
M (N.m)
6
4
2
0
Deflection Curve
Deflection (m)
0
-0.005
-0.01
-0.015
-0.02
-0.025
x (m)

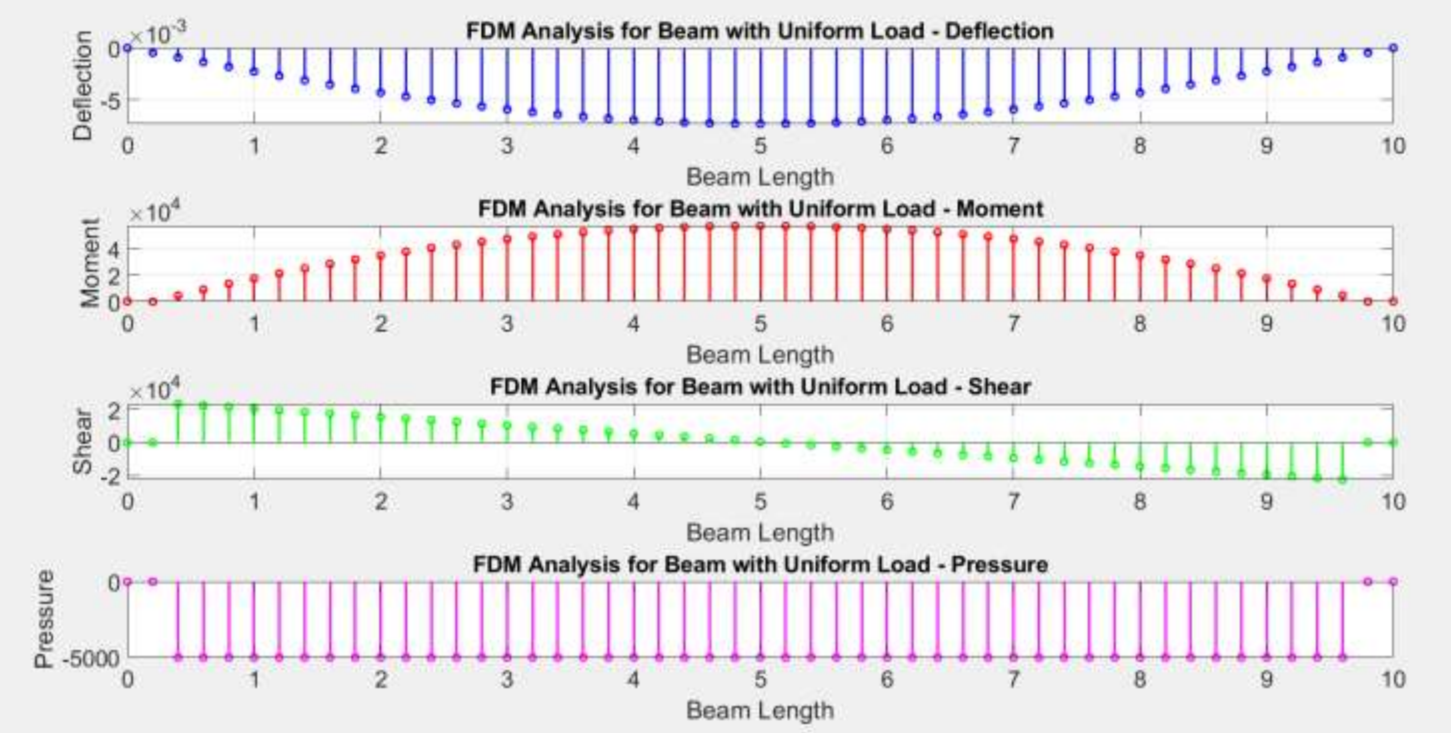


Reactions at Supports: R1 = 32000.00 N, R2 = 32000.00 N
Deflection at Midpoint: -2.1094e-02 m
Deflection at 2m from Left Support: -2.7535e-02 m
Deflection at Right Support: -1.1276e-17 m
Maximum Deflection: -1.5798e-03 m at x = 5.53 m

beamdeflection_fdm.m

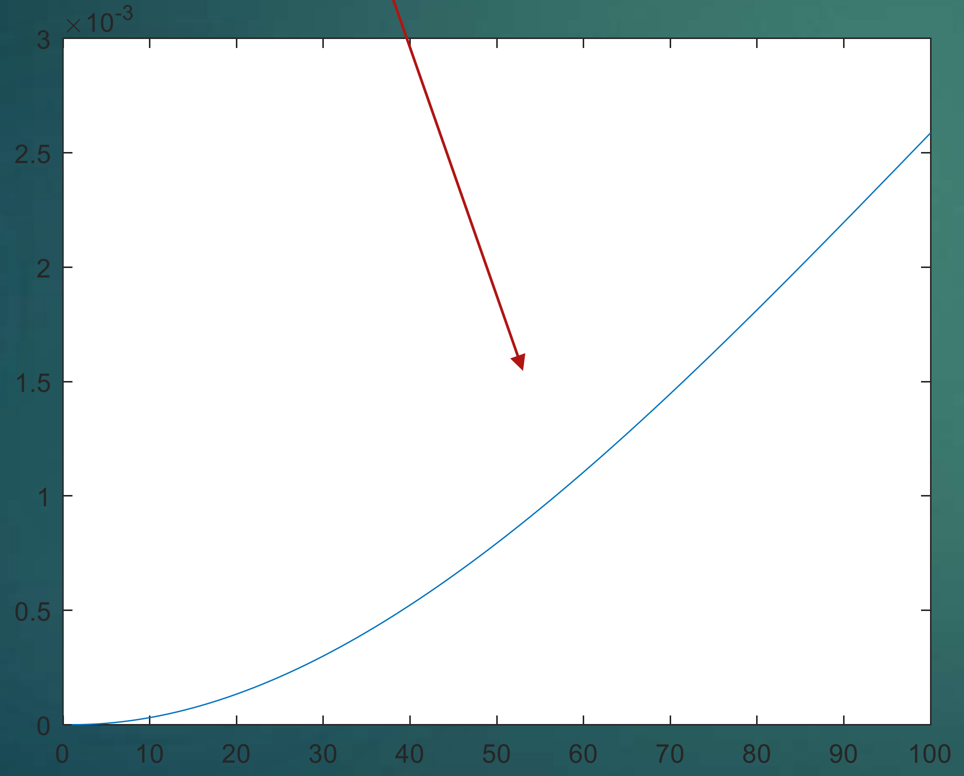
```
%Example of Realistic Parameters:
%Length of the beam, L: 10 meters
%Moment of inertia, I: 0.0004
%Elastic modulus, E: 200 GPa (200 x 10^9 N/m^2)
%Uniformly distributed load, w0: 5000 N/m
%Number of elements, ne: 50
%beamdeflection_fdm(10, 0.0004, 200e9, 5000, 50);
```

```
>> beamdeflection_fdm(10, 0.0004, 200e9, 5000, 50);
Max displacement = 0.007344
Max moment = 57400.000000
Max shear = 23500.000000
Max pressure = 5000.000000
```



beamdeflection_alternative.m

- In this example, a load force of 1500 N, Young's modulus of 210 GPa, beam length of 3 meters, beam width of 0.15 meters, and a circular cross-section are used. The function will calculate the end deflection, end slope, and the deflection vector for the beam.



A Finite Element Solution of the Beam Equation

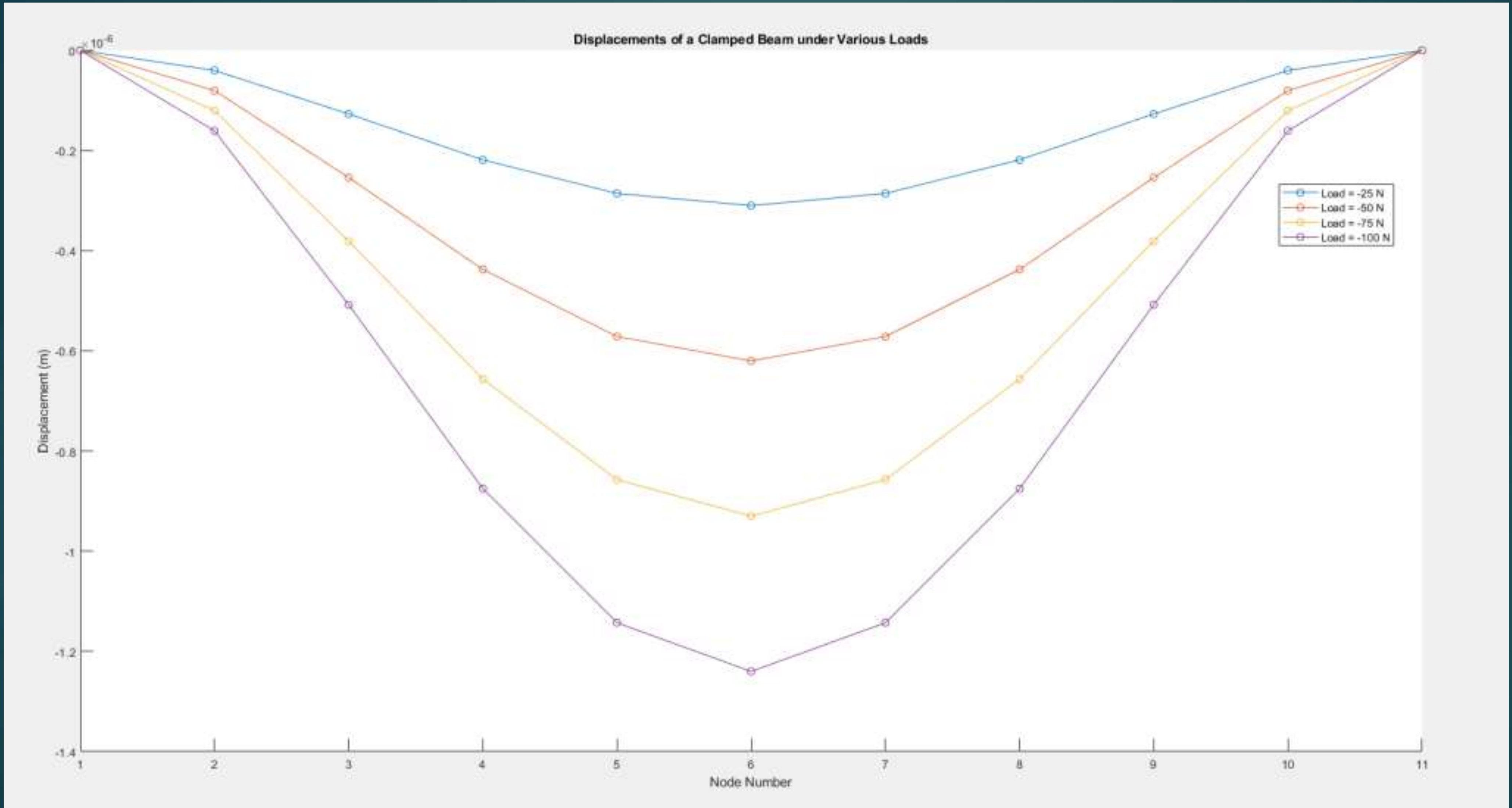
1. **Euler-Bernoulli Beam Theory**:
   - The transverse deflection $w(x)$ of the beam is governed by a fourth-order differential equation:
     $EI\frac{d^4w}{dx^4} = q(x), \quad 0 \leq x \leq L$
     where EI is the flexural rigidity (product of Young's modulus E and moment of inertia I), $q(x)$ is the transversely distributed load, and L is the length of the beam.
2. **Finite Element Method (FEM)**:
   - **Discretization**: The beam's domain is divided into finite elements with nodes.
   - **Weak Formulation**: Involves integrating the differential equation by parts using a weight function $\phi(x)$.
   - **Galerkin Approach**: The weight function $\phi(x)$ is equated to the approximating function $N(x)$, typically a cubic interpolation polynomial for third-order derivatives.

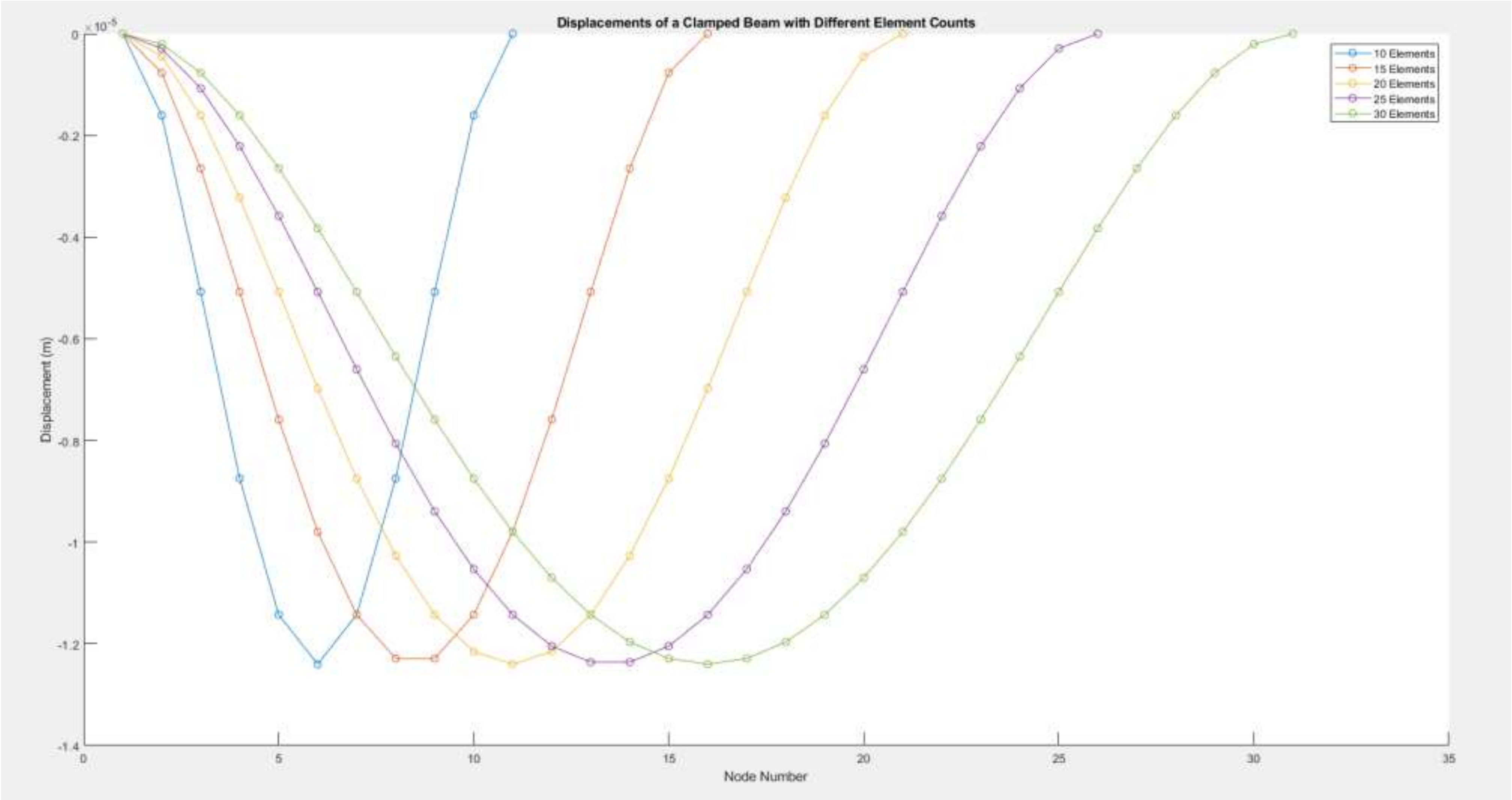
3. **Cubic Interpolation Polynomial (Hermite Cubic Interpolation)**:
   - The shape functions $N_i(x)$ are defined as cubic polynomials, ensuring continuity and differentiability up to the third order.
4. **Stiffness Matrix and Force Vector**:
   - The stiffness matrix K and force vector F are derived from the weak form of the differential equation and the shape functions.
5. **Solution of the System**:
   - The system of equations $[K][w] = [F]$ is solved using MATLAB, where $[w]$ is the displacement matrix.

Step-by-Step Symbolic Derivation

1. **Differential Equation**:
   - Start with the Euler-Bernoulli beam equation:
     $EI\frac{d^4w}{dx^4} = q(x)$
2. **Weak Formulation**:
   - Multiply by a weight function $\phi(x)$ and integrate:
     $\int_0^L EI\frac{d^4w}{dx^4}\phi(x)dx = \int_0^L q(x)\phi(x)dx$
3. **Galerkin Approach**:
   - Assume $\phi(x) = N(x)$, where $N(x)$ are cubic interpolation functions.
4. **Shape Functions (Cubic Interpolation)**:
   - Define $N_i(x)$ based on cubic polynomials.
5. **Stiffness Matrix and Force Vector**:
   - Derive K and F using the shape functions and the weak form.
6. **Solving the System**:
   - Use MATLAB to solve $[K][w] = [F]$ for the displacement $[w]$.

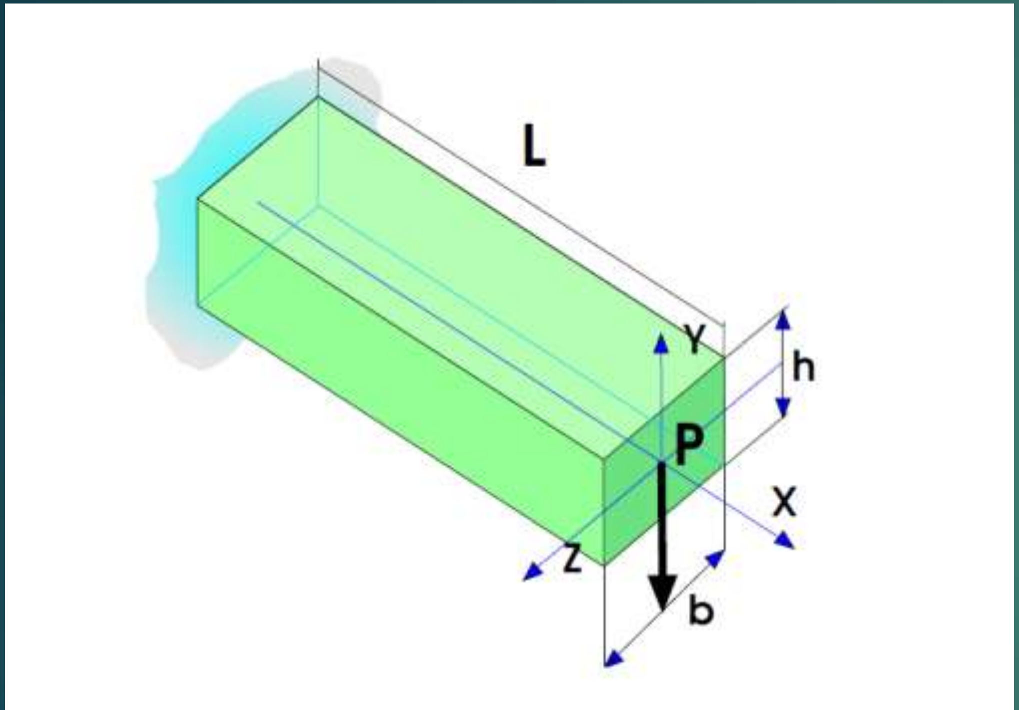
formStiffness_clampedbeam.m





Displacements of a Clamped Beam with Different Element Counts
×10⁻⁵
Displacement (m)
Node Number
10 Elements
15 Elements
20 Elements
25 Elements
30 Elements
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
0
5
10
15
20
25
30
35

Beam Vibration



Classical Euler-Bernoulli beam theory is adopted for the analysis. The beam undergoes time-and-space-dependent lateral displacement, represented as:

$$v(x,t) = \bar{v}(x)\sin(\omega t)$$

where ω is the natural frequency, and $\bar{v}(x)$ is the mode shape of the vibration

The approach consists of developing equations of motion for an infinitesimal beam segment, using linear elastic beam theory to obtain a differential equation, and solving this equation with appropriate boundary conditions at the beam ends

- Fundamental vibration frequency of a uniform cantilever beam is evaluated as:

$\omega = \sqrt{\frac{3EI}{\alpha \rho A L^3}}$, where α is a dimensionless factor, E is Young's modulus, I is the area moment of inertia, ρ is the density, A is the cross-sectional area, and L is the length of the beam

MODE SHAPE

The lowest (first mode, or fundamental) frequency is expressed as $\omega = \sqrt{\frac{2\xi^2 EI}{L^2 \rho A}}$, where ξ is obtained as the lowest positive root to the transcendental equation $1 + \cos(\xi)\cosh(\xi) = 0$. This equation defines the spectrum of natural frequencies and corresponding mode shapes

Formulating the Differential Equation

The differential equation for the mode shape $\bar{v}(x)$ and associated frequency ω is given by $\bar{v}''''(x) - \frac{\rho A \omega^2}{EI}\bar{v}(x) = 0$, where ρ is the density, A is the cross-sectional area, E is Young's modulus, and I is the area moment of inertia

Introduce the parameter β with dimensions of length^-1 by $\beta^4 = \frac{\rho \omega^2 A}{EI}$.
The differential equation then becomes $\bar{v}''''(x) - \beta^4 \bar{v}(x) = 0$

General Solution of the Differential Equation

The linear, ordinary differential equation with constant coefficients has homogeneous solutions of the form $\bar{v}(x) = C_1 \cosh(\beta x) + C_2 \sinh(\beta x) + C_3 \cos(\beta x) + C_4 \sin(\beta x)$, where C_1, C_2, C_3, and C_4 are constants to be determined

This derivation combines principles of dynamics, material mechanics, and mathematical methods to establish a fundamental understanding of beam vibrations.

The differential equation derived is crucial for analyzing the natural frequencies and mode shapes of a vibrating cantilever beam, which is a fundamental problem in structural dynamics and engineering.

- To derive the general solution for the mode shapes of a cantilever beam undergoing vibration, we'll use symbolic representation and follow a step-by-step approach.
- The derivation is based on the Euler-Bernoulli beam theory, which is a classical theory in structural mechanics.

Step 1: Establishing the Differential Equation

The Euler-Bernoulli beam theory leads to a fourth-order linear differential equation for the deflection $v(x)$ of the beam:

$$EI\frac{d^4v(x)}{dx^4} + \rho A\omega^2 v(x) = 0$$

where:

- EI is the flexural rigidity of the beam (Elastic Modulus E times Area Moment of Inertia I).
- ρ is the density of the material.
- A is the cross-sectional area.
- ω is the natural frequency of vibration.

Step 2: Introducing a Dimensionless Parameter

Introduce a dimensionless parameter β, defined as:

$$\beta^4 = \frac{\rho A \omega^2}{EI}$$

The differential equation becomes:

$$\frac{d^4 v(x)}{dx^4} - \beta^4 v(x) = 0$$

Step 3: Solving the Differential Equation

The general solution to this linear differential equation with constant coefficients can be expressed as a linear combination of exponential functions. However, for physical and practical reasons (real and bounded solutions), we use hyperbolic and trigonometric functions:

$$v(x) = C_1 \cosh(\beta x) + C_2 \sinh(\beta x) + C_3 \cos(\beta x) + C_4 \sin(\beta x)$$

where C_1, C_2, C_3, and C_4 are constants to be determined from boundary conditions.

Step 4: Applying Boundary Conditions for a Cantilever Beam

For a cantilever beam, the boundary conditions at the fixed end (at $x = 0$) are:

1. Zero deflection: $v(0) = 0$
2. Zero slope: $\frac{dv}{dx}(0) = 0$

Applying these conditions, we get:

1. $C_1 + C_3 = 0$ (from $v(0) = 0$)
2. $\beta C_2 + \beta C_4 = 0$ (from $\frac{dv}{dx}(0) = 0$)

Step 5: Simplifying the Solution

Using the boundary conditions, the general solution simplifies to:

$$v(x) = C_1(\cosh(\beta x) - \cos(\beta x)) + C_2(\sinh(\beta x) - \sin(\beta x))$$

Conclusion

This is the general form of the mode shape for a cantilever beam. The constants C_1 and C_2 are typically determined from further boundary conditions at the free end of the beam or by normalizing the mode shapes. The specific mode shapes and natural frequencies depend on the length, material properties, and geometry of the beam.

cantileverbeammodeshape.m

Script Overview

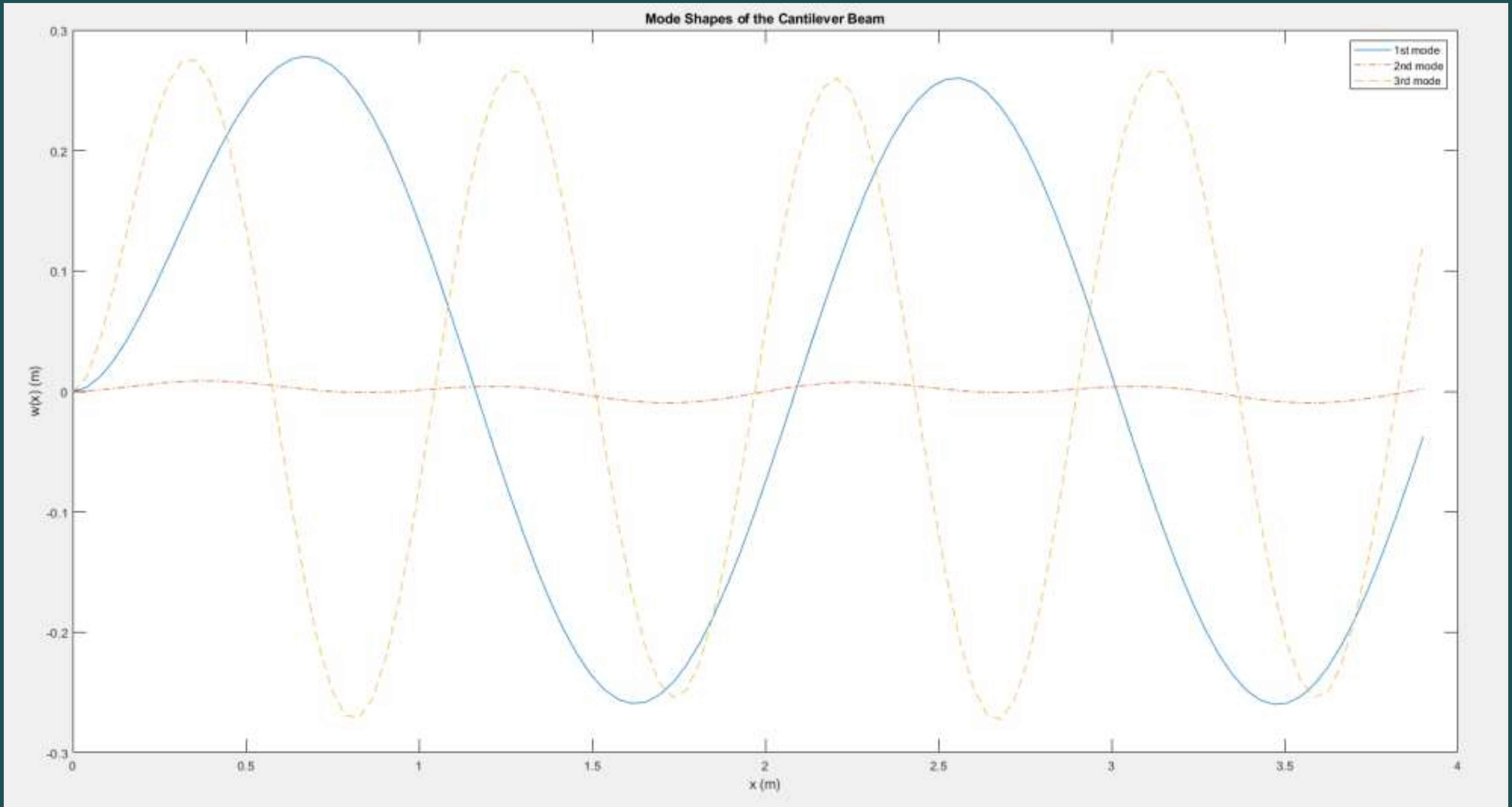
1. **Environment Setup and Parameter Definition**
   - The script starts by setting the number format to long scientific notation for precision.
   - A symbolic variable `x`, representing the spatial coordinate along the beam's length, is defined.
   - Physical parameters of the cantilever beam are defined, including Elastic Modulus (`E`), mass density (`rho`), length (`L`), cross-sectional area (`A`), and area moment of inertia (`I`).
2. **Definition of `beta`**
   - `beta` is a crucial parameter in the analysis, typically derived from the beam's properties. It's used in defining the mode shapes. The script should include a specific formula for `beta` based on the beam's characteristics.

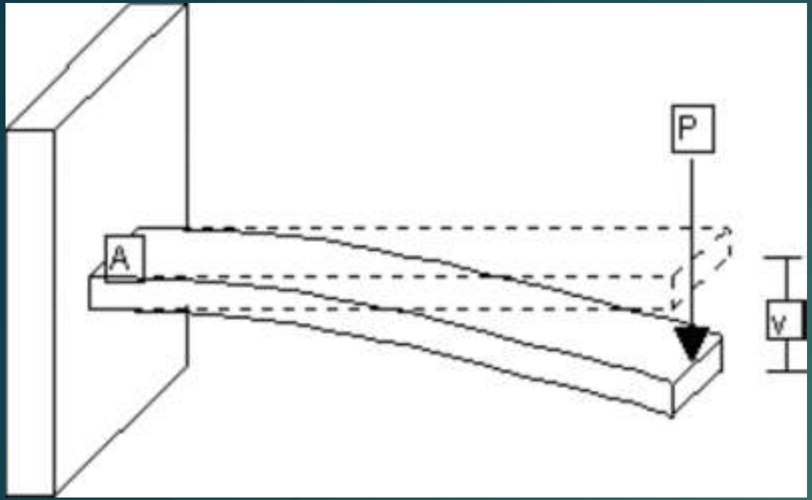
3. **Mode Shape Definition**
    - The script defines the first three mode shapes of the cantilever beam. These mode shapes are expressed as a combination of hyperbolic and trigonometric functions (`sinh`, `sin`, `cosh`, `cos`). The specific form of these functions is chosen based on the solution to the differential equation governing beam vibrations under cantilever boundary conditions.
4. **Mass and Stiffness Matrix Calculation**
    - The script calculates the mass (`M`) and stiffness (`K`) matrices by integrating the products of mode shapes and their derivatives over the beam's length. This is a standard procedure in structural dynamics for forming the equations of motion in matrix form.

5. **Eigenvalue Problem for Natural Frequencies and Mode Shapes**
   - The script solves an eigenvalue problem to find the natural frequencies and mode shapes of the beam. This is done by calculating the system matrix (`C`) as the product of the inverse of the mass matrix and the stiffness matrix, and then finding its eigenvectors (`V`) and eigenvalues (`D`). The square roots of the eigenvalues give the natural frequencies.
6. **Normalization of Eigenvectors**
   - The eigenvectors are normalized to form a modal matrix (`P`). This step ensures that the mode shapes are scaled correctly.
7. **Visualization of Mode Shapes**
   - The script visualizes the mode shapes by plotting them over the beam's length. This is achieved by evaluating the mode shapes at a series of points (`xx`) along the beam and using the normalized eigenvectors.
8. **Plotting**
   - Finally, the script plots the first three mode shapes of the cantilever beam.

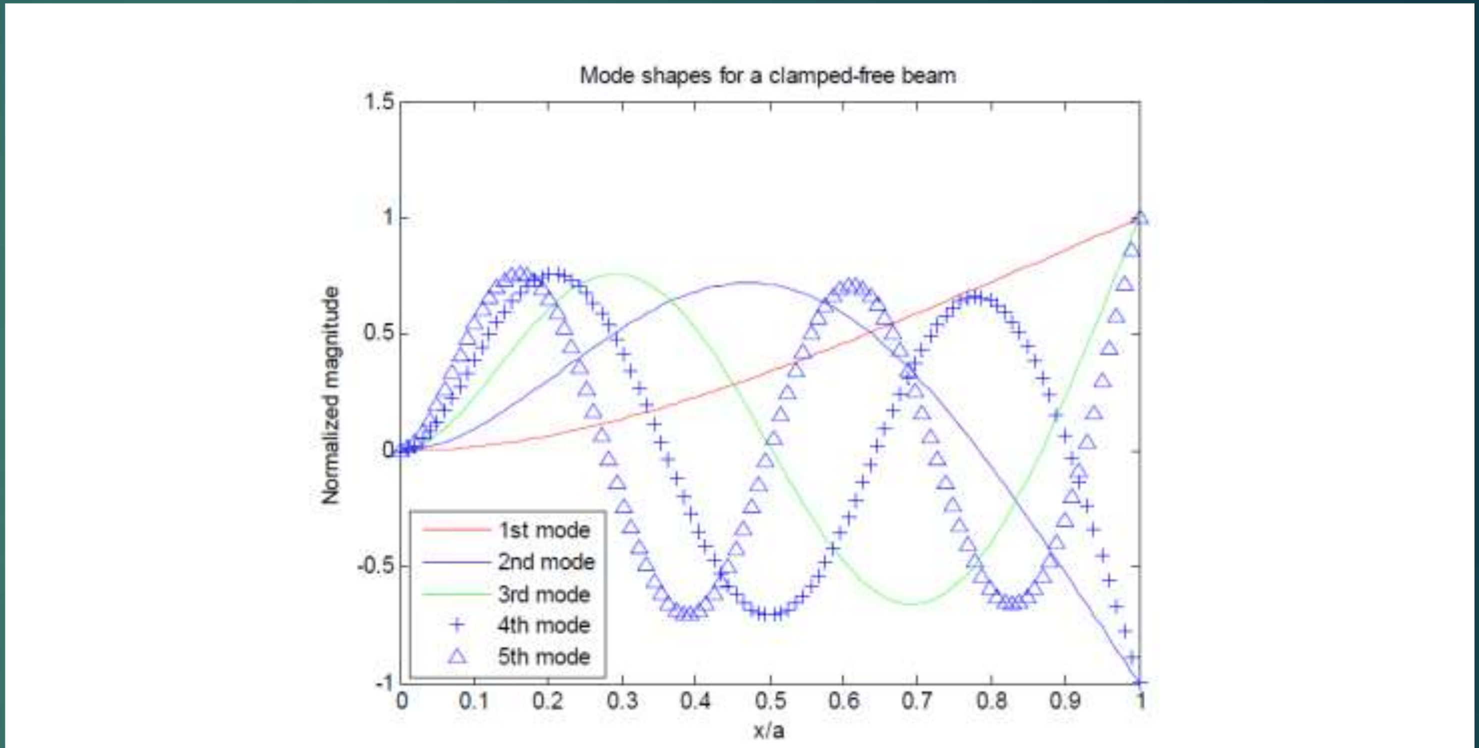
Key Points

- The script is a typical implementation of modal analysis for a cantilever beam in MATLAB.
- The accuracy of the mode shapes and natural frequencies depends on the correct definition of `beta` and the mode shape functions.
- The script combines symbolic computation (for defining mode shapes and integrating) with numerical methods (for solving the eigenvalue problem and plotting).


Mode Shapes of the Cantilever Beam
1st mode
2nd mode
3rd mode
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
w(x) (m)
0
0.5
1
1.5
2
2.5
3
3.5
4
x (m)

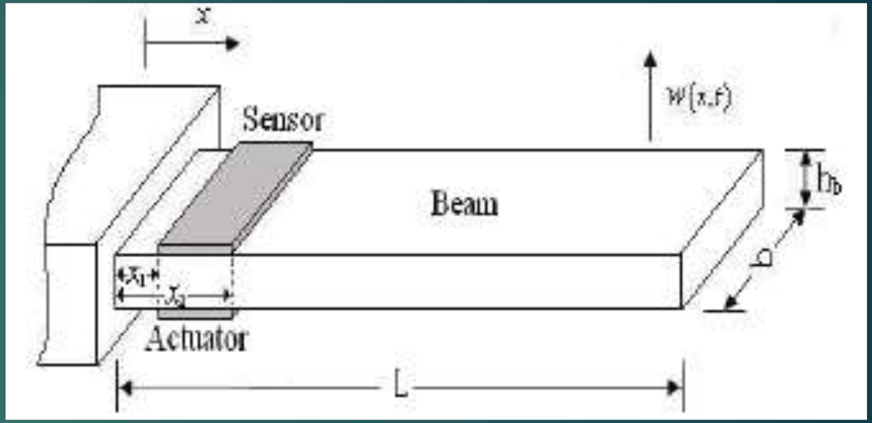


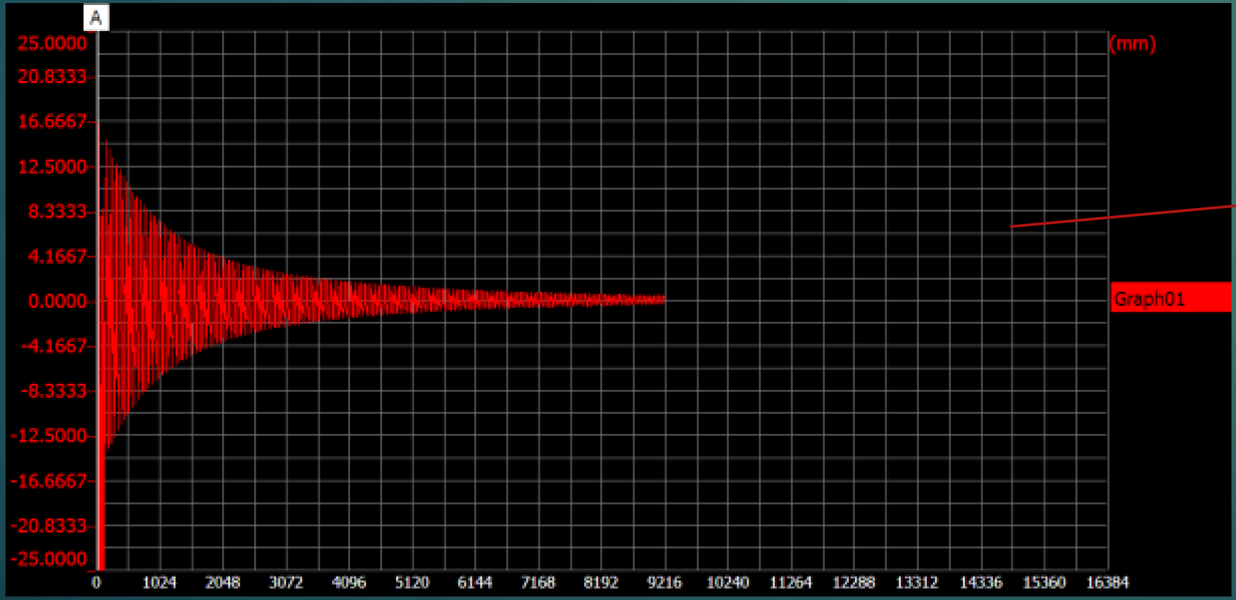
n	k_nL
1	1.875104069
2	4.694091133
3	7.854757438
4	10.99554073
5	14.13716839
6	17.27875953



Fundamental Principle: Piezoelectric Effect

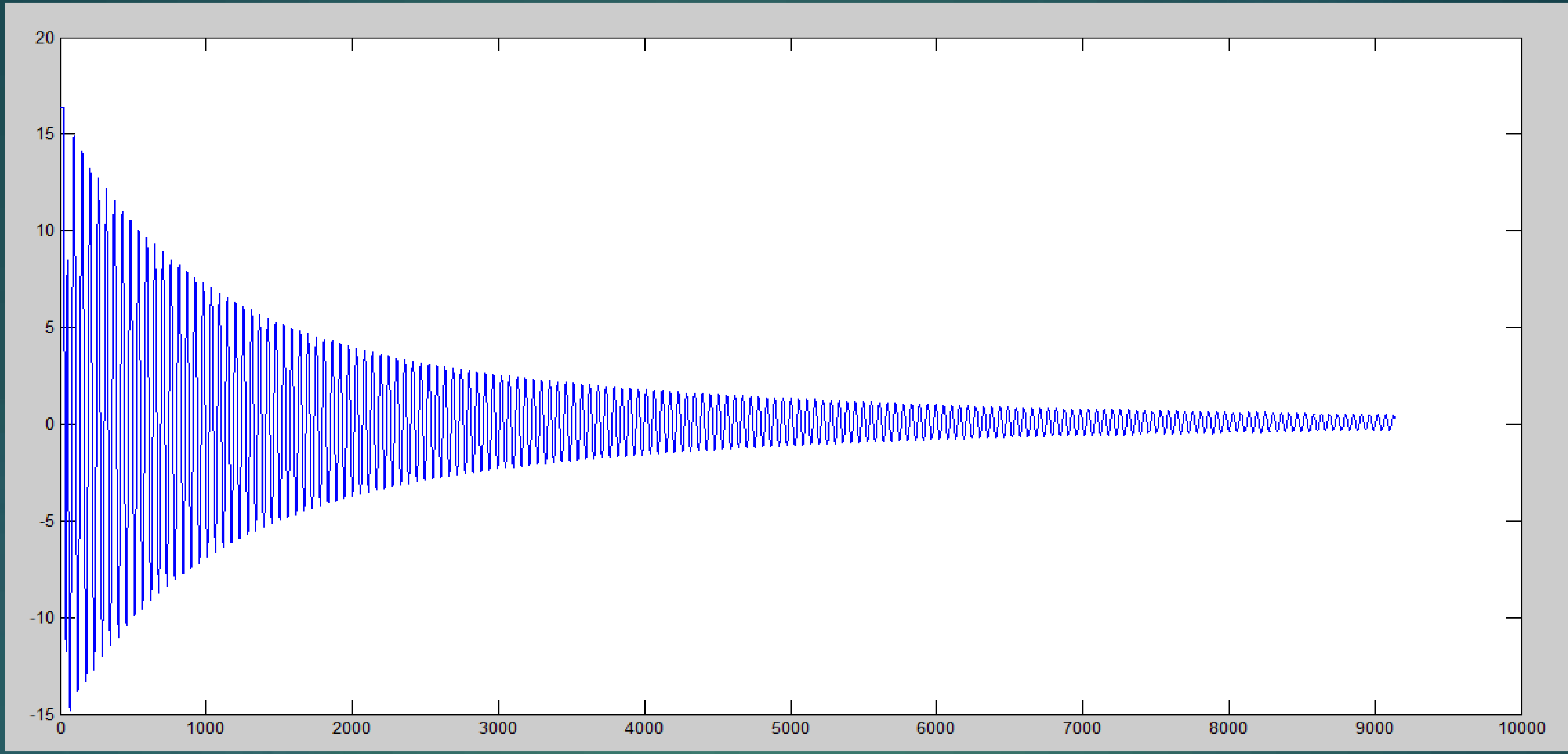
1. **Piezoelectric Effect**: Piezoelectric materials generate an electric charge in response to mechanical stress. Conversely, they can deform when an electric field is applied, which is used in actuators. In your case, the focus is on the direct piezoelectric effect, where mechanical vibration is converted into electrical energy.
2. **Cantilever Beam Dynamics**: When a force is applied to the tip of a cantilever beam, it induces vibrations along the beam. The frequency and amplitude of these vibrations depend on the beam's material properties and geometry.
3. **Energy Transfer**: These mechanical vibrations are transferred to the piezoelectric patch attached at the end of the beam.





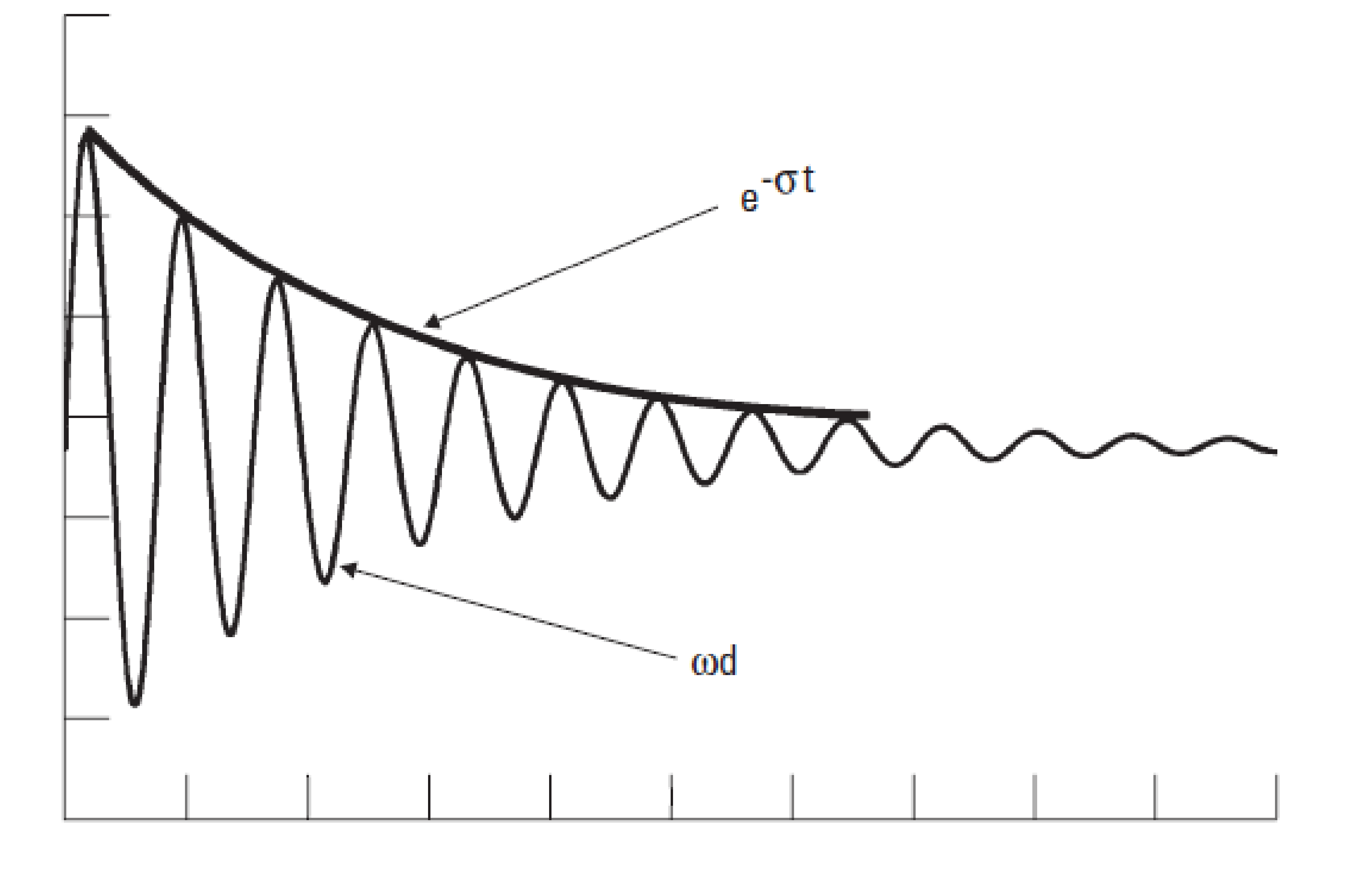
Laser Measuring

Processing Data in Matlab



Displacement data graph in Matlab

Finding Local Maximum Points



Displacement as an exponential harmonic function

Data Conversion and Import into MATLAB

1. **Data Conversion**:
   - The vibration data, initially recorded in .csv format, is converted into a .txt format. This step might be necessary due to compatibility requirements or for ease of handling in MATLAB.
2. **Data Import**:
   - The converted .txt file is then imported into MATLAB. MATLAB provides versatile tools for importing and processing various data formats.

Data Plotting and Analysis

1. **Plotting Vibration Data**:
   - Initially, the vibration data is plotted to visually assess the vibration patterns. This graphical representation can reveal key characteristics like frequency, amplitude, and damping.

Calculation of Damping Coefficient

1. **Identification of Local Maxima**:
   - The local maximum points of the vibration data represent the peak amplitudes of the oscillations over time.
   - These points are crucial for understanding how the vibration amplitude decays, which is directly related to the system's damping properties.
2. **Fitting an Exponential Curve**:
   - An exponential curve is fitted through these local maximum points. The general form of this curve is $Ae^{-\beta t}$, where A is the initial amplitude, β is the damping coefficient, and t is time.
   - This curve acts as the envelope of the damped oscillation, illustrating how the peak amplitude decreases over time.
3. **Cosine Function for Harmonic Oscillation**:
   - The actual vibration data will also exhibit a cosine-like behavior representing the harmonic oscillation of the system.
   - The complete model of the damped vibration can be represented as $y(t) = Ae^{-\beta t}\cos(\omega t + \phi)$, where ω is the angular frequency and ϕ is the phase angle.

Using Signal Processing Toolbox

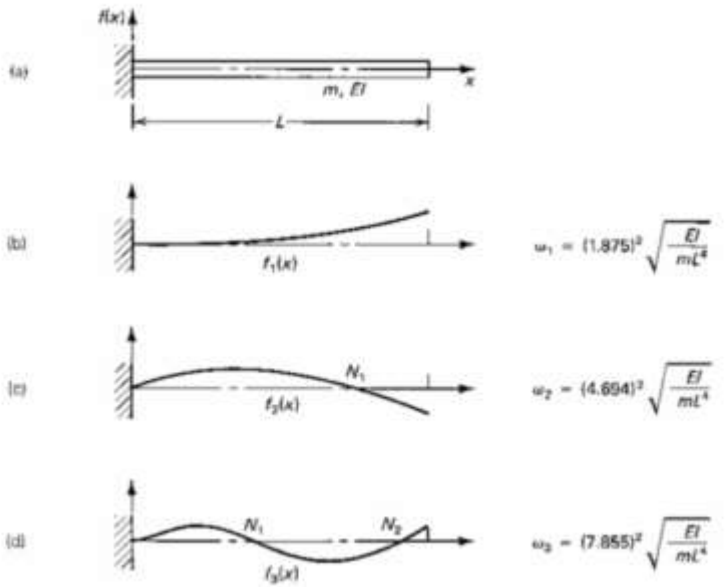
1. **Importing Data into Toolbox:**
   - MATLAB's Signal Processing Toolbox provides advanced functions and tools for analyzing and manipulating signals, including vibration data.
2. **Envelope Detection:**
   - The toolbox can be used to identify the envelope of the signal, equivalent to finding the local maximum points.
   - This step is crucial for determining the damping coefficient (β) accurately.

Conclusion

By utilizing MATLAB's Signal Processing Toolbox, you can effectively analyze the damping characteristics of your piezoelectric cantilever beam setup. The process of identifying the local maxima and fitting an exponential decay curve is central to quantifying the damping coefficient. This analysis is not only vital for understanding the dynamic behavior of the beam but also crucial for optimizing the energy harvesting efficiency and the overall performance of the system.

Modal Analysis

The frequencies and mode shapes of the cantilever beam



Natural Frequencies

- **Definition**: Natural frequencies are the frequencies at which a structure tends to vibrate when disturbed from its rest position and then allowed to vibrate freely. Each structure has its unique set of natural frequencies, depending on its material properties, geometry, and boundary conditions.
- **Significance**: These frequencies are critical because they represent the conditions under which the structure can resonate. Resonance can lead to large amplitude oscillations, potentially causing structural damage.

Damping

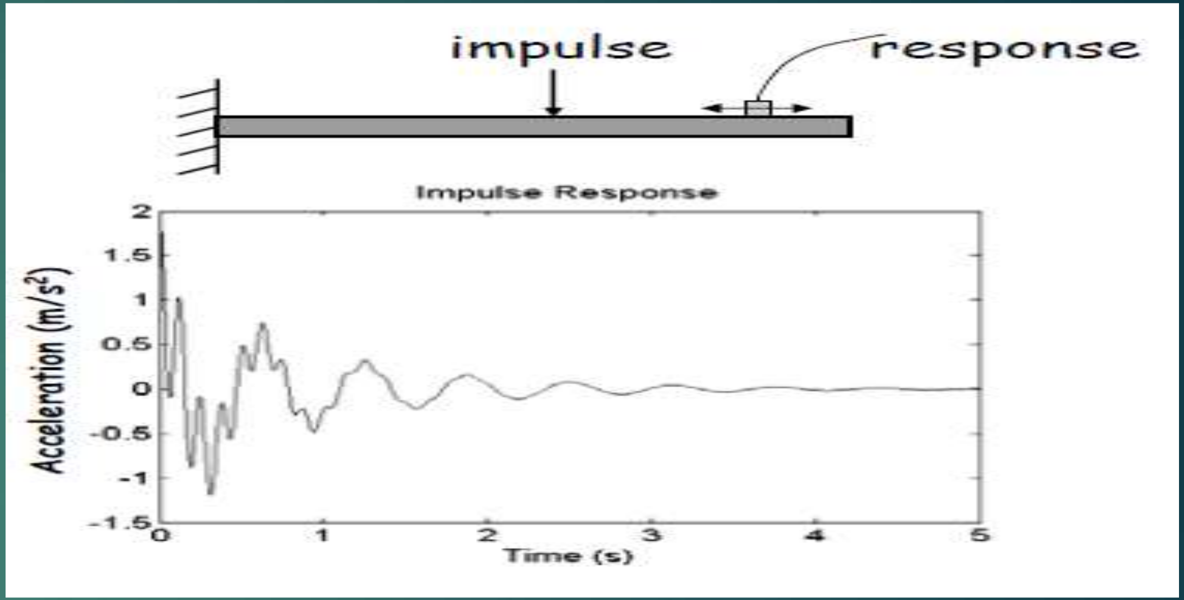
- **Concept**: Damping refers to the mechanisms that dissipate energy from a vibrating system, gradually bringing it to a rest. It's a measure of how quickly the vibrations decay after the structure has been excited.
- **Types**: Damping can be due to material properties (material damping), structural design (structural damping), or external factors like air resistance (external damping).
- **Role in Analysis**: Understanding damping is crucial for predicting how a structure will respond under dynamic loading, especially in transient conditions.

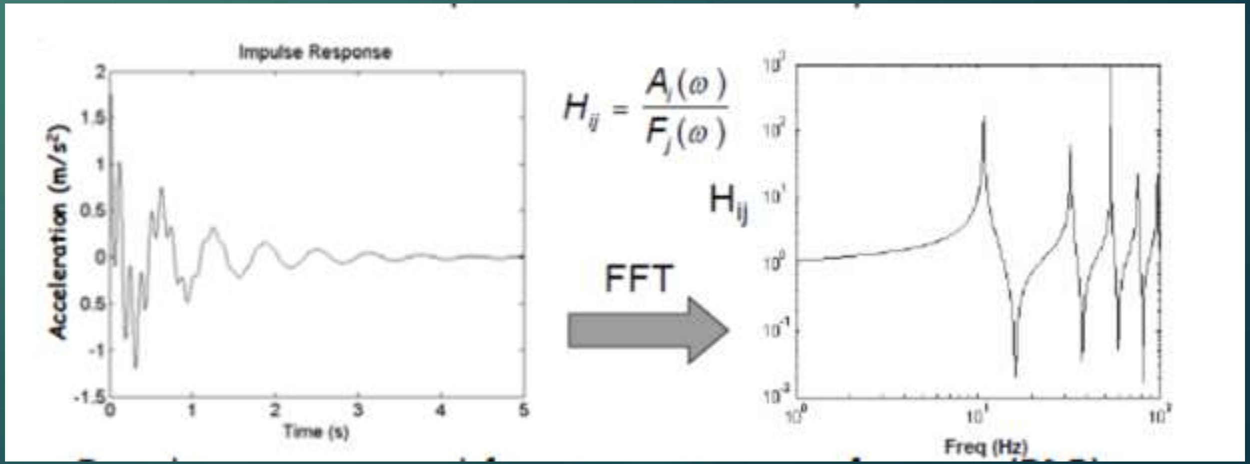
Mode Shapes

- **Description**: Mode shapes are the specific deformation patterns that a structure assumes when vibrating at its natural frequencies. Each natural frequency has a corresponding mode shape.
- **Visualization**: Mode shapes can be visualized as the pattern of deflection or deformation of the structure at specific points during its vibration cycle.
- **Application**: Analyzing mode shapes helps in identifying weak spots in the design and can guide modifications to improve structural performance and safety.

Application of Modal Analysis

1. **Design and Testing**: Modal analysis is used extensively in the design phase of structures and mechanical components to predict their dynamic behavior and ensure they are safe from resonant vibrations.
2. **Fault Diagnosis**: In existing structures and machinery, modal analysis can identify changes in dynamic properties, which may indicate damage or wear.
3. **Optimization**: Engineers use modal analysis to optimize structures for specific dynamic responses, such as reducing vibrations in sensitive equipment.
4. **Control Systems**: Understanding the modal properties is crucial in designing control systems for vibration suppression or noise reduction.





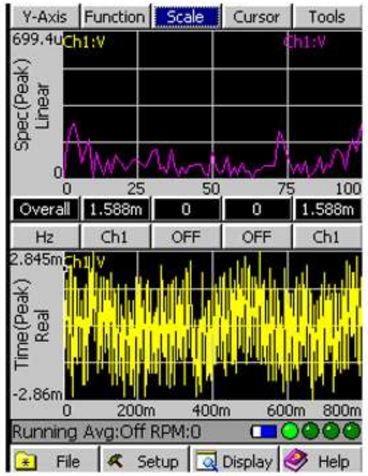
Experimental Setup

1. **Piezoelectric Cantilever Beam**:
   - **Composition**: An aluminum beam with a piezoelectric patch (such as PZT - Lead Zirconate Titanate) bonded at its end.
   - **Function**: The beam acts as a substrate that magnifies the mechanical stress experienced by the piezoelectric material.
2. **Vibration Inducement**:
   - A force is applied at the tip of the beam, either manually or through a mechanical setup like an impulse hammer or a shaker, to induce vibrations.
3. **Energy Harvesting**:
   - The piezoelectric patch, subjected to these vibrations, generates an electric charge due to the stress.
   - This charge can be captured and stored or immediately used, depending on the system design.
4. **Electrical Circuitry**:
   - An electrical circuit is connected to the piezoelectric material. This often includes rectifying components to convert AC to DC, a storage component like a capacitor, and possibly a voltage regulator.
5. **Data Acquisition and Analysis**:
   - Instruments like an oscilloscope or a data acquisition system (e.g., NI-DAQ) can be used to measure the electrical output (voltage, current) generated by the piezoelectric patch.
   - These measurements are crucial for evaluating the efficiency of energy conversion and understanding the dynamics of the system.

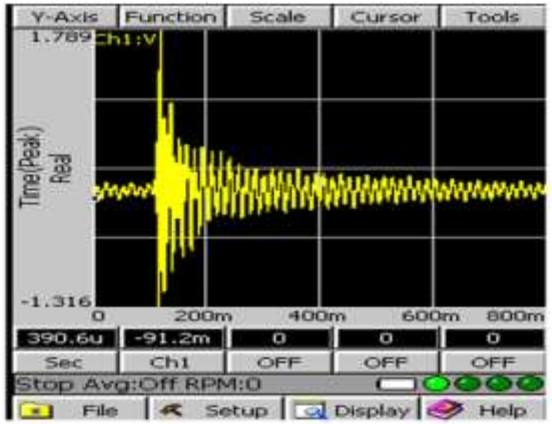
Theoretical and Practical Considerations

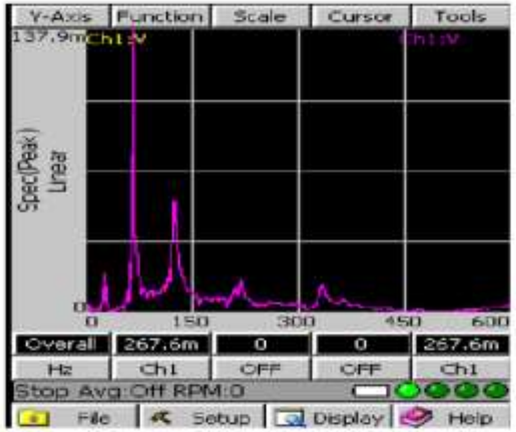
1. **Efficiency**: The efficiency of converting mechanical to electrical energy depends on several factors, including the properties of the piezoelectric material, the frequency of vibration, and the mechanical design of the beam.
2. **Resonance**: Operating at or near the resonant frequency of the beam-piezoelectric system maximizes the energy conversion, as this is where the largest amplitude of vibration occurs.
3. **Electrical Load Matching**: For optimal energy transfer, the electrical impedance of the piezoelectric material should be matched to that of the connected circuit.
4. **Practical Applications**: Such setups have applications in low-power electronics, sensor networks, and sustainable energy solutions, particularly in environments where vibrations are abundant.

Modal Analysis Experiment

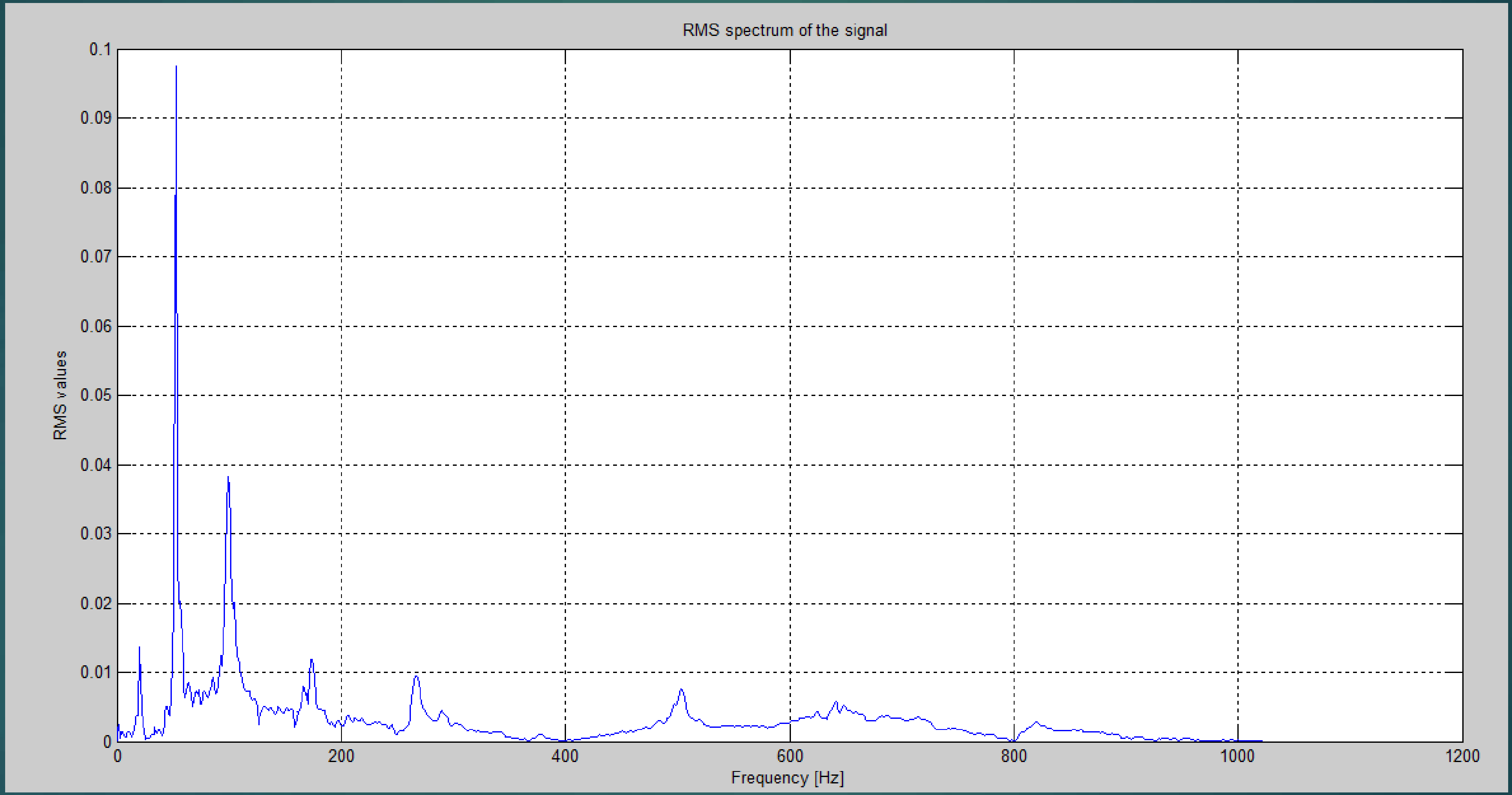


Impulse Response

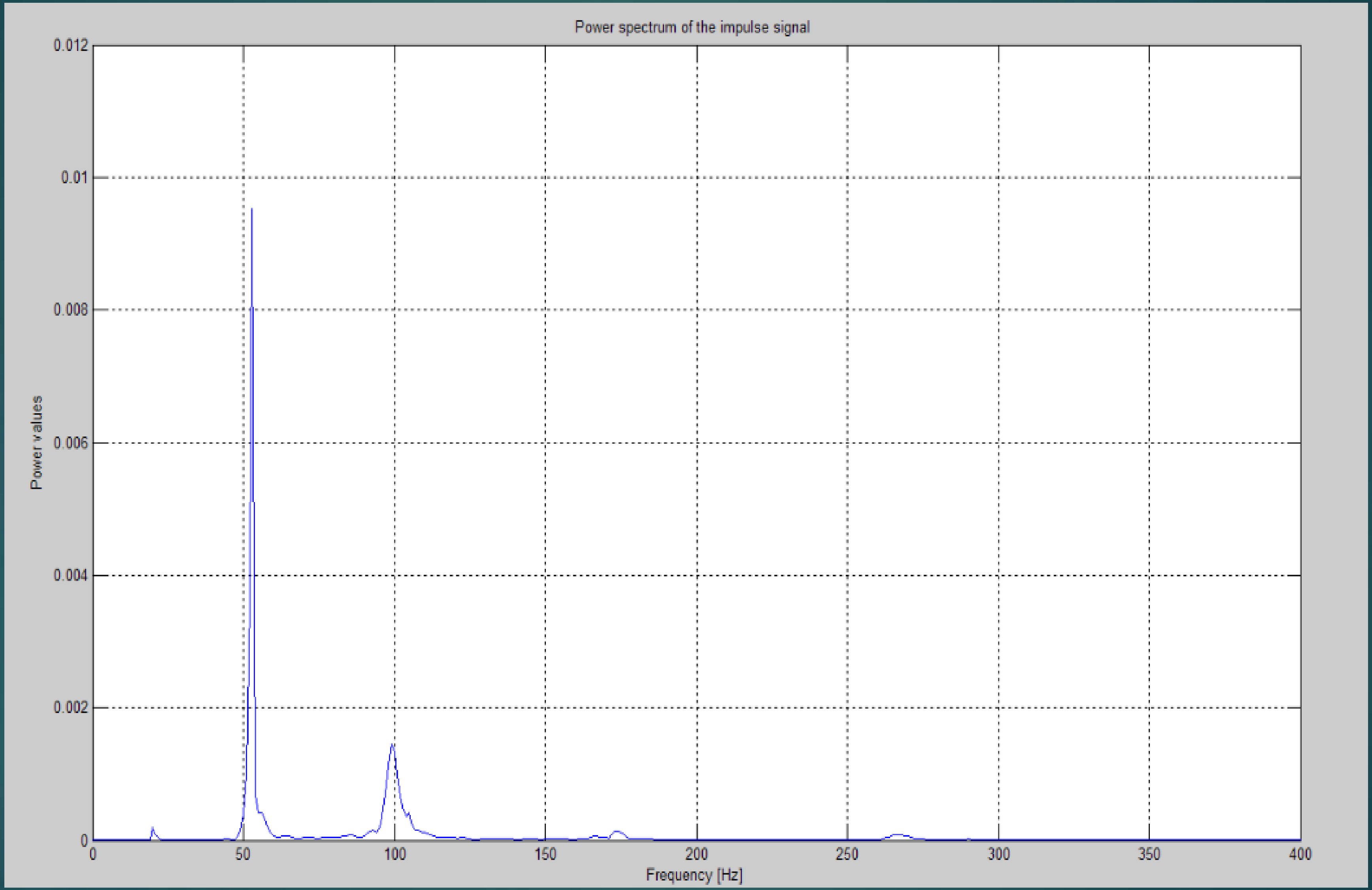




RMS Spectrum of Impulse Response



Power Spectrum of Impulse Response



Impact Hammer Testing

1. **Procedure:**
   - An impact hammer, equipped with a force transducer, is used to deliver a known force impulse to the end of the cantilever beam. This impulse excites the beam, causing it to vibrate.
2. **Benefits:**
   - This method is non-destructive and can excite a wide range of frequencies, making it ideal for identifying natural frequencies of the system.

Accelerometer Measurement

1. **Placement:**
   - An accelerometer is attached at the tip of the beam to measure the acceleration response of the beam due to the impact.
2. **Data Capture:**
   - The accelerometer captures the time-history of the beam's acceleration as it vibrates in response to the impact.

FFT Analysis

1. **Fast Fourier Transform (FFT):**
   - The time-domain acceleration data captured by the accelerometer is transformed into the frequency domain using FFT.
   - FFT is a computational algorithm used to efficiently convert time-domain data (like acceleration over time) into frequency-domain data (showing how much of the signal lies in each given frequency band).
2. **Frequency Response:**
   - The result of the FFT is a frequency response function (FRF), which shows the amplitude of vibration at each frequency.
   - Peaks in the FRF correspond to the natural frequencies of the beam, as these are the frequencies at which the structure exhibits resonant behavior.

Interpretation and Analysis

1. **Natural Frequencies Identification:**
   - By analyzing the peaks in the FRF, you can identify the natural frequencies of the beam. These are the frequencies at which the beam has a high amplitude response.
2. **Modal Parameters:**
   - Besides natural frequencies, information about damping and mode shapes can also be extracted, although extracting mode shapes typically requires more sophisticated modal analysis techniques.
3. **Further Applications:**
   - Understanding the natural frequencies is crucial for many applications, such as designing for structural integrity, avoiding resonance conditions, and for vibration control systems.

Numerical Natural Frequencies (Simulation Result)

f1=3.4625 Hz.

f2=21.6992 Hz.

f3 = 60.7584 Hz.

f4 =119.0622 Hz.

f5 =196.8183 Hz.

Experimental Natural Frequencies

f1 = 2.5 Hz.

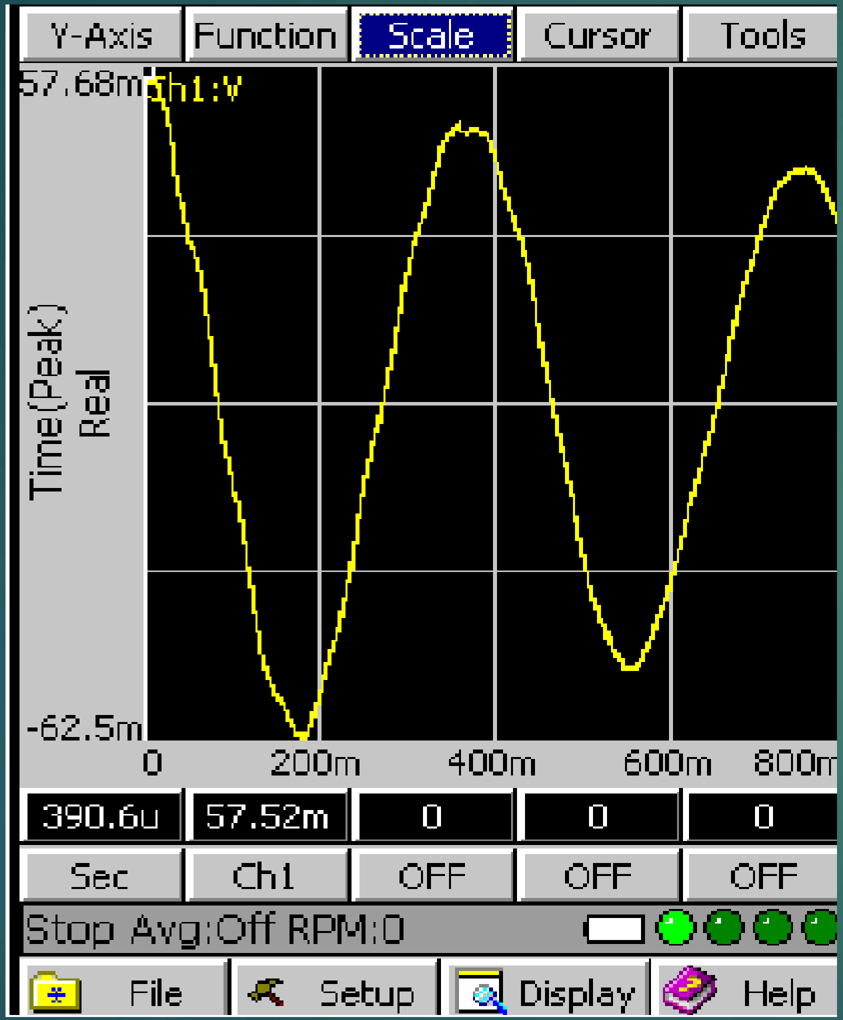
f2 = 25 Hz.

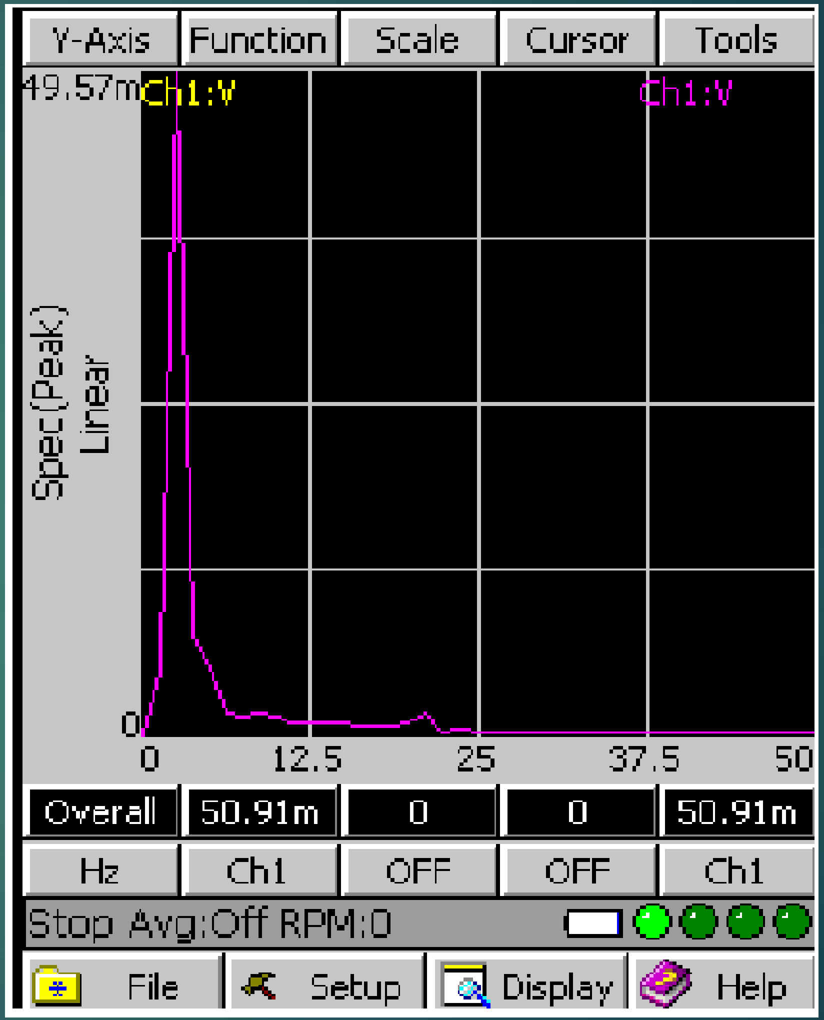
f3 = 65.75 Hz.

f4 = 123.5 Hz.

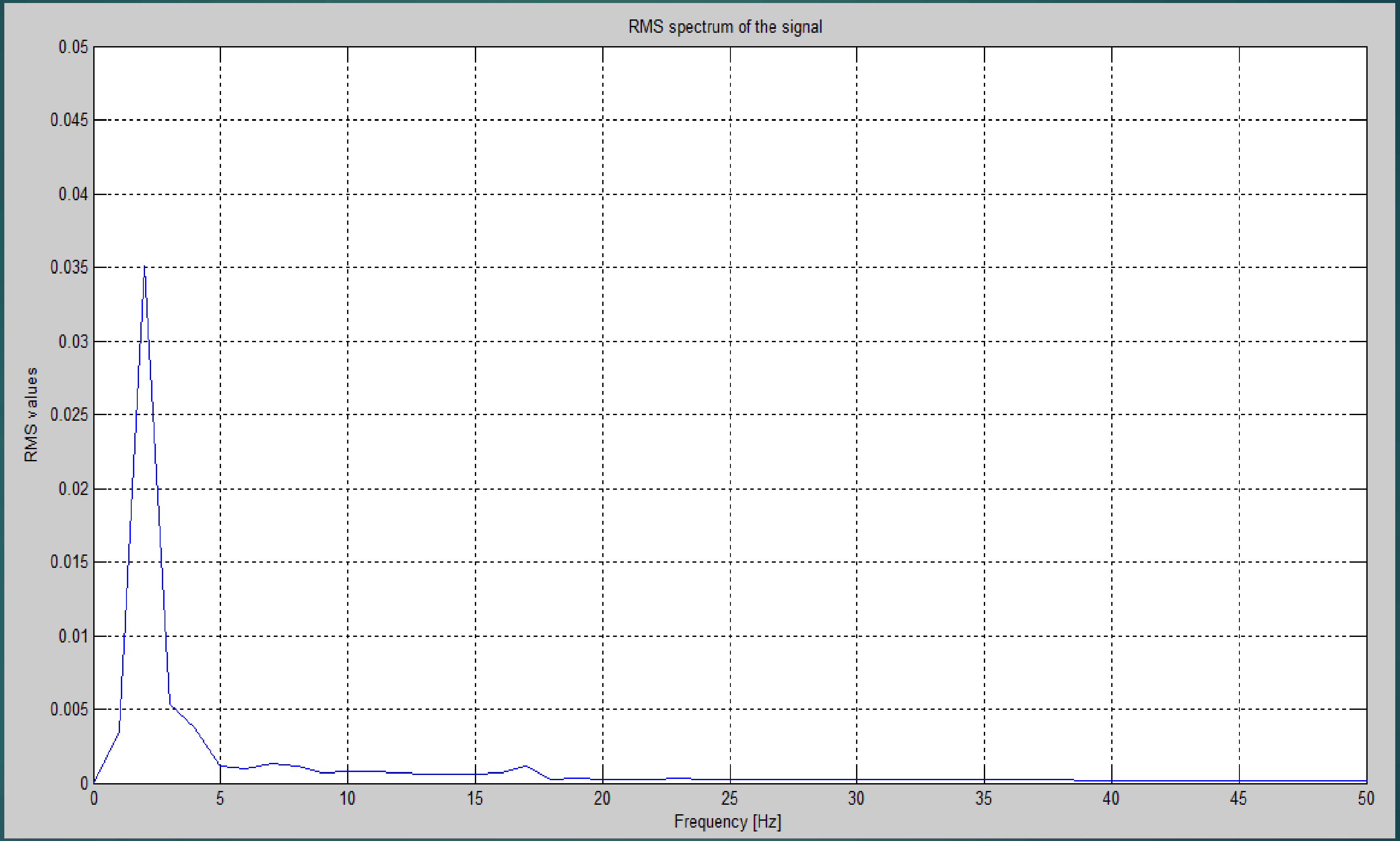
f5 = 217 Hz.

Step response without tip mass



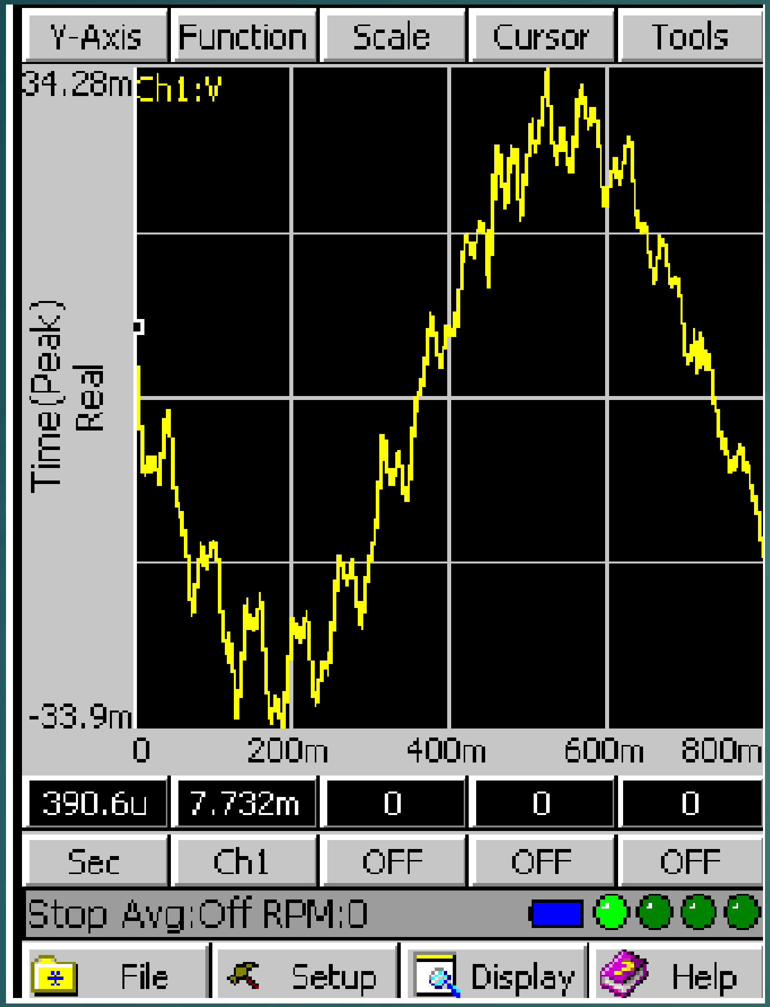


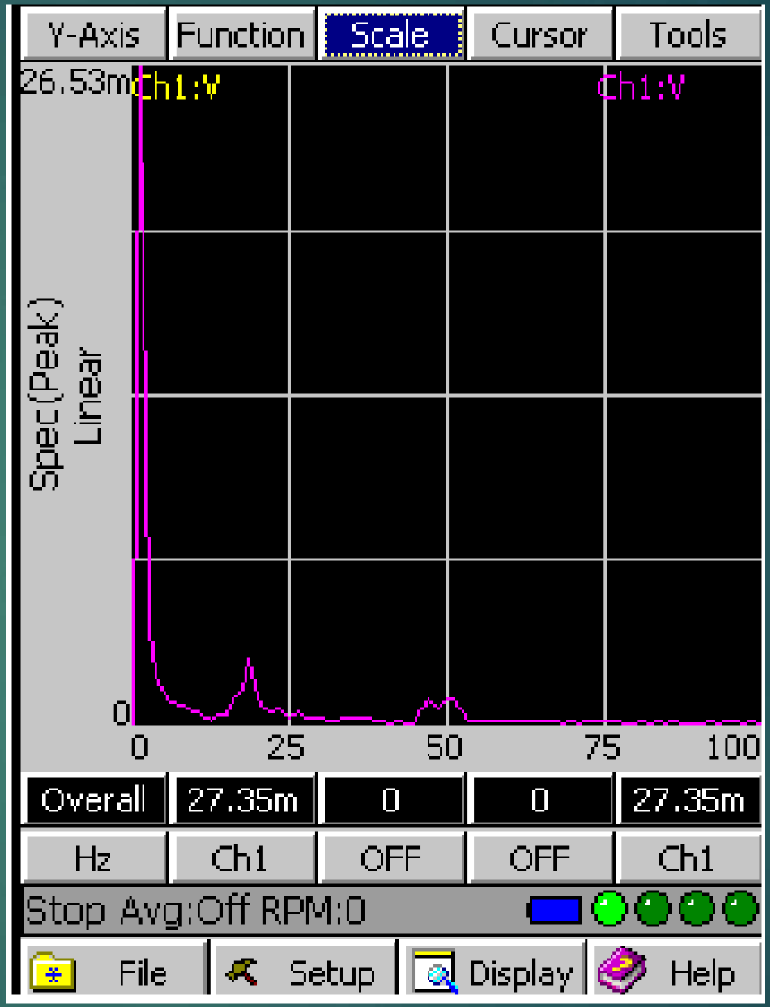
RMS spectrum of step response without tip mass



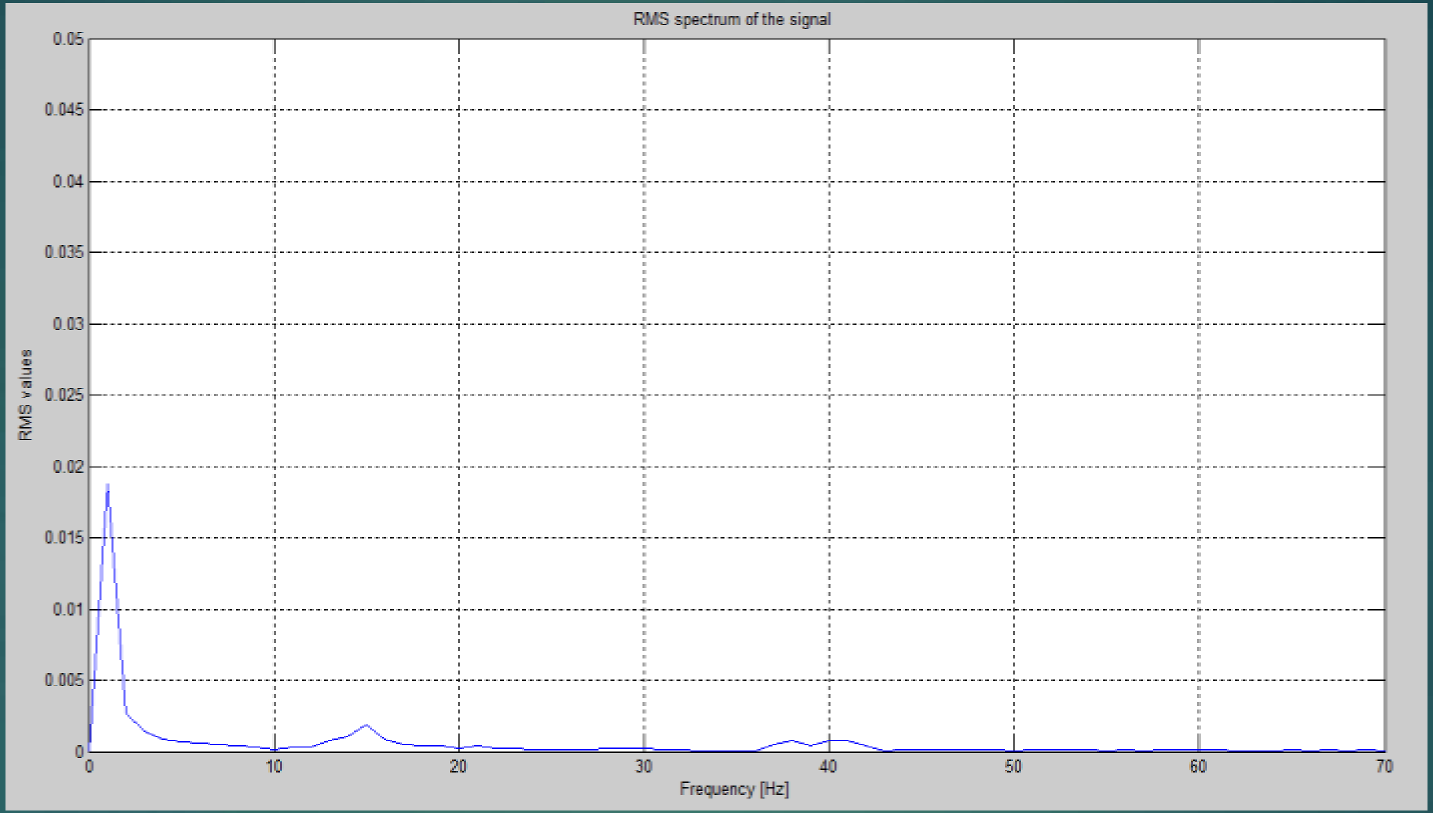
The dominant mode is first mode and f1=2.5 Hertz

Step response with tip mass





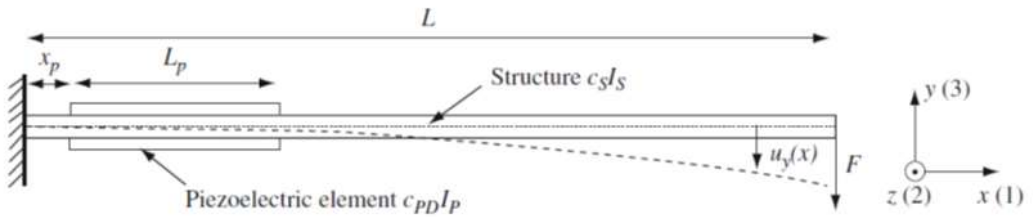
RMS spectrum of step response with tip mass

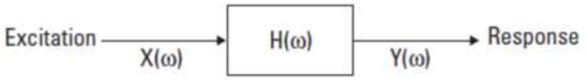


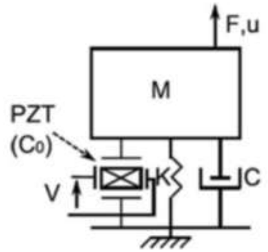
The dominant mode is first mode and f1=1 Hertz
Mass effect has made decrease the natural frequency of the beam.
The formula is :

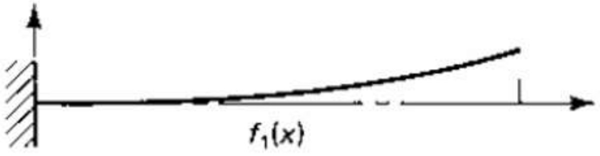
Lumped Model

The system presents one mechanical degree of freedom (u) and one electrical degree of freedom (V).









$$\omega_1 = (1.875)^2 \sqrt{\frac{EI}{mL^4}}$$

Realizing that the natural frequency is

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{(in rad/sec)}$$

and the natural frequency for the first mode of the cantilever from the continuous solution is

$$\omega_n = (1.875)^2 \sqrt{\frac{E\,I\,g}{w\,L^4}} \quad \text{(where w - weight and g – gravitational constant)}$$

allows the effective mass at the tip of the cantilever beam to be determined. This approximation allows the cantilever beam to be modeled as a single degree of freedom system since the mass and stiffness are known.

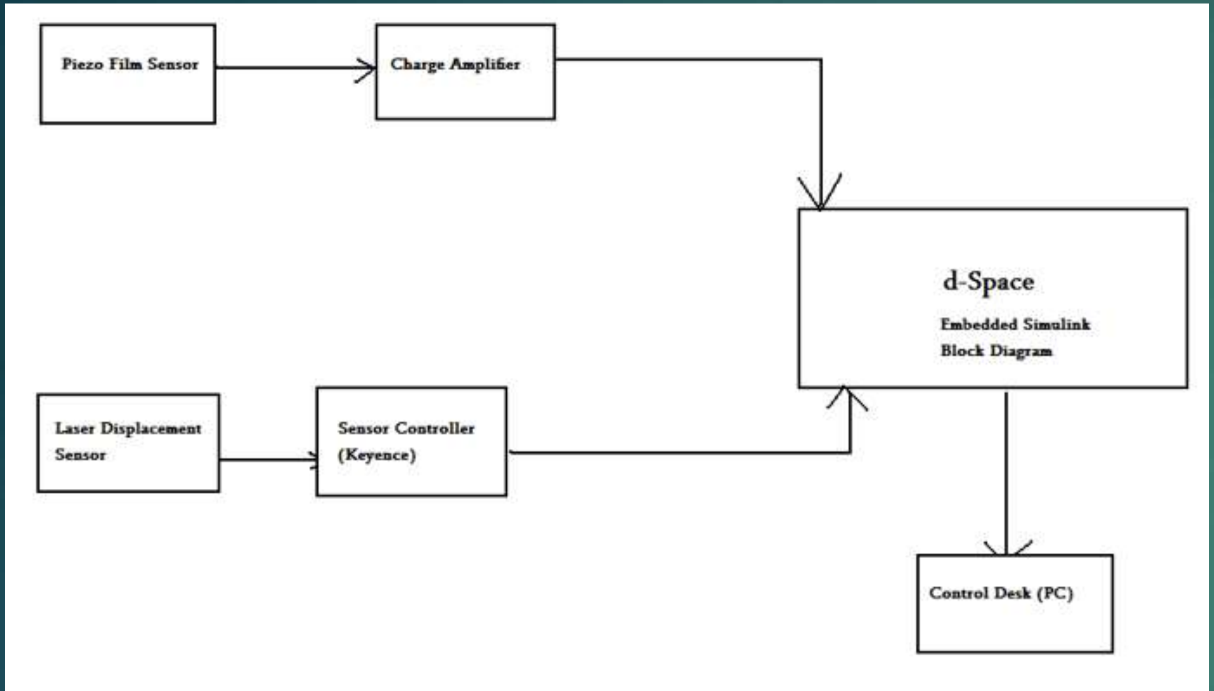
Experimental Procedure and Observations

1. **Elastical Potential Energy Release**:
   - By pulling and releasing the free end of the beam, you essentially stored and then released elastic potential energy, initiating vibrations in the beam.
2. **Accelerometer Data Processing**:
   - The acceleration data captured by the accelerometer at the free end of the beam was processed in a signal analyzer using FFT transformation.
3. **FFT Analysis and Dominant Mode**:
   - The FFT graph revealed that the dominant vibration mode was the first mode, with a natural frequency of approximately 3.4625 Hz.

Interpretations and Implications

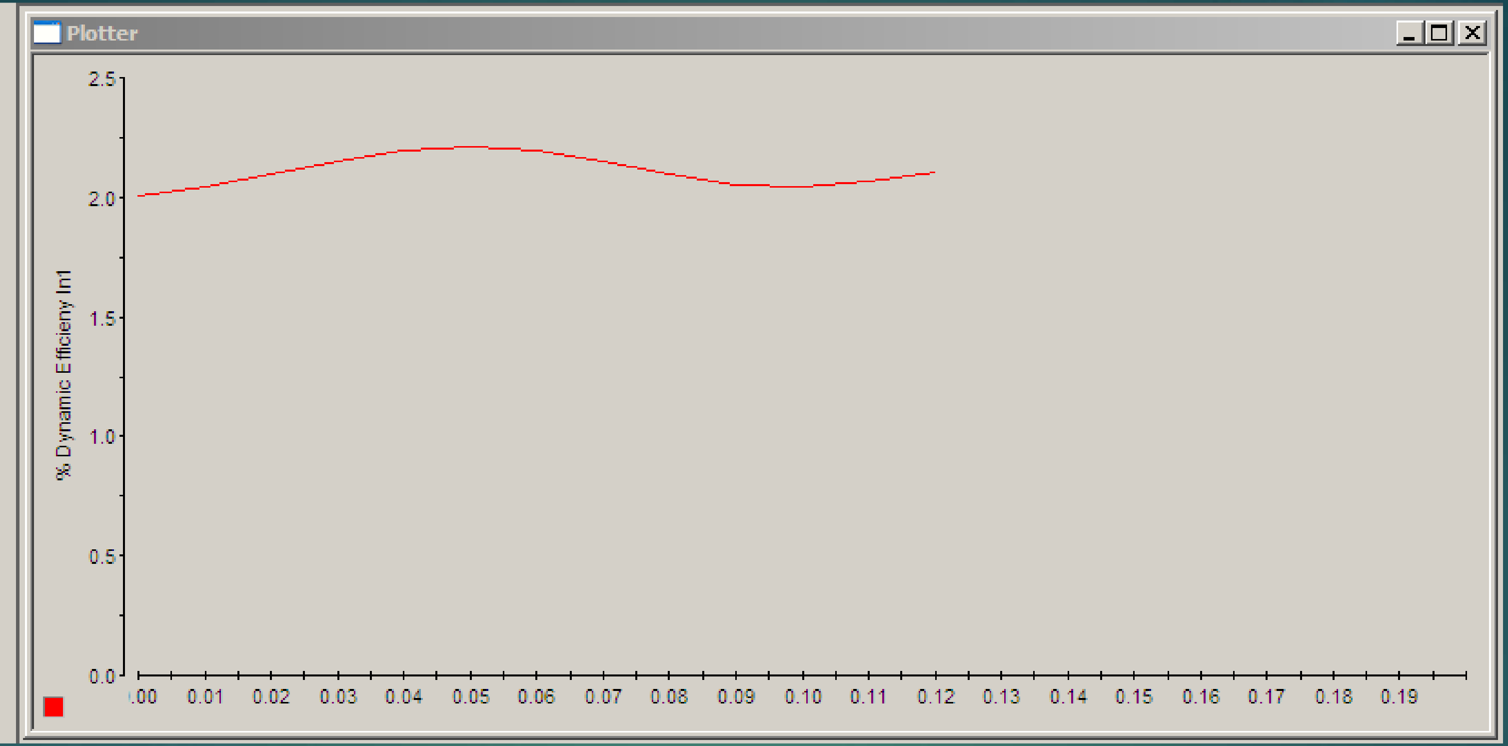
1. **Resonance Frequency and Electromechanical Behavior**:
   - The identified resonance frequency at 3.4625 Hz is critical for both vibration control and energy harvesting applications.
   - This frequency represents a realistic behavior of the electromechanical structure near its resonance, which is often where the most significant dynamic responses occur.
2. **Vibration Control Considerations**:
   - In vibration control, it's crucial to understand the resonance frequencies, as forces at these frequencies lead to the most significant displacements. Effective damping strategies often focus on these resonant modes.
3. **Energy Harvesting Efficiency**:
   - For energy harvesting, resonance frequencies are key since they represent points where the maximum energy transfer occurs.
   - The observation that the majority of harvestable energy is concentrated within resonance frequency bands is a fundamental principle in designing effective energy-harvesting systems.

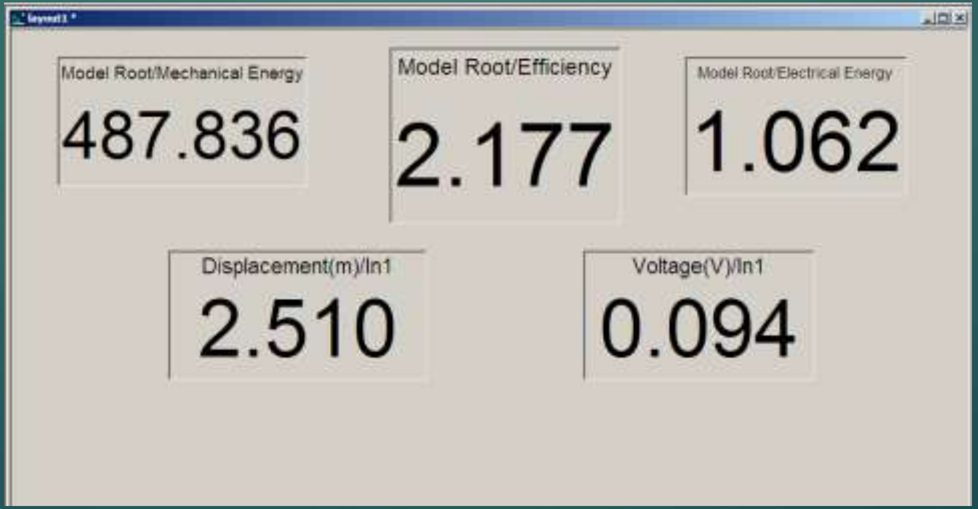
Hardware System Block Diagram



Dynamic Efficiency and Optimization Strategies

1. **Efficiency Range**:
   - You observed a dynamic efficiency range of 0% to 3.2%. This indicates that initially, when mechanical vibrations are high, the conversion to electrical energy is relatively inefficient.
2. **Efficiency Increase over Time**:
   - As the mechanical vibrations in the beam decrease, the efficiency of energy conversion increases. This might be due to the nature of the piezoelectric material and its interaction with the decreasing amplitude of vibrations.
3. **Improving Efficiency**:
   - Connecting multiple piezoelectric elements or covering the beam with piezo-film can enhance efficiency. This could be due to the increased surface area for energy conversion and the potential for capturing more mechanical energy from vibrations.


Plotter
% Dynamic Efficieny In1
2.5
2.0
1.5
1.0
0.5
0.0
.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10 0.11 0.12 0.13 0.14 0.15 0.16 0.17 0.18 0.19


Model Root/Mechanical Energy
487.836
Model Root/Efficiency
2.177
Model Root/Electrical Energy
1.062
Displacement(m)/In1
2.510
Voltage(V)/In1
0.094

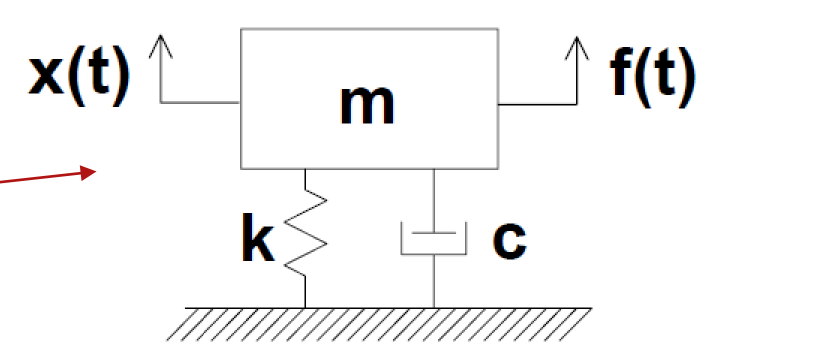
Cantilever Beam Experiment

Objective

The primary goal is to identify the dynamic characteristics of the armature, focusing on natural frequency, damping, and tip displacement/acceleration. This understanding is crucial for evaluating overall performance and implementing effective redesigns, including both active and passive damping treatments.

Methodology

Model Development: The approach involves developing both analytical and experimental models. The analytical model is based on a mass, spring, dashpot system, while the experimental model involves measurements using strain gage, eddy current probe, accelerometer, and LVDT devices.



Single Degree of Freedom Model

Single Degree of Freedom Model: The simplest analytical model is a single degree of freedom lumped mass model, represented by a second-order differential equation with constant coefficients (m, c, k representing mass, damping, and stiffness respectively). The equation of motion is given by:

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

Continuous Solution and Finite Element Analysis: For a more detailed analysis, a continuous solution for the cantilever beam or a finite element model in software like MATLAB can be used. This involves using strength of materials approximations and the continuous beam vibration equation.

Mode Shapes and Natural Frequencies: The mode shapes for a continuous cantilever beam are given by a complex equation involving trigonometric and hyperbolic functions. The natural frequencies are derived from the properties of the beam, such as its elasticity (E), moment of inertia (I), and mass per unit length (m).

Tip Deflection and Stiffness: The deflection at the tip of the cantilever beam is a function of the applied load, length of the beam, Young’s Modulus, and bending moment of inertia. The stiffness of the cantilever beam can be expressed as a function of these parameters.

Natural Frequency Approximation: The natural frequency, particularly for the first mode, can be approximated, allowing the cantilever beam to be modeled as a single degree of freedom system.

From Strength of Materials, the deflection, x, at the tip of a cantilever beam is given by

$$x = P L^3 / 3 E I$$

where

x	tip displacement
P	applied load (also referred to as F)
L	length of the cantilever beam
E	Young's Modulus of Elasticity
I	bending moment of inertia

The deflection at the end of the cantilever beam can be expressed as

$$F = k x$$

and therefore, the stiffness of the cantilever beam can be expressed as

$$k = 3 E I / L^3$$

Realizing that the natural frequency is

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{(in rad/sec)}$$

Mode Shapes

- The mode shapes of a continuous cantilever beam are mathematically expressed as follows:

$$f_n(x) = A_n \{(\sin \beta_n L - \sinh \beta_n L)(\sin \beta_n x - \sinh \beta_n x) + (\cos \beta_n L - \cosh \beta_n L)(\cos \beta_n x - \cosh \beta_n x)\}$$

- $n = 1, 2, 3, \ldots, \infty$ represents the mode number.
- $\beta_n L = n\pi$ is a parameter related to the length of the beam and the mode number.
- A_n is the amplitude of the nth mode.

This equation captures the complex nature of the mode shapes, incorporating both trigonometric and hyperbolic functions.

Natural Frequencies

The natural frequencies of the cantilever beam are given by:

$$\omega_n = \alpha_n^2 \frac{EI}{mL^4}$$

- ω_n is the natural frequency for the nth mode.
- α_n are constants for each mode (e.g., 1.875, 4.694, 7.855 for the first three modes).
- E is the Young's Modulus of Elasticity.
- I is the bending moment of inertia.
- m is the mass per unit length of the beam.
- L is the length of the cantilever beam.

This formula provides the frequencies for each mode of vibration, crucial for understanding the dynamic behavior of the beam.

Interpretation

Mode Shapes: These shapes represent the deformation patterns of the beam at different frequencies. Each mode shape corresponds to a specific pattern of vibration along the length of the beam.

Natural Frequencies: These are the frequencies at which the beam will naturally tend to vibrate if disturbed. Each mode has its unique natural frequency, and these frequencies are critical in avoiding resonance in practical applications.

1. **Equation of Motion for Euler-Bernoulli Beam**:

   $$EI\frac{d^4w}{dx^4} + \rho A\frac{d^2w}{dt^2} = 0$$

   Where EI is the flexural rigidity, ρ is the density, A is the cross-sectional area, w is the deflection.

2. **Separation of Variables**:

   $$w(x,t) = W(x) \cdot T(t)$$

   This leads to two separate ordinary differential equations for $W(x)$ and $T(t)$.

3. **Natural Frequency Equation**:

   $$\omega_n^2 = \frac{EI}{\rho A}\left(\frac{n\pi}{L}\right)^2$$

   ω_n is the natural frequency, L is the length of the beam, and n is the mode number.

beam_vibration_ode45_armature.m

$$EI\frac{\partial^4 u(z,t)}{\partial z^4} + \gamma\frac{\partial u(z,t)}{\partial t} + \rho S\frac{\partial^2 u(z,t)}{\partial t^2} = 0$$

In this equation:

- EI represents the flexural rigidity of the beam.
- γ is the damping coefficient, and $\frac{\partial u(z,t)}{\partial t}$ is the first derivative of displacement with respect to time, representing the damping effect.
- $\rho S\frac{\partial^2 u(z,t)}{\partial t^2}$ is the inertial term, with ρ being the mass density and S the cross-sectional area of the beam.

This is the main function that sets up and solves the beam vibration problem.

1. **Parameter Initialization:**
   - `E`: Elastic Modulus, affecting the stiffness of the material.
   - `rho`: Mass density of the beam.
   - `L`, `b`, `h`: Length, width, and depth of the beam, respectively.
   - `I`: Area moment of inertia, crucial for bending calculations.
   - `A`: Cross-sectional area of the beam.
2. **Matrices Definition:**
   - `M`: Mass matrix, representing the distribution of mass along the beam.
   - `K`: Stiffness matrix, showing how the beam resists deformation.
   - `C`: Damping matrix (Rayleigh damping), combining mass and stiffness to model energy loss in the system.
3. **Setup for ODE Solver:**
   - `t_span`: Time interval for the simulation.
   - `initial_conditions`: Starting condition (zero displacement and velocity).
   - `options`: Settings for the ODE solver to ensure numerical stability and accuracy.

4. **Solving the ODEs:**
   - The ODE45 solver is used to solve the vibration equations over time.
   - The `beam_ode_impulse` function defines the system of differential equations.
5. **Post-processing:**
   - Setting the displacement at the fixed end to zero.
   - Animating the response of the beam over time.

Function `beam_ode_impulse`

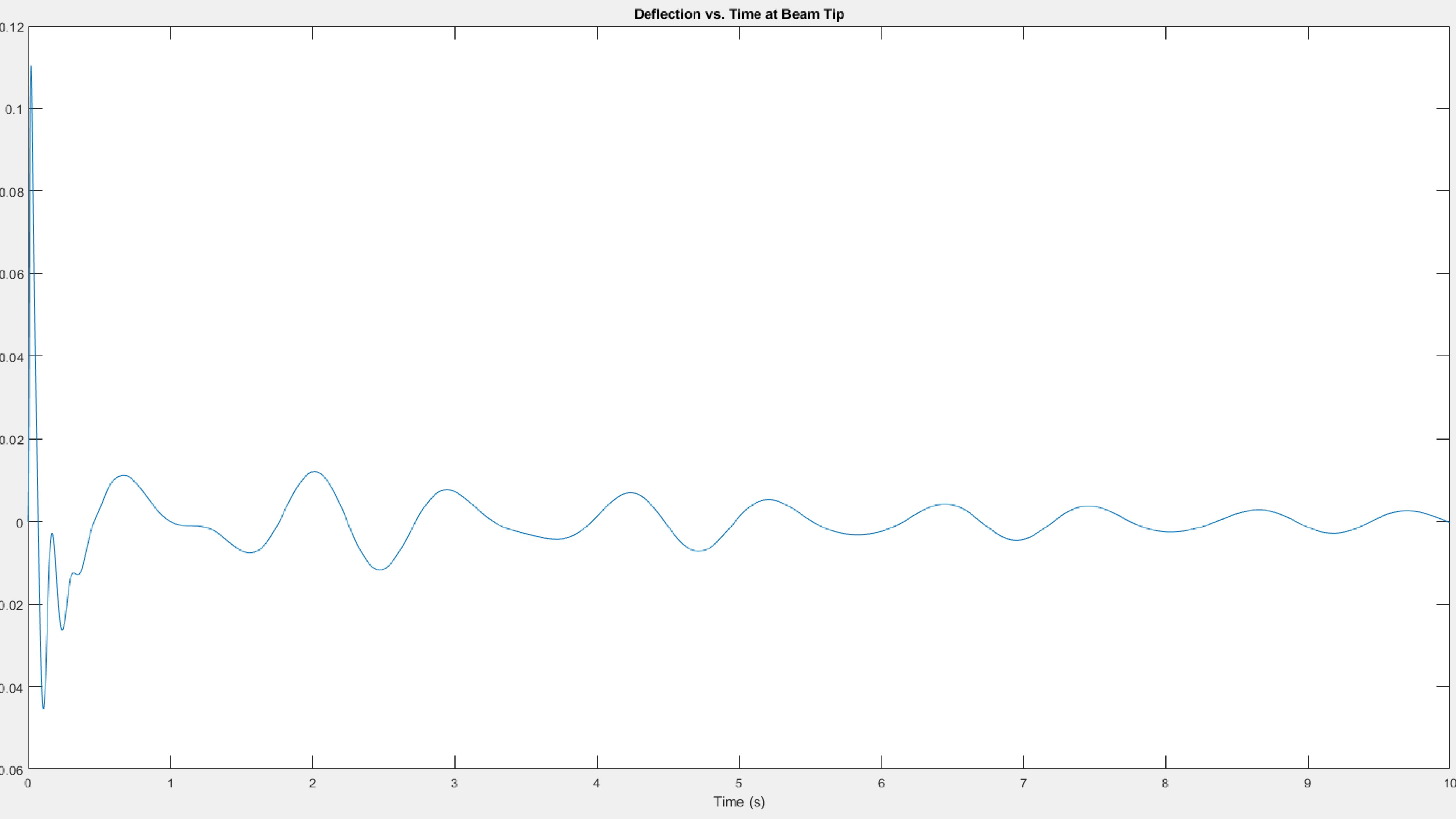
This function represents the system of differential equations for the beam under vibration.

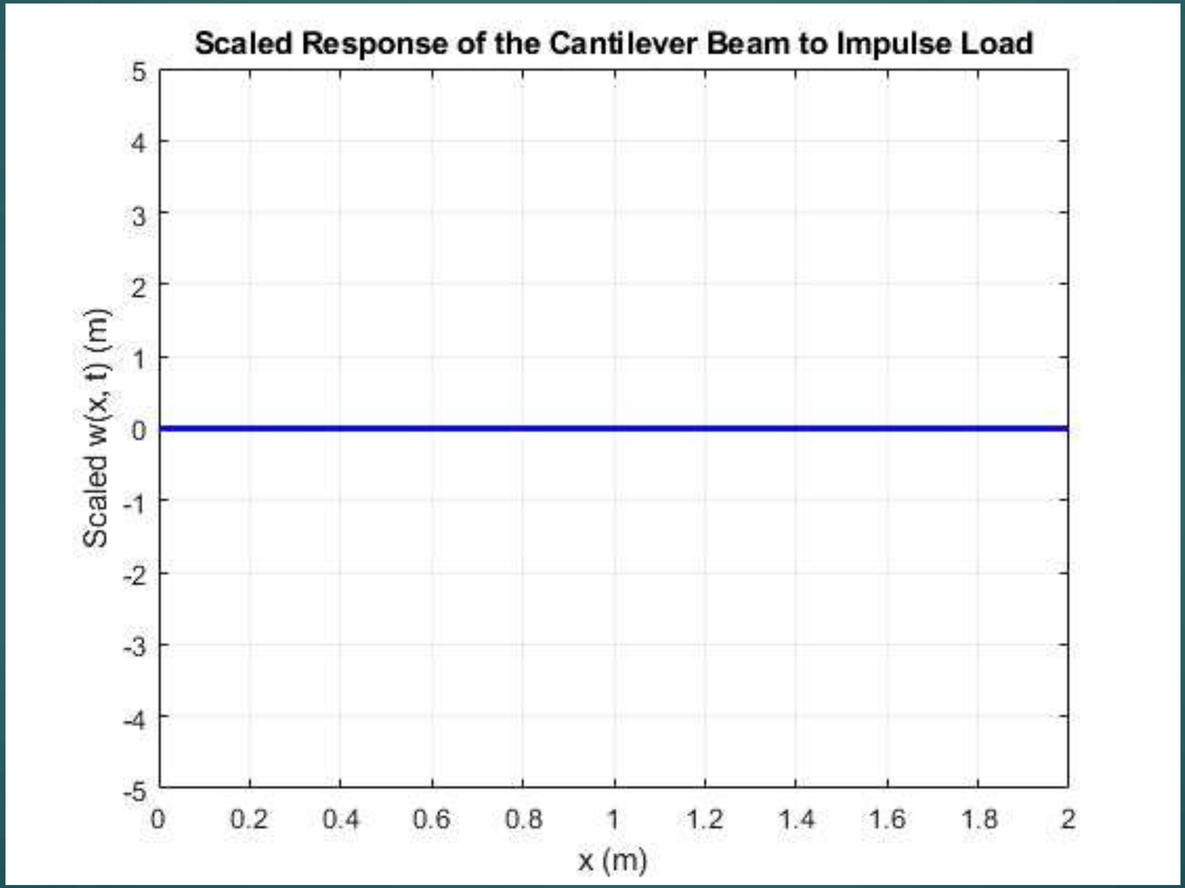
1. **State Variables:**
   - `displacement` and `velocity` of the beam.
2. **Impulse Load:**
   - Simulating an impulse load at the beam's tip.
3. **ODE Formulation:**
   - Converting the second-order ODEs into a first-order system.
   - Calculating the derivative of the state vector (`dydt`).

Function `plot_beam`

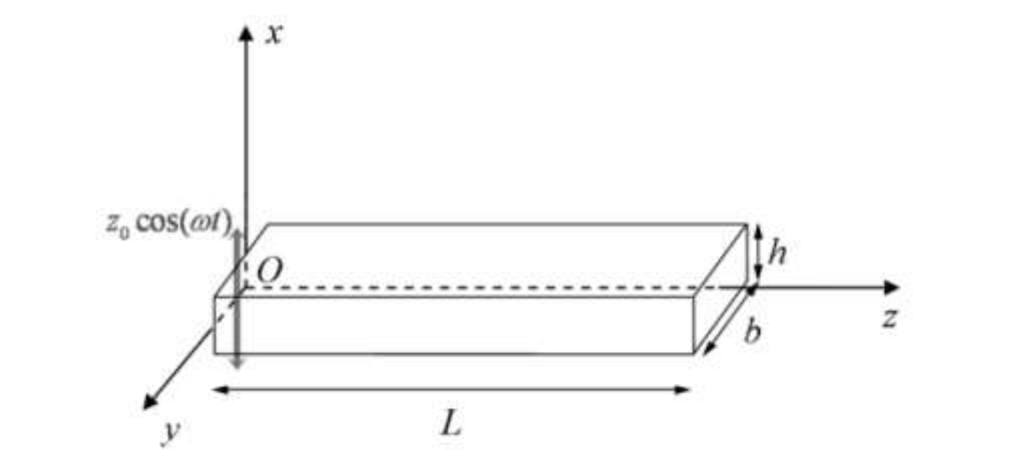
This function visualizes the beam's response.

1. **Visualization Scaling:**
   - Scaling up the displacement for easier visualization.
2. **Plotting:**
   - Creating a 2D plot to represent the beam's deflection over its length.
3. **Axes and Labels:**
   - Setting up the plot with proper labels, limits, and titles.


Deflection vs. Time at Beam Tip
0.12
0.1
0.08
0.06
0.04
0.02
0
0.02
0.04
0.06
0
1
2
3
4
5
6
7
8
9
10
Time (s)


Scaled Response of the Cantilever Beam to Impulse Load
Scaled w(x, t) (m)
x (m)
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Oscillations of the beam in the proximity of a resonance



Analysis Steps

1. **Understanding the Problem**:
   - We have a cantilever beam oscillating due to a driving device at one end.
   - The beam is uniform, linear elastic, isotropic, homogeneous, and longer than it is wide.
   - Vibrations are governed by the Euler-Bernoulli equation.
2. **Euler-Bernoulli Beam Equation**:
   - $\frac{EI}{\rho S}\frac{\partial^4 u}{\partial z^4} + \frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} = 0$
   - Parameters: $I(z)$ - second moment of inertia, E - Young's modulus, ρ - mass density, $S(z)$ - cross-sectional area, γ - damping coefficient, L - beam length.
   - $u(z,t)$ represents the displacement.
3. **Boundary Conditions**:
   - At $z = 0$: $u(0,t) = z_0 e^{iwt}$, $\frac{\partial u(0,t)}{\partial z} = 0$.
   - At $z = L$: $\frac{\partial^2 u(L,t)}{\partial z^2} = 0$, $\frac{\partial^3 u(L,t)}{\partial z^3} = 0$.
4. **Initial Conditions**:
   - $u(z,0) = F(z)$, $\frac{\partial u(z,t)}{\partial t} = G(z)$ at $t = 0$.

To find the general solution $Z(z)$ for the displacement $u(z,t)$ of a point on the beam, we start by substituting the proposed form $u(z,t) = Z(z)e^{iwt}$ into the Euler-Bernoulli beam equation:

$$EI\frac{\partial^4 u}{\partial z^4} + \frac{\partial^2 u}{\partial t^2} + \gamma\frac{\partial u}{\partial t} = 0$$

Substitution:

1. Substitute $u(z,t) = Z(z)e^{iwt}$ into the equation.
2. Calculate the derivatives:
   - $\frac{\partial^2 u}{\partial t^2} = -w^2 Z(z)e^{iwt}$
   - $\frac{\partial u}{\partial t} = iwZ(z)e^{iwt}$
   - $\frac{\partial^4 u}{\partial z^4} = \frac{d^4 Z}{dz^4}e^{iwt}$
3. Substituting these derivatives into the equation gives:
   $EI\frac{d^4 Z}{dz^4}e^{iwt} - w^2 Z(z)e^{iwt} + i\gamma w Z(z)e^{iwt} = 0$

Simplification:

1. Factor out e^{iwt} (since it's nonzero, we can divide both sides by it):
   $EI\frac{d^4Z}{dz^4} - w^2Z(z) + i\gamma wZ(z) = 0$
2. Rearrange the equation:
   $EI\frac{d^4Z}{dz^4} = (w^2 - i\gamma w)Z(z)$

Ordinary Differential Equation (ODE) for $Z(z)$:

This is now a linear, fourth-order ordinary differential equation in Z:

$$\frac{d^4Z}{dz^4} - \frac{w^2 - i\gamma w}{EI}Z = 0$$

Solving the ODE:

1. Let $k^4 = \frac{w^2 - i\gamma w}{EI}$. This simplifies the ODE to:
   $\frac{d^4Z}{dz^4} - k^4Z = 0$
2. The general solution to this type of linear homogeneous ODE is a combination of exponential functions:
   $Z(z) = Ae^{kz} + Be^{-kz} + Ce^{ikz} + De^{-ikz}$
   where $A, B, C,$ and D are constants determined by boundary conditions.

Conclusion:

The general solution $Z(z)$ for the displacement of any point on the beam in the permanent regime, where the beam oscillates with the excitation frequency w, is a combination of exponential functions as given above. The specific values of $A, B, C,$ and D would be found by applying the relevant boundary conditions of the specific beam problem.

- To find the eigenfrequencies of a cantilever beam under free oscillations, we will apply the stated boundary conditions to the general solution $Z(z)$ and perform a step-by-step derivation.
- The context is finding the eigenfrequencies of a cantilever beam under free oscillations, which involves solving a differential equation with specific boundary conditions stated in the problem.

1. Understanding the Equation for q^4:

The equation given is:

$$q^4 = \frac{\omega^2}{c^2}(1 - i\gamma/\omega)$$

where:

- ω is the angular frequency of oscillation.
- $c = \frac{EI}{\rho S}$ is a constant depending on the beam's material and geometric properties (Euler's constant EI, density ρ, and cross-sectional area S).
- γ is the damping coefficient.
- q is a complex wave number.

2. Analyzing the Expression for $Z(z)$:

The given expression for $Z(z)$ is a solution for the displacement of the beam, considering the complex wave number q and involving hyperbolic and trigonometric functions:

$$Z(z) =$$

$$z_0 \left[\frac{\cosh(qL)\cos(qL)+\sinh(qL)\sin(qL)+1}{2(1+\cosh(qL)\cos(qL))} (\cosh(qz) - \cos(qz)) - \frac{\cosh(qL)\sin(qL)+\sinh(qL)}{2(1+\cosh(qL)\cos(qL}\right.$$

$$z_0 \cos(qz)$$

Here:

- z_0 is the amplitude of oscillation.
- L is the length of the beam.
- z is the position along the length of the beam.

3. The Transcendental Equation Δ:

The transcendental equation Δ is given by:

$$\Delta = 2(1 + \cosh(qL)\cos(qL)) = 0$$

This equation is crucial for finding the eigenfrequencies. It's a condition derived from the boundary conditions of the cantilever beam.

4. Determining Eigenfrequencies:

To obtain the eigenfrequencies:

- Solve the transcendental equation $\Delta = 0$ for q.
- Use the relation between q and ω to find the eigenfrequencies.

5. Solving for Eigenfrequencies:

The solution involves:

- Numerically solving the transcendental equation for q, as it typically cannot be solved analytically.
- Substituting the values of q into the equation $q^4 = \frac{\omega^2}{c^2}(1 - i\gamma/\omega)$ to find ω.

Analysis of Amplitude Equation

1. **Amplitude Equation:** The amplitude $A(\omega)$ of the free end of the beam at $z = L$ is given by:
   $A(\omega) = |Z(L)| = \left|\frac{2z_0}{2(1+\cosh(qL)\cos(qL))}(\cos(qL) + \cosh(qL))\right|$
2. **Relation to q and ω:** Recall the relationship $q^4 = \frac{\omega^2}{c^2}(1 - i\gamma/\omega)$, where $c = \frac{EI}{\rho S}$.
3. **Resonant Frequencies:** Resonant frequencies occur where the amplitude reaches local maxima. Near these frequencies, the amplitude will significantly increase.

forcedbeamvibration.m

1. Parameter Initialization

- **Material and Beam Properties**: Young's modulus (`E`), the second moment of inertia (`I`), density (`rho`), and cross-sectional area (`S`) are defined. These parameters are fundamental in defining the beam's stiffness and mass properties.
- **Beam Dimensions and Damping**: The length of the beam (`L`), the damping coefficient (`gamma`), and the amplitude of oscillation at the fixed end (`z0`) are specified. Damping is crucial in real-world applications as it accounts for energy dissipation.
- **Forced Vibration Frequency**: The frequency of oscillation (`w`) is defined, representing an external driving frequency applied to the beam.

2. Discretization and Initial Conditions

- **Spatial and Temporal Discretization**: The beam and the simulation time are discretized into `Nz` spatial steps and `Nt` time steps, respectively. `dz` and `dt` represent the increments in space and time.
- **Initial Conditions**: Initial deflection (`Fz`) and velocity (`Gz`) profiles along the beam are defined. These conditions are crucial for dynamic analysis.

3. Solution Initialization

- The displacement field `u` over space and time is initialized as a zero matrix.
- The first column of `u` is set to the initial deflection, and a forward Euler step is used to compute the second column, setting initial motion.

4. Time-Stepping Algorithm

- The core of the simulation uses a nested loop over time (`n`) and space (`j`). It employs a central difference method for spatial derivatives and a forward Euler method for time integration.
- The displacement `u` at each point is updated based on the beam's dynamic equation, incorporating the effects of bending stiffness, damping, and inertia.

5. Boundary Conditions

- At each time step, boundary conditions are applied: The first spatial point (`u(1, n+1)`) simulates a driven end with an oscillatory motion, and the last point (`u(end, n+1)`) represents a free end.

6. Plotting the Results

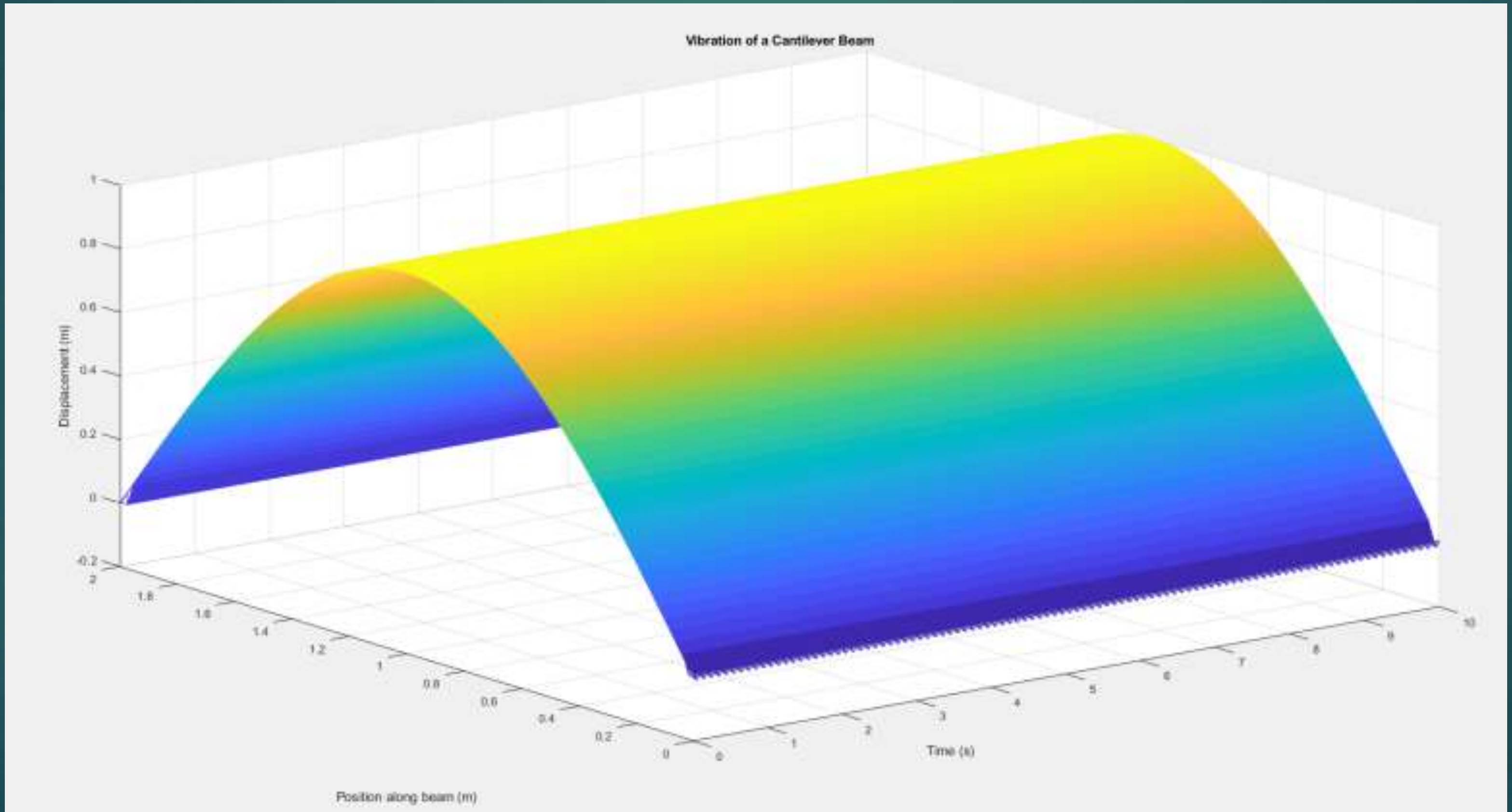
- A mesh plot visualizes the real part of the displacement field `u` over time and along the length of the beam.

7. Eigenfrequency Calculation

- **Parameter `c` Calculation**: A parameter combining stiffness and mass properties of the beam is calculated.
- **Root Finding for Transcendental Equation**: The code defines a function `Delta` representing a transcendental equation associated with the beam's vibration. Roots of this equation (`qRoots`) are found numerically.
- **Eigenfrequency Calculation**: For each root found, an eigenfrequency is calculated considering the stiffness, mass properties, and damping. These frequencies represent natural frequencies at which the system will resonate.

The calculated eigenfrequencies are displayed, providing insight into the resonant behavior of the beam.

- In summary, this code provides a comprehensive analysis of a cantilever beam's dynamic behavior under forced vibration, taking into account both spatial and temporal variations and calculating its natural frequencies of vibration. It is a typical example of a numerical approach used in structural and vibration engineering for dynamic analysis.



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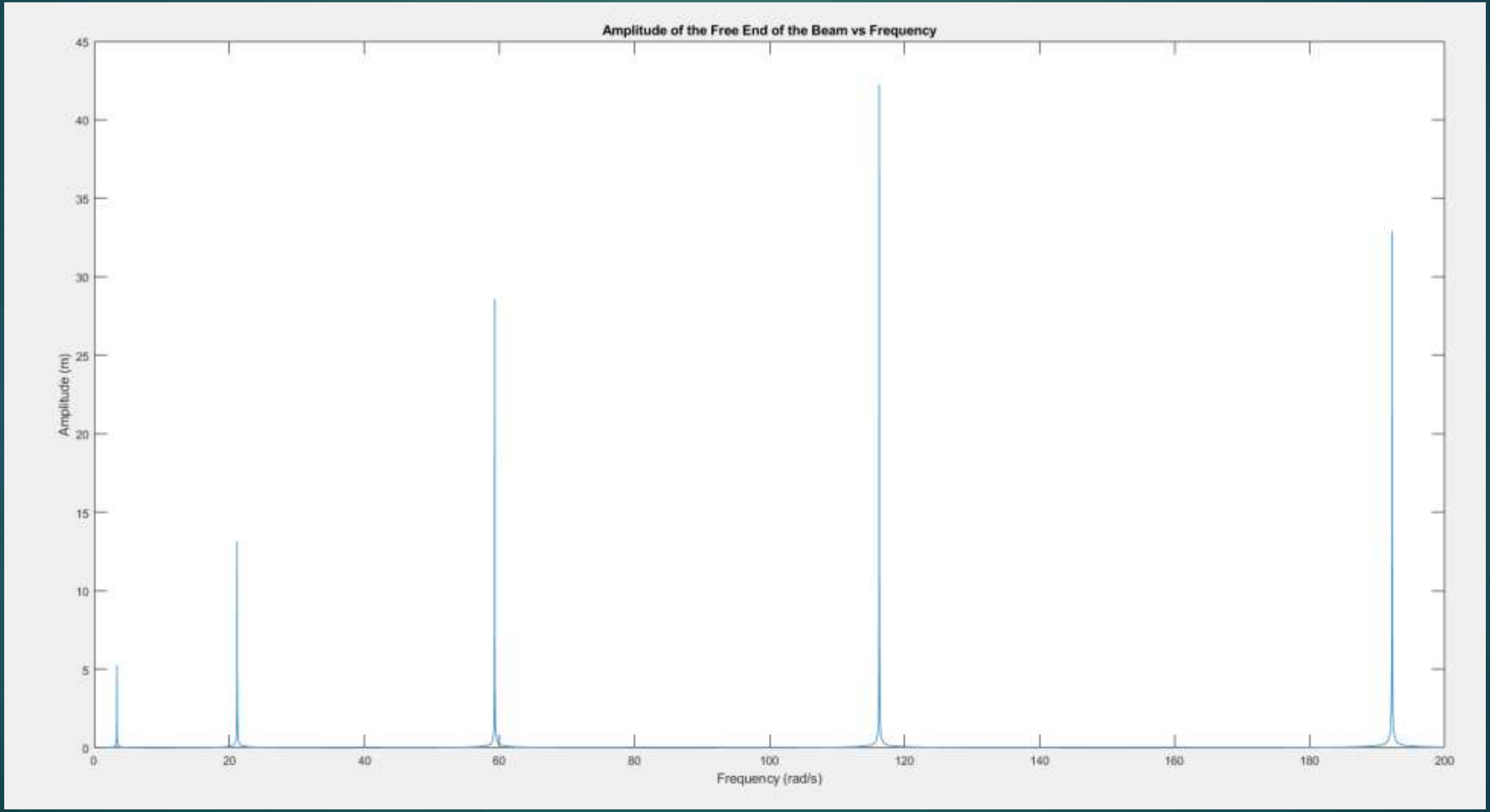


Review of Active Vibration Control (AVC)

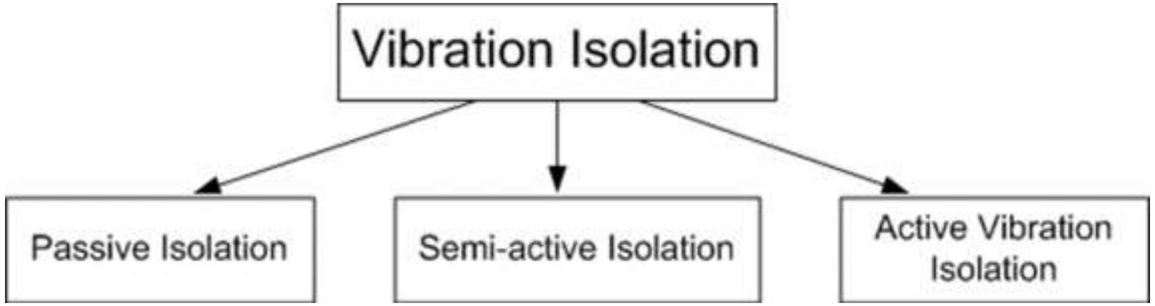
AVC is defined as the active application of force in an equal and opposite fashion to the forces imposed by external vibration. This technique is particularly important in situations where vibration is a limiting factor in the performance of industrial systems. The goal is to operate machines and structures with low levels of vibration, which leads to reduced stresses, fatigue, and noise.

Limitations of Passive Damping: The limitations occur in the passive damping, especially at lower frequencies. Passive vibration control treatments are unable to adapt or retune to changing disturbance or structural characteristics over time. AVC systems emerge as viable technologies to fill this low-frequency gap without adding excessive weight to sensitive structures.

Control Approaches: There are two primary control approaches: feedforward and feedback. Feedforward control anticipates and corrects for errors before they occur, while feedback control adjusts for errors as they happen. The paper also discusses the differences between analog and digital control systems, with digital systems offering predictive capabilities and adaptiveness.

Actuators and Sensors: Piezoelectric, electrodynamic, and hydraulic actuators. The use of piezoceramics as both sensors and actuators

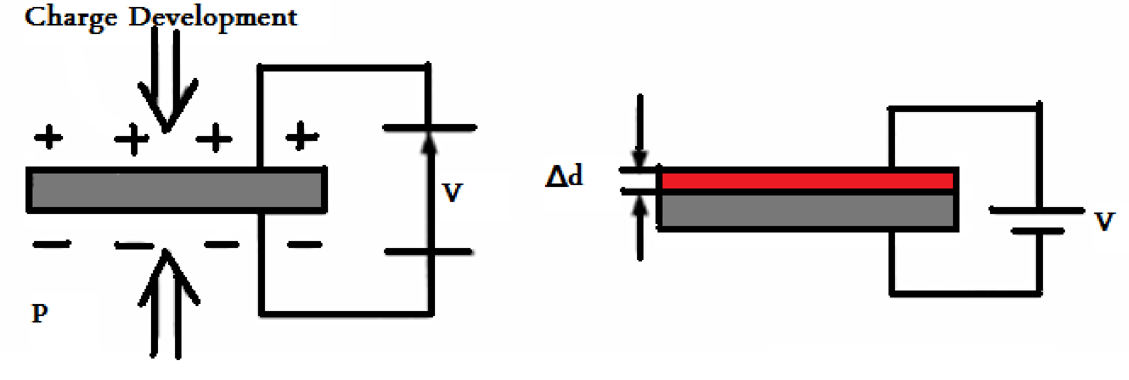
AVC is crucial for reducing vibration in various systems, particularly where passive damping is insufficient. If I were to approach this problem, I would consider the specific characteristics of the vibration (frequency, amplitude, source) and the system's constraints (weight, size, environment). The choice between feedforward and feedback control would depend on the predictability of the vibration source and the desired response time.

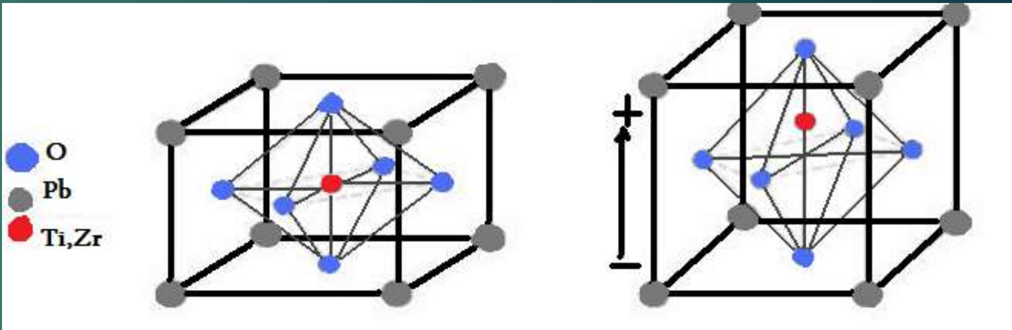


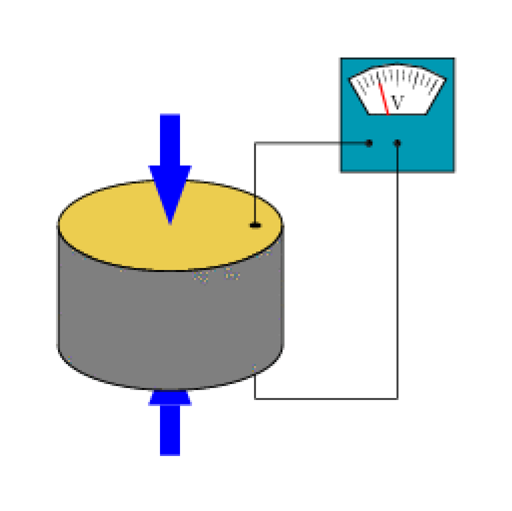
1. **Nonadaptive and Adaptive Control Approaches**: Nonadaptive methods are simpler and have higher acceptance due to their simplicity. However, adaptive methods, while potentially more complex, are effective for many applications but can have issues with convergence speed and stability. The robustness of these methods varies with the magnitude and phase delay of the plant transfer function.
2. **Disturbance Observer Approach**: This approach is based on a state observer and state feedback. It involves reconstructing the primary waveform and generating a secondary cancellation signal. The disturbance is modeled as a sum of finite number of sinusoidal signals, which are harmonically related. The goal is to attenuate the disturbance by producing an estimate of it and using this estimate, with a sign reversal, as a control signal.
3. **Feedback System with Secondary Path Modeling**: This system involves synthesizing a reference signal and updating the coefficients of a filter to minimize the residual error signal. The secondary path modeling is crucial for accurate estimation and should not intrude on the operation of the AVC system. The tradeoff between accuracy and intrusion can be solved by the Additive Random Noise Technique.
4. **Multichannel AVC System**: The goal of a multichannel system is to minimize the kinetic energy of a vibrating structure. This is achieved by using a number of accelerometers to provide an estimate proportional to the kinetic energy of the structure. Adaptive algorithms used in these systems are matrix extensions of single-channel systems.

- Comparing these methodologies with the initial conceptual solutions, we see a blend of both predictive (feedforward) and reactive (feedback) approaches. The use of adaptive algorithms and multichannel systems indicates a sophisticated approach to dealing with complex vibration patterns. The emphasis on accurate modeling and the use of secondary path modeling aligns with the need for precision in AVC

- Piezoelectricity is a phenomenon where certain materials generate an electric charge in response to applied mechanical stress. This reversible process also works in the opposite direction, where the application of an electric field causes mechanical deformation in the material. This unique interaction between electrical and mechanical states underpins various applications in sensing, actuation, and energy harvesting.







Classification of Piezoelectric Materials

1) Polymer Piezoelectric Materials

- Polymer-based piezoelectrics, such as Polyvinylidene Fluoride (PVDF), are known for their flexibility, ease of processing, and relatively good piezoelectric properties. These materials exhibit lower piezoelectric constants compared to ceramics but are favored in applications requiring large-area coverage and flexibility, like wearable sensors and energy harvesters. Their mechanical properties offer advantages in damping vibrations and can be easily integrated into composite structures.

2) Ceramic Piezoelectric Materials

- Ceramic piezoelectrics, such as Lead Zirconate Titanate (PZT), are widely used due to their high piezoelectric coefficients and electromechanical coupling factors. These materials are inherently brittle and have high permittivity, making them suitable for high-power applications, such as ultrasonic transducers and actuators. However, their brittleness and the environmental concerns regarding lead content present challenges in certain applications.

3) Natural Piezoelectric Materials

- Natural piezoelectric materials, like quartz, have been historically significant. Quartz, with its stable piezoelectric properties and high-temperature resilience, is used in precision resonators and filters. However, the relatively low piezoelectric response limits their use in high-power or high-sensitivity applications.

Usage Areas of Piezoelectric Materials

- Piezoelectric materials have diverse applications:
- **Sensing:** Utilized in accelerometers, pressure sensors, and microphones due to their ability to convert mechanical stress into electrical signals.
- **Actuation:** In applications like piezoelectric motors and precision positioning systems, the converse piezoelectric effect is used.
- **Energy Harvesting:** These materials can harvest energy from ambient vibrations, finding use in powering small electronics or wireless sensors.
- **Medical Devices:** Ultrasound imaging and therapeutic devices leverage high-power piezoelectric ceramics.

Critical Perspectives and Future Directions

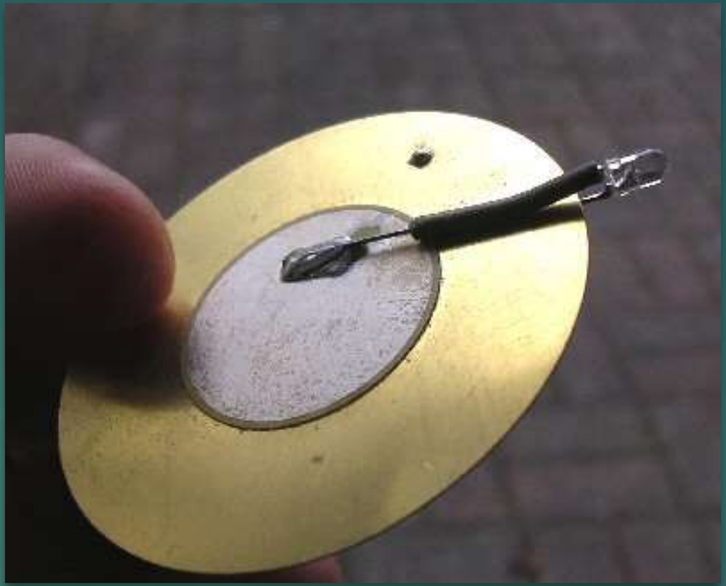
While the utility of piezoelectric materials is undeniable, several challenges and opportunities for advancement exist:

- Material Development: The quest for lead-free ceramics with comparable properties to PZT is a significant area of research, driven by environmental concerns.

- Integration and Fabrication Techniques: Advanced manufacturing techniques, such as 3D printing of piezoelectric materials, open up new application possibilities but require further research to optimize material properties and process reliability.

- Energy Harvesting Efficiency: Enhancing the efficiency of energy harvesters, especially in low-frequency and low-amplitude environments, remains a critical research area.

- Broadening Application Scope: Exploring unconventional applications, like piezoelectric energy harvesting in infrastructure or biomedical implants, could significantly impact various fields.



- **Piezoelectric Constants (d, g, and e)**

1. **d-Constant (Piezoelectric Strain Constant):** This constant relates the electric field to the mechanical strain in a material. It's a measure of how much strain (deformation) is produced under an applied electric field. Higher d-constants are desirable for sensors and actuators as they imply a strong electromechanical response. For example, PZT ceramics have high d-constants, making them efficient in converting electrical energy to mechanical energy and vice versa.

2. **g-Constant (Piezoelectric Voltage Constant):** This constant relates the mechanical stress to the electric field generated. It is crucial for applications where a material is subjected to mechanical stress, and the resulting electric field is of interest, such as in piezoelectric energy harvesters. Materials with high g-constants are excellent for harvesting energy from mechanical stresses.

3. **e-Constant (Piezoelectric Charge Constant):** It links the mechanical stress to the polarization (charge) generated. It's a direct measure of the charge produced under mechanical stress.

- **Dielectric Constant (ε)**

The dielectric constant (or permittivity) of a piezoelectric material is a measure of its ability to store electrical energy. It is particularly significant in capacitive applications like sensors and resonators. A high dielectric constant can enhance the charge storage capability but may also lead to larger dielectric losses. The optimization of the dielectric constant is essential for balancing energy storage and dissipation.

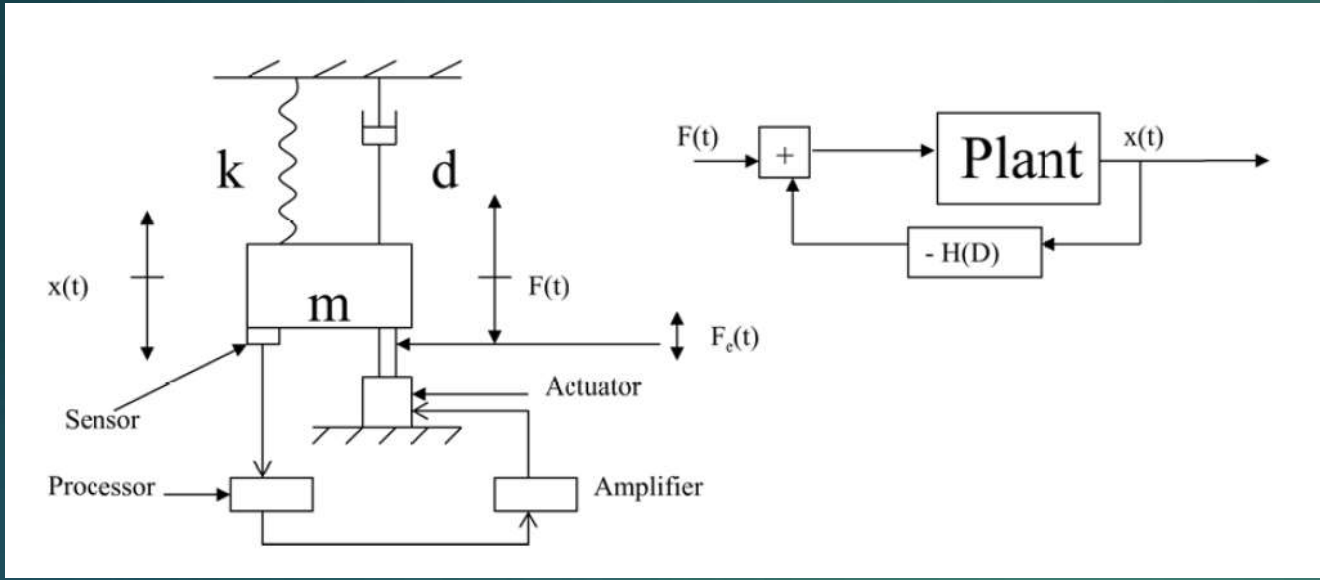
- **Elastic Compliance (S)**

Elastic compliance refers to the ability of a material to deform under mechanical stress. It's the inverse of stiffness. In piezoelectric materials, this property is critical because it influences how much strain can be induced by an applied electric field (or stress). Materials with high elastic compliance are more easily deformable, which is advantageous in applications requiring significant mechanical movement, such as actuators.

- **Coupling Factor (k)**

The coupling factor is a dimensionless parameter that measures the efficiency of conversion between electrical and mechanical energy in a piezoelectric material. A higher coupling factor indicates a more efficient energy conversion, which is desirable in both sensors and actuators. This factor is crucial for determining the applicability of a piezoelectric material in a specific application, as it directly affects the device performance.

Vibration Control for S-DOF System



In a basic spring-mass-damper system, an SDOF system, vibration control can be effectively achieved by integrating piezoelectric sensor and actuator elements. The fundamental concept involves using these elements to detect vibrations and subsequently apply an appropriate counteracting force to dampen these vibrations.

Collocated Piezoelectric Sensor/Actuator

Concept:

- **Collocation:** Placing a pair of piezoelectric elements (sensor and actuator) in close proximity on a flexible structure. This configuration ensures that the sensor directly experiences the vibrations that the actuator needs to counteract.
- **Operational Mechanism:** When the structure vibrates, the sensor piezo element detects this motion and converts it into an electrical signal (voltage). This signal, after being processed through the control system, is used to drive the actuator piezo element. The actuator, in turn, generates a mechanical force that counteracts the original vibration.

Transfer Functions:

- The system's effectiveness relies on accurately determining the transfer functions, which relate:
  1. The mechanical vibration of the structure to the electrical output of the sensor.
  2. The electrical input to the actuator to the mechanical force generated.
- These transfer functions are crucial for designing the feedback control loop.

Challenges in Practical Implementation

Feedback Control Stability:

- **Precision in Transfer Functions:** Even minor inaccuracies in the transfer functions can lead to instability in the feedback loop. This instability can manifest as an inadequate or excessive counteracting force, failing to dampen the vibration effectively or even amplifying it.
- **Phase and Amplitude Matching:** Ensuring that the sensor and actuator are perfectly collocated and that their responses are precisely 180° out of phase is challenging. Any mismatch can degrade the system's performance.

System Complexity:

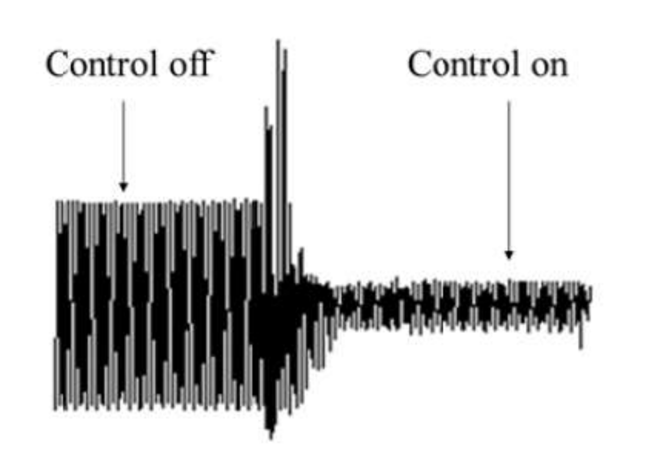
- While the conceptual model is straightforward, the practical implementation involves complexities related to mechanical, electrical, and control system integration.
- The system must be robust against various disturbances and operational variations, such as temperature changes and material aging.

Sensitivity to Environmental Factors:

- Piezoelectric materials are sensitive to environmental conditions like temperature and humidity, which can affect their performance.
- Long-term reliability and consistency in such environments can be challenging to maintain.

Experimental Setup: Cantilever Beam with Piezoelectric Sensor and Actuator

1. **Structure:**
   - **Material:** Aluminum beam.
   - **Dimensions:** $20 \times 2.0 \times 1.5$ cm.
   - **Configuration:** Cantilever beam, clamped at one end and free at the other.
2. **Piezoelectric Sensor:**
   - **Material:** PZT (Lead Zirconate Titanate).
   - **Location:** Fixed end of the beam.
   - **Function:** Converts stress in the beam into piezoelectric current.
3. **Piezoelectric Actuator:**
   - **Location:** Fixed end, where stress and bending moment are maximal.
   - **Function:** Induces mechanical stress in the beam when driven by a voltage.
4. **Signal Generation and Processing:**
   - **Function Generator:** Produces a sinusoidal, square, or triangular waveform.
   - **Amplifier:** Amplifies the weak signal from the generator to drive the exciter.
   - **DAQ System:** Converts electrical signals from the sensor for computer processing.
5. **Control System:**
   - **Input:** Electrical signal from the sensor.
   - **Output:** Voltage fed to the actuator after processing.



Dynamics of the Cantilever Beam System

The dynamic behavior of the cantilever beam can be described by the following equation of motion:

$$m\ddot{x}(t) + d\dot{x}(t) + kx(t) = F(t)$$

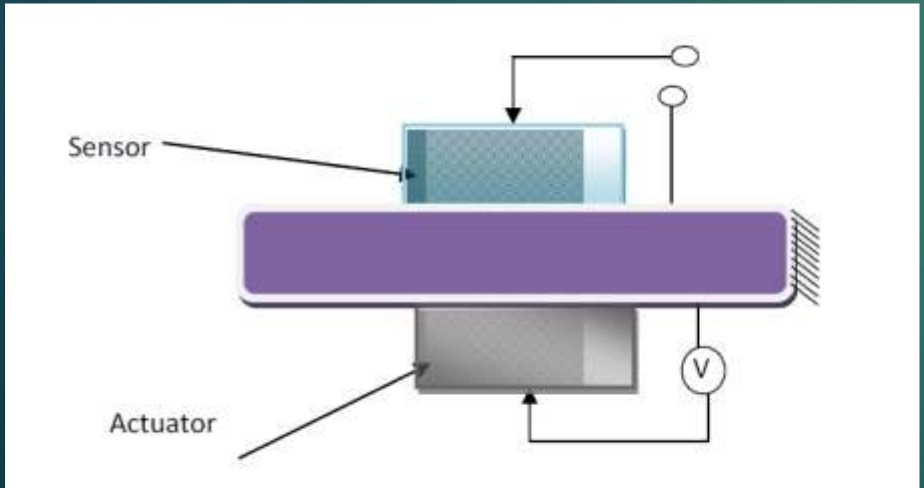
where:

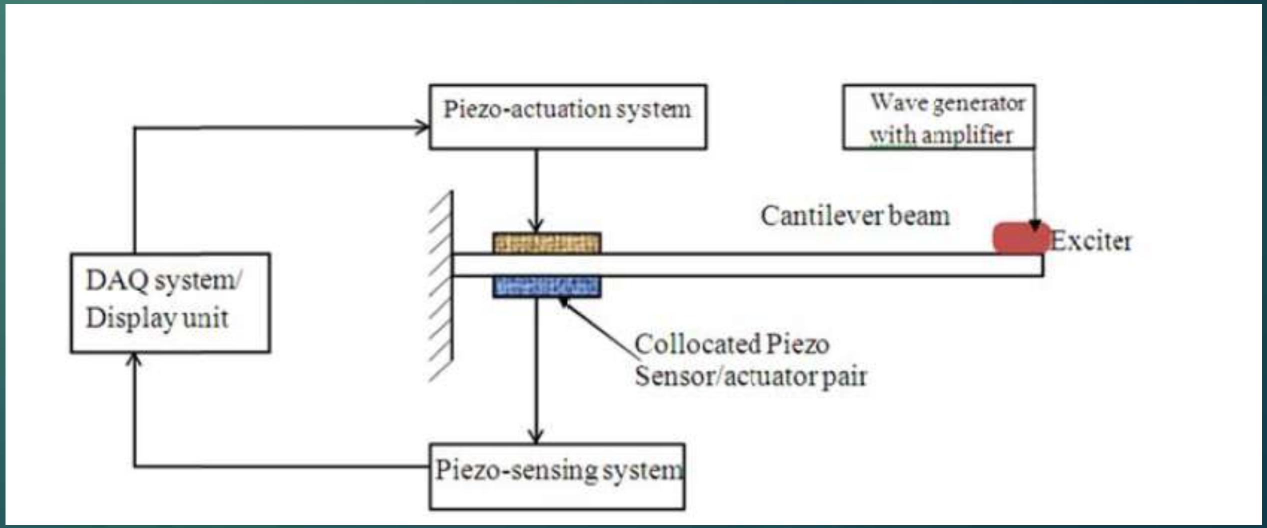
- m: Mass of the beam.
- d: Damping coefficient.
- k: Stiffness of the beam.
- $x(t)$: Displacement of the beam.
- $F(t)$: External force applied to the beam (from the actuator).

Vibration Control Process

1. **Vibration Sensing:** The sensor detects vibration-induced stress, generating a corresponding electrical signal.
2. **Signal Processing:** The electrical signal is processed, amplified, and converted to be compatible with the control system.
3. **Actuation:** The processed signal is fed to the actuator, which applies a counteracting force to the beam.
4. **Feedback Control:** The control system adjusts the actuator's output based on the sensor input, forming a feedback loop for vibration control.

- **Measurement and Analysis**
- **Laser Doppler Vibrometer:** Used to measure deflection, particularly at the free end of the beam.
- **Vibration Signatures:** Comparisons between uncontrolled and controlled vibrations are made.
- **Key Points**
- **Frequency Range:** Adjustable between 1 Hz to 1000 kHz.
- **Amplification Limit:** Must be within the safe limits of the exciter and the PZT patches.





1. Fundamental Concepts of Piezoelectricity

Piezoelectric Effect: The fundamental principle of piezoelectricity involves the generation of an electric potential in response to applied mechanical stress. Conversely, an applied electric field induces mechanical deformation. This bidirectional energy transformation is the essence of piezoelectric materials, making them indispensable in applications like MEMS (Microelectromechanical Systems).

Microstructure and Polarization: The piezoelectric effect is intimately related to the microstructure of the materials. The displacement of ionic charges within the crystal lattice under stress leads to the development of electric dipoles. This change in charge distribution under external stress or electric field is the crux of piezoelectric behavior.

2. Piezoelectric Anisotropy and Poling

Crystal Symmetry: The anisotropic nature of the unit cell in piezoelectric materials is crucial. Only materials with non-centrosymmetric crystal structures exhibit piezoelectricity. The asymmetry in the unit cell is what enables the conversion of mechanical energy to electrical energy and vice versa.

Poling Process: The poling process aligns the domains within piezoelectric ceramics, transforming a randomly oriented microstructure into a uniformly polarized material. This orientation is essential for the material to exhibit significant piezoelectric properties.

3. Mathematical Representation and Analysis

Piezoelectric Constants: The piezoelectric effect is quantified using constants that relate mechanical stress or strain to electric polarization. The equation presented, $d_{ij} = k_{ij}\sqrt{s_{jj}^{E} + \epsilon_{ii}^{T}}$, is a symbolic representation of this relationship, where:

- d_{ij} is the piezoelectric constant.
- k_{ij} relates to the material's piezoelectric sensitivity.
- s_{jj}^{E} is the elastic compliance under a constant electric field.
- ϵ_{ii}^{T} represents the dielectric permittivity under constant mechanical stress or strain.
- Indices i, j denote the directions of polarization, electrical area, and mechanical stress or strain.

This equation encapsulates the piezoelectric behavior by linking mechanical and electrical properties. Let's break it down further:

Step-by-Step Derivation

1. **Starting Point:** Begin with the fundamental piezoelectric equation $P_i = d_{ij}\sigma_j$ where P_i is the polarization and σ_j is the stress.
2. **Incorporate Material Properties:** Introduce material-specific constants k_{ij}, s_{jj}^{E}, and ϵ_{ii}^{T} to account for the material's unique response to stress and electric fields.
3. **Combine Relations:** The combination of these parameters in the given equation represents the material's response under combined mechanical and electrical influences.

$$d_{31} = k_{31}\sqrt{s_{11}{}^{E} + \varepsilon_{33}{}^{T}}$$

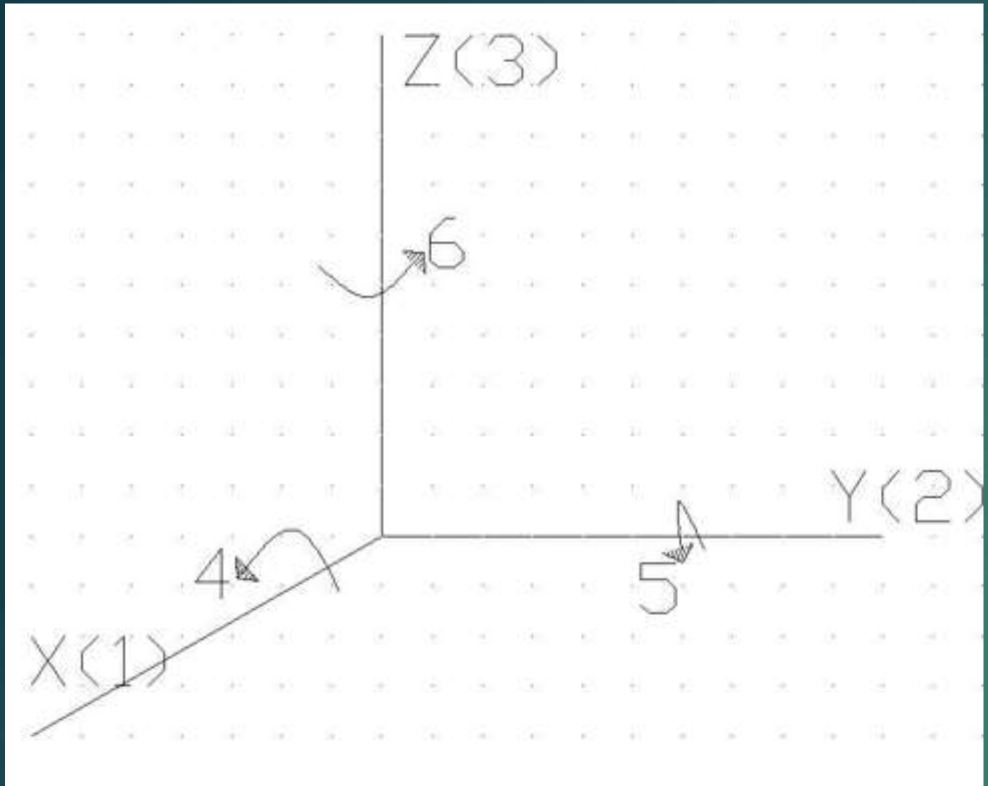
$$d_{33} = k_{33}\sqrt{s_{33}{}^{E} + \varepsilon_{33}{}^{T}}$$

$$d_{15} = k_{15}\sqrt{s_{55}{}^{E} + \varepsilon_{11}{}^{T}}$$

d	Commentary
d_{33}	The polarization at direction of 3* by unit mechanic stress at direction of 3* or the strain at direction of 3* by applying the unit electrical area at direction of 3*.
d_{31}	The polarization at direction of 3* by unit mechanic stress at direction of 1** or the strain at direction of 1** by applying the unit electrical area at 3* direction of 3*.
d_{15}	The polarization at direction of 1** by unit shear stress at direction of 2** or the unit shear stress at direction of 2** by applying the unit electrical area at direction of 1**.
	*= the same direction with polarization direction of ceramic material **= the vertical direction to polarization direction of ceramic material.

Critical Analysis

- **Completeness:** The given equation provides a comprehensive view of the piezoelectric effect, but it might oversimplify the complex interplay of mechanical and electrical domains in certain materials.
- **Applicability:** The equation is highly applicable in design and analysis of MEMS devices, but its utility in predicting behavior under non-linear or extreme conditions may be limited.
- **Modifications for Non-linear Behavior:** For materials exhibiting non-linear piezoelectric responses, higher-order terms and non-linear models would be more appropriate.



Directional Indexes and Piezoelectric Constants

Directional Indexes: The indexes (1 through 6) correspond to different stress and strain directions:

- 1, 2, 3: Normal stress in the x, y, z directions respectively.
- 4, 5, 6: Shear stress in the x, y, z directions respectively.

Piezoelectric Constant (d_{ij}) Interpretation:

- d_{ij} represents the polarization in direction 'i' due to unit mechanical stress or strain in direction 'j'.
- The first subscript 'i' indicates the direction of the electric field (or polarization), and the second subscript 'j' relates to the mechanical stress or strain direction.

Piezoelectric Voltage Constant (g_{ij})

Definition and Equation:

- $g_{ij} = \frac{d_{ij}}{\epsilon_{ii}^T}$ (Equation 2)
- It signifies the electric field in direction 'i' generated per unit mechanical stress or strain in direction 'j'.

$$g_{31} = d_{31} / \varepsilon^T{}_{33}$$
$$g_{33} = d_{33} / \varepsilon^T{}_{33}$$
$$g_{15} = d_{15} / \varepsilon^T{}_{11}$$

Interpretation:

- g_{ij} offers insights into the efficiency of electric field generation per unit mechanical stress/strain, thus serving as a crucial parameter in the design and optimization of piezoelectric devices.

Relative dielectric constant

The ratio of the permittivity of the material ε to the permittivity of free space ε_0 8,85x10^{-12} farad/meter at low frequencies well below any mechanical resonance of the structure.

$$K^T = \varepsilon^T / \varepsilon_0$$

ε	Commentary
ε^T_{11}	Dielectric shift at direction of 1** for stable mechanical stress and Permittivity of electrical area
ε^S_{33}	Dielectric shift at direction of 3* for stable mechanical stress and Permittivity of electrical area
	*= the same direction with polarization direction of ceramic material **= the vertical direction to polarization direction of ceramic material.

Coupling Factor (k_{ij})

Definition and Equation:

- $k_{ij}^2 = \frac{d_{ij}^2}{s_{ij}^E \times \epsilon_{ii}^T}$ (Equation 3)
- This dimensionless quantity measures the efficiency of energy transfer between electrical and mechanical domains.

Interpretation:

- k_{ij} is a critical factor in evaluating the performance of piezoelectric materials, especially in energy harvesting and sensor applications.

Relative Dielectric Constant (K^T)

Definition:

- K^T is the ratio of the permittivity of the material (ϵ) to the permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12}$ F/m).
- It is significant at low frequencies, below any mechanical resonance of the structure.

Implications:

- K^T influences the capacitance of piezoelectric components, affecting the electrical current required at high frequencies.
- It's a crucial parameter for understanding the behavior of piezoelectric materials in various frequency ranges.

k	Commentary
k_{33}	Coupling factor for electrical area at direction of 3* and lengthwise vibrations at direction of 3* (ceramic wire, length>10x dimension)
k_T	Coupling factor for electrical area at direction of 3* and lengthwise vibrations at direction of 3* 3* (thin disc, surface sizes are bigger; k_{33}>k_T)
k_{31}	Coupling factor for electrical area at direction of 3* and lengthwise vibrations at direction of 1** (ceramic wire)
k_p	Coupling factor for electrical area at direction of 3* and lengthwise vibrations at direction of 1** and 2**.(thin disc)
	*=the same direction with polarization direction of ceramic material **=the vertical direction to polarization direction of ceramic material

Generator and Motor Actions in Piezoelectric Materials

Generator Action: This involves converting mechanical energy into electrical energy. When mechanical stress or strain is applied to a poled piezoelectric material, it alters the dipole moment, generating a voltage. The polarity of the voltage depends on whether the applied force is compression or tension relative to the polarization direction.

Motor Action: This is the conversion of electrical energy to mechanical energy. Applying voltage in the same polarity as the poling voltage causes the piezoelectric ceramic to expand or contract. This action is useful in applications involving alternating current.

Equations for Generator and Motor Actions:

- Generator (Direct Effect): $D_i = d_{ij}T_j + \epsilon_{ij}^T E_i$
- Motor (Converse Effect): $S_j = s_{ij}^E T_j + d_{ij} E_i$

Here, D_i is the electric displacement due to applied stress T_j, S_j is the strain due to an electric field E_i, d_{ij} is the piezoelectric coefficient, s_{ij}^E is the elastic compliance, and ϵ_{ij}^T is the dielectric constant under constant stress.

Factors Affecting Piezoelectric Properties

- **Aging Rate:** Over time, piezoelectric properties can change, impacting their performance.
- **Mechanical Constraints:** External forces can influence the piezoelectric behavior.
- **Electrical and Thermal Limitations:** The performance of piezoelectric materials is affected by their electrical and thermal environments.

Piezoelectric Material Varieties and Applications

1. **Types of Piezoelectric Materials:**
   - Single crystals like quartz, lithium niobate (LiNbO3), lithium tantalate (LiTaO3).
   - Ceramic materials like barium titanate (BaTiO3, BT), lead titanate (PbTiO3, PT), lead zirconate titanate (Pb(ZrxTi1-x)O3, PZT), and various doped or composite materials.
2. **Applications:**
   - **Medical and Industrial Imaging:** Ultrasound imaging devices.
   - **Data Storage and Display:** Electro-optic materials in non-volatile memories, data storage, and displays.
   - **Surface Acoustic Wave (SAW) Devices:** Lithium niobate and doped lithium niobate crystals are useful in SAW devices for data communication.
   - **Timing Mechanisms:** Quartz crystals in watches and clocks.
   - **Piezo Actuators:** Used for ultra-precise positioning, generating high forces/pressures in various applications, including scanning tunnel microscopy and fluid pumps.

PZT Properties and Applications

Electromechanical Transformation Efficiency:

- PZT exhibits a high level of electromechanical coupling, making it highly efficient in converting electrical energy to mechanical energy and vice versa.
- Its stability across various temperature ranges enhances its applicability in diverse environments.

Applications in Different Components:

1. **Sensors:** Piezoelectric sensors, such as accelerometers and force sensors, leverage PZT's high natural frequency and excellent linearity over a wide amplitude range.
2. **Generators:** Piezoelectric generators produce electrical energy by applying mechanical energy. Techniques like multilayer construction enhance the efficiency of these generators.
3. **Actuators:** PZT-based actuators convert electrical signals to precise physical displacement. They are used in precision machining tools, lens adjustments, mirror positioning, and hydraulic valves.
4. **Transducers:** These devices convert electrical energy to vibrational sound or ultrasound energy. They benefit from the reversible piezoelectric effect and are used in applications like ultrasound imaging.

Determining Resonance Frequency:

- Depends on the ceramic material's composition and the shape and volume of the element.
- A thicker element resonates at a lower frequency compared to a thinner element of the same shape.

Electromechanical Coupling Factor (k):

- Depends on the mode of vibration and shape of the ceramic element.
- Relationships among k, f_m, and f_n vary depending on the geometry of the ceramic element (plate, disc, rod).

Piezoelectric Motors and Their Advantages

- **Electromagnetic Noise Prevention:** An additional advantage of piezoelectric motors is their ability to operate without generating electromagnetic noise, making them suitable for sensitive applications.

Resonance Frequency in Piezoelectric Ceramics

AC Electric Field Exposure:

- A piezoelectric ceramic element changes dimensions cyclically at the cycling frequency of the field.
- The resonance frequency is where the element most efficiently transforms electrical to mechanical energy.

Frequency Impedance Relationship:

- At **series resonance frequency (f_s)**, the impedance in an electrical circuit representing the element is zero, ignoring mechanical losses.
- At **parallel resonance frequency (f_p)**, the parallel resistance in the equivalent electrical circuit is infinite, again disregarding mechanical losses.
- **Minimum impedance frequency (f_m)** approximates f_s, and **maximum impedance frequency (f_n)** approximates f_p.

Specification of PZT patch

Length (mm)	76.2
Width (mm)	25.4
Thickness (mm)	2
Young's modulus (GPa)	63
Density (kg/m^3)	7500
Poisson's ratio	0.28
Damping constants	$\alpha = 0.001$ $\beta = 0.0001$
Max. Input voltage (V)	270

General Piezo Symbols

General piezo symbols

Symbol	Description	Unit	Generel/ Noliac
A	Electrode surface area	mm^2	N
c	Stiffness coefficient	N/m^2	G
C	Capacitance	F	G
d	Piezoelectric charge coefficient	C/N	G
f	Frequency	Hz	G
f_{max}	Maximum frequency	Hz	N
f_a	Anti resonance frequency	Hz	G
f_r	Resonance frequency	Hz	G
F_b	Blocking force	N	N
F_{dyn}	Dynamic Force	N	N
g	gravitational constant	$kg\ m / s^2$	G
g_{ij}	Piezoelectric voltage coefficienct	V•m/N	G
H	Height	mm	N
I	Current	A	G
I_{avg}	Average Current	A	N
I_{max}	Maximum Current	A	N
I^E	Current Closed Circuit	V	N
ID	Inner diameter	mm	N
k	Coupling factor		G
k	Stiffness of actuator	mm/N	N
K_0	Spring constant	mm/N	N
K	Relative dielectric constant	-	G
l_0	Nominal Displacement	m	N
L	Length	mm	N
L_c	Clamping length (on plate benders)	mm	N
ΔL	Maximum free displacement	µm	N
m	Mass	kg	N
m_{eff}	Effective Mass	kg	N
n	Number	-	N
N	Frequency constant	Hz•m	N
OD	Outer diameter	mm	N
OD_m	Maximum outer diameter	mm	N

P	Power	W	G
Q	Charge	C	G
Q_m	Mechanical factor	-	G
s	Elastic compliance	M^2/N	G
s	Stroke	µm	N
t	Time	s	G
t_p	Minimum rising time in pulsed operation	s	N
t_r	Period time of actuator's resonance	s	N
t_0	Arbitrary time when measurement started	s	N
tan δ	Dissipation factor	-	G
T	Thickness/ height of internal ceramic layers	µm	N
T_c	Curie temperature	°C	G
Y	Youngs modulus	N/m^2	G
U	Voltage	V	N
U_{pp}	Voltage peak to peak	V	N
U^D	Voltage Open Circuit	V	N
W	Width	mm	N
W_m	Maximum Width (including external electrodes)	mm	N
ε_0	Permittivity of free space	$8{,}854 \cdot 10^{-12}$ F/m	G
ε^T	Permittivity	F/m	G
ρ	Density	g/mm^3	G

Subscript:

31 = direction perpendicular to polarisation field
33 = direction parallel to polarisation field
m = maximum
scma = Stacked Ceramic Multilayer Actuator
cma = Ceramic Multilayer Actuator
0 = original
in = in / beginning
p = planar (f.ex. coupling factor)
t = thickness/ height (f.ex. coupling factor)
b = blocking force

Coupling factor

Ceramic sheet;

$$k_{31}^{2} = d_{31}^{2}(s^{E}{}_{31}\varepsilon^{T}{}_{33}) = \frac{(\pi/2)(f_n/f_m)\tan[(\pi/2)((f_n - f_m)/f_m)]}{\Delta x}$$

Ceramic disc;

$$k_{P}^{2} = 2d_{31}^{2}((s^{E}{}_{11} + s^{E}{}_{12})\varepsilon^{T}{}_{33}) = \sqrt{[(2.51(f_n - f_m)/f_n) - ((f_n - f_m)/f_n)^2}$$

Ceramic bar;

$$k_{33}^{2} = d_{33}^{2}(s^{E}{}_{55}\varepsilon^{T}{}_{11}) = (\pi/2)(f_n/f_m)\tan[(\pi/2)((f_n - f_m)/f_m)]$$

Here f_n is maximum impedance frequency, f_n is minimum impedance frequency.

Elastic compliance

States with s symbol and explains that unit strain as a result of unit mechanical stress at directions of 1** and 3*. S^D is elastic compliance for stable electrical shift and S^E is elastic compliance for stable electrical area. S_{ab} here first index is shows the direction of deplacement and second one is the direction of mechanical stress.

$s=1/v^2$ (v wave speed for ceramic)

$$s^D{}_{33} = 1/Y^D{}_{33}$$
$$s^E{}_{33} = 1/Y^E{}_{33}$$
$$s^D{}_{11} = 1/Y^D{}_{11}$$
$$s^E{}_{11} = 1/Y^E{}_{11}$$

S	Commentary
$s^E{}_{11}$	Elastic compliance for strain at direction of 1** under stable electrical area conditions at direction of 3*.
$s^D{}_{33}$	Elastic compliance for strain at direction of 3* under stable electrical shift conditions at direction of 3*.
	*= the same direction with polarization direction of ceramic material **= the vertical direction to polarization direction of ceramic material

Frequency constant

N_P= radial mode resonance frequency constant

$$N_P = f_S . D_\vartheta$$

N_T= thickness mode frequency constant

$$N_T = f_S . h$$

N_L=lengthwise mode frequency constant.

$$N_L = f_S . l$$

Features of Piezoelectric Actuators

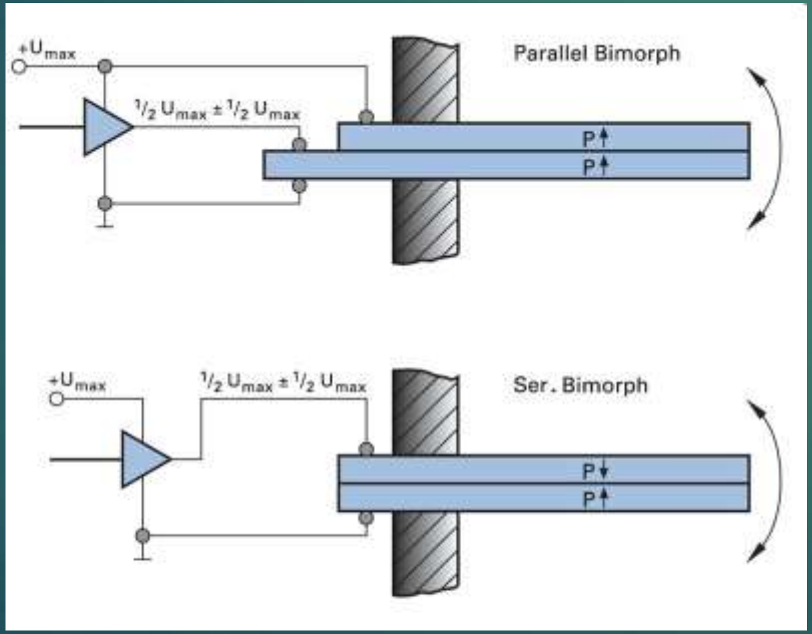
- Sub-nanometer moves at high frequencies
- Movement of high loads, up to several tons
- Virtually no power in static operation
- No moving parts

Types of piezoelectric actuators

- Bender type (bimorph) actuators
- Stack actuators
- Piezoelectric tube actuators
- Shear actuators

Bender type (bimorph) actuators

- Serial benders
- Parallel benders
- Multilayer benders



Stack actuators

- Geting greater expansions and forces

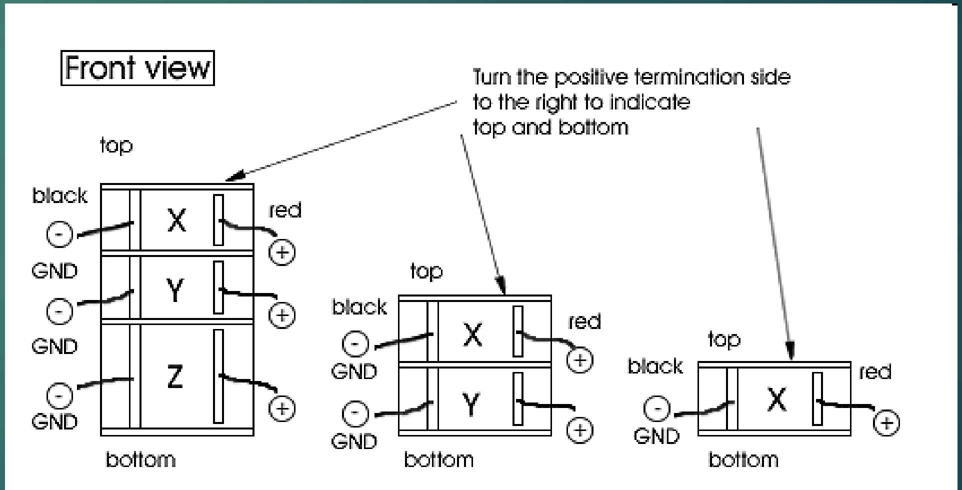
- Main applications;
  - Ultraprecise positioning
  - Generation and handling of high forces or pressures

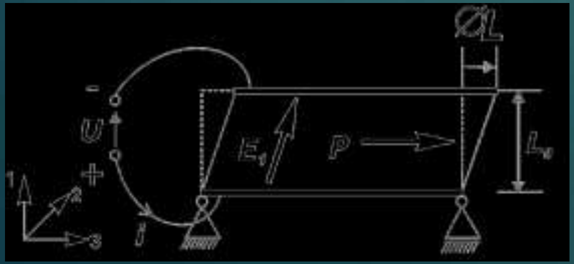
Piezoelectric tube actuators

- They can be used as actuators based on the d31 effect, resulting in
  - a contraction of tubes length
  - a contraction of the diameter of the tube
- Main applications;
  - Scanning tunnel microscopy
  - Fluid pumps
  - Modulation the optical pathlength of wound up fibers

Shear actuators

- Compact Multi-axis Actuators
- Main applications;
  - Laser tuning
  - Atomic force microscopy
  - Scanning applications
  - Micro-stepper motors





Signal Conditioning for Piezofilm Sensor

1. **Differential Charge Amplifier Design**: The decision to use a high-input-impedance differential charge amplifier is apt for this application. Piezoelectric sensors, like the piezofilm in question, have high output impedance, necessitating such an amplifier for effective signal conditioning. The use of three operational amplifiers (op-amps) in a classical instrumentation amplifier configuration enhances the precision and reliability of the signal processing.

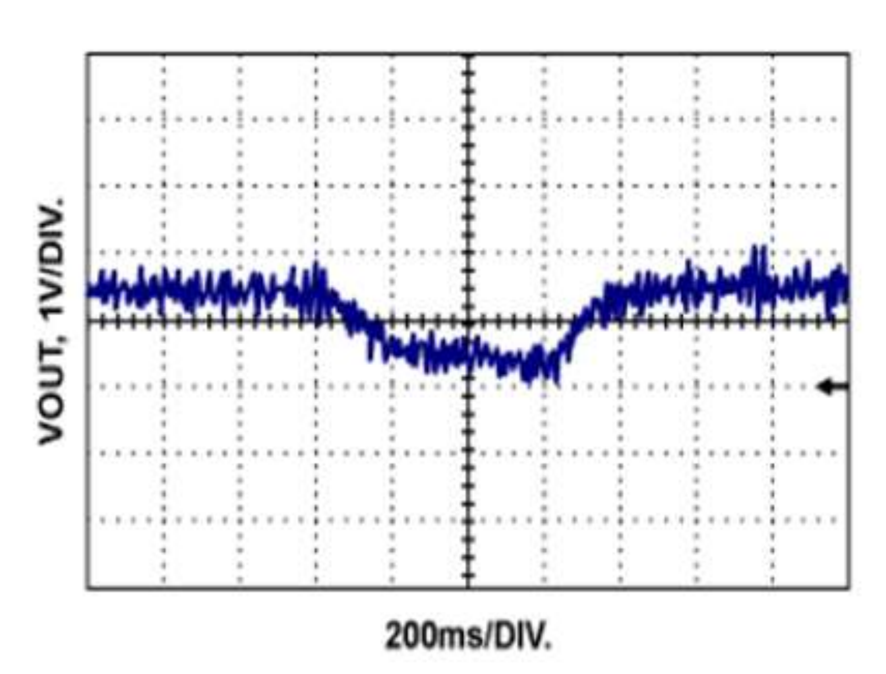
2. **Noise Rejection and Filtering**: The circuit's design to reject resistor-generated thermal noise through the differential stage is commendable. This feature, along with its function as a first-order high-pass filter, significantly improves the signal-to-noise ratio, which is crucial in high-sensitivity applications. However, the assertion that thermal noise is proportional to resistance and is, therefore, attenuated in this setup, might oversimplify the complexities of thermal noise behavior in practical scenarios.

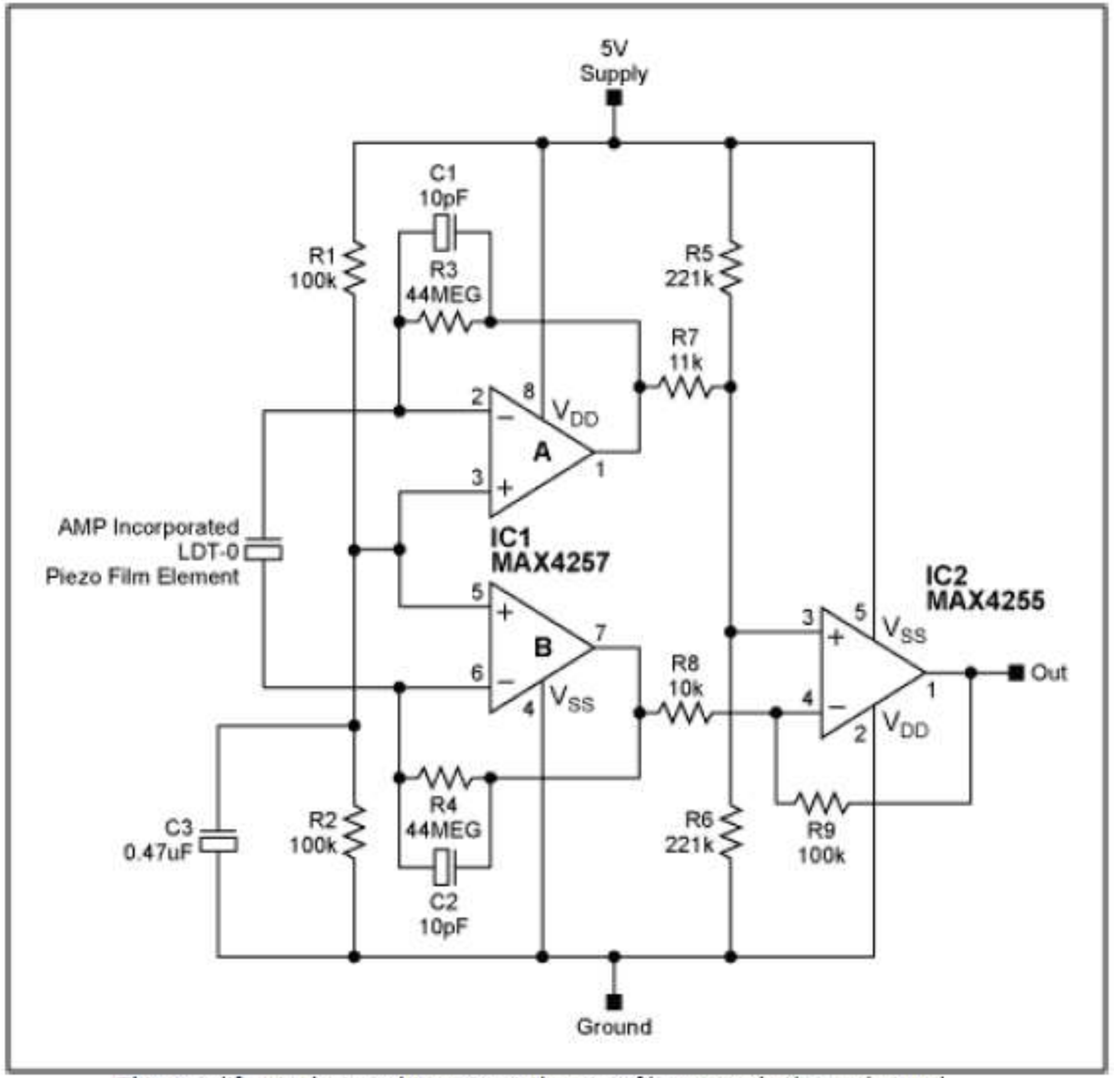
3. **Component Choices and Limitations**: The discussion on the choice of component values and the suggestion of typical values provides valuable insights for customization and optimization. The acknowledgment of circuit limitations and the proposition of alternate configurations indicate a thorough understanding of practical implementation challenges.

4. **Sensor Modeling and Gain Analysis**: Modeling the piezofilm sensor as a capacitor in series with a voltage source is an appropriate simplification for circuit analysis. The detailed examination of AC gain, considering the sensor's equivalent capacitance and series resistance, showcases a deep understanding of piezoelectric sensor characteristics. The choice of a CMOS dual op-amp for its low input leakage is a thoughtful decision, especially considering the impact on offset voltage in high-resistance scenarios.

5. **Differential-to-Single-Ended Conversion and Common-Mode Rejection**: The approach for differential-to-single-ended conversion using IC2 and associated resistors is a standard technique. However, the critique about the degradation of common-mode rejection due to the incorporation of gain in this stage is valid and warrants consideration in design optimization.

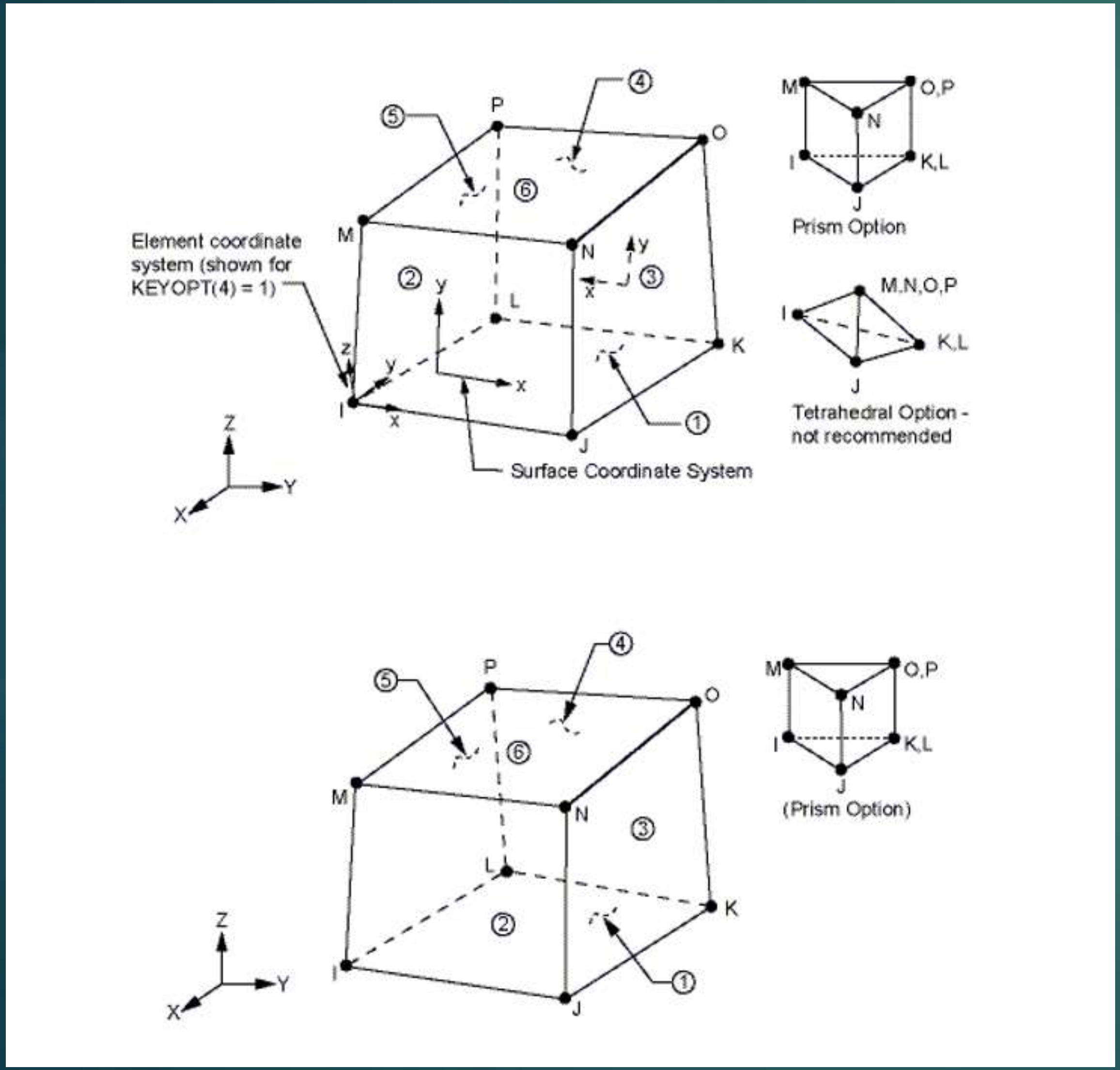
6. **Practical Application and Scope Display**: The demonstration of the circuit's pyro-electrical capability using a heated soldering iron incident is an excellent practical application scenario. It highlights the circuit's sensitivity and





This amplifier is designed to extract the piezofilm sensor's thermal signal.

Finite Element Modelling



1. **Coupled Field Analysis in Piezoelectrics**:
   - Piezoelectric Analysis: This involves solving the coupled structural and electric fields, represented symbolically as:
     $\text{Coupled Field} = \text{Structural Field} + \text{Electric Field}$
   - Objective: To solve for displacements ($\mathbf{u}$) under an applied voltage distribution (V).
2. **Analysis Methods**:
   - Indirect Method: This involves sequential analyses where the output of one field is used as input for the other, represented as:
     $\text{Field 1 Output} \rightarrow \text{Field 2 Input}$
   - Direct Method: This uses coupled-field elements, integrating both fields in a single analysis, represented as:
     $\text{Coupled Analysis} = \text{Element Matrices(Structural + Electric)}$

3. **Properties of Elements**:
    - **SOLID45**:
        - Usage: 3-D modeling of solid structures.
        - Degrees of Freedom (DoF): 3 per node ($\text{Translations in } x, y, z$).
        - Symbolic Representation:

$$\text{Node DoF} = \{u_x, u_y, u_z\}$$

        - Material Properties: Orthotropic, aligned with element coordinate directions.
    - **SOLID5**:
        - Usage: 3-D coupled field analysis (magnetic, thermal, electric, piezoelectric, structural).
        - Degrees of Freedom: Up to 6 per node.
        - Symbolic Representation:

$$\text{Node DoF} = \{u_x, u_y, u_z, \theta_x, \theta_y, \theta_z\}$$

        - Capabilities: Large deflection, stress-stiffening in structural and piezoelectric analyses.

4. **Piezoelectric Analysis Components**:
    - Permittivity (Dielectric Constants) $[\varepsilon^S]$:
        - Usage: Defines permittivity in piezoelectric materials.
        - ANSYS Command: MP command.
        - Symbolic Representation:

$$[\varepsilon^S] = \begin{bmatrix} PERX & 0 & 0 \\ 0 & PERY & 0 \\ 0 & 0 & PERZ \end{bmatrix}$$

    - Piezoelectric Matrix $[e]$:
        - Relation: Links electric field to stress.
        - ANSYS Commands: TB, PIEZ, TBDATA.
        - Symbolic Representation:

$$[e] = \begin{bmatrix} e_{31} & e_{32} & e_{33} \\ e_{24} & e_{25} & e_{26} \\ e_{15} & e_{14} & e_{16} \end{bmatrix}$$

    - Elastic Coefficient Matrix $[c]$:
        - Usage: Specifies stiffness coefficients.
        - ANSYS Commands: TB, ANEL, TBDATA.
        - Symbolic Representation:

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix}$$

Element Type: Solid45 and Solid5

Solid45

- **Description**: Solid45 is typically used in ANSYS for 3D modeling of solid structures. It supports plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. This element has eight nodes with six degrees of freedom at each node: translations in the x, y, and z directions.
- **Usage Critique**: Its application in modeling a beam suggests that the structure might be subject to complex loading conditions, where the full 3D stress state is relevant. However, for simpler beam problems, this might be an overkill, leading to increased computational cost.

Solid5

- **Description**: Solid5 is often used for piezoelectric analysis. It's a 3D, 20-node solid element, capable of exhibiting piezoelectric and dielectric behavior.
- **Usage**: Ideal for meshing the piezoelectric actuator, as it can accurately capture the electromechanical coupling effects.

Material Properties

For Aluminium (Beam Material)

- **Density (ρ) and Elastic Modulus (E)**: These are fundamental for determining the dynamic characteristics of the beam. They play a crucial role in how the beam will resonate and how it will interact with the actuator.

For Piezoelectric Actuator

- **Piezoelectric Matrix (d), Permitivity (ε), and Density (ρ)**: The piezoelectric matrix is critical for modeling the electromechanical coupling, essential for the actuator's performance. Permitivity is key in defining the dielectric behavior under an electric field.

Modeling and Meshing

Beam Dimensions

- **Importance**: Defines the structural geometry, which is crucial for analyzing stress distribution and vibration modes.

Meshing Procedure

- **Element Edge Length**: Critical for ensuring accurate stress and strain distribution. The choice to overlap actuator and beam nodes is significant for ensuring effective energy transfer and mechanical coupling.

Merging & Coupling

Node Merging

- **Process**: Ensures continuity between the beam and the actuator. This step is vital for an accurate representation of the interface where mechanical and electrical energies interact.

Coupling of Actuator Layers

- **Objective**: This is done to apply voltage across the piezoelectric actuator. It is crucial for the actuator's performance as it dictates how the electrical input is converted into mechanical energy.

Symbolic Representations and Mathematical Models

For Beam Analysis (Using Euler-Bernoulli Beam Theory)

- Beam Deflection: $y(x) = \frac{Fx^2}{6EI}(3L - x)$ for a simply supported beam with a point load F at distance x, L is the length, E is the Elastic modulus, and I is the moment of inertia.

For Piezoelectric Actuator

- Electromechanical Coupling: $D = \varepsilon E + d \cdot T$ and $S = sE + d^T \cdot E$, where D is electric displacement, E is electric field, T is stress, S is strain, s is the compliance matrix, and d is the piezoelectric matrix.

Detailed Analysis

- **Finite Element Method (FEM):** It's crucial to use FEM for solving the coupled electromechanical problem. The discretization must be fine enough to capture the dynamics accurately, especially at the interface.
- **Dynamic Analysis**: It involves computing the natural frequencies and mode shapes, which are vital for understanding the vibration characteristics of the integrated beam-actuator system.

4.1 Structural Modeling

1. **Element Selection and Material Properties Definition**:
   - Cantilever Beam and Piezoelectric Patch Modeling:
     - SOLID45 for metal.
     - SOLID5 for piezoelectric material.
     - Mass21 for tip mass.
   - Material Properties:
     - Metal: Density (`ρ_metal`) and Elastic Modulus (`E_metal`).
     - Piezoelectric Material: Density (`ρ_piezo`), Permittivity in three directions (`ε_x, ε_y, ε_z`), Piezoelectric Matrix (`[e]`), and Elastic Coefficient Matrix (`[c]`).

```
mp,ex,1,62e9        // Elastic modulus for metal
mp,dens,1,2676.15   // Density for metal
mp,nuxy,1,0.32      // Poisson's ratio for metal
mp,dens,2,7350      // Density for piezoelectric material
mp,perx,2,15.03e-9 // Permittivity in x-direction for piezoelectric
```

2. **Meshing and Boundary Conditions**:
   - Sketching using BLOCK command.
   - Meshing with specific step sizes.
   - Voltage coupling (`VOLT`) at piezoelectric patch nodes.
   - Clamped side displacement set to zero.

4.2 Closed Loop PID Control in ANSYS

1. **Control System Setup**:
   - Implemented using Ansys Parametric Design Language (APDL).
   - Modal analysis for time step size (Δt) determination:

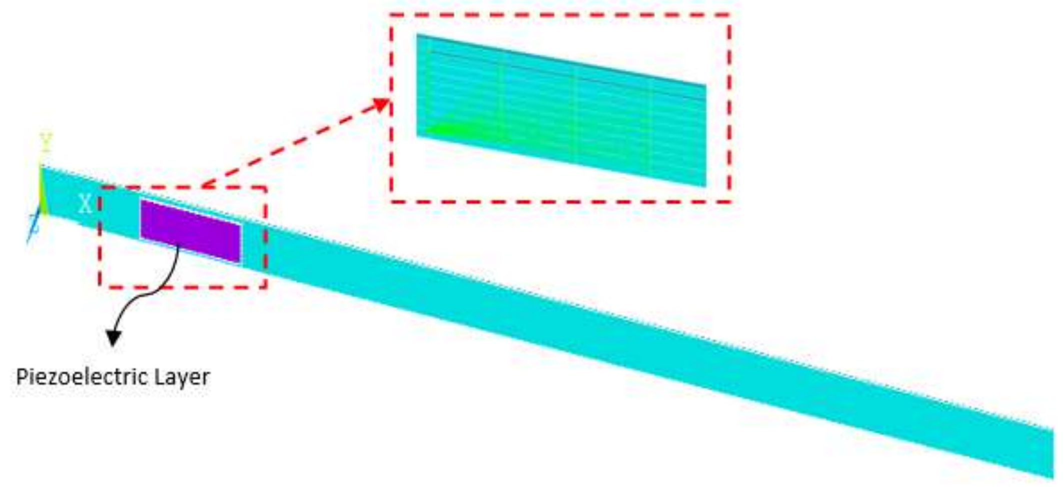
     $$\Delta t = \frac{1}{20 \times f_{\max}}$$

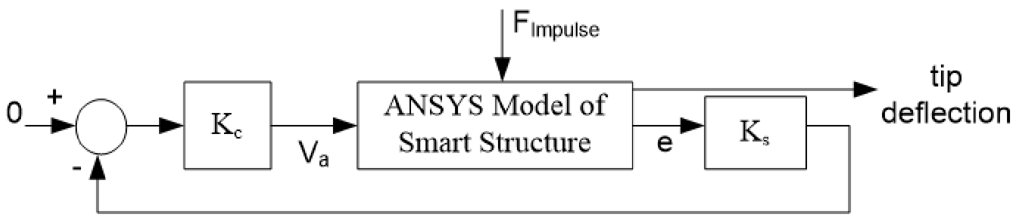
     Where `f_max` is the highest natural frequency of interest.
2. **Damping Modeling**:
   - Rayleigh Damping Coefficients (`α` and `β`):
     - `α` = Mass matrix multiplier.
     - `β` = Stiffness matrix multiplier.
   - Damping calculation:

     $$\alpha = 0, \quad \beta = \frac{\xi_i}{\pi \times f_i}$$

     Where `ξ_i` is the damping ratio and `f_i` is the natural frequency of interest.

3. **Control Algorithm**:
   - An impulse force is applied to the beam tip for a duration of Δt seconds.
   - Control action based on beam tip deflection.
   - Laser transformation coefficient (`Ks`) used to convert displacement (`m`) to voltage (`V`).


Piezoelectric Layer


F Impulse
0
+
-
K c
V a
ANSYS Model of Smart Structure
e
K s
tip deflection

Electromechanical Constitutive Equations for Linear Material Behavior:

1. **Constitutive Equations**:
   - Stress Vector ($\mathbf{T}$)
   - Electric Flux Density Vector ($\mathbf{D}$)
   - Strain Vector ($\mathbf{S}$)
   - Electric Field Vector ($\mathbf{E}$)
   - Elasticity Matrix ($[c]$)
   - Piezoelectric Stress Matrix ($[e]$)
   - Dielectric Matrix ($[\varepsilon]$)
2. **Relation Between Vectors and Matrices**:

$$\mathbf{T} = [c]\mathbf{S} + [e]^T\mathbf{E}$$
$$\mathbf{D} = [e]\mathbf{S} + [\varepsilon]\mathbf{E}$$

The Elasticity (Compliance) Matrix:

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ & & c_{33} & c_{34} & c_{35} & c_{36} \\ & \text{Symmetric} & & c_{44} & c_{45} & c_{46} \\ & & & & c_{55} & c_{56} \\ & & & & & c_{66} \end{bmatrix}$$

The Piezoelectric (Coupling) Matrix:

$$[e] = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{41} & e_{42} & e_{43} \\ e_{51} & e_{52} & e_{53} \\ e_{61} & e_{62} & e_{63} \end{bmatrix}$$

The Dielectric (Permittivity) Matrix:

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

Finite Element Discretization:

1. **Nodal Solution Variables and Element Shape Functions**:
   - Displacements in Element Domain ($\mathbf{u}_c$) in x, y, z directions.
   - Electric Potential in Element Domain (V_c).
   - Matrix of Displacement Shape Functions ($[N_u]$).
   - Vector of Electrical Potential Shape Functions (N_V).
   - Vector of Nodal Displacements ($\mathbf{u}$).
   - Vector of Nodal Electrical Potential ($\mathbf{V}$).
2. **Expansion of Definitions**:
   - N_i: Shape function for node i.
   - n: Number of nodes of the element.
3. **Strain and Electric Field Relations**:
   - $\mathbf{S} = [B]\mathbf{u}$, where $[B]$ is the strain-displacement matrix.
   - $\mathbf{E} = -[B_e]\mathbf{V}$, where $[B_e]$ is the electric field-potential matrix.

The finite element discretization is performed by establishing nodal solution variables and element shape functions over an element domain which approximate the solution.

$\{u_C\} = [N^U]^T\{u\}$

$\{V_C\} = \{N^V\}^T\{V\}$

Expanding these definitions:

$$[N^u]^T = \begin{bmatrix} N_1 & 0 & 0 & & N_n & 0 & 0 \\ 0 & N_1 & 0 & \ldots & 0 & N_n & 0 \\ 0 & 0 & N_1 & \ldots & 0 & 0 & N_n \end{bmatrix}$$

$$\{N^V\}^T = (N_1\ N_2 \ldots N_n)$$

Where N_i is the shape function for node i

$$\{u\} = \lfloor UX_1\ UY_2\ UZ_3 \ldots UX_n\ UY_n\ UZ_n \rfloor^T$$

$$\{V\} = \begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{Bmatrix}$$

Where n is the number of nodes of the element.

Then the strain {S} and electric field {E} are related to displacements and potentials,

$$\{S\} = [B_u]\{u\}$$

$$\{E\} = -[B_V]\{V\}$$

$$[B_u] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix}$$

$$[B_V] = \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{Bmatrix} \{N^V\}^T$$

After the application of the variational principle and finite element discretization, the coupled finite element matrix equation derived for a one-element model is:

$$\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}\} \\ \{\ddot{V}\} \end{Bmatrix} + \begin{bmatrix} [C] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{u}\} \\ \{\dot{V}\} \end{Bmatrix} + \begin{bmatrix} [K] & [K^z] \\ [K^z]^T & [K^d] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{V\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{L\} \end{Bmatrix}$$

[M] is the mass matrix derived from density and volume, [K] is the mechanical stiffness matrix derived from the piezoelectric matrix, and $[K^d]$ is the dielectric stiffness matrix derived from the dielectric matrix. The variables F and L are the mechanical force vector and charge vector.

Coupled Finite Element Matrix Equation:

1. **One-Element Model**:
   - Mass Matrix ($[M]$): Derived from density and volume.
   - Mechanical Stiffness Matrix ($[K]$): Derived from the piezoelectric matrix.
   - Dielectric Stiffness Matrix ($[K_d]$): Derived from the dielectric matrix.
   - Mechanical Force Vector ($\mathbf{F}$) and Charge Vector ($\mathbf{L}$).
2. **Matrix Representation**:
   - Structural Damping Matrix ($[C]$): $[C] = \alpha[M] + \beta[K]$, where α and β are Rayleigh damping coefficients.
3. **Energy Formulations**:
   - Elastic Energy, Dielectric Energy, Electromechanical Coupling Energy.
   - Potential Energy Output.

Elastic Energy:

$$U_E = \frac{1}{2}\{S\}^T[c]\{S\}$$

Dielectric Energy:

$$U_D = \frac{1}{2}\{E\}^T[\varepsilon]\{E\}$$

Electromechanical Coupling Energy:

$$U_M = -\frac{1}{2}\{S\}^T[e]\{E\}$$

Potential Energy Output:

$$E^{po} = U_E + U_D$$

Structural Mass Matrix:

$$[M] = \int_{vol} \rho [N^u][N^u]^T d(vol)$$

Structural Stiffness Matrix:

$$[K] = \int_{vol} [B^u]^T [c][B_u] d(vol)$$

Dielectric Conductivity Matrix:

$$[K^d] = -\int_{vol} [B_V]^T [\varepsilon][B_V] d(vol)$$

Piezoelectric Coupling Matrix:

$$[K^z] = \int_{vol} [B_u]^T [e][B_V] d(vol)$$

Structural damping matrix [C] can be defined as a linear combination of mass and mechanical stiffness matrices:

$$[C] = \alpha[M] + \beta[K]$$

Where α and β are the Rayleigh damping coefficients.

Structural Modelling Scheme in ANSYS

Step	Action	Description/Details
1	Element Selection	Choose SOLID5, SOLID45 for the beam and piezoelectric patch, and Mass21 for the tip mass.
2	Material Properties Definition	Define properties for metal (density, elasticity) and for piezoelectric material (density, permittivity, piezoelectric matrix, elastic coefficient matrix).
3	Sketching Geometry	Use BLOCK command in ANSYS for defining the geometry of the beam and patch.
4	Meshing	Determine meshing step sizes, ensuring appropriate overlap for piezoelectric actuator and beam nodes.
5	Boundary Conditions and Coupling	Apply voltage coupling at piezoelectric patch nodes and set displacement constraints for the clamped side.
6	Modal Analysis for Time Step Size	Perform modal analysis to determine the time step size (Δt) for dynamic simulations.
7	Damping Coefficients	Define Rayleigh Damping Coefficients (α and β) for the system.
8	Apply External Forces	Model the impulse force at the beam tip (if applicable) and integrate it within the simulation timeline.
9	PID Control Implementation	Implement PID control using Ansys Parametric Design Language (APDL), adjusting the control parameters based on beam deflection.
10	Simulation and Analysis	Run the simulation in ANSYS and analyze the results for the cantilever beam's response with the piezoelectric actuator.

Piezoelectic Beam Applications in ANSYS

The problem presented here involves a piezoelectric bimorph beam, a structure commonly used in sensors and actuators. The bimorph is composed of two piezoelectric layers with opposite polarities, enabling it to act both as an actuator and a sensor. The material used is Polyvinylidene Fluoride (PVDF), chosen for its piezoelectric properties.

Actuation Mode Analysis:
In the actuator mode, the application of an electric field (100 Volts in this case) causes one layer to contract and the other to expand, resulting in the bending of the beam. The theoretical and computational methods are consistent, showing a deflection of approximately -32.9 μm. This is a clear demonstration of the direct piezoelectric effect where electric fields induce mechanical strain.

Sensing Mode Analysis:
Conversely, in the sensor mode, the beam experiences a physical deformation (10 mm tip deflection), and the resulting electrical response (electrode voltages) is measured. This is an application of the converse piezoelectric effect. The results from the simulation are in line with the expected outcome, demonstrating the material's ability to convert mechanical stress into an electrical signal.

Material Properties and Geometric Considerations:
The problem includes detailed specifications of material properties like Young's modulus, Poisson's ratio, shear modulus, piezoelectric strain coefficients, and relative permittivity. These properties are crucial for accurately modeling the piezoelectric behavior. The geometric specifications of the beam are equally important for an accurate analysis.

Critique and Discussion:

1. **Material Choice:** PVDF is a well-chosen material for this application due to its high piezoelectric coefficients and flexibility. However, other materials like PZT could also be considered, depending on the application's specific requirements such as temperature stability or higher piezoelectric constants.
2. **Linear vs. Nonlinear Analysis:** The use of linear static analysis for the actuator mode is appropriate for small deformations. However, for larger deformations or more complex loading conditions, a nonlinear analysis might be more accurate.
3. **Sensitivity Analysis:** In the sensor mode, the large deflection static analysis is suitable. However, it would be beneficial to conduct a sensitivity analysis to understand how variations in material properties or geometrical dimensions affect the sensor's output.
4. **Finite Element Method (FEM) Application:** The problem statement details a comprehensive FEM approach, but it's important to consider the mesh density and element type for accuracy. In real-world applications, the mesh must be refined near areas of high stress gradients to capture the behavior accurately.
5. **Experimental Validation:** While the problem provides theoretical and FEM-based results, experimental validation is key in such simulations. It's important to note any discrepancies between theoretical, simulated, and experimental results to refine the model and understand the material behavior better.
6. **Practical Applications:** The discussion could be enriched by elaborating on practical applications of such bimorph beams, such as in precision positioning systems, vibration control, and sensing applications in various industries.

Problem Overview:

- **Structure**: A quartz tuning fork, modeled using SOLID226 elements, consists of two tines connected to a base. It is excited into in-plane vibration by an alternating voltage.
- **Coriolis Effect**: When rotated around the Y-axis, the Coriolis effect induces a torque proportional to the angular velocity Ω. This torque is converted to an electric signal to sense rotational velocity.

Modal Analysis and Harmonic Analysis:

- **Modal Analysis**: A QR-damped modal analysis is used to assess the shift in eigenfrequencies due to the Coriolis effect and spin-softening.
- **Harmonic Analysis**: This analysis demonstrates the impact of the Coriolis force near the 4th resonance.

Problem Specifications:

- **Geometry and Material**: Quartz properties, dimensions of the tuning fork, and operational parameters (angular velocity and frequency) are specified.
- **Elastic Coefficients, Piezoelectric Coefficients**: These are crucial for accurately modeling the quartz tuning fork's behavior.

Critique and Recommendations:

- **Angular Velocity Exaggeration**: The significantly exaggerated angular velocity (1e4 rad/s) to showcase the Coriolis effect might be unrealistic for practical gyroscopes. It would be beneficial to also present results at more typical operational speeds for a better real-world applicability assessment.
- **Experimental Validation**: As with any simulation, experimental validation is key. Comparing these results with experimental data would enhance the credibility and applicability of the findings.
- **Further Applications**: Discussing potential applications of this analysis in various types of gyroscopes and other rotational sensors would provide a broader perspective on its practical significance.

Results:

- **Eigenfrequencies**: The table shows how eigenfrequencies are affected by inertia, Coriolis effect, and spin-softening.
- **Mode Shapes**: The analysis also expands on complex mode shapes in response to these effects.

Technical Discussion:

1. **Importance of the Coriolis Effect**: The Coriolis effect is critical in gyroscopic sensors. This analysis effectively demonstrates how rotational motion can be converted into measurable electrical signals through piezoelectric materials.
2. **Material Modeling**: The choice of quartz, with its well-documented piezoelectric and elastic properties, is appropriate for high-precision sensors like gyroscopes.
3. **Geometric Precision**: The detailed geometric modeling of the tuning fork, including dimensions and element type, is vital for accurate simulation results, particularly in sensitive applications like angular velocity sensing.
4. **Spin-Softening Consideration**: The inclusion of spin-softening effects is a nuanced aspect of the analysis, indicating a comprehensive approach to the dynamic behavior of rotating systems.
5. **Harmonic Analysis**: The harmonic analysis near the 4th resonance offers insights into the vibrational behavior under operational conditions, which is crucial for sensor design.

Explanation of the Methodology: Active Vibration Control

1. Piezo Equations to Transfer Function:

- The starting point in AVC using piezoelectric (piezo) materials involves deriving the piezo equations. These equations are crucial as they represent the electromechanical interaction within piezoelectric materials.
- The primary goal is to establish a transfer function from these equations. The transfer function is a mathematical representation that maps the dynamic system's output (in this case, vibration response) to its input (electrical signal applied to the piezo materials).
- This process involves complex mathematical modeling, typically integrating principles from electromechanics, material science, and dynamics.

2. Determination of PID Coefficients in MATLAB:

- Using the transfer function, one can utilize MATLAB, a high-level technical computing language, to determine the Proportional-Integral-Derivative (PID) coefficients.
- These coefficients are fundamental in control theory and are used to design a controller that adjusts the system's response in a desired manner. The PID controller is a critical component in AVC as it dictates how effectively the system can counteract unwanted vibrations.

3. Application in Experimental Setup:

- The calculated PID coefficients are then applied in an experimental setup to achieve vibration damping. This step is crucial for validating the theoretical models and for fine-tuning the control system in real-world conditions.

Numerical Methods and Analysis Techniques

1. Numerical Methods (Finite Element, DQM, Spectral, etc.):

- Finite Element Method (FEM) is widely used for structural analysis, including vibration analysis. It's particularly effective in handling complex geometries and boundary conditions.
- Differential Quadrature Method (DQM) and Spectral methods are also prominent in solving differential equations that arise in vibration problems, offering high accuracy and efficiency in certain scenarios.

2. Transfer Function and Analytical Equations:

- The transfer function and analytical equations form the backbone of modeling in AVC. They provide a theoretical framework for understanding the dynamics of the system.

3. Free Response-Time and FFT Analysis:

- Free response analysis is essential in understanding the natural behavior of the system without external forces.
- Fast Fourier Transform (FFT) is a mathematical tool used to transform time-domain data into frequency-domain data, which is crucial for identifying dominant vibration modes and their frequencies.

Beam+Piezo: Active Vibration Control

1. Free Response, Time, FFT Analysis:

- In a beam integrated with piezo materials, the free response, time-domain analysis, and FFT are conducted to understand the dynamic characteristics of the system.
- These analyses help in identifying the resonant frequencies and the effectiveness of the vibration control strategies.

2. Active Vibration Control for Different Modes:

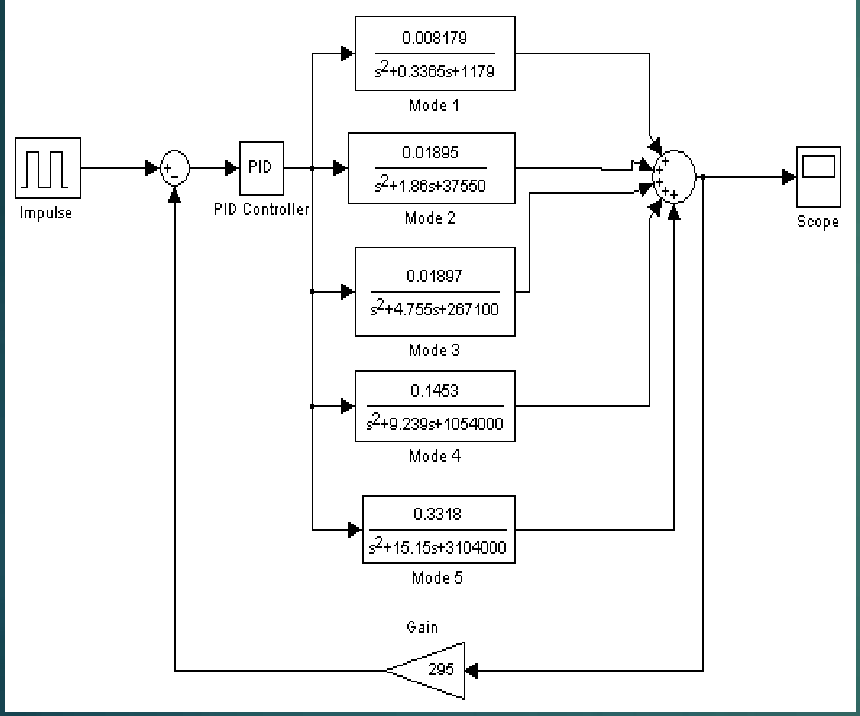
- The AVC is particularly focused on damping out specific vibration modes (1st, 2nd, 3rd, 4th, etc.). The effectiveness of the control system is often evaluated based on its ability to suppress these modes.

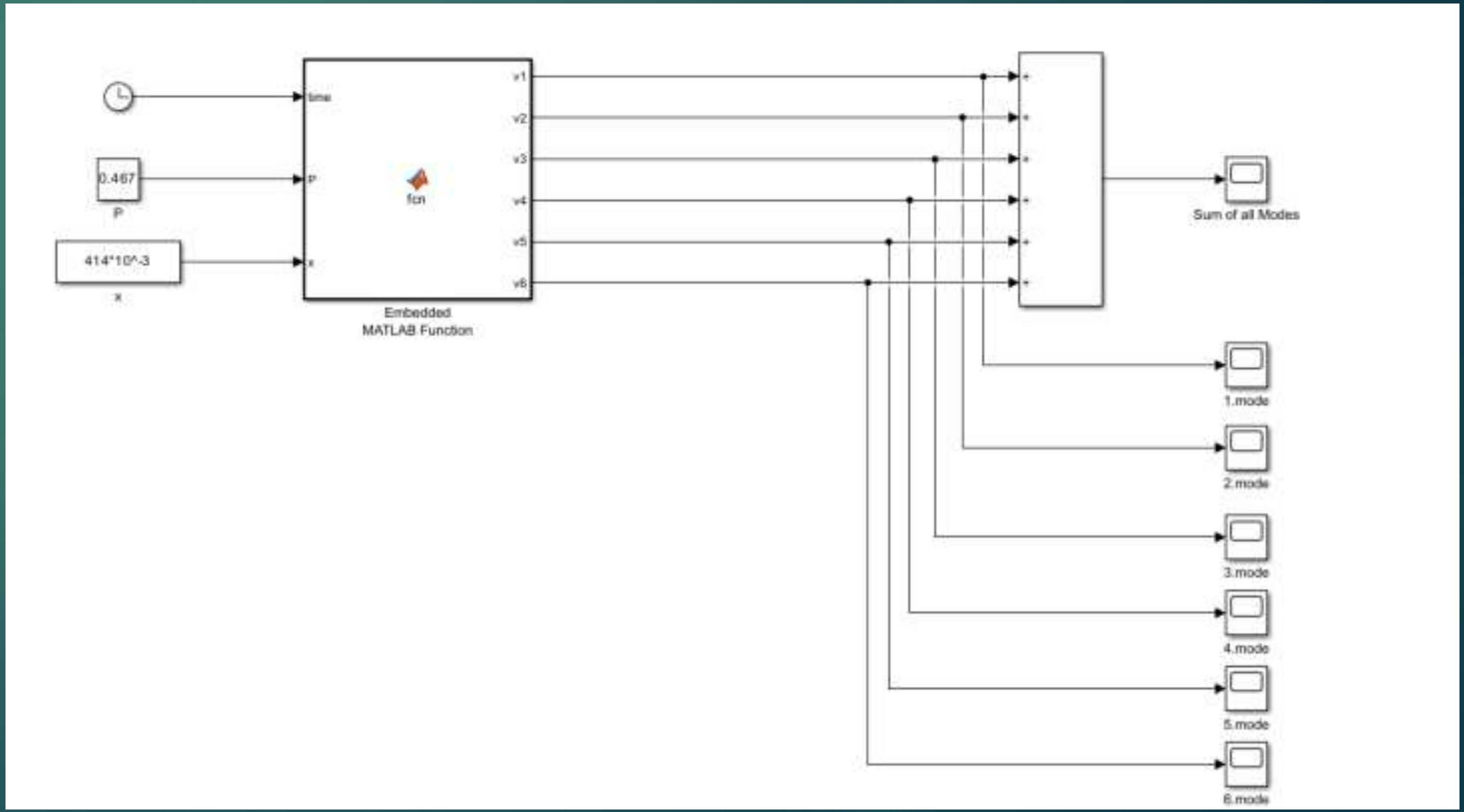
Critique and Discussion

- **Modeling Complexity vs. Real-World Application:** While the theoretical models are robust, the real-world application often presents challenges like non-linearities and environmental variations that are not fully captured in the models.
- **PID Controller Limitations:** While PID controllers are widely used, their effectiveness can be limited in systems with complex or varying dynamics. Advanced control strategies like adaptive control or robust control might be necessary.
- **Numerical Methods Selection:** The choice of numerical methods (FEM, DQM, Spectral) can significantly impact the accuracy and computational efficiency. There's a constant need for balance between model fidelity and computational resources.

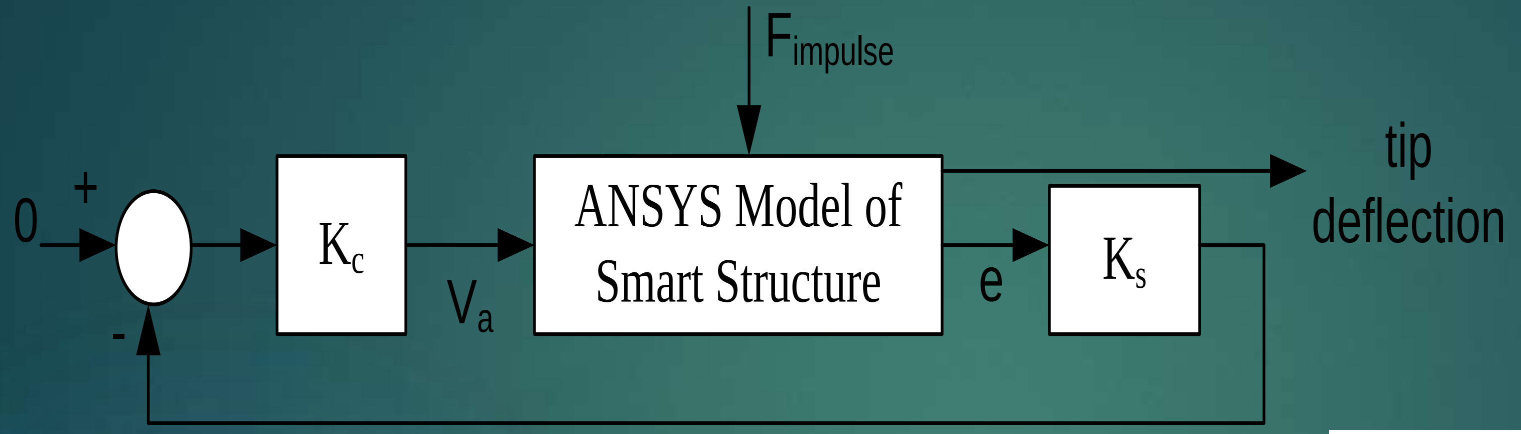
Simulink Diagram of the System with 5 Modes

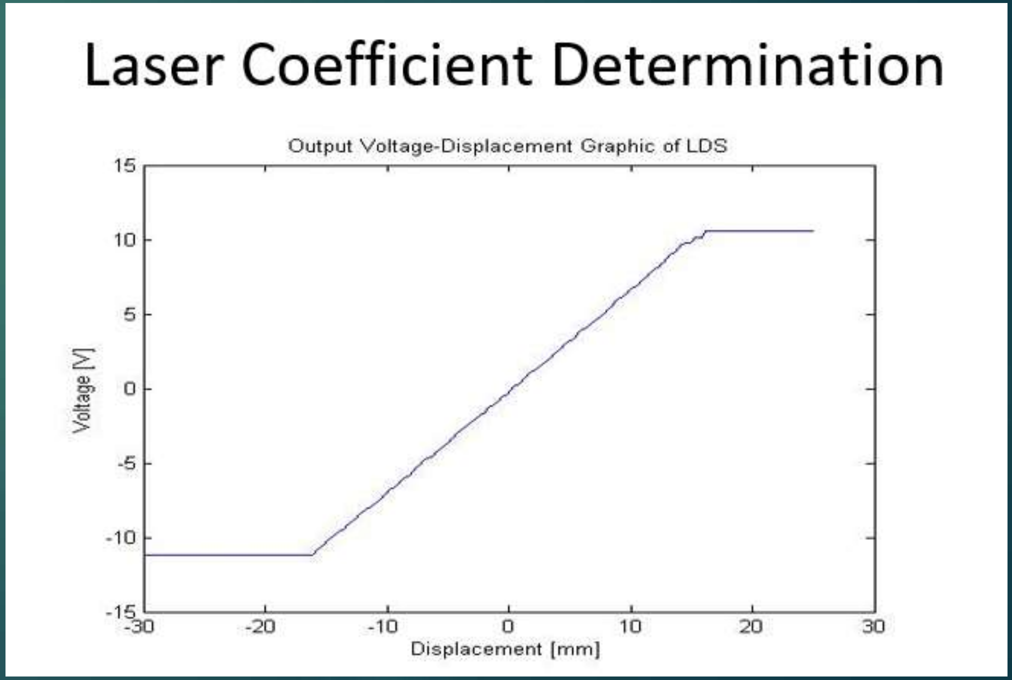
cantileverbeam_controlled.mdl





Closed Loop PID Control in Ansys





Impulse Force Application

- **Process**: An impulse force is applied at the tip of the cantilever beam. This is a common method for inducing vibrations in the beam to study its dynamic response.
- **Observation**: The response of the beam (in terms of displacement, vibration frequency, etc.) is then observed. This data is crucial for understanding the beam's dynamic characteristics.

Laser Displacement Sensor

- **Role**: It measures the displacement in the z-direction (vertical displacement) of the beam tip.
- **Conversion to Voltage**: The displacement measurement is multiplied by a laser displacement sensor coefficient to convert the displacement from millimeters to a voltage value. This conversion is essential for interfacing the mechanical response with electronic systems.

Error Calculation

- **Reference Value**: Assuming a reference value of zero displacement, the error in displacement is calculated. This is a standard practice in control systems to measure the deviation from a desired position or state.

Calibration in ANSYS

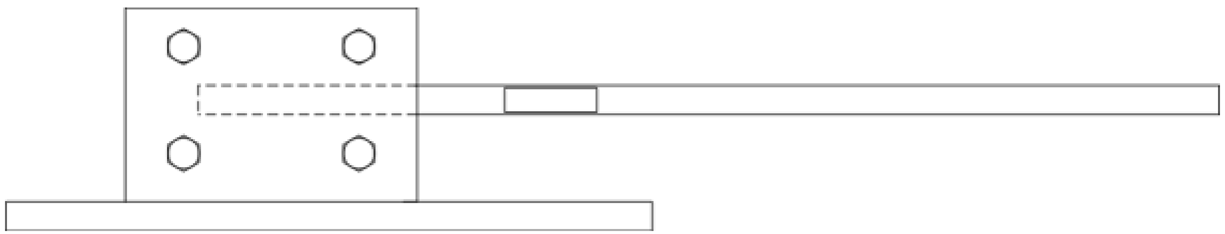
- **Procedure**: In ANSYS, the laser coefficient is determined to accurately translate the z displacement value into a voltage value. This involves a calibration process where the sensor's output is matched against known displacement values.

Recording and Plotting Voltage Values

- **Data Collection**: Voltage values corresponding to every tip displacement from zero to thirty millimeters are recorded.
- **Graph Plotting and Analysis**: These values are then plotted, presumably against the displacement values.
- **Slope Determination**: The slope of this graph, identified as 295 V/mm, is crucial. It represents the sensitivity of the laser displacement sensor, indicating how much voltage change corresponds to a unit change in displacement.

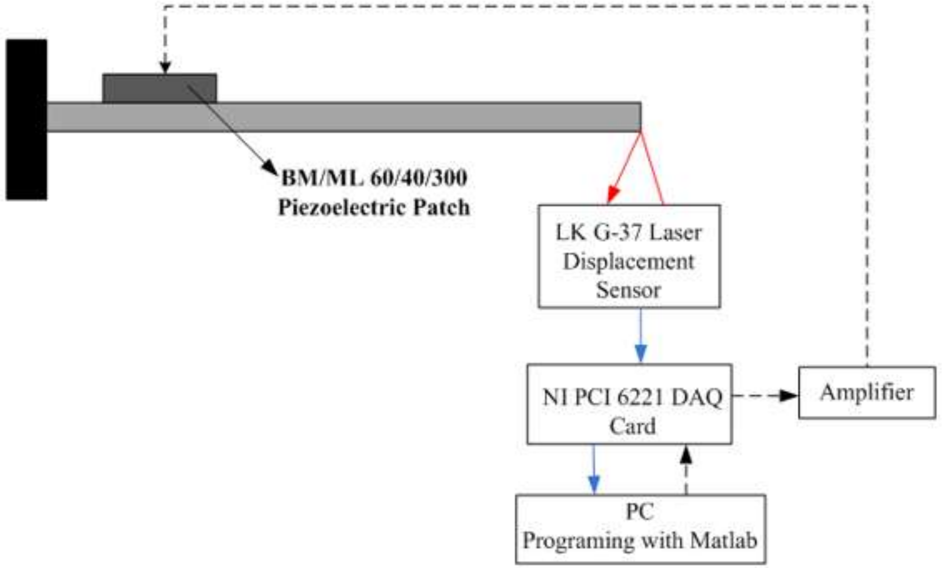
Implications in Structural Analysis

- **Sensitivity Analysis**: The slope of 295 V/mm is a high sensitivity, implying that even small displacements will result in significant voltage changes. This is beneficial for detecting and analyzing minute vibrations or displacements.
- **Feedback Loop in Control Systems**: In a typical control system involving a piezoelectric actuator, this voltage signal can be used as a feedback. By comparing it against a desired (reference) value, it's possible to calculate the necessary adjustments to be made by the actuator to correct any deviations from the desired state.
- **Dynamic Response Analysis**: The method outlined is particularly useful in dynamic response analysis, where understanding the relationship between physical displacement and the resulting electrical signal is key. This data is essential for designing and tuning control systems that rely on piezoelectric actuators for precision movements or vibration damping.



	Length	Width	Thickness
Beam	L= 400 mm	B= 12 mm	H=1 mm
Piezo multilayer bender	Lp= 40 mm	Bp= 12 mm	Hp= 1 mm
Piezo location	La=40 mm		

Experimental Setup



Experimental Setup Components

1. Aluminium Beam

- **Role**: Serves as the primary structure for the experiment. Its mechanical properties, such as elasticity and density, directly affect the vibration characteristics.
- **Objective**: The main goal is to dampen the tip deflection of the beam, a common challenge in structures subjected to dynamic loads.

2. Piezoelectric Multilayer Bender (Actuator)

- **Function**: Acts as the actuator in the system. Utilizing the piezoelectric effect, it converts electrical signals into mechanical energy, enabling active vibration control.
- **Placement**: Typically attached to the beam, where it can effectively influence the beam's vibration characteristics.

3. Laser Displacement Sensor

- **Purpose**: Measures the tip deflection of the aluminum beam. This is crucial for understanding the beam's dynamic response and for providing feedback in a control loop.
- **Accuracy**: The precision of the sensor is essential for ensuring the fidelity of the vibration data and the effectiveness of the subsequent control actions.

4. NI-DAQ Card

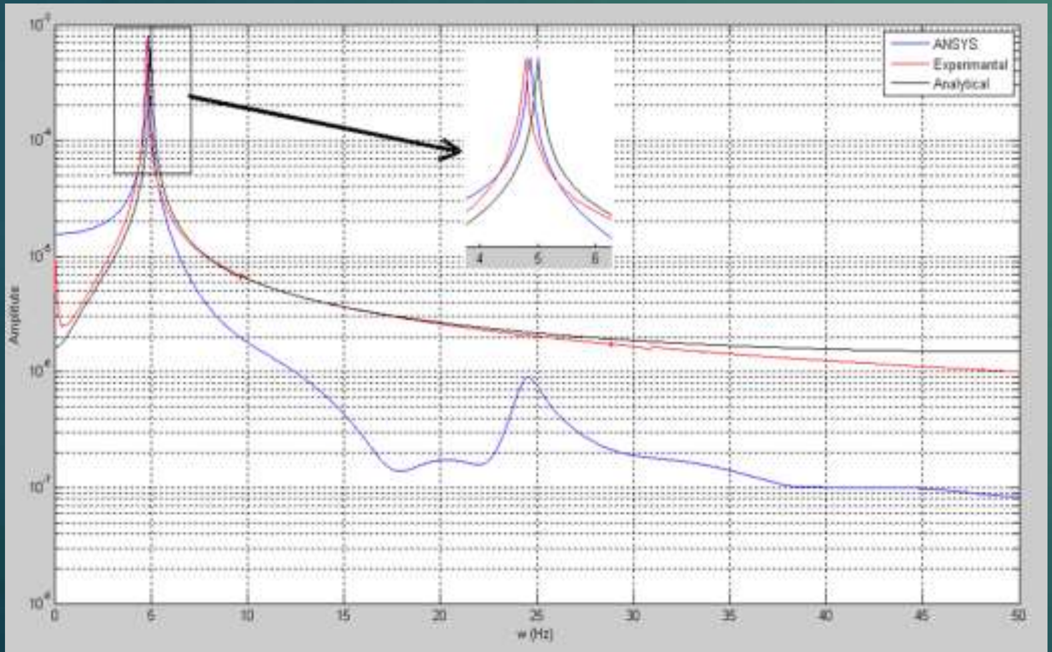
- **Use in Data Acquisition**: This card is used to capture the data from the laser displacement sensor. It serves as an interface between the sensor and the computer.
- **Future Application**: In upcoming studies, it is planned to use the NI-DAQ card for sending actuation signals from the computer to the amplifier, which then drives the piezoelectric actuator.

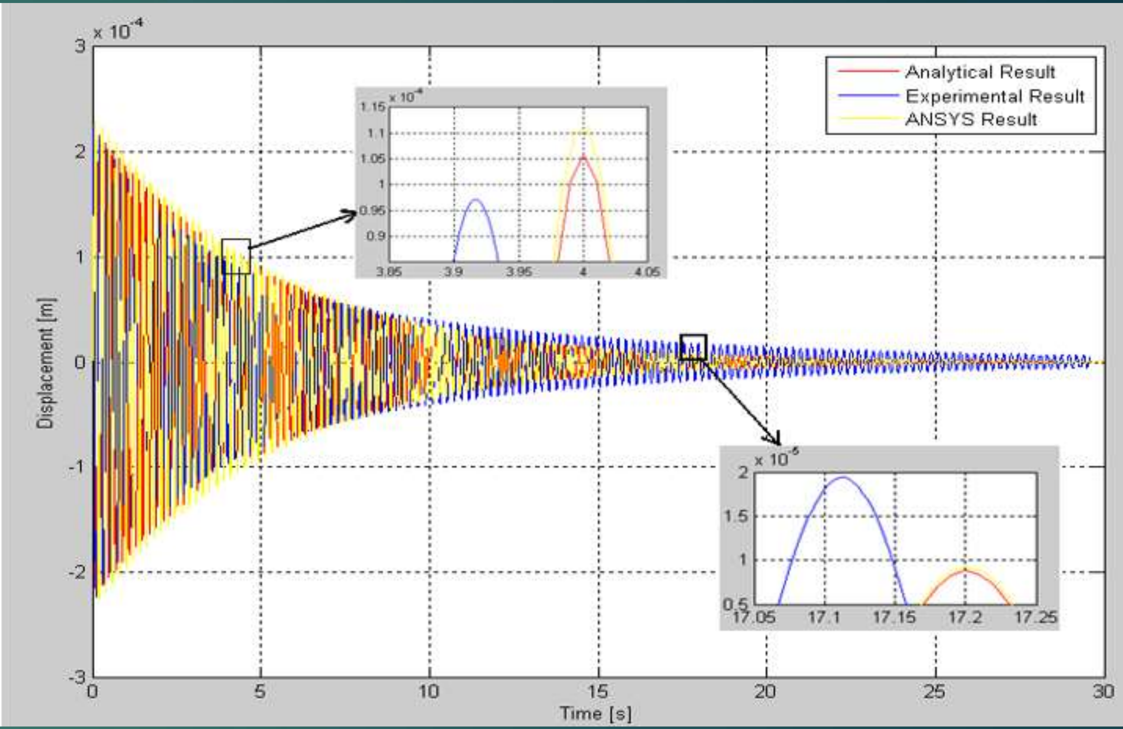
5. Personal Computer with MATLAB

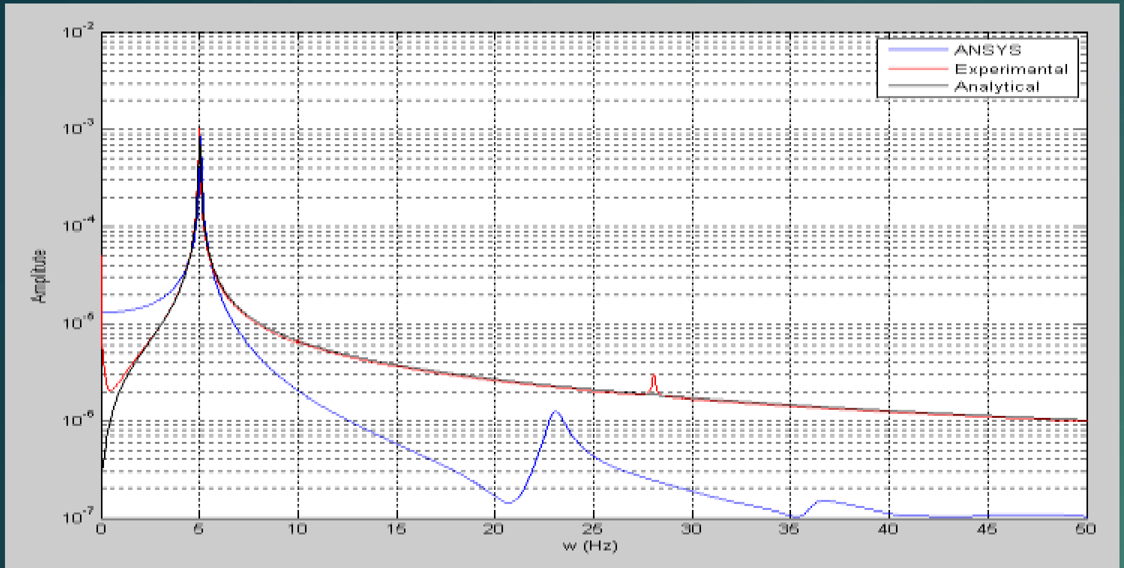
- **Software**: MATLAB is used for data processing, which includes analyzing the sensor data and generating control signals for the actuator.
- **Control Algorithm Development**: MATLAB's powerful computational and graphical tools enable the development and testing of sophisticated control algorithms.

System Integration and Operation

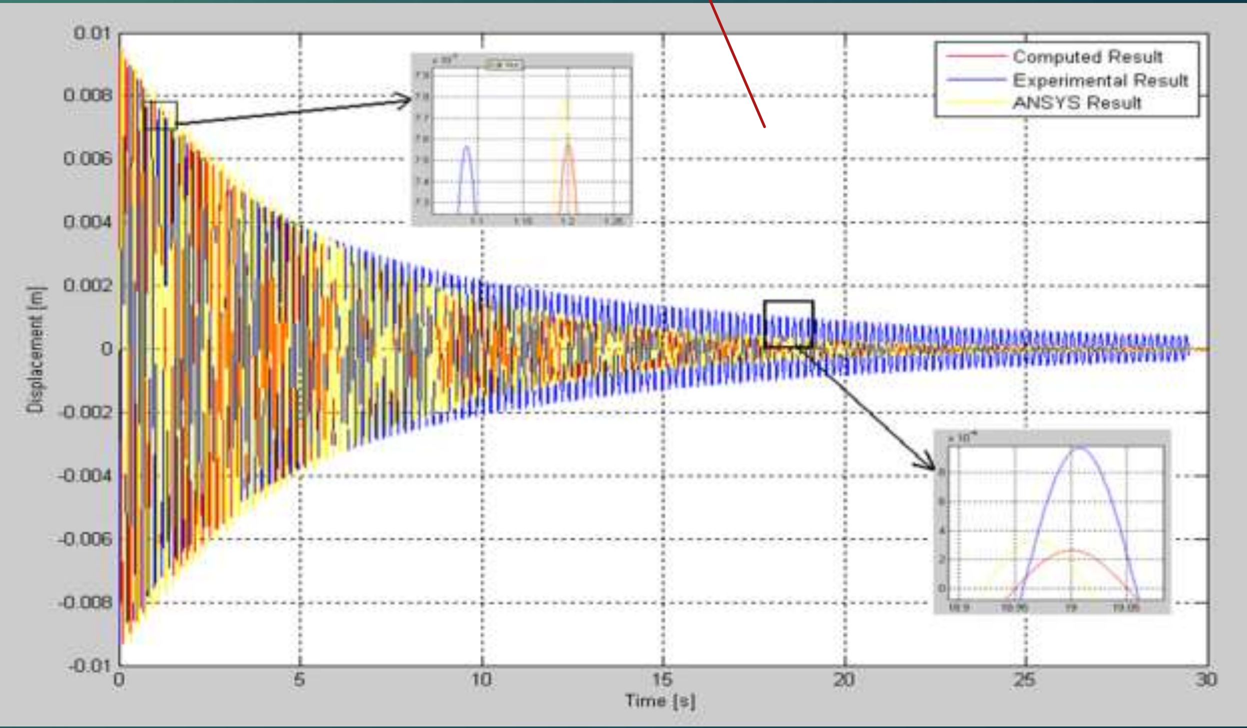
1. **Data Acquisition and Processing**:
   - The laser displacement sensor captures the beam's tip displacement.
   - The NI-DAQ card transmits this data to the PC.
   - MATLAB processes this data, analyzing the beam's dynamic behavior.
2. **Feedback Loop and Control**:
   - The control algorithm in MATLAB calculates the required actuator response based on the displacement data.
   - In future setups, the actuator signal will be sent back to the piezoelectric bender through the NI-DAQ card and an amplifier.
3. **Actuation and Damping**:
   - The piezoelectric actuator, upon receiving the signal, actuates to counteract the beam's vibrations.
   - The aim is to reduce or eliminate unwanted tip deflection, achieving effective vibration damping.

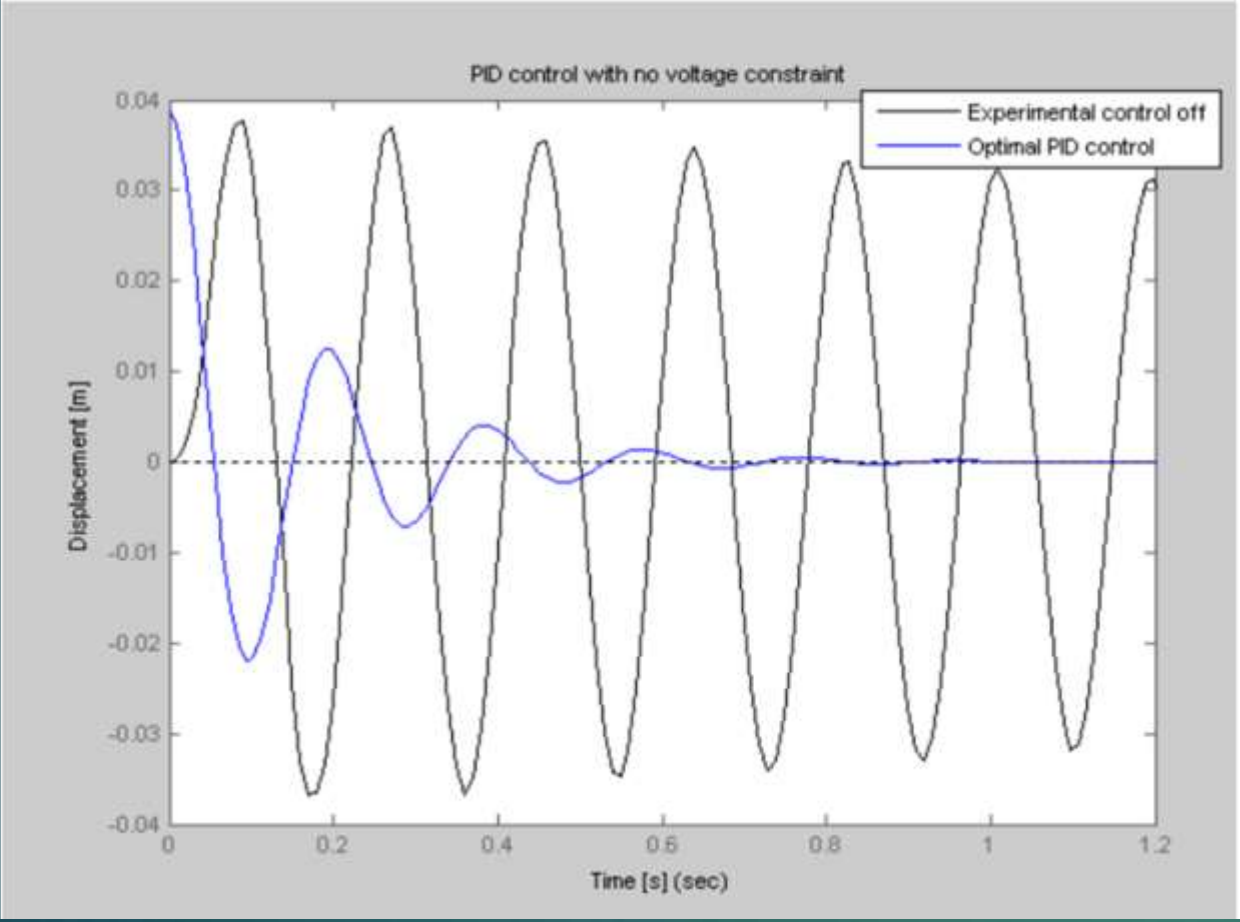


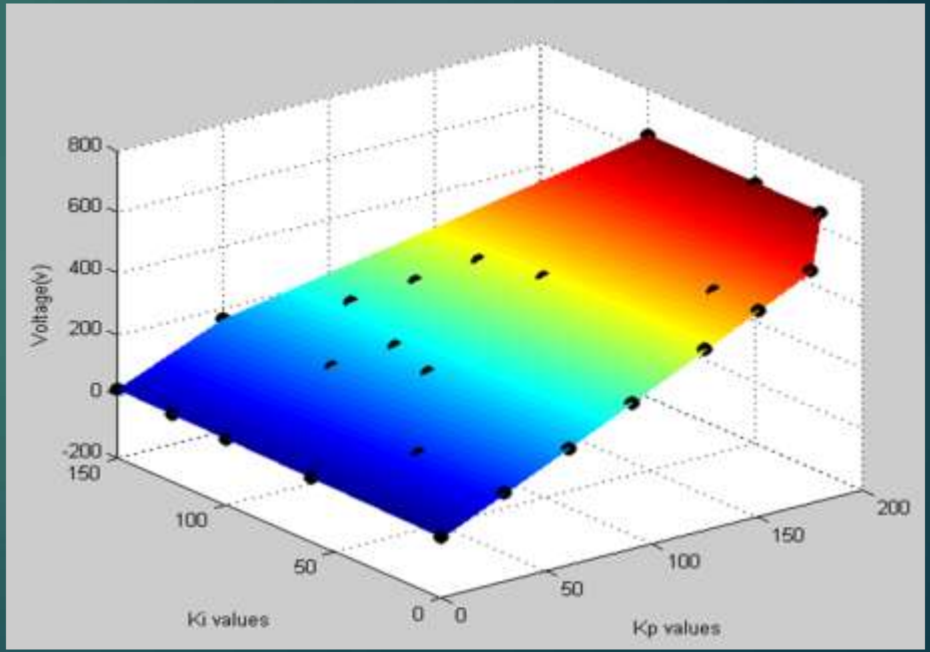


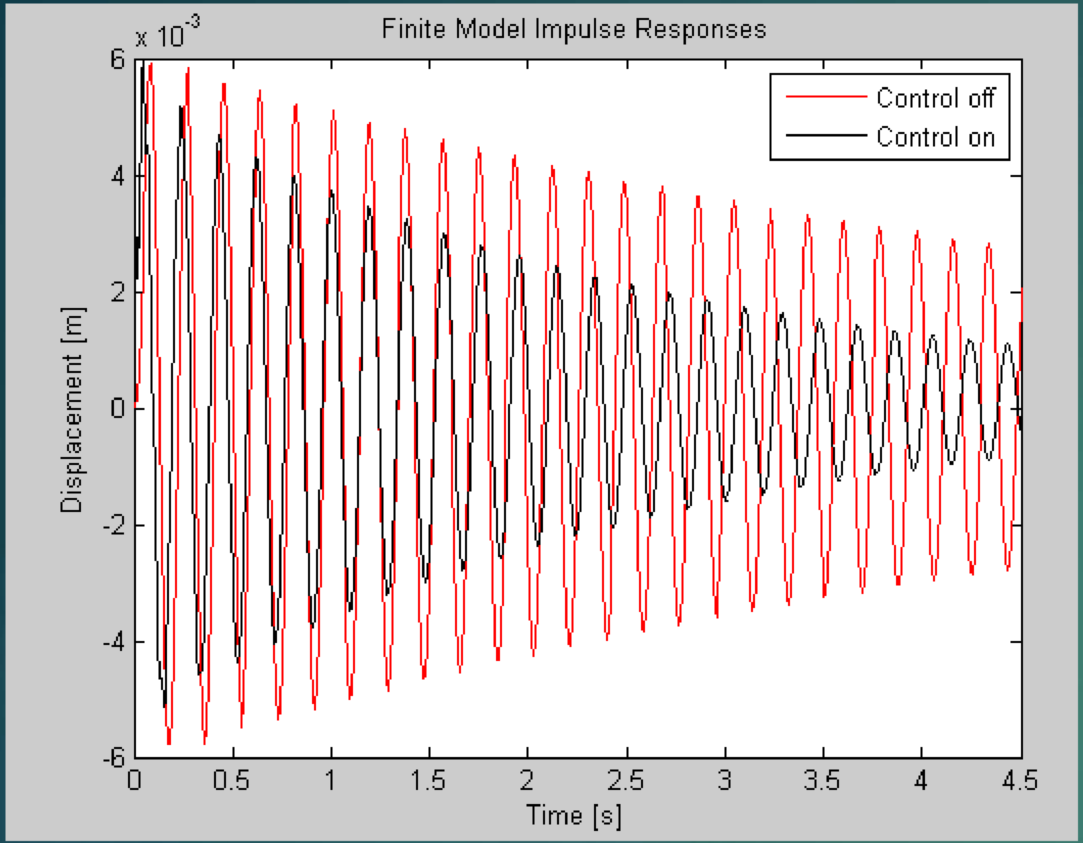


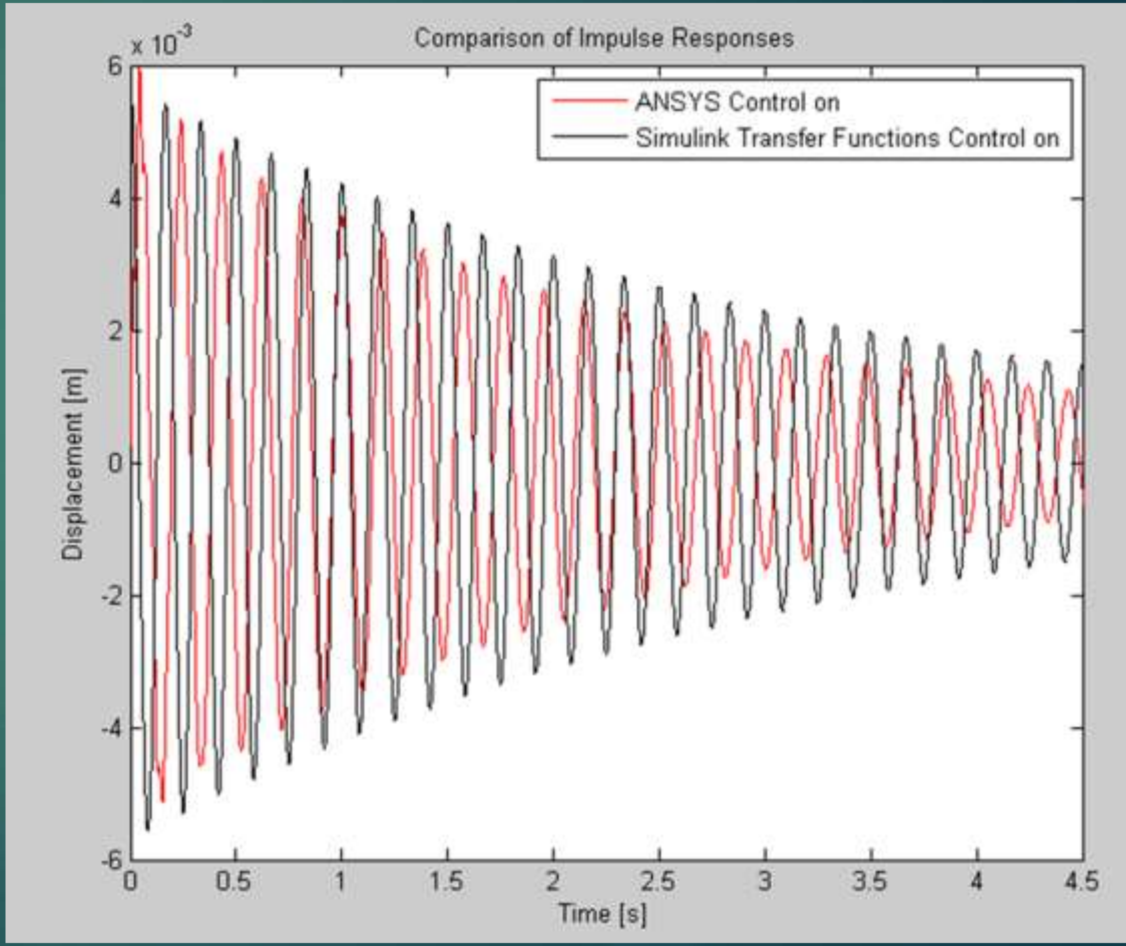
Response with Piezoelectric Actuator



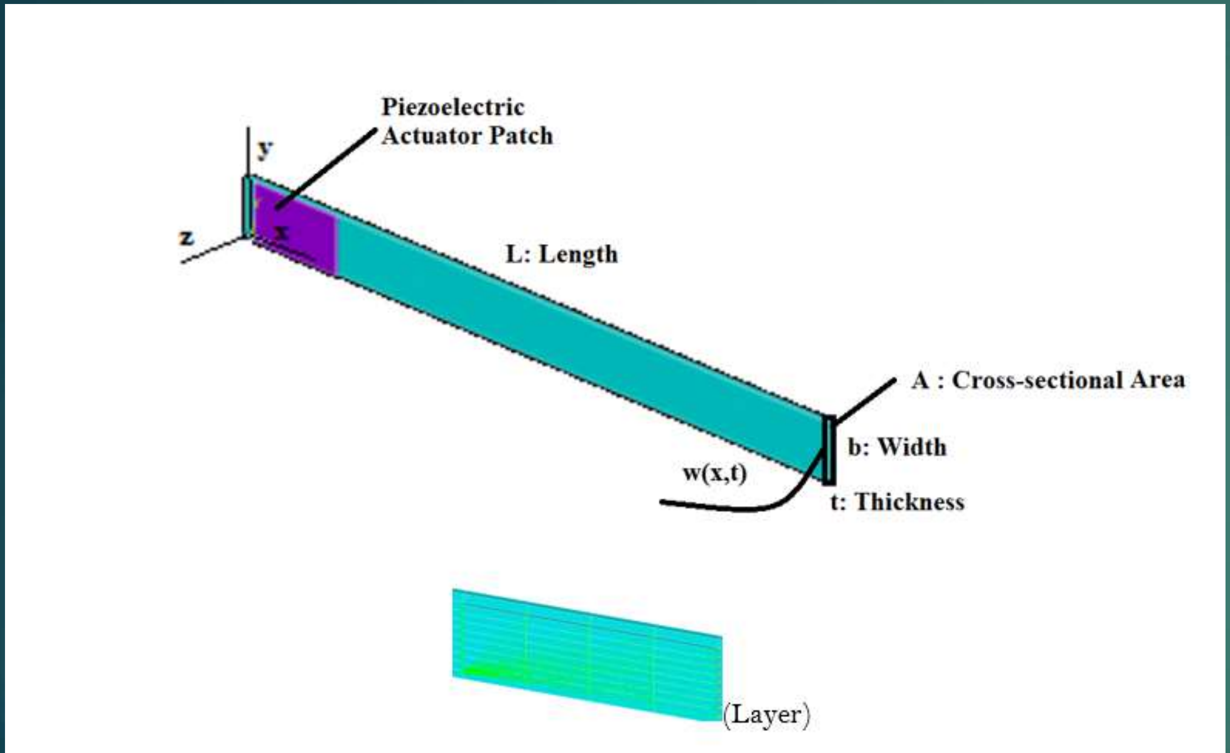

PID control with no voltage constraint
Experimental control off
Optimal PID control
Displacement [m]
Time [s] (sec)
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
0
0.2
0.4
0.6
0.8
1
1.2


Voltage(v)
800
600
400
200
0
-200
Ki values
150
100
50
0
Kp values
0
50
100
150
200


Finite Model Impulse Responses
x 10^-3
Control off
Control on
Displacement [m]
Time [s]
6
4
2
0
-2
-4
-6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5


Comparison of Impulse Responses
x 10^-3
ANSYS Control on
Simulink Transfer Functions Control on
Displacement [m]
Time [s]
6
4
2
0
-2
-4
-6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

Analytical Modeling



Euler-Bernoulli Beam Equation

The general representation of the Euler-Bernoulli beam equation, as stated

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} + r_a\frac{\partial w(x,t)}{\partial t} = f(x,t) + u(t)$$

where $w(x,t)$ represents the transverse deflection, EI the flexural rigidity, ρ the density, A the cross-sectional area, r_a the damping ratio, $f(x,t)$ the external force, and $u(t)$ the control input from the piezoelectric actuator.

Key Concepts:

1. **Flexural Rigidity (EI)**: This is a crucial parameter in beam theory, influencing the beam's resistance to bending. It's a product of the modulus of elasticity (E) and the moment of inertia (I) of the beam's cross section.
2. **Damping Ratio (r_a)**: Represents energy dissipation in the system, crucial for understanding the dynamic response of the beam, especially in vibration suppression.
3. **Piezoelectric Actuator**: A sophisticated component that uses piezoelectric effect to produce a mechanical displacement or force when an electric field is applied.

Analytical Modeling Assumptions

1. **Layer Composition**: The layers are either piezoelectric or purely elastic. This assumption simplifies the analysis by defining clear mechanical properties for each layer.
2. **Static Equilibrium**: Each element of the beam is considered in static equilibrium, a standard assumption in Euler-Bernoulli beam theory.
3. **Thickness vs. Radius of Curvature**: The beam's thickness is less than its radius of curvature under various effects, validating the use of linear beam theory.
4. **Stress Orientation**: Stresses occurring in the x-z plane align with classic beam theory assumptions.
5. **Layer Interface**: The assumption of continuous and non-slip interfaces is crucial for the accurate transfer of stresses and strains between layers.
6. **Poling Direction**: Aligning the poling direction of the piezoelectric material with the transverse displacement direction is key for effective actuation and sensing.

Solution Approach

The free response of the beam, given by $w(x,t) = X(x)T(t)$, separates the problem into spatial and temporal components, a common approach in vibration analysis.

Derivation of Spatial Component (X(x))

Substituting $w(x,t)$ into Eq.1, and considering free response, we get:

$$\frac{\partial^4 X(x)}{\partial x^4} - k^4 X(x) = 0$$

where $k^4 = \frac{\omega^2}{EI\rho A}$.

Applying Laplace transforms, we derive the general solution for $X(x)$ as shown in

$$X(x) = C_p(kx) * X(0) + \frac{1}{k} S_p(kx)\frac{dX(0)}{dx} + \frac{1}{k^2} C_n(kx)\frac{d^2X(0)}{dx^2} + \frac{1}{k^3} S_n(kx)\frac{d^3X(0)}{dx^3}$$

where

$$C_p(kx) = \frac{1}{2}[\cosh(kx) + \cos(kx)]$$

$$S_p(kx) = \frac{1}{2}[\sinh(kx) + \sin(kx)]$$

$$C_n(kx) = \frac{1}{2}[\cosh(kx) - \cos(kx)]$$

$$S_n(kx) = \frac{1}{2}[\sinh(kx) - \sin(kx)]$$

with following derivatives

$$\frac{dX(x)}{dx} = kS_n(kx)X(0) + C_p(kx)\frac{dX(0)}{dx} + \frac{1}{k} S_p(kx)\frac{d^2X(0)}{dx^2} + \frac{1}{k^2} C_n(kx)\frac{d^3X(0)}{dx^3}$$

$$\frac{d^2X(x)}{dx^2} = k^2 C_n(kx)X(0) + kS_n(kx)\frac{dX(0)}{dx} + C_p(kx)\frac{d^2X(0)}{dx^2} + \frac{1}{k} S_p(kx)\frac{d^3X(0)}{dx^3}$$

$$\frac{d^3X(x)}{dx^3} = k^3 S_p(kx)X(0) + k^2 C_n(kx)\frac{dX(0)}{dx} + kS_n(kx)\frac{d^2X(0)}{dx^2} + C_p(kx)\frac{d^3X(0)}{dx^3}$$

Boundary Conditions

The boundary conditions for a cantilever beam (clamped at one end, free at the other) are crucial for solving the differential equation:

1. **Clamped Side**: $X(0) = 0$, $\frac{dX(0)}{dx} = 0$.
2. **Free Side**: $\frac{d^2X(L)}{dx^2} = 0$, $\frac{d^3X(L)}{dx^3} = 0$.

Critique and Discussion

The Euler-Bernoulli beam theory, while foundational, has limitations especially for beams with significant thickness, where shear deformation and rotary inertia effects become non-negligible. This limitation is addressed in more advanced theories like Timoshenko beam theory.

The assumption of non-slip interfaces between layers can be overly idealistic, especially in the presence of manufacturing imperfections or material degradation.

The use of piezoelectric actuators in this context is highly relevant for modern adaptive structures, allowing for active vibration control and shape morphing. However, the control strategies (represented by $u(t)$) need to be sophisticated, especially in dynamic environments.

- **Boundary Conditons**

$X = 0$

$$Z(0,t) = 0 \rightarrow X(0) = 0$$

$$\frac{dZ}{dx}(0,t) = 0 \rightarrow \frac{dX}{dx}(0) = 0$$

$X = L$

$$\frac{d^2 Z}{dx^2}(l,t) = 0$$

$$\frac{d^3 Z}{dx^3}(l,t) = 0$$

- **Initial Conditions**

$$\phi(0) = 0$$

$$\dot{\phi}(0) = \phi_0$$

Characteristic Equation in Matrix Form

By implementing the boundary conditions (BCs) , we derive the characteristic equation in matrix form:

$$\begin{bmatrix} C_p(kL) & \frac{1}{k}S_p(kL) \\ kS_n(kL) & C_p(kL) \end{bmatrix} \begin{bmatrix} \frac{d^2X(0)}{dx^2} \\ \frac{d^3X(0)}{dx^3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The determinant of the matrix must be zero for a nontrivial solution, leading to the characteristic equation of the clamped-free bending bar.

Characteristic Equation of Clamped-Free Bending Bar

The characteristic equation becomes:

$$1 + \cos(kL) + \cosh(kL) = 0$$

This equation cannot be solved analytically in closed form, necessitating numerical methods to find the characteristic zeros $k_m L$ for $m = 1, 2, \ldots, \infty$. The parameter m refers to the natural propagatable mode of the beam.

Natural Frequencies

The natural frequencies of a clamped-free beam are given by:

$$\omega_m = \left(\frac{(k_m L)^2}{L^2}\right)\sqrt{\frac{EI}{\rho A}}$$

These frequencies correspond to the distinct modes of vibration that the beam can sustain.

Eigenmode of Space-Dependent Solution

The eigenmode for the space-dependent solution can be determined as:

$$X_m(x) = \frac{1}{k_m^2}\frac{d^2X_m}{dx^2}(0)\left[C_n(k_m x) + \frac{1}{k_m}S_n(k_m x)\frac{\frac{d^3X_m(0)}{dx^3}}{\frac{d^2X_m(0)}{dx^2}}\right]$$

where the ratio of the third and second derivatives at $x = 0$ is:

$$\frac{\frac{d^3X_m(0)}{dx^3}}{\frac{d^2X_m(0)}{dx^2}} = -k_m\frac{C_p(k_m L)}{S_p(k_m L)}$$

Superposition Principle

Considering the superposition principle, the transverse displacement function $w(x,t)$ is reconstructed as a superposition of the propagatable modes:

$$w(x,t) = \sum_{m=1}^{\infty} X_m(x) T_m(t)$$

Analysis and Implications

1. **Numerical Solutions**: The necessity of numerical computing highlights the complexity in real-world applications. Each characteristic zero corresponds to a specific vibrational mode, crucial for understanding the dynamic behavior of the beam.
2. **Physical Meaning of m**: The integer m represents different vibrational modes, with each mode having its distinct frequency and shape. This is fundamental in designing systems for vibration control or energy harvesting using piezoelectric materials.
3. **Mode Shapes and Frequencies**: Determining $X_m(x)$ and ω_m is essential for predicting how the beam will respond to various inputs, especially in dynamic environments.
4. **Application in Control Systems**: Understanding these vibrational characteristics is vital for designing control systems, particularly in structures where vibration suppression or tuning is required. Piezoelectric actuators can be effectively used to alter these modes and frequencies.
5. **Engineering Considerations**: The analysis emphasizes the importance of material properties (like EI and ρA), geometric parameters (like beam length L), and boundary conditions in influencing the vibrational characteristics of the beam.

Time-Dependent Equation

The time-dependent equation is given as:

$$\frac{\partial^2 T_m(t)}{\partial t^2} + 2\xi_m\omega_m \frac{\partial T_m(t)}{\partial t} + \omega_m^2 T_m(t) = 0$$

where ξ_m is the dimensionless damping constant, related to the damping ratio r_a and mass per unit length ρA by $\frac{r_a}{\rho A} = 2\xi_m\omega_m$.

General Homogeneous Solution

The general homogeneous solution for $T_m(t)$ is:

$$T_m(t) = e^{-\omega_m\xi_m t}(A_m\cos(\omega_{dm}t) + B_m\sin(\omega_{dm}t))$$

where $\omega_{dm} = \omega_m\sqrt{1-\xi_m^2}$ represents the damped natural frequency.

Time and Space-Dependent Representation

Combining Eqs. the complete solution for the deflection of the beam, considering specific initial conditions (ICs), is given as:

1. For $T(0) = A_m = T_0$ and $\frac{dT(0)}{dt} = B_m = 0$:
   $w(x,t) = T_0\sum_{m=1}^{\infty} X_m(x)e^{-\omega_m\xi_m t}\left(\cos(\omega_{dm}t) + \frac{\xi_m}{\sqrt{1-\xi_m^2}}\sin(\omega_{dm}t)\right)$
2. For $T(0) = 0$ and $\frac{dT(0)}{dt} = \dot{T}_0$:
   $w(x,t) = \sum_{m=1}^{\infty} X_m(x)\frac{\dot{T}_0}{\omega_{dm}}e^{-\omega_m\xi_m t}\sin(\omega_{dm}t)$

Spatial Component $X_m(x)$

Further detail on $X_m(x)$ is provided:

$$X_m(x) = \left(\tfrac{1}{2}\cosh(k_m x) - \tfrac{1}{2}\cos(k_m x)\right) - \left(\tfrac{1}{2}\sinh(k_m x) - \tfrac{1}{2}\sin(k_m x)\right) \times \frac{\left(\frac{1}{2}\cosh(k_m L)+\frac{1}{2}\cos(k_m L)\right)}{\left(\frac{1}{2}\sinh(k_m L)+\frac{1}{2}\sin(k_m L)\right)}$$

Analysis and Implications

1. **Damped Natural Frequency**: ω_{dm} reflects the frequency of oscillation in the presence of damping, which is crucial in designing control and damping strategies for the beam.
2. **Initial Conditions**: The solution changes significantly based on the initial conditions, showcasing the system's sensitivity to initial states. This is particularly important in practical scenarios where initial conditions can vary.
3. **Damping Impact**: The presence of ξ_m in the exponential term signifies the impact of damping on the amplitude of vibrations, reducing it over time.
4. **Mode Shape Contribution**: Each term $X_m(x)$ represents a mode shape, and the superposition of these shapes gives the complete response of the beam. This superposition is key to understanding complex vibrational behaviors.
5. **Engineering Applications**: In practical applications, especially in structures and materials engineering, this analysis aids in predicting how the beam will behave under various loading conditions and helps in designing systems for vibration control using piezoelectric actuators.

bernoullieuler_beamvibration_FDM.m

Parameters Initialization

- **Beam Characteristics**: Length (`L`), width (`wd`), thickness (`t`), modulus of elasticity (`E`), and linear mass density (`ro`) of the beam are defined.
- **Moment of Inertia (`I`)**: Calculated using the beam's width and thickness.
- **Cross-Sectional Area (`A`)**: Calculated from the beam's width and thickness.
- `lamda`: Product of cross-sectional area and linear mass density.
- `ksi`: Damping ratio of the beam.
- `P`: Tip payload (force applied at the beam's end).
- `nt`, `nx`, `tf`: Define the length of the time domain (`nt`), the length of the space domain (`nx`), and the total time of the simulation (`tf`).

Mesh Spacing and Time Step Calculation

- `dx`, `dt`: Calculate the spatial step size (`dx`) and the time step size (`dt`) based on the beam length, number of spatial segments, and simulation time.

Coordinates for Drawing

- Generate linearly spaced vectors `x` (for space) and `t` (for time) used for plotting the results.

Data Storage Arrays

- `w`: Matrix to store the vibration displacement at different positions (`x`) and times (`t`).
- `d`: Vector to store the boundary disturbance over time.
- `f`: Matrix to store the distributed disturbance over space and time.

Natural Frequencies Calculation

- `k1` to `k6`: Constants for calculating the first six natural frequencies of the beam.
- `w1` to `w6`: The first six natural frequencies of the beam are calculated.

Boundary Disturbance Calculation

- Populate the `d` vector with a combination of sinusoidal functions over time.

Distributed Disturbance Calculation

- Calculate a spatially and temporally varying distributed disturbance `f`.

Initial Conditions

- Set the initial displacement of the beam (`w(i,1)`) using the tip payload and beam properties.

Damping Coefficient

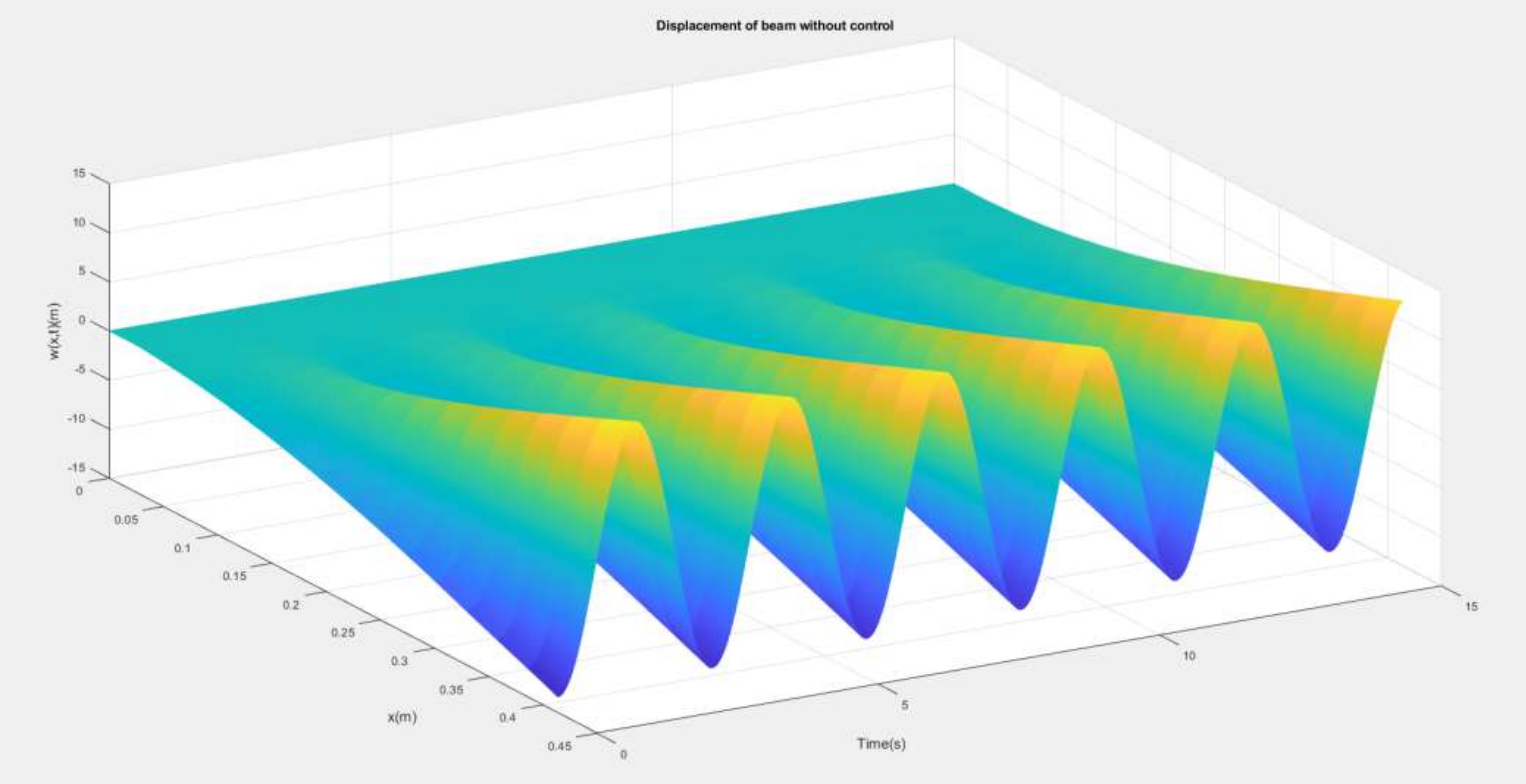
- `alpha`: Calculate a damping coefficient based on the damping ratio, first natural frequency, and beam properties.

Boundary Conditions

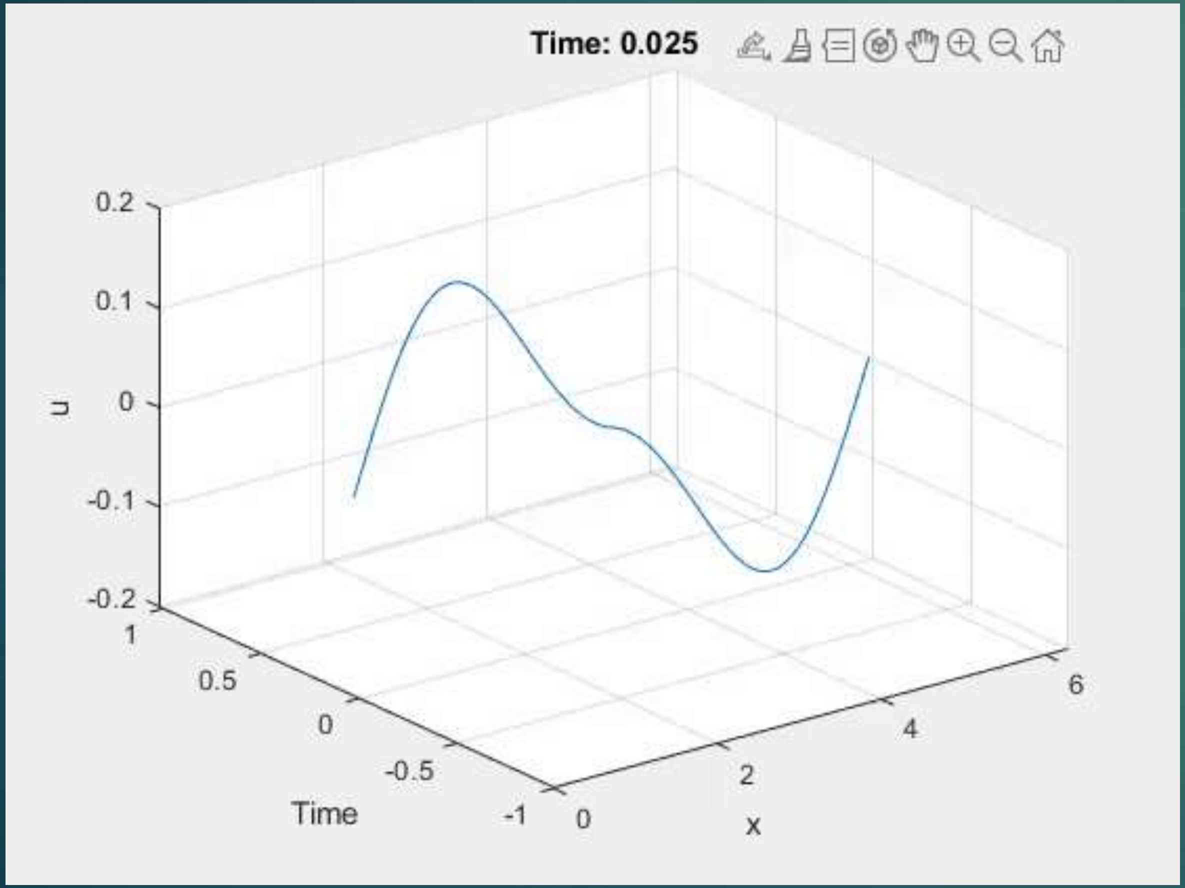
- `T`, `Ms`: Tension in the beam and mass of the tip payload are defined.
- Set the boundary conditions for the displacement at the first two positions of `w`.

Main Simulation Loop

- **Finite Difference Method**: The beam's displacement (`w`) is updated at each time step using a finite difference approximation of the beam's differential equation.
- **Boundary Conditions**: Apply specific boundary conditions at the end of the beam.


Displacement of beam without control
w(x,t)(m)
15
10
5
0
-5
-10
-15
x(m)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Time(s)
0
5
10
15

allen_cahn_1D_spectral_crank_nicolson_optimized.m



The provided code is a MATLAB script that simulates and animates the one-dimensional Allen-Cahn equation using the spectral Crank-Nicolson method. It also includes damping effects and saves the simulation as an MP4 video.

Cantilever Beam with Piezoelectric Patch

Differential Strain and Longitudinal Stress

1. **Differential Strain**

   $\sigma_{11} = -C_{11} t_b \frac{d^2 w(x,t)}{dx^2} - e_{31} \frac{V_a}{t_a}$

   where t_a is the thickness of the piezoelectric actuator, V_a is the electrical potential, e_{31} is the piezoelectric constant relating electric field to stress, and C_{11} is the modulus of elasticity measured at a constant electrical potential.

2. **Moment due to Longitudinal Stress**

   $M = \frac{\sigma_{11} I_a}{(0.5t_a + t_b)}$

   where I_a is the inertia of the piezoelectric actuator to the midplane of the beam.

Inertia of the Piezoelectric Actuator

$I_a = \frac{b_a t_a^3}{12} + t_a b_a (0.5t_a + t_b)^2$

Modeling the Piezoelectric Actuator's Moment

By substituting the moment M is obtained as:

$$M = \left(-C_{11} t_b \frac{d^2 w(x,t)}{dx^2} - \frac{e_{31} V_a}{t_a}\right) \left[\frac{b_a t_a^3}{12(0.5 t_a + t_b)} + t_a b_a (0.5 t_a + t_b)\right] [H(x - x_{a_2}) - H(x - x_{a_1})]$$

Here, $H(.)$ is the Heaviside function, and x_{a_1} and x_{a_2} are the ends of the piezoelectric actuator according to the clamped side.

Transformed Euler-Bernoulli Beam Equation

The modified form of the Euler-Bernoulli beam equation with the coupled piezoelectric actuator is given by:

$$EI \frac{d^4 w(x,t)}{dx^4} + \rho A \frac{d^2 w(x,t)}{dt^2} + r_a \frac{\partial w(x,t)}{\partial t} = f(x,t) + \frac{d^2 M}{dx^2}$$

Where:

$$\frac{d^2 M}{dx^2} = -C_{11} t_b Q \left[H(x - x_{a_2}) - H(x - x_{a_1})\right] \frac{d^4 w(x,t)}{dx^4} - e_{31} \frac{V_a}{t_a} Q \left[H(x - x_{a_2}) - H(x - x_{a_1})\right]$$

And $$Q = \frac{b_a t_a^3}{12(0.5 t_a + t_b)} + t_a b_a (0.5 t_a + t_b)$$

Analysis and Implications

1. **Coupled System Dynamics**: The introduction of the piezoelectric actuator couples the electrical and mechanical responses of the system. The modified beam equation incorporates the effect of both external forces and the piezoelectric actuator's moment.
2. **Heaviside Function Modeling**: The Heaviside functions in the moment equation indicate the localized application of the piezoelectric actuator's effect. This is essential for accurately representing the influence of the actuator on specific regions of the beam.
3. **Variable Parameters**: Parameters like $C_{11}, e_{31}, t_a, t_b, V_a$, and b_a represent material properties and dimensions, emphasizing the need for careful material selection and design considerations in piezoelectric applications.
4. **Control and Actuation**: The ability to control the beam's behavior through the applied voltage V_a showcases the potential for active vibration control and shape deformation, essential in precision engineering and adaptive structures.

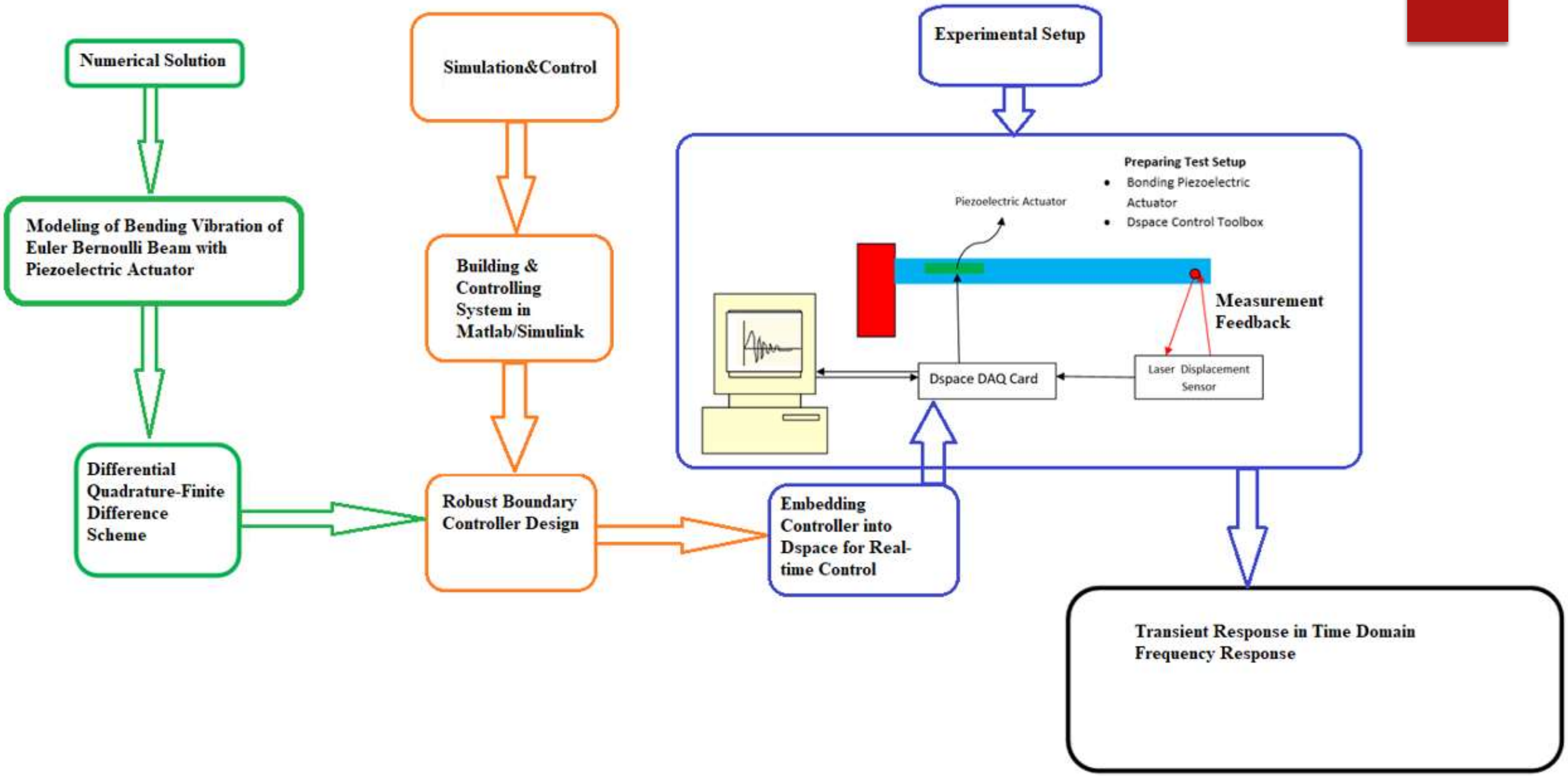
Flexural Rigidity Modification

The flexural rigidity EI in the original equation is modified to account for the influence of the piezoelectric actuator. The new flexural rigidity is given by:

$EI + C_{11} t_b I_a$

Implications

1. **Combined Structural and Piezoelectric Effects**: The modification in flexural rigidity reflects the combined effects of the structural beam and the presence of the piezoelectric actuator. This unified parameter accounts for both mechanical stiffness and electrical interactions.
2. **Consistent Solution Procedure**: Despite the introduction of additional terms in the equation, the solution procedure remains consistent with the original Euler-Bernoulli beam equation. This allows for the utilization of existing methods and techniques in structural analysis.
3. **Dynamic Response Control**: The ability to control the beam's dynamic response through the applied voltage V_a provides a means of actively adjusting the behavior of the structure. This is particularly valuable in applications where dynamic response needs to be finely tuned.


Numerical Solution
Modeling of Bending Vibration of Euler Bernoulli Beam with Piezoelectric Actuator
Differential Quadrature-Finite Difference Scheme
Simulation&Control
Building & Controlling System in Matlab/Simulink
Robust Boundary Controller Design
Experimental Setup
Preparing Test Setup
• Bonding Piezoelectric Actuator
• Dspace Control Toolbox
Piezoelectric Actuator
Measurement Feedback
Dspace DAQ Card
Laser Displacement Sensor
Embedding Controller into Dspace for Real-time Control
Transient Response in Time Domain
Frequency Response

Boundary Controller Design

Objective and Problem Statement

The objective is to synthesize a vibration controller to mitigate or eliminate the vibrations of a flexible beam described by a nonhomogeneous partial differential equation (PDE). The Euler-Bernoulli beam model is expressed by the governing equation along with specific boundary conditions (BCs) and initial conditions (ICs). The system is subject to external disturbances represented by $f(x,t)$ and $d(t)$.

Governing Equation and Boundary Conditions

The Euler-Bernoulli beam model is represented by the equation:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} + r_a\frac{\partial w(x,t)}{\partial t} = f(x,t)$$

Subject to the following BCs and ICs:

Boundary Conditions:

$$w(0,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad \frac{\partial^2 w(L,t)}{\partial x^2} = 0$$
$$-EI\frac{\partial^3 w(L,t)}{\partial x^3} + T\frac{\partial w(L,t)}{\partial x} = u(t) + d(t) - \frac{P}{g}\frac{\partial^2 w(L,t)}{\partial t^2}$$

Initial Conditions:

$$w(x,0) = -\frac{PL}{EI}\left(\frac{x^2}{2} - \frac{x^3}{6L}\right), \quad \frac{\partial w(x,0)}{\partial t} = 0$$

External Disturbances

1. **Payload Disturbance ($d(t)$):**

   $$d(t) = 0.1 + 0.1\sin(\pi t) + 0.1\sin(2\pi t) + 0.1\sin(3\pi t)$$

2. **Spatially and Temporally Varying Dynamic Force ($f(x,t)$):**

   $$f(x,t) = (1 + \sin(0.1\pi xt) + \sin(0.2\pi xt) + \sin(0.3\pi xt))\,\frac{x}{20L}$$

Vibration Controller Synthesis

The primary goal is to design a Lyapunov-based boundary control $u(t)$ at the right boundary of the flexible beam using a piezo actuator patch. The controller aims to counteract the effects of the external disturbances $f(x,t)$ and $d(t)$ to achieve the elimination or suppression of beam vibrations.

Challenges and Considerations

1. **Lyapunov-Based Control:** The design of the control strategy involves utilizing Lyapunov stability theory to ensure the stability of the closed-loop system.
2. **Unknown Disturbances:** The disturbances $f(x,t)$ and $d(t)$ are unknown, requiring a robust control approach to handle uncertainties.
3. **Actuator Placement:** The control is applied at the right boundary via a piezo actuator patch, emphasizing the importance of precise actuator placement for effective vibration control.
4. **Closed-Loop Stability:** An essential aspect is to investigate the closed-loop stability of the system under the influence of the designed control strategy.

Robust Boundary Control with a Disturbance Observer

Robust Boundary Control Strategy

The robust boundary control strategy is designed to suppress the effects of unknown disturbances in a flexible beam system. The control input $u(t)$ is expressed by Equation (1*), where $\hat{d}(t)$ is the estimated tip payload disturbance, and k_1, k_2, k are control gains.

$$u(t) = -EI\frac{\partial^3 w(L,t)}{\partial x^3} + T\frac{\partial w(L,t)}{\partial x} - \frac{P}{g}\left[k_1\frac{\partial^2 w(L,t)}{\partial x \partial t} - k_2\frac{\partial^4 w(L,t)}{\partial x^3 \partial t}\right] - k\left[\frac{\partial w(L,t)}{\partial t} - k_2\frac{\partial^3 w(L,t)}{\partial x^3} + k_1\frac{\partial w(L,t)}{\partial x}\right] - \hat{d}(t)$$

Equation 1*

This expression represents the boundary control input $u(t)$ for the flexible beam system, considering various terms related to beam dynamics, control gains (k_1, k_2, k), and the estimated tip payload disturbance $\hat{d}(t)$.

System Parameter Vector and Partial Derivatives Vector

The system parameter vector ϕ is defined as $\phi = [EI, T, P/g]^T$, and the vector $P(t)$ including partial derivatives is given by:

$$P(t) = \left[\frac{\partial^3 w(L,t)}{\partial x^3}, -\frac{\partial w(L,t)}{\partial t}, k_1 \frac{\partial^2 w(L,t)}{\partial x \partial t} - k_2 \frac{\partial^4 w(L,t)}{\partial x^3 \partial t}\right]$$

Equation 2*

$$u(t) = -P(t)\phi - k u_a(t) - \hat{d}(t)$$

where:

- $P(t) = \begin{bmatrix} \frac{\partial^3 w(L,t)}{\partial x^3} \\ -\frac{\partial w(L,t)}{\partial t} \\ k_1 \frac{\partial^2 w(L,t)}{\partial x \partial t} - k_2 \frac{\partial^4 w(L,t)}{\partial x^3 \partial t} \end{bmatrix}$ is a vector containing partial derivatives of the deflection $w(x,t)$ with respect to x and t.
- $\phi = \begin{bmatrix} EI \\ T \\ \frac{P}{g} \end{bmatrix}$ is a vector representing system parameters (flexural rigidity, tension, and tip payload).
- $u_a(t) = \frac{\partial w(L,t)}{\partial t} - k_2 \frac{\partial^3 w(L,t)}{\partial x^3} + k_1 \frac{\partial w(L,t)}{\partial x}$ is a control input term related to beam dynamics.
- k is a control gain.
- $\hat{d}(t)$ is the estimate of the tip payload disturbance.

This expression captures the boundary control input $u(t)$ as a combination of the system parameters, control input $u_a(t)$, and the estimated disturbance $\hat{d}(t)$. The control gain k plays a role in shaping the control action based on the system dynamics.

Disturbance Observer Dynamics

The disturbance observer dynamic is described by Equation (3*), where γ and ζ_d are positive constants.

$$\frac{d(\hat{d}(t))}{dt} = \gamma u_a(t) - \zeta_d \gamma \hat{d}(t) \text{ (3*)}$$

The time derivative of the disturbance observer error is given by Equation (4*):

$$\frac{d(\tilde{d}(t))}{dt} = -\gamma u_a(t) + \zeta_d \gamma \hat{d}(t) \text{ (4*)}$$

The proposed controller utilizes a disturbance observer to suppress the effect of unknown disturbances, providing robustness to changes in these disturbances.

Lyapunov-Based Stability Analysis

A Lyapunov function candidate $V_0(t)$ is considered as:

$V_0(t) = V_1(t) + V_2(t) + \Delta(t) + \frac{1}{2\gamma}\tilde{d}^2(t)$ (5*)

where $V_1(t), V_2(t)$, and $\Delta(t)$ are energy terms derived from the system dynamics.

$$V_1(t) = \frac{k_2\beta}{2}\rho\int_0^L \left(\frac{\partial w(x,t)}{\partial t}\right)^2 dx + \frac{k_2\beta}{2}EI\left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2 dx + \frac{k_2\beta}{2}T\int_0^L \left(\frac{\partial w(x,t)}{\partial x}\right)^2 dx$$

$$V_2(t) = \frac{1}{2}\frac{P}{g}u_a^2(t)$$

$$\Delta(t) = \alpha\rho\int_0^L x\frac{\partial w(x,t)}{\partial t}\frac{\partial w(x,t)}{\partial x}dx$$

The Lyapunov function candidate is constrained as:

$0 \leq \lambda_1(V_1(t) + V_2(t) + \tilde{d}^2(t)) \leq V_0(t) \leq \lambda_2(V_1(t) + V_2(t) + \tilde{d}^2(t))$ (6*)

where λ_1 and λ_2 are positive constants.

The time derivative of the Lyapunov function candidate is limited as:

$\dot{V}_0(t) \leq -\lambda V_0(t) + \epsilon_0$ (7*)

where $\lambda > 0$ and $\epsilon_0 > 0$.

Stability and Conclusion

The closed-loop beam system is shown to be uniformly bounded given limitations on BCs, ICs, and control signals. The stability analysis, facilitated by the Lyapunov function candidate, indicates that the steady-state system response $w(x, t)$ can be confined to a small band when controller parameters are appropriately chosen. The design parameters must be regulated to satisfy transient performance and control action, ensuring the effectiveness of the proposed controller in robustly suppressing vibrations in the flexible beam system.

Note:

1. **System of Equations**: The Crank-Nicolson scheme leads to a system of linear equations, which needs to be solved at each time step. Depending on the complexity of your PDE and boundary conditions, this might require the use of matrix inversion or iterative solvers like GMRES or conjugate gradient methods.
2. **Boundary Condition Implementation**: Ensure that the boundary conditions are consistent with the implicit method. This might require adjusting how the boundary values are computed.
3. **Testing and Validation**: After implementing these changes, it's essential to test the simulation for stability and accuracy. Compare the results with known solutions or run test cases to ensure the method behaves as expected.
4. **Computational Resources**: Be aware that implicit methods, while more stable, can be more computationally intensive, especially for large systems or very fine discretizations.

1. **Semi-Implicit Method**: For the second-order time derivative, a Crank-Nicolson type scheme is used. This treats the second-order spatial derivative $\frac{\partial^2 w}{\partial x^2}$ semi-implicitly, which should improve stability.
2. **Improved Boundary Derivative**: The third spatial derivative at the boundary is computed using a backward difference scheme, which is more stable for boundary conditions.
3. **Control Input**: The control input $u(t)$ at the boundary is calculated using the improved derivative approximations. Make sure to adjust the control law according to your specific requirements.
4. **Time Step**: The time step `dt` is reduced for enhanced stability.

Cantilever Beam with Piezoelectric Actuator Solution

$$Z(x,t) = \sum_{1}^{\infty} X_m(x) * \phi(t)$$

$$X_m(x) = c_2(k_m x) - s_2(k_m x)\frac{C_1(k_m l)}{S_1(k_m l)}$$

$$\phi_m^h(t) = e^{-\omega_m \zeta_m t}\left[A_m \cos(\omega_{dm} t) + B_m \sin(\omega_{dm} t)\right]$$

$$Z(x,t) = \phi_0 * \sum_{m=1}\{[\frac{1}{2} * Cosh(k_m x) - \frac{1}{2} * Cos(k_m x)] - [\frac{1}{2} Sinh(k_m x) - Sin(k_m x)] * \frac{[\frac{1}{2} Cosh(k_m x) - Cos(k_m x)]}{[\frac{1}{2} * Sinh(k_m x) - Sin(k_m x)]}\} * \{A_m[Cos(\omega_{dm} t) + B_m Sin(\omega_{dm} t)]\}$$

$$k^4 = \frac{\omega^2}{C}\mu \quad C = E * I + C_{11} * h * L \quad \mu = \rho * A \quad \omega_d = \omega * \sqrt{1 - \varsigma^2} \quad \omega_m = \frac{(k_m l)^2}{l^2}\sqrt{\frac{C}{\mu}}$$

Cantilever Beam with Piezoelectric Actuator and Tip Mass

$$Z(x,t) = \sum_{1}^{\infty} X_m(x) * \phi(t)$$

$$X_m(x) = c_2(k_m x) - s_2(k_m x)\frac{C_1(k_m l)}{S_1(k_m l)}$$

$$\phi_m^h(t) = e^{-\omega_m \zeta_m t}\left[A_m \cos(\omega_{dm} t) + B_m \sin(\omega_{dm} t)\right]$$

$$Z(x,t) = \phi_0 * \sum_{m=1} \{[\frac{1}{2} * Cosh(k_m x) - \frac{1}{2} * Cos(k_m x)] - [\frac{1}{2} Sinh(k_m x) - Sin(k_m x)] * \frac{[\frac{1}{2} Cosh(k_m x) - Cos(k_m x)]}{[\frac{1}{2} * Sinh(k_m x) - Sin(k_m x)]}\} * \{A_m[Cos(\omega_{dm} t) + B_m Sin(\omega_{dm} t)]\}$$

$$k^4 = \frac{\omega^2}{C}\mu \quad C = E * I + C_{11} * h * L \quad \mu = \rho * A \quad \omega_d = \omega * \sqrt{1 - \varsigma^2} \quad \omega_m = \sqrt{\frac{K_e}{M_e}}$$

$$K_e = \frac{(3EI)}{l^3}$$

$$M_e = (33/140)\rho A l + M$$

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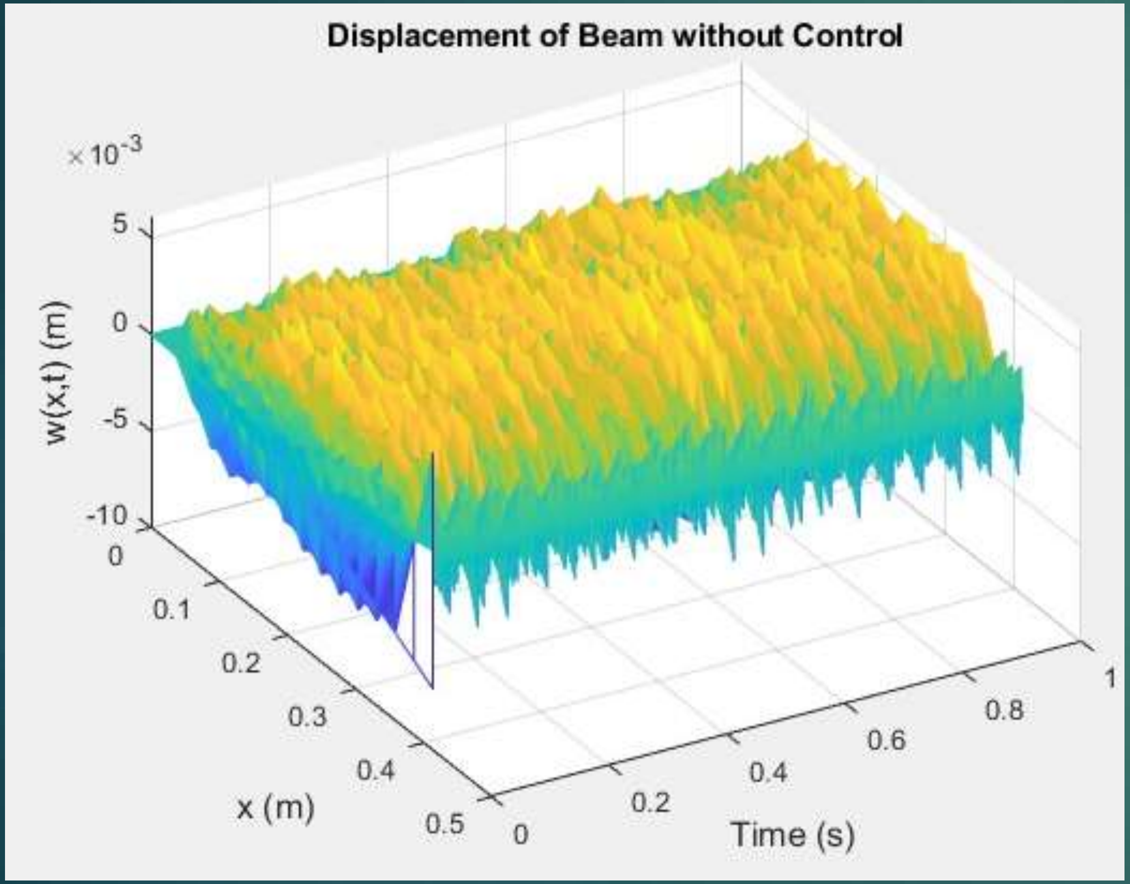

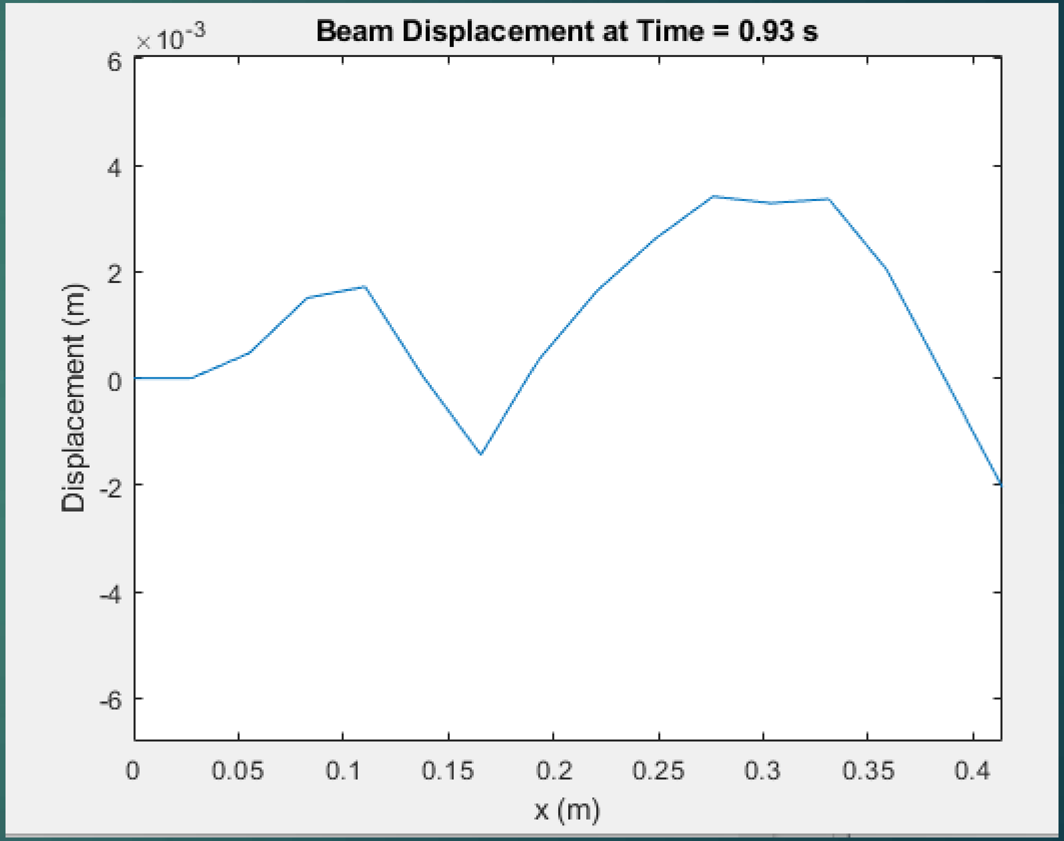

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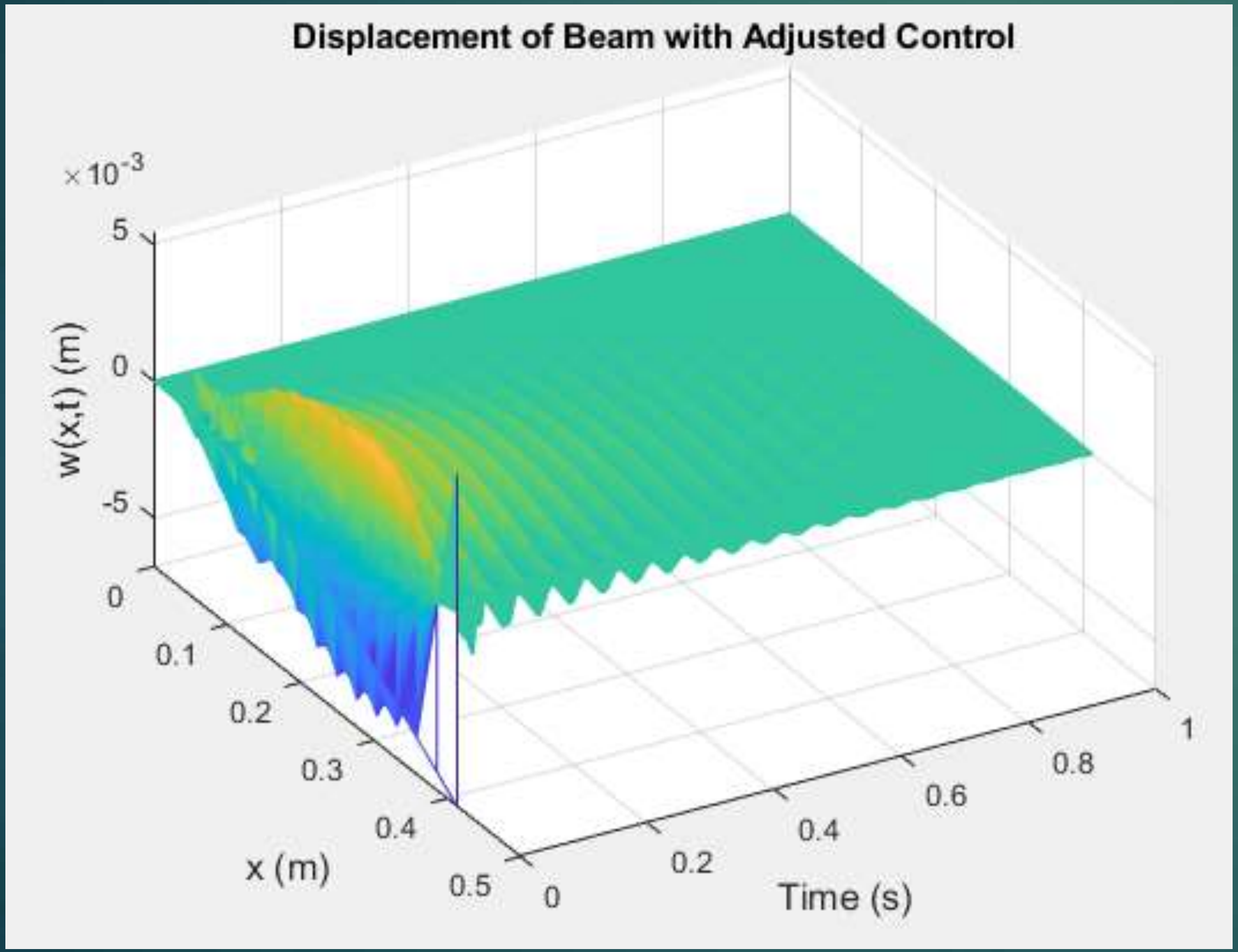
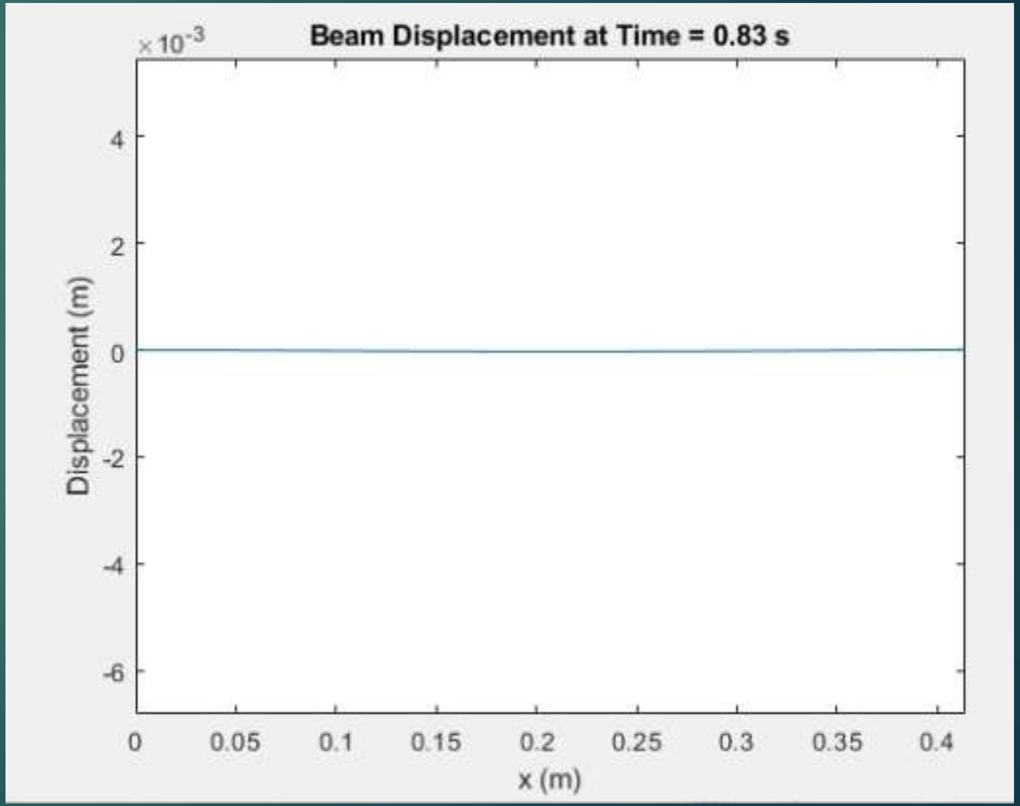
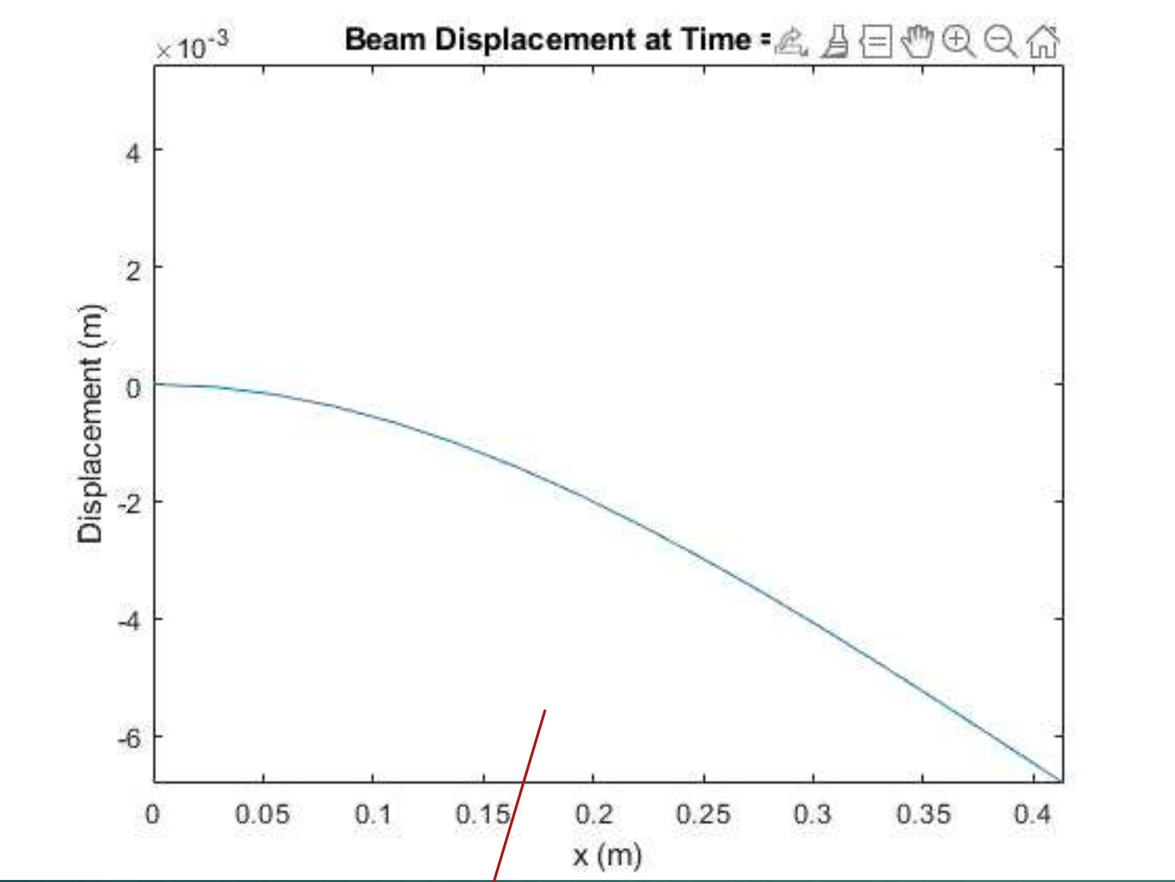
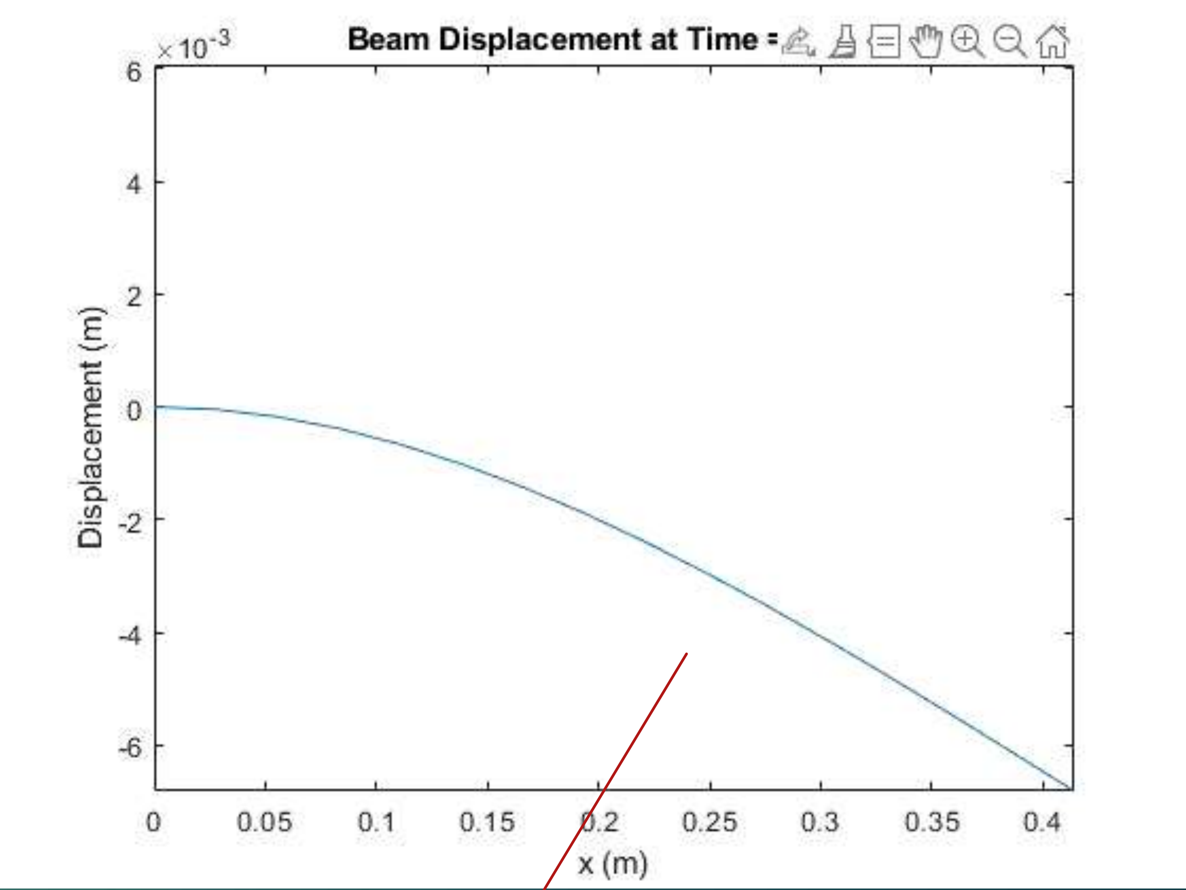




***Animations



Robust Boundary Control



Uncontrolled Case

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