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Sem-II

Practical: 1

1.) $$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$=\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$=\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$=\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$=\frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$=\frac{1}{3}\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$=\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$=\frac{1}{3}\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$=\frac{2}{3\sqrt{3}}$$

2.)

2.) $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$= \lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \dfrac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right)$

$= \lim_{y\to 0} \dfrac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$= \lim_{y\to 0} \dfrac{y}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a}\,(a+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a}\,(2\sqrt{a})}$

$= \dfrac{1}{2a}.$

3.) $\lim_{x\to \pi/6} \dfrac{\cos x - \sqrt{3}\,\cancel{\lim}\sin x}{\pi - 6x}$

By substituting $x - \dfrac{\pi}{6} = h.$

$\therefore x = h + \dfrac{\pi}{6}.$

when $h \to 0$

$= \lim_{h\to 0} \dfrac{\cos\left(h+\dfrac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\dfrac{\pi}{6}\right)}{\pi - 6\left(h+\dfrac{\pi}{6}\right)}$

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$$= \lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \cdot \sin\frac{\pi}{6} - \sqrt{3}\sinh\cos\frac{\pi}{6} + \cosh\sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{2}\right)}{\pi - 6h + \pi}$$

$$= \lim_{h\to 0} \frac{\cancel{\cos\frac{\sqrt{3}}{2}h} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cancel{\cos\frac{\sqrt{3}}{2}h}}{-6h}$$

$$= \lim_{h\to 0} \frac{\cancel{-}\sin\frac{4}{2}h}{\cancel{-}6h}$$

$$= \lim_{h\to 0} \frac{\sin 4h}{3/2\, h}$$

$$= \frac{1}{3}\lim_{h\to 0} \frac{\sinh}{h}$$

$$= \frac{1}{3}\times 1 = \boxed{\frac{1}{3}}$$

4) $$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$$

By Rationalizing Numerator and Denominator both.

$$= \lim_{x\to\infty}\left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$= \lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)\left(\sqrt{x^2+3} + \sqrt{x^2+1}\right)}{(x^2+3-x^2-1)\left(\sqrt{x^2+5} + \sqrt{x^2-3}\right)}\right]$$

$$\lim_{x\to\infty} \frac{8}{2}\frac{\left(\sqrt{x^2+3} + \sqrt{x^2+1}\right)}{\left(\sqrt{x^2+5} + \sqrt{x^2-3}\right)}$$

$$= 4\lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit
we get,

$= \underline{4.}$

5.) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$
for $\pi/2 < x < \pi$ $\Big\}$ at $x = \pi/2$.

$f[\pi/2] = \dfrac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \frac{\pi}{2}$ define

(ii) $\lim\limits_{x\to\pi/2} f(x) = \lim\limits_{x\to\pi/2} \dfrac{\cos x}{\pi - 2x}$.

By Substituting Method.

$\cancel{x - \frac{\pi}{2} = h}$.

$x = h + \frac{\pi}{2}$

where $h \to 0$.

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$$= \lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$$

$$= \lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(\frac{2h + \pi}{2}\right)}$$

$$= \lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$$

$$= \lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \cdot \sin\frac{\pi}{2}}{-2h}$$

$$= \lim_{h \to 0} \cos h \cdot 0 - \frac{\sin h}{-2h}$$

$$= \lim_{h \to 0} \frac{-\sin h}{-2h}$$

$$= \frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \boxed{\frac{1}{2}}$$

(i) $\lim_{x\to\pi/2}$ $f(x) = \lim_{x\to\pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$= \lim_{x\to\pi/2^-} \frac{2\sin x\cdot\cos x}{\sqrt{2\sin^2 x}}$

$= \lim_{x\to\pi/2^-} \frac{2\cancel{\sin x}\cdot\cos x}{\sqrt{2}\,\cancel{\sin x}}$

$= \lim_{x\to\pi/2^-} \frac{2\cos x}{\sqrt{2}}$.

$= \frac{2}{\sqrt{2}} \lim_{x\to\pi/2^-} \cos x$.

$=$ L.H.L $\neq$ R.H.L.

$\therefore$ f is not continuous at $x=\pi/2$.

5(ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \le x \le 9$

at $x = 3$ and $x = 6$

at $x = 3$.

(i) $f(3) = \frac{x^2-9}{x-3} \neq 0$.

f at $x=3$ define.

(ii) $\lim_{x\to 3} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6.$

f is define at $x=3$.

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6.$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)}$$

$\therefore$ L.H.L = R.H.L.

f is continous at $x=3$.

for $x=6$

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3.$$

(iii) $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$$= \lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$= \lim_{x\to 6^+} (x-3) = (6-3) = 3$$

$$= \lim_{x\to 6^-} x+3 = 3+6 = 9.$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ function is not continuous.

6.

(i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\quad x < 0$
$\quad\quad x = 0$ } at $x = 0$.

Sol^n: f is continuous at $x = 0$.

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = K$$

$$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = K$$

$$2\lim_{x \to 0} \frac{\sin^2 x}{x^2} = K$$

$$2\lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = K$$

$$2(2)^2 = K$$

$$K = 8.$$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$\quad = K$ $\quad x = 0$ } at $x = 0$.

Sol^n: $f(x) = (\sec^2 x)^{\cot^2 x}$.

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$.

We know that,

$\lim_{x\to 0} (1+px)^{1/px} = e$.

$= e$.

$\therefore K = e$.

(ii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x = \pi/3$

$= K$ $\quad x = \pi/3$

at $x = \dfrac{\pi}{3}$.

$x = h + \dfrac{\pi}{3}$.

where $h \to 0$.

$f(\pi/3 + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$.

$\lim_{h\to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$$= \lim_{h\to 0} \frac{\dfrac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3}\cdot\tan h}}{}}{\pi - \pi - 3h}$$

$$= \lim_{h\to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3}\cdot\tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3}\cdot\tan h}}{-3h}$$

$$= \lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3}\times\sqrt{3}\cdot\tan h) - (\sqrt{3} + \tan h)}{1 - \tan\pi/3\cdot\tan h}}{-3h}$$

$$= \lim_{h\to 0} \frac{\left(\dfrac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3}\tan h}\right)}{-3h}$$

$$= \lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$= \lim_{h\to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$= \frac{4}{3}\lim_{h\to 0} \frac{\tan h}{h} \cdot \lim_{h\to 0} \frac{1}{1 - \sqrt{3}\tan h} \qquad \frac{\tan h}{h} = 1$$

$$= \frac{4}{3} \cdot \frac{1}{(1 - \sqrt{3}(0))}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right)$$

$$= \frac{4}{3}$$

7.(i) $f(x) = \frac{1-\cos 3x}{x\tan x}$ $\left.\begin{matrix} x \neq 0 \\ x = 0 \end{matrix}\right\}$ at $x=0$.

$= a$

$f(x) = \frac{1-\cos 3x}{x\tan x}$.

$\lim_{x \to 0} \frac{2\sin^2 3/2 x}{x\tan x}$

$$\lim_{x \to 0} \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \cdot \frac{\tan x}{x^2} \times x^2}$$

$= 2 \lim_{x \to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = 2 \times \frac{9}{4} = \frac{9}{2}$.

$= \lim_{x \to 0} f(x) = \frac{9}{2}$ $\quad a = f(0)$

$\therefore f$ is not continuous at $x=0$.

Redefine function

$$f(x) = \left\{\begin{matrix} \frac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{matrix}\right\}.$$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

8.)

8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $\quad x = 0$

is continuous at $x = 0$.

given, f is continuous at $x = 0$.

$\lim_{x \to 0} f(x) = f(0)$.

$= \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$.

$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$.

$= \lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$.

$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$

$= \cancel{\log e} + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$.

$= \log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$

Multiply with 2 on Num and Denominator.

$= \frac{1}{} + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$.

9.) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$, $x \neq \dfrac{\pi}{2}$.

f(x) is continuous at $x = \pi/2$.

$= \lim_{x \to \pi/2} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$.

$= \lim_{x \to \pi/2} \dfrac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \dfrac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \dfrac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \dfrac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \dfrac{1}{2(\sqrt{2} + \sqrt{2})}$

$= \dfrac{1}{2(2\sqrt{2})} = \dfrac{1}{4\sqrt{2}}$.

$$\boxed{\therefore f(\pi/2) = \dfrac{1}{4\sqrt{2}}.}$$

Practical: 2

Derivatives.

1) Soln:

(i) $\cot x$ -

$f(x) = \cot x$.

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}.$$

$$= \lim_{x \to a} \frac{\cancel{co} 1/\tan x - 1/\tan a}{x - a}.$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x - a)\tan x \cdot \tan a}.$$

put $x - a = h$, $x = a + h$, as $x \to a$, $h \to 0$.

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}.$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}.$$

formula: $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \cdot \tan B)$.

$$= \lim_{h \to 0} \frac{\tan(a - a - h) - (1 + \tan a + \tan(a+h))}{h \times \tan(a+h)\tan a}.$$

$$= \lim_{h\to 0} \frac{-\tanh}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$Df(a) = -\cos^2 o$

$\therefore f$ is differentiable $\forall a \in R$.

(ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \cdot \sin x}$$

put $x-a=h$, $x=a+h$ as $\to x\to a$, $h\to 0$.

$$Df(n) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$Df(n) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times 1/2 \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2\cos\left(\frac{2a}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a}$$

$$= -\cot a \cdot \operatorname{cosec} a.$$

Q.2) Soln:

L.H.D.

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}.$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}.$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}.$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)}$$

$$= 4.$$

$$\therefore Df(2^-) = 4$$

R.H.D.:- $Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$.

$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$.

$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$

$= 2+2$
$= 4$.

$Df(2^+) = 4$.

$\therefore$ R.H.D = L.H.D.

$\therefore$ f is differentiable at $x=2$.

Q.3) Soln: R.H.D.:

$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3-3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$.

$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$.

$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$.

$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)}$

$= 9$.

$\therefore Df(3^+) = 9$.

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L.H.D $= Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)}.$$

$\therefore Df(3^+) = 4.$

$\therefore$ RHD $\neq$ LHD.

$\therefore$ f is not differentiable at $x = 3$.

Q.4) Solⁿ:- $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

R.H.D :

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}.$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}.$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}.$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$= 3 \times 2 + 2 = 8.$

$\therefore Df(2^+) = 8.$

L.H.D:-

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}.$$

$$= \lim_{x \to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$= Df(2^-).$

$= 8.$

$\because$ LHD = RHD.

$\therefore f$ is differentiable at $x = 3$.

AK

07/9/2020

Practical: 3

Topic:- Application of Derivative.

(1) Find the intervals in which function is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$

$\rightarrow f'(x) = 3x^2 - 5$

f is increasing if and only if $f'(x) > 0$

$3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > \frac{5}{3}$

$x > \pm\sqrt{\frac{5}{3}}$.

+   -   +

$-\infty$ $-\sqrt{5/3}$ $5/\sqrt{3}$ ∞

$\therefore x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

$\therefore$ f is decreasing if f $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3x^2 < 5$

$x^2 < 5/3$

$x < \pm\sqrt{\frac{5}{3}}$.

+   -   +

$-\infty$ $-\sqrt{5/3}$ $\sqrt{5/3}$

$\therefore x \in (-\sqrt{5/3}, \sqrt{5/3})$

(ii) $f(x) = x^2 - 4x$.

$\rightarrow$ $f'(x) = 2x - 4$

$\therefore$ f is increasing iff $f'(x) > 0$

$2x - 4 > 0$

$= 2(x-2) > 0$

$= x - 2 > 0$

$x > 2$.

$\therefore x \in (2, \infty)$

$\therefore$ f is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x < 2$.

$\therefore x \in (-\infty, 2)$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$.

$\rightarrow$ $f'(x) = 6x^2 + 2x - 20$.

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 3x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0$

$x \in (-2, 10/6)$

+   −   +

−2   10/6

(iv) $f(x) = x^3 - 27x + 5$.

$\rightarrow$ $f'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

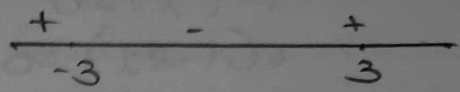
f is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

$\therefore x \in (-3, 3)$



(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$\rightarrow f(x) = -24 - 18x + 6x^2$

i.e $6x^2 - 18x - 24$

$6(x^2 - 3x - 4) > 0$

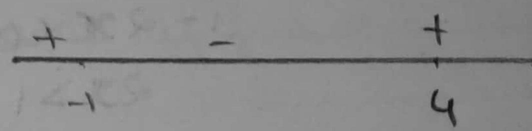
$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$



f is decreasing iff $f'(x) < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

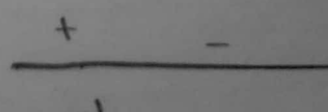
$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4)$

$(x+1)(x-4) < 0$

$\therefore x \in (1, 4)$



Q.2.

(i) $y = 3x^2 - 2x^3$

Soln: Let $f(x) = y = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$= 6(1 - 2x)$

$f''(x)$ is concave upwards if f

$f''(x) > 0$

$6(1-2x) > 0$

$1 - 2x > 0$

$-2x > -1$

$2x > 1$

$x < \frac{1}{2}$

$x \in (-\infty, 1/2)$

$f''(x)$ is concave downwards if f,

$f''(x) < 0$

$6(1-2x) < 0$

$1 - 2x < 0 = -2x < -1$

$2x > 1$

$x > 1/2$

$\therefore x \in (1/2, \infty)$

(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$.

Soln: $f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$.

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$.

$\therefore f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$.

$\therefore f''(x)$ is concave upwards iff

$f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$.

$x^2 - x - 2x + 2 > 0$.

$x(x-1) - 2(x-1) > 0$

$(x-2)(x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

+   −   +

1   2

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$12(x^2 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$x^2 - x - 2x + 2 < 0$

$x(x-1) - 2(x-2) < 0$

$(x-2)(x-1) < 0$

$\therefore x \in (1, 2)$

+   −   +

1   2

(iii) $y = x^3 - 27x + 5$.

Solⁿ: Let,

$f(x) = y = x^3 - 27x + 5$.

$f'(x) = 3x^2 - 27$.

$f''(x) = 6x$

$f''(x)$ is concave upwards if f.

$f''(x) > 0$

$6x > 0$

$x > 0$.

$x \in (0, \infty)$

$f''(x)$ is concave downwards if f.

$f''(x) < 0$.

$6x < 0$.

$x < 0$

$\therefore x \in (-\infty, 0)$.

) $y = 69 - 24x - 9x^2 + 2x^3$.

Solⁿ: Let

$f(x) = y = 69 - 24x - 9x^2 + 2x^3$.

$\therefore f'(x) = \cancel{-18 + 12x}$. $-24 - 18x + 6x^2$.

$f''(x) = -18 + 2x$.

$f''(x)$ is concave upwards iff

$f''(x) > 0$

$-18 + 12x > 0$

$12x > 18$.

$x > 18/12$.

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$\therefore x \in (3/2, \infty)$

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$-18 + 12x < 0$

$12x < 18$

$x < \frac{18}{12}$.

$\therefore x \in (-\infty, 3/2)$

(v) $y = 2x^3 + x^2 - 20x + 4$.

Soln: Let,

$f(x) = y = 2x^3 + x^2 - 20x + 4$.

$f'(x) = 6x^2 + 2x - 20$.

$f''(x) = 12x + 2$

$= 2(6x+1)$

$\therefore f''(x)$ is concave upwards iff,

$f''(x) > 0$

$2(6x+1) > 0$

$6x + 1 > 0$

$6x > -1$

$x > -1/6$.

$x \in (-1/6, \infty)$

$\therefore f''(x)$ is concave downward iff,

$f''(x) < 0$

$2(6x+1) < 0$

$6x + 1 < 0$

$6x < -1$

$x < -1/6$.

$\therefore x \in (-\infty, -1/6)$

20/12/19

Practical: 4

Topic:- Application of derivative and Newton's Method.

Q.1) Find maximum and minimum value of following:

(i) $f(x) = x^2 + \frac{16}{x^2}$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

(iii) $f(x) = x^3 - 3x^2 + 1 \quad [-1/2, 4]$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$

Q.2) Find the root of the following equation by Newton's (Take 4 iteration only) correct upto 4 decimal.

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

(ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Q.1.)

(i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$.

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/\cancel{2}$.

$x^4 = 16$

$x = \pm 2$

$\therefore f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/24$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$.

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at x

$x = -2$.

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$f(x) = 3-5x^3+3x^5$

$\therefore f'(x) = -15x^2+15x^4$

consider, $f'(x)=0$

$\therefore -15x^2+15x^4=0$

$\therefore 15x^4 = 15x^2$

$\therefore x^2=1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x+60x^3$

$f''(1) = -30+60$

$= 30>0$ $\therefore f$ has minimum value at $x=1$.

$\therefore f(1) = 3-5(1)^3+3(1)^5$

$= 6-5$

$= 1.$

$\therefore f''(-1) = -30(-1)+60(-1)^3$

$= 30-60$

$= -30<0$ $\therefore f$ has maximum value at $x=-1$.

$\therefore f(-1) = 3-5(-1)^3+3(-1)^5$

$= 3+5-3 = 5$

$\therefore f$ has maximum value 5 at $x=-1$ and has the minimum value 1 at $x=1$.

(iii) $f(x) = x^3-3x^2+1$

$\therefore f'(x) = 3x^2-6x$

consider, $f(x)=0$

$\therefore 3x^2-6x=0$

$\therefore 3x(x-2)=0$

$\therefore 3x=0$ or $x-2=0$

$\therefore x=0$ or $x=2.$

$\therefore f''(x) = 6x-6$

$f''(0) = 6(0)-6$

$= -6<0$

$\therefore f$ has maximum value at $x=0$.

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$.

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$.

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$.

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$.

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore$ f has maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$.

$\therefore$ f has maximum value 8 at $x = -1$ and f has minimum value -19 at $x = 2$.

Q.2)

(i)

Q2) $f(x) = x^3 - 3x^2 - 55x + 9.5$

(i) $f'(x) = 3x^2 - 6x - 55$ $\qquad x_0 = 0 \rightarrow$ given

By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727.$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829.$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.3062 - 55$

$= -55.9467.$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467.$

$= 0.1712.$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5.$

$= 0.0011.$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393.$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712.$

$\therefore$ The root of the equation is $\underline{0.1712}$.

(ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 23 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9.$

$\therefore f(13) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6.$

Let $x_0 = 3$ be the initial approximation.

$\therefore$ By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n).$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392.$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9.$

$= 20.5528 - 10.9568 - 9$

$= 0.596.$

$f'(x_1) = 3(2.7392)^2 - 4.$

$= 22.5096 - 4$

$= 18.5096.$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1).$

$= 2.7392 - 0.596/18.5096.$

$= 2.7071.$

$\therefore f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102.$

$\therefore f'(x_2) = 3(2.7071)^2 - 4.$

$= 21.9851 - 4$

$= 17.9851.$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 00056 = 2.7015.$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9$
$= 0.0901$

$f(3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
$= 17.8943.$

$x_4 = 2.7015 + 0.901/17.8943$
$= 2.7015 + 0.0056$
$= 2.7065.$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2.$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17$
$= -2.2.$

Let $x_0 = 2$ be initial approximation. By Newton's Method -

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$
$= 2 - 2.2/5.2$
$= 2 - 0.4230.$
$= 1.577.$

$\therefore f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.6755.$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10.$
$= 7.4608 - 5.6772 - 10$
$= -8.2164.$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.577 + 0.0822$
$= 1.6592.$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204.$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143-$

$x_3 = x_2 - f(x_2)/f'(x_2)$
$= 1.6592 + 0.0204/7.7143$
$= 1.6592 + 0.0026$
$= 1.6618.$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -7.6977.$

$\therefore x_4 = x_3 - f(x_3)/f'(x_3)$

$= 1.6618 + \dfrac{0.0004}{-7.6977.}$

$= 1.6618$

Practical: 5

Topic: Integration.

(i) $\int \frac{1}{x^2+2x-3} dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} dx.$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2=(a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

Substitute put $x+1=t.$

$dx = \frac{1}{t} \times dt$ where $t=1$, $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

Using: # $\frac{1}{\sqrt{x^2-a^2}} dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4})$

$t = x+1.$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$t = x+1$

$= \ln(|x+1| + \sqrt{(x+1)^2 - 4|})$

$= \ln(|x+1| + \sqrt{x^2+2x-3}|)$

$= \ln(|x+1| + \sqrt{x^2+2x-3}|) + c.$

2.) $\int (4e^{3x}+1)\,dx.$

$= \int 4e^{3x}\,dx + \int 1\,dx.$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x.$

$= \frac{4e^{3x}}{3} + x + c.$

3.) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx.$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2 dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx.$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\sqrt{x}}{3} + c.$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c.$

4.) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx.$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx.$

Split the Denominator.

$$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} \, dx.$$

$$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}} \, dx$$

$$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} dx$$

$$= \frac{x^{5/2+1}}{5/2+1}$$

$$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c.$$

5.) $\int t^7 \times \sin(2t^4)\, dt$

Put $u = 2t^4$

$du = 2 \times 4t^3$

$$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3} \, du.$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4} \, du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{8} du$$

$$= \frac{t^4 \times \sin(2t^4)\, du}{8}$$

Substitute t^4 with $u/2$.

$$= \int \frac{u/2 \times \sin(u)\, du}{8}.$$

$$= \int \frac{u \times \sin(u)}{2} / 8 \, du.$$

$$= \int \frac{u \times \sin(u)}{16} du$$

$$= \frac{1}{16} \int u \times \sin(u) du$$

$\int u dv = uv - \int v du.$

where $u = u$.

$dU = \sin(u) \times du. \qquad V = -\cos(u)$

$du = 1\, du$

$= \frac{1}{16}\left(u \times (-\cos(u)) - \int -\cos(u)\, du\right.$

$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u)\, du$

$\int \cos x\, dx = \sin(x).$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

Return the substitution on $u = 2t^4$

$= \frac{1}{16} \times \left(2t^4 \times (-\cos(2t^4)) + \sin(2t^4)\right)$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c.$

(vi) $\int \sqrt{x}\,(x^2-1)\, dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$

$= \int x^{5/2} - x^{1/2}\, dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C.$

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\, dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$

put $t = \sin(x)$.

$t = \cos x$.

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)} dt$

$= \frac{1}{\sin(x)^{3/2}} dt$.

$= \frac{1}{t^{2/3}} dt$.

$= \frac{-1}{(2/3 - 1)\, t^{2/3-1}}$

$= \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}} - \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$.

Return Substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + C$.

(k) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$.

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$.

put $x^3 - 3x^2 + 1 = dt$.

$\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x} dt$.

$= \int \frac{x^3 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x} dt$.

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)} dt.$$

$$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3} dt$$

$$= \int \frac{1}{3(x^3-3x^2+1)} dt$$

$$= \int \frac{1}{3t} dt.$$

$$= \frac{1}{3} \int \frac{1}{t} dt$$

$$= \frac{1}{3} \ln|t + c.$$

$$= \frac{1}{3} \ln|x^3-3x^2+1|+c.$$

03/01/2020

Practical: 6

Topic: Application of integration and Numerical Integration.

Q.1) Find the Length of the following curve:

i) $x = t\sin t$, $y = 1 - \cancel{\cot}\cos t$ to $[0, 2\pi]$
for t belong to $[0, 2\pi]$

Sol^n:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$x = t - \sin t$$

$$\frac{dx}{dt} = t - \cos t$$

$$y = 1 - \cos t$$

$$\frac{dy}{dt} = 0 - (-\sin t)$$

$$\frac{dy}{dt} = \sin t.$$

$$L = \int_0^{2\pi} \sqrt{(t - \cos t + \sin t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - \cos t + 1}\quad dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2\left|\sin\frac{t}{2}\right| dt \qquad \therefore \sin^2\frac{t}{2} = \frac{1-\cos t}{2}$$

$$= \int_0^{2\pi} 2\sin\frac{t}{2}\, dt$$

$$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi}$$

$$= (-4\cos\pi) - (-4\cos 0)$$

$$= 4+4$$

$$= 8.$$

(11) $y = \sqrt{4-x^2} \qquad x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$$

$$= \frac{-x}{2\sqrt{4-x^2}}$$

$$I = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_{-2}^{2} \sqrt{1+\frac{x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4x^2+x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$$

$$= 2\left[\sin^{-1}(x/2)\right]_{-2}^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

(iii) $y = x^{3/2}$ $\quad x \in [0,4]$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2}x^{1/2}$$

$$L = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1+\frac{9}{4}x}\, dx$$

$$= \frac{1}{2}\int_0^4 4+9x\, dx$$

$$= \frac{1}{2}\left[\cancel{\frac{[4+9x]^{3/2}}{3/2}} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{2}\left[(4+9x)^{3/2}\right]$$

$$= \frac{1}{27}\left[[4+0]^{7/2} - [4+31]^{7/5}\right]$$

$$= \frac{1}{27}\left(70^{3/2} - 8\right) \text{units}$$

(ii) $x = 3\sin t$, $y = 3\cos t$

$\Rightarrow \frac{dx}{dt} = 3\cos t$ $\quad \frac{dy}{dt} = -3\sin t$

$$I = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3[2\pi - 0]$$

$= 6$ units

Q.2) (i) $\int_0^2 e^{x^2} dx$ with $n=4$.

$\Rightarrow l = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$.

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.4877	54.5982
	y_0	y_1	y_2	y_3	y_4

$$\int_0^2 e^{x^2} dx = \frac{2}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{0.5}{3}\left[(1+54.5982)+5(1.284+9.487)+2.823885\right]$$

$$= \frac{0.5}{3}\left[55.5982+43.0866+5.436\right].$$

$$\int_0^2 e^{x^2} dx = 17.3535.$$

(ii) $\int_0^4 x^2 dx \quad n=4$

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2 dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$$

$$= \frac{1}{3}\left[0+16+4(1+9)+2\times 4\right]$$

$$=\frac{1}{13}[16+4(10)+8]$$

$$=\frac{64}{3}$$

$$\int_0^4 x^2 dx = 21.3333.$$

(iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n=6.$

$$h = \frac{\frac{\pi}{3}-0}{6} = \pi/18.$$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$	$(\pi/3)$
y	0	0.4167	0.4585	0.7071	0.8017	0.8752	0.9306	
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$=\frac{\pi/18}{3}[0.4167+0.9306+4(0.4167+0.7071+0.915) + 2(0.5848+0.8017)]$$

$$=\frac{\pi}{54}[1.3473+4(1.999)+2(1.3865)]$$

$$=\frac{\pi}{54}[\cancel{1.3473+7.996+2.773}]$$

$$=\frac{\pi}{54}\times 12.1163$$

$$\int^{\pi/3} \sqrt{\sin x}\, dx = 0.7049.$$

Practical: 7

Topic: Differential Equations.

Q.1) (i) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$(x) = 1/x$ and $(x) = \frac{e^x}{x}$

$I.F = e^{\int 1/(x)} dx$

$= e^{\int 1/x \, dx}$

$= e^{\ln x}$

$\therefore I.F = x$

$y(I.F) = \int Q(x)(I.F) \, dx + c$

$= \int \frac{e^x}{x} \cdot x \, dx + c$

$= \int e^x dx + c$

$xy = e^x + c$

Q.2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\Rightarrow \frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $\qquad Q(x) = e^{-x}$

$\int P(x)\,dx$

$I.F = e^{\int 2dx}$

$= e^{2x}$

$y = (I.F) = \int Q(x)(I.F)\,dx + C$

$y.e^{2x} \int e^{-x} + 2x\,dx + C$

$= \int e^{x}\,dx + C$

$y.e^{2x} = e^{x} + C.$

(iii) $x \cdot \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore x \cdot \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x)$ $\qquad Q(x) = \frac{\cos x}{x^2}$

$I.F = e \int P(x)\,dx.$

$\cancel{= e \int 2 \int x\,dx}$

$= e^{2/x}\,x$

$= \ln x^2.$

$I.F = x^2.$

$y(I.F) = \int Q(x)(I.F)\,dx + C.$

$= \int \cos x + C.$

$Q.2(i)$ e

$\Rightarrow$

$x^2y = \sin x + C.$

(iv) $x \cdot \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}.$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ $(\div \text{ by } x)$

$P(x) = 3/x \quad Q(x) = \sin x/x^3$

$= e^{\int P(x)dx}$

$= e^{\int 3/x \, dx}$

$= e^{\ln x^3}$

$I.F = x^3$

$= \frac{3V+3}{V+2}$

$= \frac{3(V+1)}{V+2}.$

$\int \frac{(V+2)}{(V+1)} dV = 3dx$

$\int \frac{V+1}{V} dx + \int \frac{1}{V+1} dV = 3x.$

$V + \log|x| = 3x + C.$

$2x + 3y + \log|2x+3y+1| = 3x + C.$

$3y = x - \log|2x+3y+1| + C$

Q.2(i) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\Rightarrow \frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$

$\therefore \frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\therefore \frac{dy}{dx} + 2y = e^{-x}$

$v(x) = 2 \qquad Q(x) = e^{-x}$

$\int p(x)\,dx$

$I.F = e^{\int 2dx}$

$= e^{2x}$

$y = (I.F) = \int Q(x)\,(I.F)dx + c$

$y \cdot e^{2x} = \int e^{-x+2x} dx + c$

$= \int e^x\,dx + c$

$y \cdot e^{2x} = e^x + c$

(V) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

$\Rightarrow$ put $2x+3y = v$

$\cancel{2 + \frac{3dy}{dx} = \frac{dv}{dx}}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

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$$\frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v + 2}.$$

(vi) $\sec^2 x \cdot \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\Rightarrow \sec^2 x \cdot \tan y\, dx = -\sec^2 y \cdot \tan x\, dy$

$$\frac{\sec^2 x\, dx}{\tan x} = \frac{-\sec^2 y}{\tan y} dy$$

$$\int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\therefore \log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c.$

AK
10/01/2020

Practical : 8

Topic: Euler's Method

(i) $\frac{dy}{dx} = y + e^{x} - 2$ $\quad y(0) = 2, h = 0.5$, find $y(2)$

Sol^n:

$f(x, y) = y + e^{x} - 2$, $y(0) = 2$, $h = 0.5$ ∴ (given)

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.~~4875~~5	2.487	3.57435
2	1	3.5743	4.2925	5.3615
3	1.5	5.3615	7.8431	9.2830
4	2	9.2831		

∴ By Euler's formula,

$$y(2) = 9.2831.$$

(ii) $\frac{dy}{dx} = 1 + y^2$

$f(x, y) = 1 + y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula.

$$\therefore y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	0.104	0.408
2	0.4	0.408	1.1605	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8630	1.2942
5	1	1.2942		

∴ By Euler's formula,

$y(1) = 1.2942$

3.) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $\quad y(0)=1$, $x_0 = 0$. $h = 0.2$

Using Euler's iteration formula,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

a) $\frac{dy}{dx} = 3x^2+1$ $\quad y_0=2,\ x_0=1,\ h=$

for $h=0.5$

using Euler's iteration formula

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1.0	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Euler's Formula

$\therefore y(2) = 28.5$

b) for $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

By Euler's Formula

$\therefore y(2)$ at $h = 0.25 = 8.9048$

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5.) $\frac{dy}{dx} = \sqrt{xy+2}$, $y_0 = 1$, $x_0 = 1$, $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

∴ By Euler's formula

∴ $y(1.2) = 1.6$.

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Practical: 9

Limits and Partial Order Derivative

1. (i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$\Rightarrow$ At $(-4,-1)$ denominator $\neq 0$

By applying limit

$= \dfrac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$

$= \dfrac{-64+3+1-1}{4+5}$

$= \dfrac{-61}{9}$

(ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$\rightarrow$ At $(2,0)$, denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(0+1)((2)^2-0-4(2))}{2+0}$

$= \dfrac{1(4+0-8)}{2}$

$= -4/2$

$= -2$

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$$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2}{x^3-x^2y}$$

$\Rightarrow$ At(1, 1, 1) Denominator = 0

$$=\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2}{x^3-x^2y}$$

$$=\lim_{(x,y,z)\to(1,1,1)} \frac{(x-yz)(x+yz)}{x^2(x-yz)}$$

$$=\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

On Applying limit

$$=\frac{(1+1)}{1^2}$$

$$=2$$

Q.2)

(i) $f(x,y) = xye^{x^2}+y^2$

$$f_x = \frac{\partial\,(f(x,y))}{\partial x}$$

$$=\frac{\partial\,(xye^{x^2}+y^2)}{\partial x}$$

$$=ye^{x^2+y^2}(2x)$$

$$=2xye^{x^2}+y^2$$

$$\therefore f_y = \frac{\partial\,(f(x,y))}{\partial y}$$

$$=\frac{\partial\,(xye^{x^2}+y^2)}{\partial y}$$

$$= xe^{x^2+y^2}(2y)$$

(ii) $f(x,y) = e^x \cos y$

$$f_x = \frac{\partial (f(x,y))}{\partial x}$$

$$= \frac{\partial (e^x \cos y)}{\partial x}.$$

$$f_x = e^x \cos y$$

$$f_y = \frac{\partial f((x,y))}{\partial y}.$$

$$= \frac{\partial}{\partial y} (e^x \cos y)$$

$$f_y = -e^x \sin y.$$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$$f_x = \frac{\partial (f(x,y))}{\partial x}$$

$$= \frac{\partial}{\partial x} (x^3y^2 - 3x^2y + y^3 + 1)$$

$$f_x = 3x^2y^2 - 6xy.$$

$$f_y = \frac{\partial (f(x,y))}{\partial y}.$$

$$\cancel{= \frac{\partial (x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}.}$$

$$f_y = 2x^3y - 3x^2 + 3y^2$$

Q.3) →

3) $f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2-0}{(1+y^2)^2}$$

$$= \frac{2(1+y^2)}{(1+y^2)(1+y^2)}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2.$$

$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0.$$

Q.4)

(i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right) \quad \cdots (iii)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{x^2 \frac{\partial}{\partial x}(2y-x) - (2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2} \quad \cdots (iv)$$

from (iii) and (iv)

$f_{yy} = f_{yx}$.

5)

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\Rightarrow f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$.

$$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

$$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$$

$$= \frac{y}{\sqrt{x^2+y^2}}$$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

(ii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\qquad$ $f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\qquad$ $f_y(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x-1+y-1$

$= x+y-2$

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Practical: 10

3) Find the directional derivative of the following function at given points in the direction of given vector.

(i) $f(x,y) = x + 2y - 3$ $\quad a(1,-1)$ $\quad u = 3i - j$

Here, $u \cdot 3i - j$ is not a unit vector.

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a) = f(1,-1) = (1) + 2(1) - 3$$
$$= 1 - 2 - 3$$
$$= -4$$

$$\cancel{f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)}$$
$$\cancel{= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)}$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$- 2 - \frac{2h}{\ldots} - 3$$

$\therefore f(a+hu) = -4 + \frac{h}{\sqrt{10}}$.

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$.

$= \lim_{h \to 0} \frac{4 + h/\sqrt{10} + 4}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$.

(ii) $f(x) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i + 5j$

Here $u = i + 5j$ is not a unit vector.

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$.

Unit Vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$.

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$.

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}}}{h}$$

$$= \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

(iii) $2x + 3y$ $\quad a = (1, 2)$, $u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$$f(a) = f(1, 2) = 2(1) + 3(2) = 8$$

$$f(a + hu) = f(1, 2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$$

$$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= 8 + \frac{18h}{5}$$

$$\therefore D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{+\frac{18h}{5}}{h}$$

$$= \frac{18}{5}.$$

Q.2.)

(i) $f(x,y) = x^y + y^x \qquad a = (1,1)$

$$f_x = y(x^{y-1}) + y^x \log y$$

$$f_y = x(y^{x-1}) + x^y \log x$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= (yx^{y-1} + y^x \log y,\ xy^{x-1} + x^y \log x)$$

$\nabla f(x,y)$ at $(1,1)$

$$= (1 \times (1)^0 + 1^1 \log(1),\ 1(1)^{(1-1)} + 1 \log 1)$$

$$= (1,1)$$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2 \qquad a = (1,-1)$

$$f_x = y^2 \left(\frac{1}{1+x^2}\right) = \frac{y^2}{1+x^2}$$

$$f_y = 2y \tan^{-1} x.$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= \left(\frac{y^2}{1+x^2},\ 2y \tan^{-1} x\right)$$

$\nabla f(x,y)$ at $(1,-1)$

$$= \left(\frac{(-1)^2}{1+(1)^2},\ 2(-1)\tan^{-1}(1)\right)$$

$$= \left(\frac{1}{2}, -\frac{\cancel{2}\pi}{\cancel{4}}\right)$$

$$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$$

(iii) $f(x,y,z) = xyz - e^{x+y+z}$ $\qquad a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0}, 1(0) - e^{1-1+0}, 1(-1) - e^{1-1+0})$

$= (0-1, 0-1, -1-1)$

$= (-1, -1, -2)$

Q:3)

(i) $x^2 \cos y + e^{xy} = 2$ $\qquad$ at $(1,0)$

$f(x,y) = x^2 \cos y + e^{xy} - 2$

$f'x = 2x \cos y + ye^{xy}$

$f_y = -x^2 \sin y + xe^{xy}$

$(x_0, y_0) = (1, 0)$

f_x at $(1,0) = 2(1)\cos 0 + 0$

$= 2.$

fy at $(1,0) = -(1)^2 \sin(0) + 1(e)^{1(0)}$
$= 1.$

$fx(x-x_0) + fy(y-y_0) = 0$
$2(x-1) + 1(y-0) = 0$
$\therefore 2x - 2 + y = 0$
$\therefore 2x + y - 2 = 0 \rightarrow$ Equation of Tangent.

Now,
for Equation of Normal
$bx + ay + d = 0$
$\therefore x + 2y + d = 0$

$\therefore (1) + 2(0) + d = 0$ At $(1,0)$
$1 + d = 0$
$d = -1$

$\therefore x + 2y - 1 = 0 \rightarrow$ Eqn of Normal.

(ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(x,-2)$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2.$ fx at $(2,-2) = 2(2) - 2$
$fx = 2x + 0 - 2 + 0 + 0$ $= 2.$
$= 2x - 2.$

$fy = 0 + 2y - 0 + 3 + 0$ fy at $(2, x-2) = 2(-2) + 3$
$= 2y + 3$ $= -1.$

Eq^n of tangent,

$f_x(x-x_0)+f_y(y-y_0)=0$

$2(x-2)+(-1)(y+2)=0$

$2x-4-y-2=0$

$2x-y-6=0 \longrightarrow$ Eq^n of tangent

for Eq^n of Normal,

$bx+ay+d=0$

$-x+2y+d=0$

$-(2)+2(-2)+d=0$

$-2+4+d=0$

$d=+6.$

$-x+2y+6=0 \longrightarrow$ Eq^n of Normal.

Q.4)

(i) $x^2-2yz+3y+x+z=7$ at $(2,1,0)$

$f(x,y,z)=x^2-2yz+3y+xz-7$ $\quad f_x$ at $(2,1,0)=2(2)+0$

$=4$

$f_x=2x-0+0+z-0$

$=2x+\cancel{z}$

$f_y=-2z+3+0-0$

$=-2z+3$

f_y at $(2,1,0)=-2(0)+3$

$=3.$

$f_z=0-2y+0+x-0$

$=-2y+x$

f_z at $(2,1,0)=-2(1)+2$

$=0.$

Eq^n of tangent,

$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$

$4(x-2)+3(y-1)+0(z-0)=0$

$4x-8+3y-3=0$

$\therefore 4x+3y-11=0 \longrightarrow$ Eq^n of tangent.

Eq^n of normal

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z-0}{0} \longrightarrow \text{Eq}^n \text{ of normal}$$

(ii) $3xyz = x-y+z=-4$ at $(1,-1,2)$

$f(x,y,z) = 3xyz - x - y + z + y$

$f_x = 3yz-1-0+0+0$
$= 3yz-1$

$f_y = 3xz-0-1+0+0$
$= 3xz-1$

$f_z = 3xy-0-0+-0$
$= 3xy+n$

f_x at $(1,-1,2) = 3(-1)(2)-1$
$= -7$

f_y at $(1,-1,2) = 3(1)(2)-1$
$= 5$

f_z at $(1,-1,2) = 3(1)(-1)+1$
$= -2.$

Eq^n of tangent;

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$-7(x-1) + 5(y+1) + (-2)(z-2) = 0$

$-7x+7+5y+5-2z+4=0$

$-7x+5y-2z+16=0 \longrightarrow$ Eq^n of tangent.

Eq^n of normal,

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}.$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2} \longrightarrow \text{Eq}^n \text{ of normal}.$$

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Q.5) (i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$\therefore f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$ ... (i)

$f_y = 2y - 3x + 0 - 4$
$= 2y - 3x - 4$ ... (ii)

$f_x = 0$
$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ ... (iii)

$f_y = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4$ ... (iv)

Multiplying (iii) by 2 and subtracting (iv) from (iii)

$4x - 2y = -4$
$2y - 3x = 4$
$7x = 0$
$x = 0$

Substituting value of x in (iii)
$2(0) - y = -2$
$-y = -2$
$y = 2$

$\therefore$ Critical points are $(0, 2)$

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