The student t-distribution with mean zero and $n-1$ degrees of freedom is the distribution of a random variable that is computed as follows

$$\sqrt{n}\frac{\bar{X}-\mu}{S}$$

where $\bar{X}$ is the sample average of n observations and S is their sample standard deviation, and the observations are samples of iid gaussian distributed random variables. I.e.

$$\bar{X} = \frac{\sum X_i}{n} \tag{1}$$

$$S = \sqrt{\frac{\sum(X_i - \bar{X})^2}{n-1}} \tag{2}$$

$$X_i \sim \mathcal{N}(\mu, \sigma^2) \tag{3}$$

The variance of the t-distribution with ν d.o.f. is $\frac{\nu}{\nu-2}$ for $\nu > 2$. Which means that when the t-distribution arises from $n > 3$ samples then its variance is $\frac{n-1}{n-3}$ otherwise its infinity.
The parameteric (closed-form) definition of the t-distribution is given through its PDF.

$$t_\nu(x) = \frac{1}{\sqrt{\nu\pi}}\frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)}\left(1 + x^2/\nu\right)^{-(\nu+1)/2} \tag{4}$$

More generally when we divide a gaussian RV by the square root of a independent χ^2 distributed RV then the t-distribution arises.
The tricky part is in showing that $\bar{X}$ and S which are statistics on the same observations are actually independent random variables under the assumption that the samples are normal iid.

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January 23, 2021