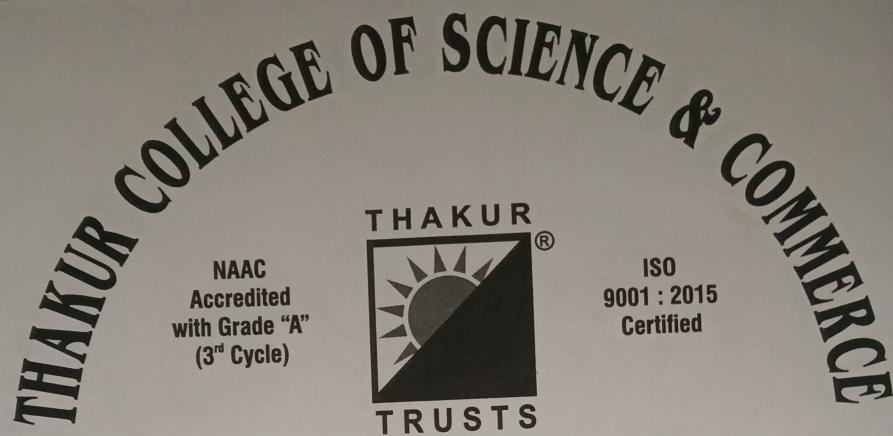


PERFORMANCE

Term	Remarks	Staff Member's Signature
I	Not present in some practices	AM 18/10/19
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CERTIFICATE

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AIK
06/01/2020
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Head of Department

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Examiner

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PRACTICAL No.1

Topic-

Limits and Continuity.

$$1. \lim_{x \rightarrow a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$\lim_{x \rightarrow a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$\lim_{x \rightarrow a} \frac{(a+2x-3x)}{(3a+x-4x)} \times \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$$

$$\lim_{x \rightarrow a} \frac{(a-x)}{(3a-3x)} \times \frac{(\sqrt{3a-x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \lim_{x \rightarrow a} \frac{(a-x)}{(a-x)} \times \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+x} + \sqrt{3x})}$$

$$= \frac{1}{3} \lim_{x \rightarrow a} \frac{(a-x)}{(a-x)} \times \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(a-a)(\sqrt{2x+a} + \sqrt{3x})}$$

$$= \frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{2a+a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}}$$

Q8.

$$\lim_{y \rightarrow 0} \left[\frac{\sqrt{ay} - \sqrt{a}}{y \sqrt{ay}} \right]$$

$$\lim_{y \rightarrow 0} \left[\frac{\sqrt{ay} - \sqrt{a}}{y \sqrt{ay}} \times \frac{\sqrt{ay} + \sqrt{a}}{\sqrt{ay} + \sqrt{a}} \right]$$

$$\lim_{y \rightarrow 0} \frac{a + y - a}{y \sqrt{ay} (\sqrt{ay} + \sqrt{a})}$$

$$\lim_{y \rightarrow 0} \frac{y}{y \sqrt{ay} (\sqrt{ay} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a} + a (\sqrt{a} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a} (\sqrt{a} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a} (2\sqrt{a})} = \frac{1}{2a}$$

$$\lim_{x \rightarrow \pi/6} \frac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$$

$$\text{By substituting } x - \pi/6 = h \\ x = h + \pi/6$$

where $h \rightarrow 0$

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$$\lim_{h \rightarrow 0} \frac{\cos(h + \pi/6) + \sqrt{3} \sin(h + \pi/6)}{\pi - 6(h + \pi/6)}$$

$$\lim_{h \rightarrow 0} \frac{\cosh \cdot \cos \pi/6 - \sinh \sin \pi/6 - \sqrt{3} \sinh \cos \pi/6 + \cosh \sin \pi/6}{\pi - 6 \left(\frac{\pi}{6} + h \right)}$$

$$\lim_{h \rightarrow 0} \frac{\cosh \cdot \frac{\sqrt{3}}{2} - \sinh 1/2 - \sqrt{3} (\sinh \sqrt{3}/2 + \cosh \frac{1}{2})}{\pi - 6h + \pi}$$

$$\lim_{h \rightarrow 0} \frac{(\cosh \frac{\sqrt{3}}{2} + \sinh 1/2) - \sqrt{3} (1/2 \cosh - \sqrt{3}/2 \sinh)}{6h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{\sqrt{3}}{2} \cosh + \frac{\sinh}{2} - \frac{\sqrt{3}}{2} \cosh + \frac{3 \sinh}{2}}{6h}$$

$$\lim_{h \rightarrow 0} \frac{48 \sinh}{2(6h)}$$

$$\lim_{h \rightarrow 0} \frac{48 \sinh}{12h}$$

$$= 1/3$$

$$4) \lim_{x \rightarrow 0} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2+3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$$

By Rationalizing Numerator & Denominator both.

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2+3}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right] \times \left[\frac{\sqrt{x^2+5} + \sqrt{x^2+3}}{\sqrt{x^2+5} + \sqrt{x^2+3}} \right] \times \left[\frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right]$$

$$\lim_{x \rightarrow 0} \left[\frac{(x^2+5-x^2-3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})} \right]$$

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$$\lim_{\infty} \frac{1}{2} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2+3})}$$

$$\text{Then } \lim_{\infty} \frac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+\frac{5}{x^2})} + \sqrt{x^2(1-3/x^2)}}$$

After applying limit we get,

= 4

$$5. i) f(x) = \begin{cases} \frac{\sin 2x}{\sqrt{1-\cos 2x}}, & \text{for } 0 < x \leq \pi/2 \\ \frac{\cos x}{\pi - 2x}, & \text{for } \frac{\pi}{2} < x < \pi \end{cases} \quad \left. \begin{array}{l} \text{at } x = \pi/2 \\ \text{at } x = \frac{\pi}{2} \text{ defined} \end{array} \right\}$$

$$ii) \lim_{x \rightarrow \pi/2^+} f(x) = \lim_{x \rightarrow \pi/2^+} \frac{\cos x}{\pi - 2x}$$

by substituting method,
 $x - \pi/2 = h$

where $h \rightarrow 0$

$$\lim_{h \rightarrow 0} \frac{\cos(h + \pi/2)}{\pi - 2(-h + \pi/2)}$$

$$\lim_{h \rightarrow 0} \frac{\cos(h + \pi/2)}{\pi - 2\left(\frac{2h + \pi}{2}\right)}$$

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$$\lim_{h \rightarrow 0} \frac{\cos(h + \pi/2)}{-2h}$$

$$\lim_{h \rightarrow 0} \frac{\cosh \cdot \cos \pi/2 - \sinh \sin \pi/2}{-2h}$$

$$\lim_{x \rightarrow 0} \frac{\cosh \cdot 0 - \sinh}{-2h}$$

$$\lim_{h \rightarrow 0} \frac{-\sinh}{-2h}$$

$$b) \lim_{x \rightarrow \pi/2} f(x) = \lim_{x \rightarrow \pi/2} \frac{\sin 2x}{\sqrt{1-\cos^2 x}}$$

$$\lim_{x \rightarrow \pi/2} \frac{2\sin x \cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x \rightarrow \pi/2} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x \rightarrow \pi/2} \cos x$$

LHS \neq RHS

$\therefore f'$ is not continuous at $x = \pi/2$

$$5. ii) f(x) = \begin{cases} x^2 - 9 & 0 < x < 3 \\ 2-3 & \\ 2+3 & 3 \leq x < 6 \\ \frac{x^2-9}{x+3} & 6 \leq x < 9 \end{cases}$$

at $x = 3$
and $x = 6$

$$x=3$$

$$\text{i) } f(3) = \frac{x^2 - 9}{x-3} = 0$$

'f' at $x=3$ defined

$$\text{ii) } \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} x + 3$$

$$f(3) = x + 3 = \lim_{x \rightarrow 3} x + 3$$

'f' is defined at $x=3$,

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3^+} (x + 3) = 6$$

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2 - 9}{x-3} = \frac{(x-3)(x+3)}{x-3}$$

LHL = RHL.

'f' is continuous at $x=3$.

$$\text{2. } \lim_{x \rightarrow 6} \frac{x^2 - 9}{x+3}$$

$$\lim_{x \rightarrow 6^+} \frac{(x-3)(x+3)}{x+3}$$

$$\lim_{x \rightarrow 6^+} (x-3) = 6-3 = 3$$

$$\lim_{x \rightarrow 6^+} x+3 = 3+6 = 9$$

$LHL \neq RHL$

Functions is not continuous.

$$\text{i) } f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & x \neq 0 \\ k, & x=0 \end{cases} \quad \text{at } x=0.$$

f is continuous at $x=0$

$$\lim_{x \rightarrow 0} \frac{1-\cos 4x}{x^2} = k$$

$$\lim_{x \rightarrow 0} \frac{2\sin^2 2x}{x^2} = k.$$

$$2 \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2} = k.$$

$$2(2)^2 = k$$

$$k=8.$$

$$\text{ii) } f(x) = (\sec^2 x)^{\cot^2 x}$$

$$\begin{array}{ll} x \neq 0 & f(x) \\ x=0 & \end{array}$$

$$f(x) = (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \rightarrow 0} (1 + \tan^2 x)^{1/\tan^2 x}.$$

We know that,

$$\lim_{x \rightarrow 0} (1 + px)^{1/px} = e.$$

$$\therefore e$$

$$\therefore x = e.$$

$$f(x) = \frac{\sqrt{3} - \tan x}{x - 3x}$$

$$= k$$

$$x - \pi/3 = h$$

$$x = h + \pi/3$$

where,
 $h \rightarrow 0$.

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi/3 - (\pi/3 + h)}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi/3 - (\pi/3 + h)}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{3} - \tan \pi/3 + \tanh h}{1 - \tan \pi/3 \tanh h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{3} (1 - \tan \pi/3 \tanh h) - (\tan \pi/3 + \tanh h)}{1 - \tan \pi/3 \tanh h}$$

$$-3h$$

$$\lim_{h \rightarrow 0} \frac{(\sqrt{3} - \sqrt{3} \cdot \sqrt{3} \tanh h) - (\sqrt{3} + \tanh h)}{1 - \tan \pi/3 \tanh h}$$

$$-3h$$

$$x \neq \pi/3$$

$$x = \pi/3$$

at $x = \pi/3$

$$\lim_{h \rightarrow 0} \frac{(\sqrt{3} - 3 \tanh h) - (\sqrt{3} + \tanh h)}{1 - \sqrt{3} \tanh h}$$

$$-3h$$

$$\lim_{h \rightarrow 0} \frac{-4 \tanh h}{-3h (1 - \sqrt{3} \tanh h)} - \frac{4}{3} \lim_{h \rightarrow 0} \left(\frac{\tanh h}{h} \right) \left(\frac{1}{1 - \sqrt{3} \tanh h} \right)$$

$$= \frac{4}{3} \frac{1}{1 - \sqrt{3}/0} - \frac{4}{3}$$

i) $f(x) = \begin{cases} \frac{1 - \cos 3x}{x \tan x} & x \neq 0 \\ 2 & x = 0 \end{cases}$ at $x = 0$

$$f(x) = \frac{1 - \cos x}{x \tan x}$$

$$= 2 \sin^2 \frac{3}{2} x / x \tan x$$

$$= 2 \sin^2 \frac{3x}{2} x^2 / \frac{x \tan x}{x^2}$$

$$= 2 \lim_{x \rightarrow 0} \left(\frac{3}{2} x \right)^2 / 1$$

$$= \cancel{2} \times \frac{9}{4} = \frac{9}{2}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{9}{2} \quad g = f(0)$$

$\therefore f$ is not at $x = 0$

Redline function

$$f(x) = \frac{1 - \cos 3x}{x \tan x} \quad x \neq 0$$

$$\frac{9}{2} \quad x = 0$$

88.

$$\text{Now, } \lim_{x \rightarrow 0} f(x) = f(0)$$

f' has removable discontinuity at $x=0$

$$\text{i)} f(x) = \begin{cases} \frac{(e^{3x}-1) \sin x}{x^2} & x \neq 0 \\ \pi/6 & x=0 \end{cases} \quad \text{at } x=0$$

at $x=0$

$$\lim_{x \rightarrow 0} (e^{3x}-1) \sin \left(\frac{\pi x}{180}\right) / x^2$$

$$3 \lim_{x \rightarrow 0} \frac{e^{3x}-1}{3x} \cdot \lim_{x \rightarrow 0} \frac{\sin \pi x / 180}{x}$$

$$3 \log e \frac{x}{180} = \frac{x}{60} = f(0)$$

f' is continuous at $x=0$

$$\text{ii)} f(x) = \frac{e^{x^2} - \cos x}{x^2} \quad x=0$$

is continuous at $x=0$

f' is continuous at $x=0$

$$\lim_{x \rightarrow 0} f(x) = f(0)$$

$$= \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$(e^{x^2}-1) + (1 - \cos x) / x^2$$

$$\frac{e^{x^2}-1}{x^2} + \lim_{x \rightarrow 0} 2 \frac{\sin^2 x / 2}{x^2}$$

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$$= \log e + 2 \left(\frac{\sin x / 2}{x} \right)_2$$

Multiplying with 2 on Numerator & Denominator

$$= 1 + 2x \frac{1}{4} = \frac{3}{2} = f(0)$$

$$\text{iii)} f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \quad x \neq \pi/2$$

$$x = \pi/2$$

f' is continuous at

$$\begin{aligned} & \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}} \\ &= \frac{2 - 1 - \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})} \\ &= \frac{1 - \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})} \\ &= \frac{1}{2\sqrt{2+\sqrt{2}}} \\ &= \frac{1}{2\sqrt{2}\sqrt{2}} = \frac{1}{4\sqrt{2}} \end{aligned}$$

$$\bullet \quad \text{iv)} f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq ? \text{ at } x=0$$

$$= k \quad x=0$$

$$f(x) = (\sec^2 x)^{\cot^2 x}$$

Using, $\sec^2 x \cdot \tan^2 x - \sec^2 x = 1$

$$\therefore \sec^2 x = 1 + \tan^2 x$$

and, $\cot^2 x = 1 / \tan^2 x$

$$\lim_{x \rightarrow 0} (1 + \tan^2 x) / \tan^2 x$$

We know,

$$\lim_{x \rightarrow 0} (1+px) / p x = e$$

18.

PRACTICAL NO. 2

Topic - Derivation.

Q.1 Show following function defined from R to R :-

- 1) $\cot x$
- 2) $\operatorname{cosec} x$
- 3) $\sec x$

$$1) \cot x = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{\tan a - \tan x}{(x - a) \tan x \tan a}$$

put $x - a = h$, $x = a + h$ as $x \rightarrow a$, $h \rightarrow 0$

$$Df(h) = \lim_{h \rightarrow 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$$

$$= \lim_{h \rightarrow 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

$$\tan A - \tan B = \tan(A-B)(1 + \tan A \cdot \tan B)$$

$$= \lim_{h \rightarrow 0} \frac{\tan(a-h) - (1 + \tan a \cdot \tan(a+h))}{h \times \tan(a+h) \tan a}$$

$$= \lim_{h \rightarrow 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h) \tan a}$$

$$= -1 \times \frac{\tan^2 a + 1}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a} = -\frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$$Df(a) = -\cos^2 a$$

f is differentiable $\forall a \in R$.

2) $\operatorname{cosec} x$

$$f(x) = \operatorname{cosec} x$$

$$Df(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{\sin a - \sin x}{(x - a) \sin a \sin x}$$

put $x - a = h$, $x = a + h$ as $x \rightarrow a$, $h \rightarrow 0$

$$Df(h) = \lim_{h \rightarrow 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \sin(a+h)}$$

$$= \lim_{h \rightarrow 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \sin(a+h)}$$

$$= \lim_{h \rightarrow 0} \frac{2\cos\left(\frac{a+h}{2}\right) \sin\left(\frac{a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \rightarrow 0} -\frac{\sin h/2 \times 1/2}{h/2} \times 2\cos\left(\frac{a+h}{2}\right) / \sin a \sin(a+h)$$

$$= -\frac{1}{2} \times 2\cos\left(\frac{a}{2}\right) / \sin(a+h)$$

$$= -\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

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3. $\sec x$

$$f(x) = \sec x$$

$$Df(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{\sec(x) - \sec(a)}{x - a}$$

$$\lim_{x \rightarrow a} \frac{1/\cos x - 1/\cos a}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{\cos a - \cos x}{(x-a) \cos a \cos x} = \frac{x-a-h}{x-a+h}$$

$$Df(h) = \lim_{h \rightarrow 0} \frac{\cos a - \cos(a+h)}{h \cdot \cos a (\cos(a+h))}$$

$$= \lim_{h \rightarrow 0} -2\sin\left(\frac{a+h}{2}\right) \sin\left(\frac{a-h}{2}\right)$$

$$= \lim_{h \rightarrow 0} -2\sin\left(\frac{a+0+h}{2}\right) \sin\left(\frac{a-0-h}{2}\right)$$

$$\cos a \cdot \cos(a+h) \approx h/2$$

$$= \tan a \sec a$$

$$2. Df: f(x) = 4x+1 \quad x \leq 2$$

$$= x^2 + 5 \quad x > 0 \quad \text{at } x=2, \text{ then}$$

find f' is differentiable or not

$$Df: f(x) = 4x+7 \quad x < 3$$

$$= x^2 + 3x + 1 \quad x > 3 \quad \text{at } x=3 \text{ then,}$$

find f' is differentiable or not.

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$$Q4. Df: f(x) = 8x-5 \quad x \leq 2$$

$= 3x^2 - 4x + 7 \quad x \leq 2 \quad \text{at } x=2, \text{ then,}$
find f is differentiable or not?

Q2) LHD

$$Df(2^-) = \lim_{x \rightarrow 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \rightarrow 2^-} \frac{4x-1 - (4 \cdot 2 + 1)}{x-2}$$

$$= \lim_{x \rightarrow 2^-} \frac{4x-9}{x-2}$$

$$= \lim_{x \rightarrow 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \rightarrow 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$$Df(2^+) = 4$$

$$RHD = Df(2^+) = \lim_{x \rightarrow 2^+} x^2 + 5 - 9 / x - 2$$

$$= \lim_{x \rightarrow 2^+} \frac{x^2 + 4}{x-2}$$

$$= \lim_{x \rightarrow 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$$= 2+2$$

$$= 4$$

RHD = LHD.

'f' is differentiable at $x=2$.

Q3. RHD

$$Df(3^+) = \lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \rightarrow 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 - 3 + 1)}{x - 3}$$

$$= \lim_{x \rightarrow 3^+} \frac{x^2 + 3x - 18}{x - 3}$$

$$= \lim_{x \rightarrow 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

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$$= \lim_{x \rightarrow 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$$

$$= \lim_{x \rightarrow 3^+} \frac{(x+6)(x-3)}{(x-3)}$$

$$= 3+6 = 9$$

$$Df(3^+) = 9$$

$$LHD = Df(3^-)$$

$$= \lim_{x \rightarrow 3^-} f(x) - f(3)/x-3$$

$$= \lim_{x \rightarrow 3^-} \frac{4x^2 + 7 - 19}{x-3} = \lim_{x \rightarrow 3^-} \frac{4x^2 - 12}{x-3} = \lim_{x \rightarrow 3^-} 4(x-3)/x-3$$

$$Df(3^-) = 4$$

$$RHD \neq LHD$$

f' is not differentiable.

$$q.4) f(2) = 8 \cdot 2 - 5 = 16 - 5 = 11$$

$$RHD = Df(2^+) = \lim_{x \rightarrow 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \rightarrow 2^+} \frac{3x^2 - 4x - 4}{x-2}$$

$$= \lim_{x \rightarrow 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$$

$$= \lim_{x \rightarrow 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \rightarrow 2^+} (3x+2)(x-2)/x-2$$

$$= 3 \cdot 2 + 2 = 8$$

$$LHD = Df(2^-) = \lim_{x \rightarrow 2^-} \frac{f(x) - f(2)}{x-2}$$

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$$\lim_{x \rightarrow 2^-} \frac{8x^2 - 5 - 11}{x-2}$$

$$\lim_{x \rightarrow 2^-} \frac{8(x-2)}{x-2}$$

$$Df(2^-) = 8$$

$$LHD = RHD$$

f' is differentiable at $x=2$

PRACTICAL No.3.

Topic - Application of Derivative.

1) Find the intervals in which function is increasing or decreasing.

$$f(x) = x^3 - 5x - 11 \quad ; \quad f'(x) = 3x^2 - 5$$

$$f'(x) = 2x^3 + x^2 - 20x + 4$$

$$f(x) = x^3 - 27x + 5$$

$$f(x) = 69 - 24x - 9x^2 + 2x^3$$

Sqn

$$f(x) = x^3 - 5x - 11$$

$$f'(x) = 3x^2 - 5$$

 f' is increasing iff $f''(x) > 0$

$$3x^2 - 5 > 0$$

$$3(x^2 - 5/3) > 0$$

$$(x - \sqrt{5}/3)(x + \sqrt{5}/3) > 0$$

$$\begin{array}{c} + + + + + - \\ \hline -\sqrt{5}/3 \qquad \qquad \qquad \sqrt{5}/3 \end{array} \quad x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$$

and f is decreasing iff $f'(x) < 0$

$$3x^2 - 5 < 0$$

$$3(x^2 - 5/3) < 0$$

$$(x - \sqrt{5}/3)(x + \sqrt{5}/3) < 0$$

$$\begin{array}{c} + + + + + + + - \\ \hline -\sqrt{5}/3 \qquad \qquad \qquad \sqrt{5}/3 \end{array} \quad x \in (-\sqrt{5}/3, \sqrt{5}/3)$$

$$2) f(x) = x^2 - 4x$$

$$f'(x) = 2x - 4$$

 $f(x)$ is increasing iff $f'(x) > 0$

$$2x - 4 > 0$$

$$2(x - 2) > 0$$

$$x - 2 > 0$$

$$x \in (2, \infty)$$

and f' is decreasing iff $f'(x) < 0$

$$2x - 4 < 0$$

$$2(x - 2) < 0$$

$$x - 2 < 0$$

$$x \in (-\infty, 2)$$

$$3) f(x) = 2x^3 + x^2 - 20x + 4$$

$$f'(x) = 6x^2 + 2x - 20$$

 f is increasing iff $f'(x) > 0$

$$6x^2 + 2x - 20 > 0$$

$$3x^2 + x - 10 > 0$$

$$3x^2 + 6x - 5x - 10 > 0$$

$$3x(x+2) - 5(x+2) > 0$$

$$(x+2)(3x-5) > 0$$

$$\begin{array}{c} + + + + + + + - \\ \hline -2 \qquad \qquad \qquad 5/3 \end{array} \quad x \in (-\infty, -2) \cup (5/3, \infty)$$

and f is decreasing iff $f'(x) < 0$

$$6x^2 + 2x - 20 < 0$$

$$2(3x^2 + x - 10) < 0$$

$$3x^2 + x - 10 < 0$$

$$3x^2 + 6x - 5x - 10 < 0$$

$$3x(x+2) - 5(x+2) < 0$$

$$(x+2) - 5(x+2) < 0$$

$$(x+2)(3x-5) < 0$$

$$\begin{array}{c} + + + + + + + + + + + + + - \\ \hline -2 \qquad \qquad \qquad 5/3 \end{array} \quad x \in (-2, 5/3)$$

$$2) f(x) = x^3 - 27x + 5$$

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$$f'(x) = 3x^2 - 27$$

f is decreasing iff $f'(x) < 0$

$$3(x^2 - 9) < 0$$

$$(x-3)(x+3) < 0$$

$$\begin{array}{c|cc} + & + & + \\ \hline -3 & - & + \\ \hline -3 & & + \end{array}$$

$$x \in (-\infty, -3) \cup (3, \infty)$$

and f' is decreasing iff $f'(x) < 0$

$$3x^2 - 27 < 0$$

$$3(x^2 - 9) < 0$$

$$(x-3)(x+3) < 0$$

$$\begin{array}{c|cc} + & + & + \\ \hline -3 & - & + \\ \hline -3 & & + \end{array}$$

$$x \in (-3, 3)$$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$$f'(x) = 6x^2 - 18x - 24$$

$\therefore f'$ is increasing iff $f'(x) > 0$

$$6x^2 - 18x - 24 > 0$$

$$6(x^2 - 3x - 4) > 0$$

$$x(x-4) + 1(x-4) > 0$$

$$(x-4)(x+1) > 0$$

$$\begin{array}{c|cc} + & - & + \\ \hline -1 & & + \\ \hline & & + \end{array}$$

$$x \in (-\infty, -1) \cup (4, \infty)$$

and f' is decreasing iff $f'(x) < 0$

$$6x^2 - 18x - 24 < 0$$

$$6(x^2 - 3x - 4) < 0$$

$$x^2 - 4x - x - 4 < 0$$

$$(x-4)(x+1) < 0$$

$$\begin{array}{c|cc} + & - & + \\ \hline -1 & + & + \\ \hline & & + \end{array} \quad x \in (-1, 4)$$

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2) Find intervals in which functions is concave upwards

$$y = 3x^2 - 2x^3 ; y = x^3 - 27x + 5$$

$$y = x^4 - 6x^3 + 12x^2 - 15x + 7 ; y = 69 - 24x - 9x^2 + 2x^3$$

$$y = 2x^3 + x^2 - 20x + 4$$

soln 1) $y = 3x^2 - 2x^3 = f(x)$

$$f'(x) = 6x - 6x^2$$

$$f''(x) = 6 - 12x$$

f is continuous upward if $f''(x) > 0$

$$(6 - 12x) > 0$$

$$12(6/12 - x) > 0$$

$$x - 1/2 > 0 \therefore x > 1/2$$

$$\therefore f''(x) > 0 \quad x \in (y_2, \infty)$$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$$f'(x) = 4x^3 - 18x^2 + 24x + 5$$

$$f''(x) = 12x^2 - 36x + 24$$

f is concave upward if $f''(x) > 0$

$$12x^2 - 36x + 24 > 0$$

$$12(x^2 - 3x + 2) > 0$$

$$x^2 - 2x - x + 2 > 0$$

$$x(x-2) - 1(x-2) > 0$$

$$(x-2)(x+1) > 0$$

$$\begin{array}{c|cc} + & - & + \\ \hline -1 & + & + \\ \hline & & + \end{array} \quad x \in (-\infty, -1) \cup (2, \infty)$$

3) $y = x^3 - 27x + 5$

$$f'(x) = 3x^2 - 27$$

$$f''(x) = 6x$$

f is concave upward iff $f''(x) > 0$

$$6x > 0$$

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$$x \in (0, \infty)$$

9) $y = 6x - 24x - 9x^2 + 2x^3$
 $f'(x) = 6x^2 - 18x - 24$

$$f''(x) = 12x - 18$$

f' is concave upwards iff $f''(x) > 0$

$$12x - 18 > 0$$

$$12(x - \frac{18}{12}) > 0$$

$$x - \frac{3}{2} > 0 \rightarrow x > \frac{3}{2}$$

$$x \in (\frac{3}{2}, \infty)$$

5) $y = 2x^3 + 2x^2 - 20x + 4$
 $f'(x) = 6x^2 + 2x - 20$

$$f''(x) = 12x + 12$$

f' is concave upward iff $f''(x) > 0$

$$12x + 12 > 0$$

$$12(x + \frac{2}{12}) > 0$$

$$x + \frac{1}{6} > 0$$

$$x < -\frac{1}{6}$$

$$f''(x) > 0$$

There exist interval $(-\frac{1}{6}, \infty)$

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PRACTICAL No. 4.

Topic : Application of derivative & Newton's Method.

a) Find max & min value of following

$$i) f(x) = x^2 + \frac{16}{x^2}$$

$$f(x) = 3 - 5x^3 + 3x^5$$

$$f(x) = x^3 - 3x^2 + 1 \quad [-\frac{1}{2}, 4]$$

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

$$[-2, 3]$$

b) Find the root of following equation by Newton's (Iteration) correct upto 4 decimal.

$$f(x) = x^3 - 3x^2 - 5.5x + 9.5 \text{ (take } x_0 = 0)$$

$$f(x) = x^3 - 4x - 9 \text{ in } [2, 3]$$

$$f(x) = x^3 - 18x^2 - 10x + 17 \text{ in } [1, 2]$$

9) i) $f(x) = x^2 + 16/x^2$

$$f'(x) = 2x - 32/x^3$$

Now consider, $f'(x) = 0$

$$2x - 32/x^3 = 0$$

$$2x = 32/x^2$$

$$2x = 32/2$$

$$x = \pm 2$$

$$f'(x) = 2 + 96/x^4$$

$$f''(x) = 2 + \frac{96}{24} = 2 + 96/16$$

$$= 2 + 6$$

$$= 8 > 0$$

f' has min. value at $x = 2$

$$f(2) = 2^2 + 16/2^2$$

$$= 4 + 16/4$$

$$= 4 + 4 = 8$$

$$f''(-2) = 2 + 96$$

$$= 2 + 96/16$$

$$= 2 + 6$$

$$= 8 > 0$$

f' has minimum value at $x = 2$

Functions reaches minimal value at $x=2$, and $x=-2$

$$f(x) = 3 - 5x^3 + 3x^5$$

$$f'(x) = -15x^2 + 15x^4$$

$$\text{Consider, } f'(x) = 0$$

$$-15x^2 + 15x^4 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$f''(x) = -30x + 60x^3$$

$$f(1) = -30 + 60$$

$= 30 > 0$ has min value at $x=1$

$$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$$

$$= 6 - 5 = 1$$

$$f''(-1) = -30(-1) + 60(-1)^3$$

$$= 30 - 60$$

$= -30 < 0$ has max value at $x=-1$

$$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$$

$$= 3 + 5 - 3 = 5$$

f has max value 5 at $x=-1$, and has the min value 1 at $x=1$

$$f(x) = x^3 - 3x^2 + 1$$

$$f'(x) = 3x^2 - 6x$$

$$\text{Consider, } f'(x) = 0$$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$3x > 0 \text{ OR } x-2 = 0$$

$$x = 0$$

$$f''(x) = 6x - 6$$

$$f''(0) = 0$$

$= -6 < 0$ has max value at $x=0$

$$3) f(x) = x^3 - 3x^2 + 1$$

$$f'(x) = 3x^2 - 6x$$

$$\text{Consider, } f'(x) = 0$$

$$3x^2 - 6x = 0$$

$$3x^2(x-2) = 0$$

$$3x=0 \text{ OR } x=2$$

$$x=0$$

$$f''(0) = 6x - 6$$

$= -6 < 0$ $\therefore f$ has max value at $x=0$

$$f(0) = 0^3 - 3(0)^2 + 1 = 1$$

$$f''(2) = 6(2) - 6 = 12 - 6 = 6 \quad \text{min value at } x=2$$

$$f(2) = (2)^3 - (3)(2)^2 + 1$$

$$= 8 - 3(4) + 1 = -3 \quad \text{max value at } x=2 \text{ and } f \text{ has}$$

min value -3 at $x=2$.

$$4) f(x) = 2x^3 - 3x^2 - 12x + 1$$

$$f'(x) = 6x^2 - 6x - 12$$

$$\text{Consider, } f'(x) = 0$$

$$6x^2 - 6x - 12 = 0$$

$$x^2 - x - 2 = 0$$

$$x^2 + x - 2x - 2 = 0$$

$$x(x+1) - 2(x+1) = 0$$

$$x=2 \text{ OR } x = -1$$

$$f''(x) = 12x - 6$$

$$f''(2) = 12(2) - 6$$

$= 18 > 0$ min value at $x=2$

$$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$$

$$= -19$$

$$f''(-1) = 12(-1) - 6$$

$$= -12 - 6 = -18 < 0$$

f has max value at $x=-1$

$$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$$

$$= 8$$

f has max min value -1 at $x=-1$

$$2) f(x) = x^3 - 4x^2 - 55x + 95$$

[2,3]

$$\begin{aligned} f'(x) &= 3x^2 - 8x - 55 \\ f(2) &= 23 - 4(2) - 9 \\ &= 8 - 8 - 9 \\ &= 9 \end{aligned}$$

Let $x_0 = 3$ be initial approx.

By Newton's Method,

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

$$\begin{aligned} x_1 &= 3 - f(3)/f'(3) \\ &= 2.7392 \end{aligned}$$

$$\begin{aligned} f(x_1) &= (2.7392)^3 - 4(2.7392) - 9 \\ &= 0.596 \end{aligned}$$

$$\begin{aligned} f(x_1) &= 3(2.7392)^2 - 4 \\ &= 22.5096 - 4 \\ &= 18.5096 \end{aligned}$$

$$\begin{aligned} x_2 &= x_1 - f(x_1)/f'(x_1) \\ &= 2.7071 \end{aligned}$$

$$\begin{aligned} f(x_2) &= (2.7071)^3 - 4(2.7071) \\ &= 0.0102 \end{aligned}$$

$$\begin{aligned} f'(x_2) &= 3(2.7071)^2 - 4 \\ &= 17.9851 \end{aligned}$$

$$= 2.7071 - 0.0102 = 2.7015$$

$$\begin{aligned} f(x_3) &= (2.7015)^3 - 4(2.7015) - 9 \\ &= 19.758 - 10.806 - 9 \\ &= -0.0091 \end{aligned}$$

$$f'(x_3) = 3(2.7015)^2 - 4 = 17.8943$$

$$\begin{aligned} f(4) &= 2.7015 + 0.0091 / 17.8943 \\ &= 2.7065 \end{aligned}$$

$$3) f(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1, 2]$$

$$\begin{aligned} f'(x) &= 3x^2 - 3.6x - 10 \\ &= (1)^3 - 1.8(1)^2 - 10(1) + 17 \\ &= 6.2 \end{aligned}$$

$$q2) i) f(x) = x^3 - 3x^2 - 55x + 95 \quad x_0 = 0 \rightarrow \text{given}$$

By Newton's Method,

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

$$x_1 = x_0 - f(x_0)/f'(x_0)$$

$$x_1 = 0 + 95/55$$

$$x_1 = 0.1727$$

$$\begin{aligned} f(x_1) &= 0.0051 - 0.0895 - 9.4985 + 9.5 \\ &= -0.0829 \end{aligned}$$

$$\begin{aligned} f'(x) &= 3(0.1727)^2 - 6(0.1727) - 55 \\ &= -55.9467 \end{aligned}$$

$$\begin{aligned} x_2 &= x_1 - f(x_1)/f'(x_1) \\ &= 0.1712 \end{aligned}$$

$$\begin{aligned} f(x_2) &= 0.0050 - 0.0879 - 9.416 + 9.5 \\ &= 0.0011 \end{aligned}$$

$$\begin{aligned} f'(x_2) &= 3(0.1712)^2 - 6(0.1712) - 55 \\ &= -55.9393 \end{aligned}$$

$$\begin{aligned} x_3 &= x_2 - f(x_2)/f'(x_2) \\ &= 0.1712 \end{aligned}$$

$$2) f(x) = x^3 - 3x^2 - 55x + 95 \quad x_0 \rightarrow 0 \rightarrow \text{given}$$

$$f'(x) = 3x^2 - 6x - 55$$

By Newton's Method,

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

$$x_1 = x_0 + 95/55$$

$$x_1 = 0.1727$$

Q4.

$$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17 \\ = -2.2$$

Let $x_0 = 2$ be initial approx.

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \\ = x_0 - \frac{f(x_0)}{f'(x_0)} \\ = 2 - \frac{-2.2}{-8.2164} \\ = 1.577$$

$$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17 \\ = 0.6755$$

$$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10 \\ = -8.2164$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \\ = 1.577 + \frac{0.6755}{-8.2164} \\ = 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17 \\ = 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10 \\ = -7.7143$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10 \\ = -7.6977$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \\ = 1.6618$$

PRACTICAL No.5

Topic - Integration

$$1) \int \frac{dx}{\sqrt{x^2 + 2x - 3}}$$

$$= \int \frac{1}{\sqrt{x^2 + 2x - 4}} dx$$

$$= a^2 + 2ab + b^2 = (a+b)^2$$

$$= \int \frac{1}{\sqrt{(x+1)^2 - 4}} dx$$

Substitute put $x = 1 = t$

$$dx = \frac{1}{t} dt \quad \text{where } t-1, t=x+1$$

$$\int \frac{1}{\sqrt{t^2 - 4}} dt$$

$$\text{using } \int \frac{1}{\sqrt{t^2 - a^2}} dt = \ln(t + \sqrt{t^2 - a^2})$$

$$= \ln(t + \sqrt{t^2 - 4})$$

$$= \ln(t + \sqrt{t^2 + 2t - 3})$$

$$= \ln(t + \sqrt{t^2 + 2t - 3}) + C$$

$$2) \int (4e^{3x} + 1) dx \\ = 4 \int e^{3x} dx + \int 1 dx \quad \# \quad \int e^{ax} = \frac{1}{a} x e^{ax}$$

$$= \frac{4e^{3x}}{3} + x + C$$

$$\begin{aligned}
 3) & \int 2x^2 - 3\sin(x) + 5\sqrt{x} \, dx \\
 &= \int 2x^2 - 3\sin(x) + 5^{1/2} \, dx \\
 &= \int 2x^2 \, dx - \int 3\sin(x) \, dx + \int 5\sqrt{x} \, dx \\
 &= \frac{2x^3}{3} + \cos(x) + \frac{10\sqrt{x}}{3} + C \\
 &= 2x^3 + 10\sqrt{x}/3 + 3\cos(x) + C
 \end{aligned}$$

$$\begin{aligned}
 4) & \int x^3 + 3x + 4/\sqrt{x} \, dx \\
 &\text{# split the denominator.} \\
 &= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} \, dx \\
 &= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}} \, dx \\
 &= \int x^{5/2} \, dx + \int 3x^{1/2} \, dx + \int \frac{4}{x^{1/2}} \, dx
 \end{aligned}$$

$$\begin{aligned}
 5) & \int t^7 x \sin(2t^4) \, dt \\
 &\text{put } u = 2t^4 \\
 &du = 8t^3 \, dt
 \end{aligned}$$

$$\begin{aligned}
 & \int t^7 x \sin(2t^4) \lambda \frac{1}{2x^4} \times \frac{1}{2x^4} \, du \\
 &= \int t^9 \sin(2t^4) \times \frac{1}{8} \, du - \left(\frac{t^9 x \sin(2t^4)}{8} \right) \, du
 \end{aligned}$$

Substitute

$$\begin{aligned}
 & \int \frac{4/2 x \sin(u)}{2} / 8 \, du \\
 & \int \frac{4x \sin(u)}{16} \, du.
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{16} \int 4x \sin(u) \, du \\
 & \quad \int u \, dv \\
 &= \frac{1}{16} (ux(-\cos(u)) - \int -\cos(u) \, du) \\
 &= \frac{1}{16} x(4x(-\cos(u))) + \int \cos(u) \, du \\
 &= \frac{1}{16} x(4x(-\cos(u))) + \sin(u)
 \end{aligned}$$

Return the sub. $u = -2t^4$

$$\begin{aligned}
 &= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4)) \\
 &= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4) + C}{16}
 \end{aligned}$$

$$6) \int \sqrt{x}(x^2 - 1) \, dx$$

$$\begin{aligned}
 &= \int \sqrt{x}x^2 - \sqrt{x} \, dx \\
 &= \int x^{5/2} - x^{1/2} \, dx
 \end{aligned}$$

$$= \int x^{5/2} \, dx - \int x^{1/2} \, dx$$

$$\begin{aligned}
 J_1 \frac{x^{5/2} + 1}{5/2 + 1} &= \frac{2x^{3/2}\sqrt{x}}{7} & J_2 = \frac{x^{1/2} + 1}{1/2 + 1} &= \frac{2x^{3/2}}{3/2} &= \frac{2\sqrt{x}}{3}
 \end{aligned}$$

$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x}}{3} + C$$

$$\int \frac{\cos x}{\sqrt[3]{\sin(x^2)}} \, dx$$

$$= \int \left(\frac{\cos x}{\sin(x^2)^{1/3}} \right) \, dx$$

$$\begin{aligned}
 &\text{put } t = \sin(x) \\
 &t = \cos x.
 \end{aligned}$$

$$\int \frac{\cos(x)}{\sin(x)^{1/3}} \times \frac{1}{\cos x(t)} \, dt$$

14.

$$= \frac{1}{\sin x^{3/2}} dt$$

$$= \frac{1}{t^{2/3}} dt$$

$$I = \int \frac{1}{t^{2/3}} dt = \frac{-1}{(2/3-1)} t^{2/3-1}$$

$$= \frac{-1}{(2/3+1)} t^{2/3-1}$$

$$= 3J$$

$$x) \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$$

$$\text{put } x^3 - 3x^2 + 1 = dt$$

$$J = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3x + 2x} dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3} dt$$

$$= \int \frac{1}{3(x^3 - 3x^2 + 1)} dt = \int \frac{1}{3t} dt$$

$$= 1/3 \int 1/t dt = 1/3 \ln|t| + C_1$$

$$= 1/3 \ln|x^3 - 3x^2 + 1| + C_1$$

?

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PRACTICAL NO.6.

Aim-

Application of Integration and Numerical Integration.

Q1) Find the length of the following curve-

$$1) x = t \sin t, y = 1 - \cos t \quad t \in [0, 2\pi]$$

$$2) y = \sqrt{4x^2} \quad x \in [-2, 2]$$

$$3) y = 3^{1/2} \text{ in } [0, 4]$$

$$4) x = 3t \sin t, y = 3 \cos t \quad t \in [0, 2\pi]$$

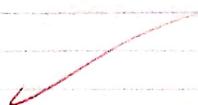
$$5) x = \frac{1}{6} y^3 + \frac{C}{2y} \text{ on } y \in [1, 2]$$

Q2) Using Simpson's Rule solve the following

$$1) \int_0^2 e^{x^2} dx \text{ with } n=4$$

$$2) \int_0^4 x^2 dx \text{ with } n=4$$

$$3) \int_0^{1/3} \sqrt{\sin x} dx \text{ with } n=6.$$



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$$1) y = \sqrt{4-x^2} \quad x \in [-2, 2]$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \cdot (-2x)$$

$$= \frac{-2x}{\sqrt{4-x^2}}$$

$$L = \int_{-2}^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_{-2}^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2} dx$$

$$= \int_{-2}^2 \sqrt{\frac{4-x^2+x^2}{4-x^2}} dx$$

$$= \int_{-2}^2 \sqrt{\frac{4}{4-x^2}} dx$$

$$= \int_{-2}^2 \frac{2}{\sqrt{4-x^2}} dx$$

$$= 2 \left[\sin^{-1}(x/2) \right]_{-2}^2$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= 2 \left[\pi/2 - (\pi/2) \right]$$

$$= 2 \left[\pi/2 + \pi/2 \right]$$

$$= 2\pi$$

$$2) \text{ if } x = t^{3/2} \text{ in } [0, 4]$$

$$\frac{dy}{dx} = \frac{3}{2} t^{1/2} - 1$$

$$L = \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_0^4 \sqrt{1 + \left(\frac{3\sqrt{t}}{2}\right)^2} dt$$

$$= \int_0^4 \sqrt{(1+9t/4)} dt$$

$$= \int_0^4 \sqrt{4+9t} dt$$

$$= \frac{1}{2} \left[(4+9t)^{1/2+1} \right]_0^{1/2+1}$$

$$= \frac{1}{2} \left[\frac{(4+9t)^{3/2}}{3/2} \times \frac{1}{9} \right]_0^4$$

$$= \frac{1}{2} \left[(4+9t)^{3/2} \right]_0^4$$

$$= \frac{1}{2} (4+0)^{3/2} - (4+36)^{3/2}$$

$$= \frac{1}{2} (4)^{3/2} - (40)^{3/2}$$

$$= \frac{1}{2} (4)^{3/2} - 8 \text{ units}$$

$$3) x = 3\sin t \quad y = 3\cos t$$

$$\frac{dx}{dt} \frac{dy}{dt} = 3\cos t \quad ; \quad \frac{dy}{dx} = -3\sin t$$

$$= \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2} dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t} dt$$

Q2

$$\begin{aligned}
 &= 2\int_0^{\pi} \sqrt{9(1)} dt \\
 &= 4\int_0^{\pi} 3dt \\
 &= 3\int_0^{2\pi} dt \\
 &= 3[2x]_0^{2\pi} \\
 &= 3(2\pi - 0) \\
 &= 6\pi.
 \end{aligned}$$

2) $\frac{dx}{dy} = \frac{1}{t} \frac{d}{dy}(y^3) - 1 \cdot \frac{1}{2} \frac{dx}{dy} \left(\frac{1}{y}\right)$

$$\begin{aligned}
 &= \frac{y^2}{2} - \frac{1}{2}y^2 \\
 &= \frac{y^2 - 1}{2y^2}
 \end{aligned}$$

$$l = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$\int_1^2 \sqrt{1 + \left(\frac{y^2 - 1}{2y^2}\right)^2} dy$$

$$\int_1^2 \sqrt{1 + \left(\frac{y^2 - 1}{2y^2}\right)^2} dy$$

$$\int_1^2 \sqrt{\frac{(y^2 + 1)^2}{4y^4}} dy$$

$$\int_1^2 \frac{\sqrt{(y^2 + 1)^2}}{(2y^2)^2} dy$$

$$\begin{aligned}
 &= \int_1^2 \frac{y+1}{2y^2} dy \\
 &= \int_1^2 \frac{y+1}{2y^2} dy = \int_1^2 \frac{1}{2y^2} dy \\
 &= \frac{1}{2} \int_1^2 y dy - \frac{1}{2} \int_1^2 \frac{1}{y^2} dy \\
 &= \frac{1}{2} \left[\frac{y^2}{2} - \frac{1}{y} \right]_1^2 \\
 &= \frac{1}{2} \left[\frac{2}{3} - \frac{1}{2} - \frac{1}{3} + 1 \right] \\
 &= \frac{1}{2} \left[\frac{7}{3} + \frac{1}{2} \right] = \frac{17}{12}.
 \end{aligned}$$

5) Curve length $\int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

$$\begin{aligned}
 dt &= \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2} dt \\
 &= \int_0^{2\pi} \sqrt{1-2\cos t+\cos^2 t + \sin^2 t} dt \\
 &= \int_0^{2\pi} \sqrt{2-2\cos t} dt \\
 &= \int_0^{2\pi} 2\sin \frac{1}{2} dt \\
 &= \left[-2\cos \frac{1}{2} \right]_0^{2\pi} \\
 &= (-4\cos \pi) + 4\cos 0 \\
 &= 8 \text{ units.}
 \end{aligned}$$

9.2) $\int_0^2 e^{x^2} dx$ with $n=4$.

$$a=0, b=2, n=4$$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

x	0	0.5	1	1.5	2
f(x)	1.284	2.718	9.487	54.598	

$$2) \int_0^2 x^2 dx = \frac{h}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

$$= \frac{0.5}{3} [0 + 4(1.284) + 2(2.718) + 4(9.487)]$$

$$+ 54.598$$

$$= \frac{0.5}{3} [104.118]$$

$$= 17.353$$

$$3) \int_0^4 x^2 dx, n=4$$

$$a=0, b=4, n=4$$

$$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
f(x)	0	1	4	9	16

$$\int_0^4 x^2 dx = \frac{h}{3} [0 + 4(1) + 2(4) + 4(9) + 16]$$

$$= \frac{1}{3} [0 + 4 + 8 + 36 + 16]$$

$$= \frac{1}{3} [64]$$

$$= 21.333$$

$$5) \int_0^{\pi/3} \sqrt{\sin x} dx \text{ with } n=6$$

$$h = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi/3}{6} = \frac{\pi}{18}$$

$$x \quad 0 \quad \frac{\pi}{18} \quad \frac{3\pi}{18} \quad \frac{4\pi}{18} \quad \frac{5\pi}{18} \quad \frac{6\pi}{18} \quad \left(\frac{\pi}{3}\right)$$

$$y \quad 0 \quad 0.4167 \quad 0.7071 \quad 0.8017 \quad 0.8752 \quad 0.9306$$

$$y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5$$

$$\int_0^{\pi/3} \sqrt{\sin x} dx = \frac{h}{3} [0.4167 + 0.9306 + 4(0.4167 + 0.7071 + 0.8017) + 2(0.8752 + 0.9306)]$$

$$= \frac{\pi}{18} [1.3473 + 4(1.999) + 2(1.3865)]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$= \frac{\pi}{18} \int \sqrt{\sin x} dx = 0.7049.$$

PRACTICAL No.7

Topic - Differential Equation

Q.1) Solve the following equation.

$$1) \frac{dy}{dx} + y = e^x$$

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$$

$$I.F = \frac{1}{x} \int e^x dx$$

$$= e^{\ln x}$$

$$D.P = x$$

$$y(DP) = \int \phi(x) (I.F) dx + c$$

$$= \int e^x x dx + c$$

$$= \int e^x dx + 1$$

$$xy = e^x + c_1$$

$$2) e^x \frac{dy}{dx} + 2e^x y = 1$$

$$\frac{dy}{dx} + 2e^x y = \frac{1}{e^x} \quad (\div by e^x)$$

$$\frac{dy}{dx} + 2y = \frac{1}{e^x}$$

$$\frac{dy}{dx} + 2y = e^{-x}$$

$$P(x) = 2 \quad \phi(x) = e^{-x}$$

$$\int P(x) dx$$

$$I.F = c \int e^{2x} dx$$

$$y(DP) = \int \phi(x) (I.F) dx + c$$

$$ye^{2x} / e^{-x} + 2x dx + c$$

$$= \int e^x dx + c$$

$$y \cdot e^{2x} = e^x + c$$

$$3) \frac{xdy}{dx} = \frac{\cos x}{x} - 2y$$

$$x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\frac{dy + 2y}{dx} = \frac{\cos x}{x^2}$$

$$P(x) = 2/x \cdot \phi(x) = \cos x / x^2$$

$$D.P = \int P(x) dx$$

$$= c \int 2/x dx$$

$$= e^{2/x} x$$

$$= \ln x^2$$

$$D.P = x^2$$

$$y(DP) = \int \phi(x) (I.F) dx + c$$

$$= \int \frac{\cos x}{x^2} x^2 dx + c$$

$$= \int \cos x dx + c$$

$$x^2 y \downarrow \sin x + c$$

$$4) \frac{xdy}{dx} + 3y = \frac{\sin x}{x^2}$$

Ex

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\cos x}{x^2} \quad (\div \text{ by } x \text{ on LHS})$$

$$P(x) = \frac{3}{x}, \quad q(x) = \frac{\cos x}{x^2}$$

$$= c \int P(x) dx$$

$$= c \int \frac{3}{x} dx$$

$$= c \ln x$$

$$= e^{c \ln x}$$

$$I.P. = x^3$$

$$y(I.P.) = \int q(x)(I.P.) dx + C$$

$$= \int \frac{\cos x}{x^2} x^3 dx + C$$

$$= \int \cos x dx + C$$

$$x^3 y = -\cos x + C$$

$$5) \quad e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$$P(x) = 2, \quad q(x) = 2x/e^{2x} = 2xe^{-2x}$$

$$I.P. = c \int P(x) dx$$

$$= c \int 2 dx$$

$$= c^2 x$$

$$y(I.P.) = \int q(x)(I.P.) dx + C$$

$$= \int 2xe^{-2x} e^{2x} dx + C$$

$$= \int 2x dx + C$$

$$ye^{2x} = x^2 + C$$

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$$7) \sec^2 x \tan x dy + \sec^3 y \tan x dy = 0$$

$$\sec^2 x \tan x dy = -\sec^3 y \tan x dy$$

$$\frac{\sec^3 x dy}{\tan x} = -\frac{\sec^3 y dy}{\tan y}$$

$$\int \frac{\sec^3 x dy}{\tan x} = - \int \frac{\sec^3 y dy}{\tan y}$$

$$\log |\tan x| = -\log |\tan y| + C$$

$$\log |\tan x| - \log |\tan y| = C$$

$$\tan x \cdot \tan y = e^C$$

$$7) \frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

$$\text{Put } 2x+3y = v$$

$$2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3} \left(\frac{dv}{dx} - 2 \right)$$

$$\frac{1}{3} \left(\frac{dv}{dx} - 2 \right) = \frac{1}{3} \left(\frac{v-1}{v+2} \right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{v+2}$$

$$\int \left(\frac{v+2}{v+1} \right) dv = 3dx$$

$$= \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = 3x$$

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$$v + \log|x| = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c$$

$$8) \frac{dy}{dx} = \frac{2x+3y+1}{6x+9y+3} \sin^2(x-y+1)$$

$$\text{Put } x-y+1=v$$

Diff w.r.t. x

$$x-y+1=v$$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$-\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

AR
10/12/2020

PRACTICAL NO. 8

Aim-

Using Euler's Method find following

$$1) \frac{dy}{dx} = y + e^{x-2}, y(0) = 2, h=0.5 \text{ find } y(2)$$

$$2) \frac{dy}{dx} = 1+y^2, y(0) = 0, h=0.2 \text{ find } y(0)$$

$$3) \frac{dy}{dx} = \sqrt{\frac{x}{y}}, y(0) = 1, h=0.2 \text{ find } y(0)$$

$$4) \frac{dy}{dx} = 3x^2 + 1, y(1) = 2, \text{ find } y(2)$$

for $h=0.5$, & $h=0.25$.

$$5) \frac{dy}{dx} = \sqrt{xy} + 2, y(0), \text{ find } y(1.2) \text{ with } h=0.2.$$

$$\frac{dy}{dx} = 3x^2 + 2$$

$$f(x,y) = y + x^2 + 2 \quad , y_0=2 \quad , x_0=0 \quad , h=0.5$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2		2.5
1	0.5	2.5	2.487	3.57435
2	1	3.57435	4.2225	5.3615

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.28305		

By Euler's formula,

$$y(2) \approx 9.2831$$

$$\frac{dy}{dx} = 1+y^2$$

$$f(x, y) = 1+y^2 \quad , y_0=0 \quad , x_0=0 \quad , h=0.2$$

Using Euler's Iteration formulae.

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8503	1.2942
5	1	1.2942		

By Euler's formulae

$$y(1) = 1.2942$$

$$3) \frac{dy}{dx} = \sqrt{\frac{x}{y}} \quad y(0)=1 \quad , x_0=0 \quad , h=0.2$$

Using Euler's Iteration formulae,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

$$3) \frac{dy}{dx} = 3x^2 + 1 \quad y(0)=2 \quad , x_0=1 \quad , h=0$$

For $h=0.5$

Using Euler's Iteration Method,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	2.85
1	1.5	4	49	
2	2	28.5	99	

By using Euler's Method,

$$y(2) = 28.5$$

For $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

By Euler's Formula-

$$y(2) = 8.9048$$

$$5) \frac{dy}{dx} = \sqrt{xy} + 2 \quad y_0 = 1, x_0 = 1, h = 0.2$$

Using Euler's Iteration formula,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

By Euler's formula,

$$y(1.2) = 1.6$$

PRACTICAL NO. 9

Topic - Limit & partial order derivative

i) Evaluate the following limits.

$$\lim_{(x,y) \rightarrow (-4, -1)} (2, 0) \frac{-x^3 - 3y + y^2 + 1}{xy + 5} \frac{(y+1)(x^2 + y^2 - 4x)}{x+3y}$$

Applying limit.

$$\frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{(-4)(-1) + 5}$$

$$\frac{(0+1)(2)^2 + (0)^2 - 4(2)}{2 + 3(0)}$$

$$\frac{1(4-8)}{2} = -\frac{4}{2} = -2$$

$$\text{i)} \lim_{(x,y) \rightarrow (-4, -1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$$

Applying limit,

$$\frac{(0+1)(2)^2 - (0)^2 - 4(2)}{2 + 3(0)}$$

$$\frac{1(4-8)}{2} = -\frac{4}{2} = -2$$

$$\text{ii)} \lim_{(x,y,z) \rightarrow (1,1,1)} \frac{x^2 - y^2 - z^2}{x^2 - xy - yz}$$

$$\lim_{(x,y,z) \rightarrow (1,1,1)} \frac{(x)^2 - (yz)^2}{x^2 - xy - yz}$$

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$$\lim_{(x,y,z) \rightarrow (1,1,1)} \frac{(x+yz)(x-zy)}{x^2(x-yz)} \quad (\because (a^y - b^y) = (a+b)(a-b))$$

$$\lim_{(x,y,z) \rightarrow (1,1,1)} \frac{x+yz}{x} \\ \frac{1+(1)(1)}{(1)^2} = 2$$

2) Find f_x, f_y for each of the following

$$f(x,y) = xy e^{x^2+y^2}$$

$$f(x) = \frac{\partial f}{\partial x}$$

$$= \frac{\partial (xy e^{x^2+y^2})}{\partial x}$$

$$= y \frac{\partial (xe^{x^2+y^2})}{\partial x}$$

$$= y \left[x \cdot \frac{\partial}{\partial y} (e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{\partial}{\partial x} (x) \right]$$

$$[\because \frac{\partial (uv)}{\partial x} = uv' + u'v]$$

$$= y \left[x \cdot e^{x^2+y^2} \cdot 2x + e^{x^2+y^2} (1) \right]$$

$$= y \cdot e^{x^2+y^2} [2x + 1]$$

Now,

~~$$f(y) = \frac{\partial f}{\partial y}$$~~

$$= \frac{\partial (xy e^{x^2+y^2})}{\partial y}$$

$$= x \cdot \frac{\partial}{\partial y} (y \cdot e^{x^2+y^2}) \\ = x \left[y \cdot \frac{d}{dy} (e^{x^2+y^2}) + e^{x^2+y^2} \frac{d}{dy} (y) \right] [\because \frac{d(uv)}{du}] \\ = x [2y \cdot e^{x^2+y^2} + e^{x^2+y^2}] \\ = x \cdot e^{x^2+y^2} [2y^2 + 1]$$

$$(i) f(x,y) = e^x \cos y$$

$$f(x) = e^x \cos y$$

$$f(y) = e^x \frac{\partial}{\partial y} (\cos y)$$

$$= e^x \frac{\partial}{\partial y} (-\sin y)$$

$$= -e^x \sin y$$

$$(ii) f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$$

$$f(x) = e^x (-\sin y)$$

$$f(y) = e^x (-\sin y)$$

$$= -e^x (-\sin y)$$

$$(iii) f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial (x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$$

$$= 3x^2y^2 - 3(2x)y$$

$$= 3x^2y^2 - 6xy$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial (x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial (x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$$

$$f(y) = x^3(2y) - 3(1)x^2 + 3y^2 \\ = 2x^3y - 3x^2 + 3y^2$$

Q.1.

(iii) Using def'n find value of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

$$f_x(a,b) = \lim_{h \rightarrow 0} \frac{f(a+h,b) - f(a,b)}{h}$$

where $(a,b) = (0,0)$

$$f_x(0,0) = \lim_{h \rightarrow 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h - 0}{2} = 2$$

Similarly,

$$f_y(0,0) = \lim_{h \rightarrow 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0 - 0}{h} = 0$$

$$f_x = 2, f_y = 0$$

$$f(x)(0,0) = \lim_{h \rightarrow 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h - 0}{h} = 2$$

$$f(x)(0,0)$$

Q.4) Find all second order pd of f . Also verify whether $f_{xy} = f_{yx}$

$$i) f(x,y) = y^2 - xy/x^2$$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(y^2 - xy)}{\partial x}$$

$$= x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy) \cdot \frac{d}{dx}(x^2)$$

$$= (x^2)^2 \left[\because \frac{d}{dx}\left(\frac{u}{v}\right) \right]$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

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$$f(x) = \frac{-xy - 2xy^2 + 2x^2y}{x^4} = \frac{x(-zy - 2y^2)}{x^4}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(y^2 - xy)}{\partial y}$$

$$= \frac{\partial}{\partial y} \left(\frac{y^2}{x^2} - \frac{xy}{x^2} \right) / \frac{\partial y}{\partial y}$$

$$= \frac{\partial}{\partial y} \left(\frac{y^2}{x^2} - \frac{y}{x} \right) / \frac{\partial y}{\partial y}$$

$$= \frac{1}{x^2} \cdot 2y - \frac{1}{x}$$

$$f(y) = \frac{2y - x}{x^2}$$

$$f_{xx}(xy) = \frac{\partial^2(fy)}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial(fy)}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{\partial}{\partial x} \left(\frac{\partial(fy)}{\partial y} \right) \right) / \frac{\partial x}{\partial x}$$

$$= x^3(y) - (xy)y^2/(3x^3)/6$$

$$= \frac{6x^3y^2 - 3x^3y}{2x^6} = \frac{x^2(6y^2 - 2xy)}{x^6}$$

$$= 6y^2 - 2xy/x^4$$

$$f_{yy}(yy) = \frac{\partial^2(fy)}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial(fy)}{\partial y} \right) = \frac{1}{x^2}(2) = \frac{2}{x^2}$$

$$f_{xy}(xy) = \frac{\partial^2(fy)}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial}{\partial x} \left(\frac{\partial(fy)}{\partial y} \right) \right) = \frac{1}{x^2}(2) = \frac{2}{x^2}$$

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$$f(yx) = \partial \left(\frac{y}{x^2} - \frac{2y^2}{x^3} \right) / \partial y$$

$$= \frac{1}{x^2} - \frac{1}{x^3} \partial(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3 - 4y}{x^4}$$

$$f(yx) = \partial \left(\frac{2y}{x^2} - \frac{x}{x^2} \right) / \partial x$$

$$= \partial \left(\frac{2y}{x^2} - \frac{1}{x} \right) / \partial x$$

$$= 2y \left(-\frac{2}{x^3} \right) - \left(-\frac{1}{x^2} \right)$$

$$= -\frac{4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{x^2(2-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$f(x,y) = f(yx) = \frac{x-4y}{x^4}$$

Hence verified

(ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f(x) = \frac{\partial F}{\partial x} = \partial \left(x^3 + 3x^2y^2 - \log(x^2+1) \right) / \partial x$$

$$f(x) = 3x^2 + 6xy^2 - 2x / x^2 + 1$$

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$$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f(y) = \frac{\partial f}{\partial y} = \partial \left(x^3 + 3x^2y^2 - \log(x^2+1) \right) / \partial y$$

$$= 0 + 3(2y)(x^2) + 0$$

$$= 6x^2y$$

$$f(xy) = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left(3x^2 + 6xy^2 - \frac{2x}{x^2+1} \right)$$

$$= 6x + 6y^2(1) - 2 \left[x^2 + 1(1) - \frac{x(2x)}{x^2+1} \right]$$

$$\left[\because \frac{\partial y}{\partial x} \right]$$

$$= 6x + 6y^2 - 2 \left(\frac{x^2 + 1 - 2x^2}{(x^2+1)^2} \right)$$

$$= 6x + 6y^2 - 2 \left(\frac{-x^2 + 1}{(x^2+1)^2} \right)$$

$$f(yg) = \frac{\partial f}{\partial y} = \frac{\partial(6x^2y)}{\partial y}$$

$$= 6x^2(1) = 6x^2$$

$$f(yx) = \partial \left(3x^2 + 6xy^2 - \frac{2x}{x^2+1} \right) / \partial y$$

$$= 0 + 6x(2y)$$

$$= 12xy$$

$$f(xy) = f(yx) = 12xy$$

Hence verified

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$$

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$$= \cos(2y)(y) + e^{x+y}(1)$$

$$xy \cos(2y) + e^{x+y}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial (\cos(2y) + e^{x+y})}{\partial y}$$

$$= 2x \cos(2y) + e^{x+y}$$

$$f(xy) = \frac{\partial f}{\partial x} = \frac{\partial (y \cos(2y) + e^{x+y})}{\partial x}$$

$$= y \cos(2y)(y) + e^{x+y}(1)$$

$$= y^2 \cos(2y) + e^{x+y}$$

$$f(yx) = \frac{\partial f}{\partial y} = \frac{\partial (x \cos(2y) + e^{x+y})}{\partial y}$$

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$$f(xy) = \frac{\partial f}{\partial y} = \frac{\partial (y \cos(2y) + e^{x+y})}{\partial y}$$

$$+ y[\sin(2y)(x) + \cos(2y)(1)] + e^{x+y}(1)$$

$$\left[\because \frac{d}{du}(uv) \right]$$

$$= -xy \sin(2y) + \cos(2y)x + e^{x+y}$$

$$f(y,x) = \frac{\partial f}{\partial x} = \frac{\partial (x \cos(2y) + e^{x+y})}{\partial x}$$

$$f(y,x) = \frac{\partial f}{\partial x} = \frac{\partial (x \cos(2y) + e^{x+y})}{\partial x}$$

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$$= -\cos(2y)(1) + \partial(-\sin(2y))(y) + e^{x+y}$$

$$= -2y \sin(2y) + \cos(2y) + e^{x+y}$$

$$f(xy) = f(yx) = -xy \sin(2y) + \cos(2y) + e^{x+y}$$

i) Find the linearization of $f(x,y)$ at given.

- $f(x,y) = \sqrt{x^2+y^2} \cdot \cot(1,1)$

$$= \sqrt{(1)^2+(1)^2}$$

$$= \sqrt{2}$$

$$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \quad \therefore 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$\therefore f_x(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \frac{2+\sqrt{2}}{\sqrt{2}} + \frac{(y-1)}{\sqrt{2}}$$

$$= \frac{2+\sqrt{2}+2}{\sqrt{2}} = \frac{2+\sqrt{2}}{\sqrt{2}}$$

- $f(x,y) = 1-x+y \sin x$ at $(\frac{\pi}{2}, 0)$

$$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2}$$

$$f(x) = -1 + y \cos x$$

$$= 1$$

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$$f(x, \frac{x}{2}, 0) = -1 + 0 \cos\left(\frac{\pi}{2}\right)$$

$$= -1$$

$$\begin{aligned} L(x, y) &= f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) \\ &= 1 - \frac{x}{2} - x + \frac{x}{2} + y \end{aligned}$$

$$= y - x + 1$$

iii) $f(x, y) = \log x + \log y$ at $(1, 1)$
 $\rightarrow f(1, 1) = \log(1) + \log(1)$
 $= 0+0$

$$f(x) = \frac{1}{x} \quad f(y) = \frac{1}{y}$$

$$f_x(1, 1) = 1$$

$$f_y(1, 1) = 1$$

$$\begin{aligned} L(x, y) &= f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) \\ &= 0 + 1(x-1) + 1(y-1) \\ &= x-1+ty-1 \\ &= x+ty-2. \end{aligned}$$

PRACTICAL NO. 10

Q1 Find derivatives (directional) of following functⁿ at given point & in the direction of given vector.

$$\Rightarrow f(x, y) = x + 2y - 3 \quad a = (1, -1), v = 3i-j$$

Here,

$v = 3i-j$ is not a unit vector

$$\text{Unit Vector along } v \text{ is } \frac{v}{|v|} = \frac{1}{\sqrt{10}} \cdot (3, -1) \\ \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right)$$

$$f(athu) = f(1, -1) + h \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right)$$

$$f(a) = f(1, -1) = 1 + 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$\begin{aligned} f(athu) &= f(1, -1) + h \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right) \\ &= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{1}{\sqrt{10}}\right) \end{aligned}$$

$$\begin{aligned} f(athu) &= \left(1 + \frac{3}{\sqrt{10}}\right) + 2 \left(-1 - \frac{1}{\sqrt{10}}\right) - 3 \\ &= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2}{\sqrt{10}} - 3 \end{aligned}$$

$$f(athu) = -4 + \frac{h}{\sqrt{10}}$$

$$\Delta u_f(a) = \lim_{h \rightarrow 0} \frac{f(athu) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4 + h\sqrt{10} + 4}{h}$$

$$= \frac{1}{\sqrt{10}}$$

$$i) f(x) = y^2 - 4x + 1 \quad a = (3, 4) \cdot 4 = i + 5j$$

Here $u = i + 5j$ is not a Unit Vector

$$\|u\| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

$$\text{Unit vector along } u \text{ is } \frac{u}{\|u\|} = \frac{1}{\sqrt{26}} = (1, 5)$$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right)$$

$$f(a) = f(3, 4) = (4)^2 - 4(3) + 1 = 5.$$

$$f(a+hu) = f(3, 4) + h \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f(a+hu) = 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$Df(a) = \lim_{h \rightarrow 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

$$ii) 2x + 3y \quad a = (1, 2) \quad u = (3i + 4j)$$

Here $u = 3i + 4j$ is not a Unit Vector

$$\|u\| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$= \left(\frac{3}{5}, \frac{4}{5} \right)$$

$$f(a) = f(1, 2) = 2(1) + 3(2) = 8$$

$$f(a+hu) = f(1, 2) + h \left(\frac{3}{5}, \frac{4}{5} \right)$$

$$f(a+hu) = 2\left(\frac{1+3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5} = \frac{18h}{5} + 8$$

$$Df(a) = \lim_{h \rightarrow 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= \frac{18}{5}$$

ii) Find gradient vector for following function at given point

$$i) f(x, y) = xy + y^x \Rightarrow a(1, 1)$$

$$fx = y \cdot x^{y-1} + y^x \log y$$

$$fy = x^y \log x + x^y x^{-1}$$

$$\nabla f(x, y) = (fx, fy)$$

$$= (y x^{y-1} + y^x \log y, x^y \log x + x^y x^{-1})$$

$$f(1, 1) = (1+0, 1+0)$$

$$= (1, 1)$$

$$ii) f(x, y) = (\tan^{-1} x)y^2 \Rightarrow a(1, -1)$$

$$fx = \frac{1}{1+x^2} \cdot y^2$$

~~$$fy = 2y \tan^{-1} x$$~~

~~$$\nabla f(x, y) = (fx, fy)$$~~

~~$$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x \right)$$~~

$$f(1, -1) = \left(\frac{1}{2}, \frac{1}{4}(-2) \right)$$

$$= \left(\frac{1}{2}, -\frac{1}{2} \right)$$

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$$\text{iii) } f(x, y, z) = xyz - e^{x+y+z} \quad a = (1, -1, 0)$$

$$fx = yz - e^{x+y+z}$$

$$fy = xz - e^{x+y+z}$$

$$= xy - e^{x+y+z}$$

$$\nabla f(x, y, z) = fx, fy, fz$$

$$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$$

$$y(1, -1, 0) = (-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{1+(-1)+0}$$

$$(1)(-1)e^{1+(-1)+0}$$

$$= (0 - e^0, 0 - e^0, -1 - e^0) = (-1, -1, 2)$$

Q.3) Find equation of tangent & normal to each of the following curves at given points.

$$\text{i) } x^2 \cos y + e^{xy} = 2 \text{ at } (1, 0)$$

$$fx = \cos y \cdot 2x + e^{xy} y$$

$$dy = x^2(-\sin y) + e^{xy} \cdot x$$

$$(x_0, y_0) = (1, 0) \therefore x_0 = 1, y_0 = 0$$

equation of tangent.

$$f(x)(x - x_0) = \cos 0 \cdot 2(1) + e^0 D$$

$$= 1(2) + 0 = 2$$

$$fy(x_0, y_0) = (1)^2 f(1, 0) + e^0 \cdot 1$$

$$= 0 + 1 = 1$$

$$\cdot 2(x-1) + (y-0) = 0$$

$$2x - 2 + y = 0$$

$$2x + y - 2 = 0 \quad \text{required of tangent}$$

eq of Normal

$$ax + by + c = 0$$

$$(1) 1 + 2(y) + d = 0$$

$$1 + 2y + d = 0 \quad \text{at } (1, 0)$$

$$1 + 2(0) + d = 0$$

$$d + 1 = 0$$

$$d = -1,$$

$$\text{ii) } x^2y^2 - 2x + 3y + 2 = 0 \quad \text{at } (2, -2)$$

$$fx = 2x + 0 - 2 \rightarrow 0 + 0$$

$$= 2x - 2$$

$$fy = 0 + 2y - 0 + 3 = 0$$

$$= 2y + 3$$

$$(x_0, y_0) = (2, -2) \therefore x_0 = 2, y_0 = -2$$

$$fx(x_0, y_0) = 2(-2) + 3 = -1$$

$$= 2(-2) - 2 = 2$$

eq of tangent,

$$fx(x - x_0) + fy(y - y_0) = 0$$

$$2x - 2 - 4 = 0$$

$2x - 4 = 0 \rightarrow \text{eq of tangent}$

eq of Normal

$$= ax + by + c = 0$$

$$bx + ay + d = 0$$

$$= -1(x) + 2(y) + d = 0$$

$$-x + 2y + d = 0 \quad \text{at } (2, -2)$$

$$-2 - 4 + d = 0$$

$$-6 + d = 0$$

$$d = 6$$

Q) Find the eqn of tangent & normal line to each of the following surface.

i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$$fx = 2x - 0 + 0 + z$$

$$fx = 2x + z$$

$$fy = 0 - 2z + 3 + 0$$

$$= 2z + 3$$

$$fz = -2y + x$$

$$(x_0, y_0, z_0) = (2, 1, 0) = x_0 = 2, y_0 = 1, z_0 = 0$$

$$fx(x_0, y_0, z_0) = 2(2) + 0 = 4$$

$$fy(x_0, y_0, z_0) = 2(0) + 3 = 3$$

$$fz(x_0, y_0, z_0) = -2(1) + 2 = 0$$

eqn of tangent,

$$fx(x_0 - x_0) + fy(y_0 - y_0) + fz(z_0 - z_0) = 0$$

$$\therefore 4x - 8 + 3y - 3 = 0$$

$$\therefore 4x + 3y - 11 = 0 \rightarrow \text{required eqn of tangent.}$$

Eqn of normal at $(4, 3, -1)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$

$$= \frac{x - 2}{4} = \frac{y - 1}{3} = \frac{z + 1}{0}$$

ii) $3xyz = -x - y + z = -4$ at $(1, -1, 2)$

$$3xyz - x - y + z + 4 = 0 \text{ at } (1, -1, 2)$$

$$fx = 3yz - 1 - 0 + 0 + 0 = 3yz - 1$$

$$fy = 3xz - 0 - 1 + 0 + 0 = 3xz - 1$$

$$fz = 3xy - 0 - 0 + 1 + 0 = 3xy + 1$$

$$(x_0, y_0, z_0) = (1, -1, 2) \quad x_0 = 1, y_0 = -1, z_0 = 2$$

$$fx = 3(-1)(2) - 1 = 7$$

$$fy = 3(1)(2) - 1 = 5$$

$$fz = 3(1)(-1) + 1 = -2$$

$$\text{eqn of tangent}$$

$$-7(x-1) + 5(y+1) - 2(z-2) = 0$$

$$-7x + 7 + 5y + 5 - 2z + 4 = 0$$

$$-7x + 5y - 2z + 16 = 0 \quad \text{required eqn of tangent}$$

Eqn of normal at $(-7, 5, 2)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$

$$= \frac{x + 7}{-7} = \frac{y - 5}{5} = \frac{z - 2}{2}$$

Q) Find the local maxima & minima for the following function

i) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$$fx = 6x + 0 - 3y + 6 - 0 = 6x - 3y + 6$$

$$fy = 2y - 3x - 4$$

$$fx = 0, 6x - 3y + 6 = 0$$

$$2x - y + 2 = 0$$

$$2x - y - 2 = 0 \quad (1)$$

$$fy = 0$$

$$2y - 3x - 4 = 0$$

$$2y - 3x = 4 \quad (2)$$

Multiplying eqn (1) by (2)

$$2y - 3x = 0.4$$

$$x = 0.1$$

Substituting value of x' in eq (1)

$$2(0) - y = -2$$

$$y = 2$$

Critical point are $(0, 2)$

$$g = f_{xx} = 6$$

$$t = f_{yy} = 2$$

$$s = f_{xy} = -3$$

$$\text{Here } g > 0 \Rightarrow 6(2) - (-3)^2 \\ = 3 > 0$$

f' has maximum at $(0, 2)$

$$3x^2 + y^2 - 3xy + 6x - 4y \text{ at } (0, 2)$$

$$0 + 4 - 0 - 0 - 8 = -4,$$

ii) $f(x, y) = 2x^4 + 3x^2y - y^2$

$$f_y = 3x^2 - 2y$$

$$f_x = 8x^3 + 6xy$$

$$fx = 0$$

$$8x^3 + 6xy = 0$$

$$2x(4x^2 + 3y) = 0$$

$$4x^2 + 3y = 0 \quad (1)$$

$$fy = 0$$

$$3x^2 - 2y = 0 \quad (2)$$

Multiplying eq² (1) with 3, (2) with 4

$$12x^2 + 9y = 0$$

$$-12x^2 - 8y = 0$$

$$y = 0,$$

Substitute value of y in eq (1)

$$4x^2 + 3(0) = 0$$

$$x = 0,$$

Critical point is $(0, 0)$

$$g = f_{xx} = 24x^2 + 6x$$

$$t = f_{yy} = 0 - 2 = -2$$

$$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$$

$$\text{at } (0, 0), 24(0) + 6(0) = 0$$

$f(x, y)$ at $(0, 0)$

$$2(0)^4 + 3(0)^2(0) - (0)$$

$$= 0,$$

$$xt - s^2 = 0(-2) - (5)^2$$

$$= 0$$

$$x = 0 \quad \& \quad xt - s^2 = 0$$

iii) $f(x, y) = x^2 - y^2 + 2x + 8y - 70$

$$fx = 2x + 2$$

$$fy = -2y + 8$$

$$fx = 0 \quad \therefore 2x + 2 = 0$$

$$x = -1$$

$$fy = 0 \quad -2y + 8 = 0$$

$$y = 4$$

Critical point is $(-1, 4)$

$$g = f_{xx} = 2$$

$$t = f_{yy} = -2$$

$$s = f_{xy} = 0$$

$$g > 0$$

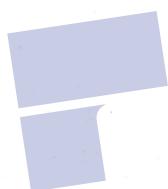
Q8

$$\begin{aligned}
 g(t-s^2) &= 2(-2) - (0)^2 \\
 &= -4 - 0 \\
 &= -4 \\
 &= -4 < 0
 \end{aligned}$$

$f(x,y)$ at $(-1, 4)$

$$\begin{aligned}
 (-1)^2 - (4)^2 + 2(-1) + 8(4) - 70 \\
 1 - 16 - 2 + 32 - 70 \\
 = 17 + 30 - 70 \\
 = \cancel{30} - 47 - \cancel{10} \\
 = -33
 \end{aligned}$$

AB
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pdfelement