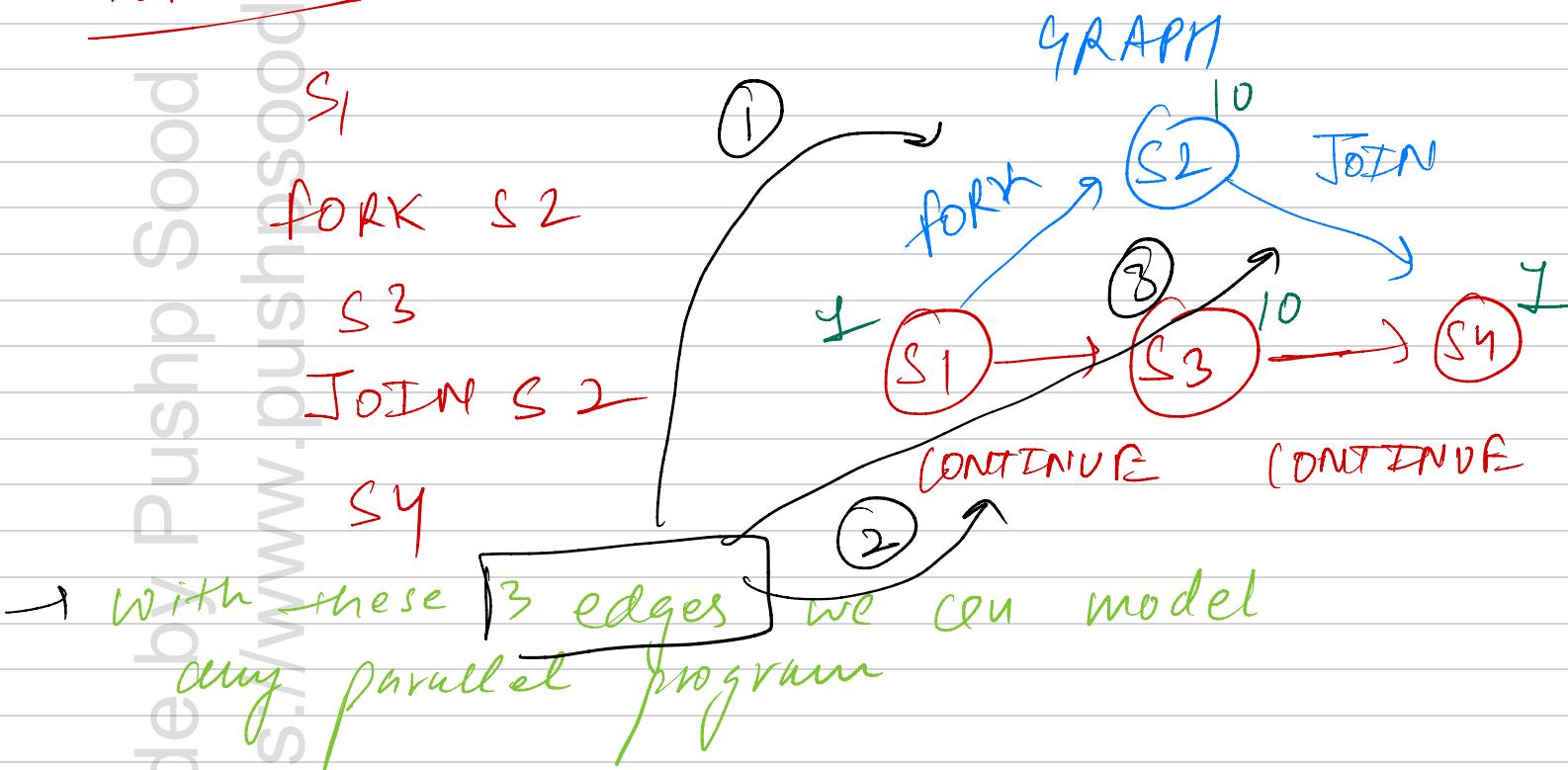




Task List



→ If there is a path directly b/w two nodes in the graph they cannot run in parallel for eg (S_2) & (S_3) can run in parallel , but (S_2) & (S_4) cannot

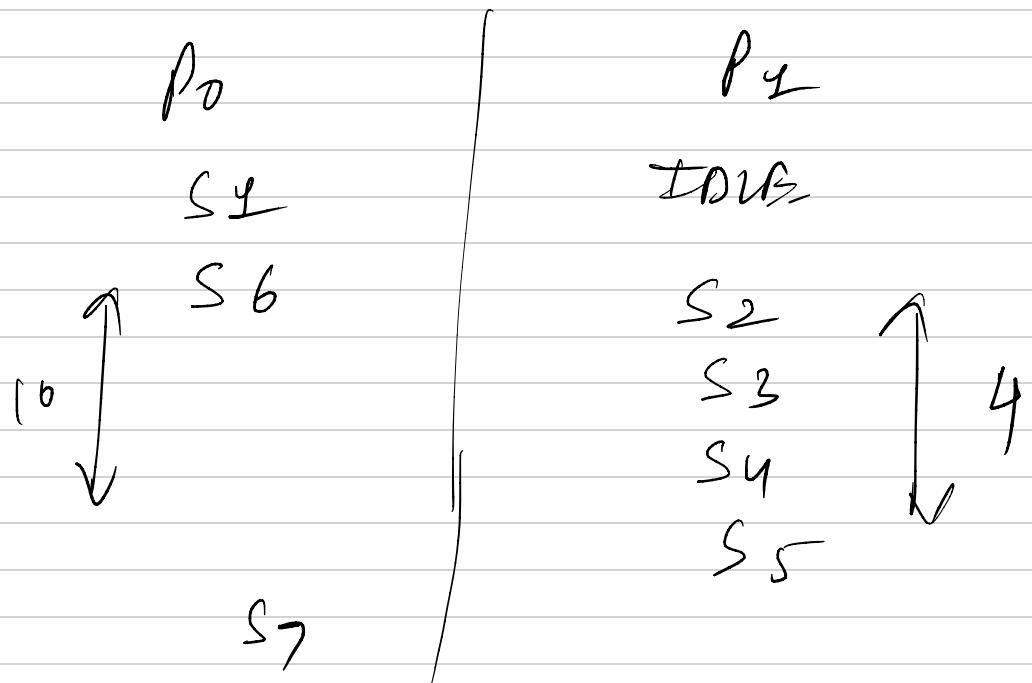
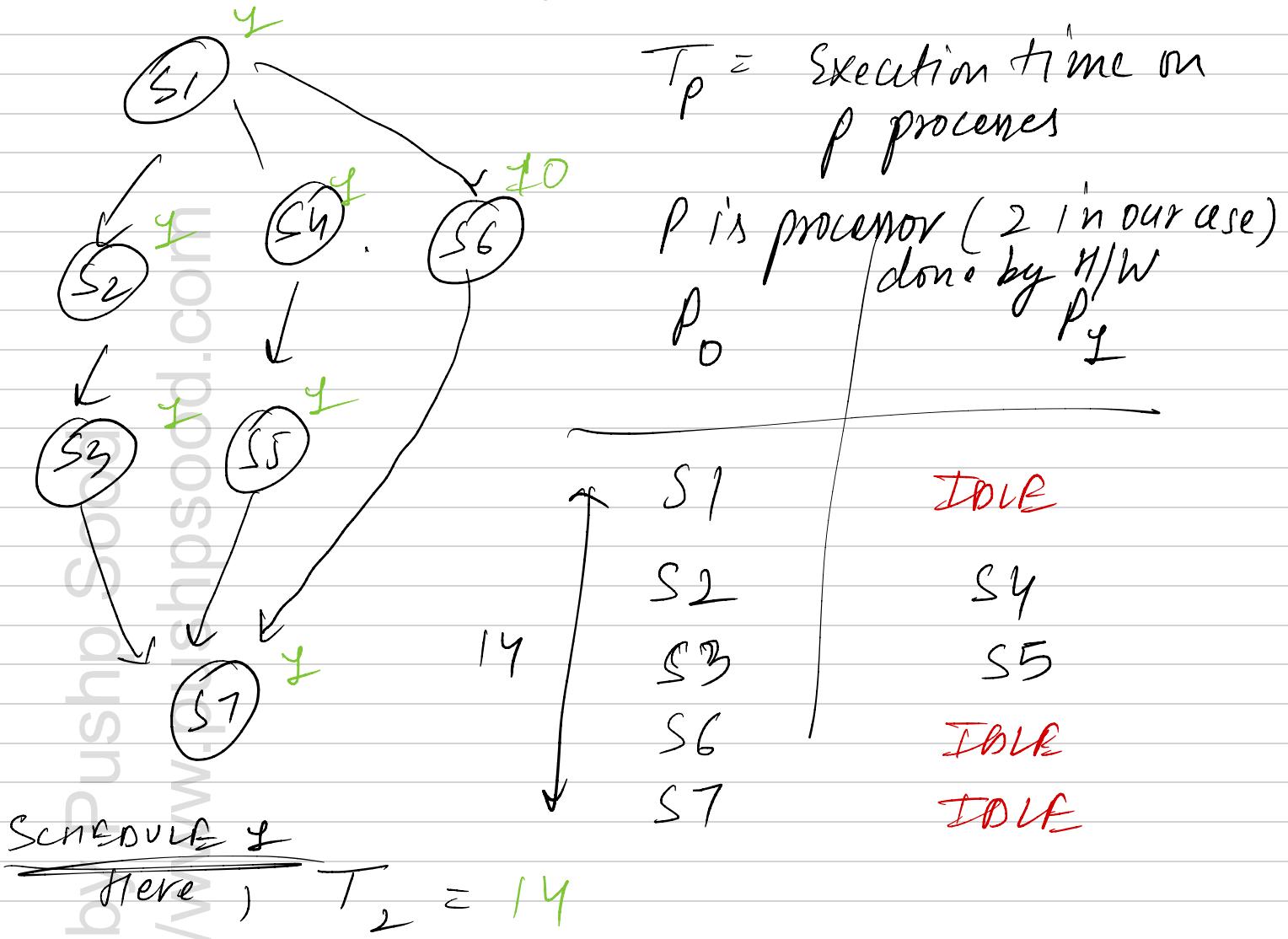
HEAP \rightarrow WORK = 22
(Total sum of all nodes)

→ SPAN = 12
(sum of longest path)

Ideal Paralitism

$$\frac{\text{WORK}}{\text{SPAN}} = \frac{22}{12}$$

Multiprocessor Scheduling, parallel speedup



$$\text{Here } T_2 = 12$$

If $P = 1$, $P \overline{T}_1 = \text{WORK}$ $\overline{T}_1 = 16$
(1 processor)

Here P is

$\overline{T}_\infty = \text{SPAN}$
(length of longest path) = 12

$$\boxed{\overline{T}_\infty \leq \overline{T}_P \leq \overline{T}_1}$$

SPEEDUP = $\frac{\overline{T}_1}{\overline{T}_P}$ (How much faster the parallel version was able to run)
BOUND ON SPEEDUP

$$\text{SPEEDUP} \leq P$$

$$\text{SPEEDUP} \leq \frac{\text{WORK}}{\text{SPAN}}$$

= IDEAL PARALLELISM

Amdahl's law

(Interseting observation made by Amdahl 50 yrs back)

$$\text{SPEEDUP} \leq \frac{\text{WORK}}{\text{SPAN}}$$

$q = \text{FRACTION SEQUENTIAL}$

$$\text{SPEEDUP} \leq \frac{1}{q}$$

$$q = 0.5 \Rightarrow \text{SPEEDUP} \leq 2$$



No Need for large parallelism

$$q = 0.1 \Rightarrow \text{SPEEDUP} \leq 10$$

PROOF

$q \times \text{WORK}$
must be included
in the computational
graph (Upper bound)

$\text{SPAN} > q \times \text{WORK}$

$$\text{SPEEDUP} \leq \frac{\text{WORK}}{\text{SPAN}}$$

$$\text{SPEEDUP} \leq \frac{\text{WORK}}{q \times \text{WORK}}$$

futures

$$A = f(B)$$



$$\begin{aligned} C &= g(A) \\ D &= h(A) \end{aligned}$$

only dependent
on A.

$$FA = \text{FUTURE}(f(B))$$

$$FC = \text{FUTURE}(g(f(A).get()))$$

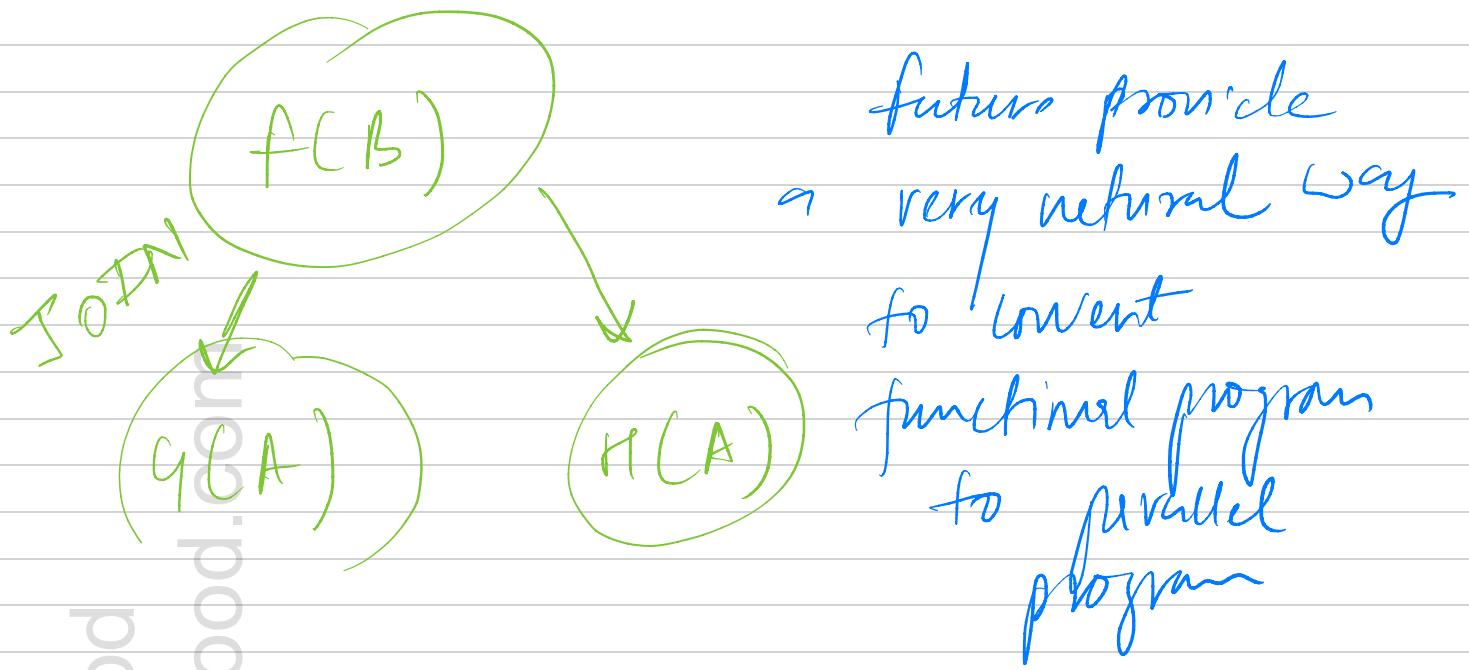
Made by Pushp Sood

<https://www.pushpsood.com>

$$FD = \text{FUTURE}$$

$$(h(f(A).get()))$$

BLOCK
WAIT
until A
is available



$fSUM1 = \text{FUTURE } \{ \text{lower } y \}$

$fSUM2 = \text{FUTURE } \{ \text{upper } y \}$

$\text{SUM} = fSUM1.get() + fSUM2.get()$

futures in JAVA FORK & JOIN

* NOT RECURSIVE ACTION

CLASS ASUM EXTENDS RECURSIVETASK

FIELDS : ARR, LO, HI

COMPUTE() {

START ↓
↓ LAUNCH if (low > HI) RETURN 0;

else if (low == HI) RETURN
ARR[low]

else {

MID = (low + HI) / 2

Join for
future implementation
L = NEW ASUM (low, MID,
ARR)

R = NEW ASUM (MID + 1, HI,
ARR)

R. FORK(); // FUTURE



RETURN

L.COMPUTEC() +

R.JOIN();

Memoization

$$y_1 = g(x_1)$$

INSERT (g, x_1, y_1)

$$y_2 = g(x_2)$$

$$y_3 = \cancel{g(x)} \rightarrow$$

LOOKUP (g, x_1)

$f(y) = \text{FUTURE } \{$

$g(x_1) \}$

INSERT ($g, x_1, f(y)$)

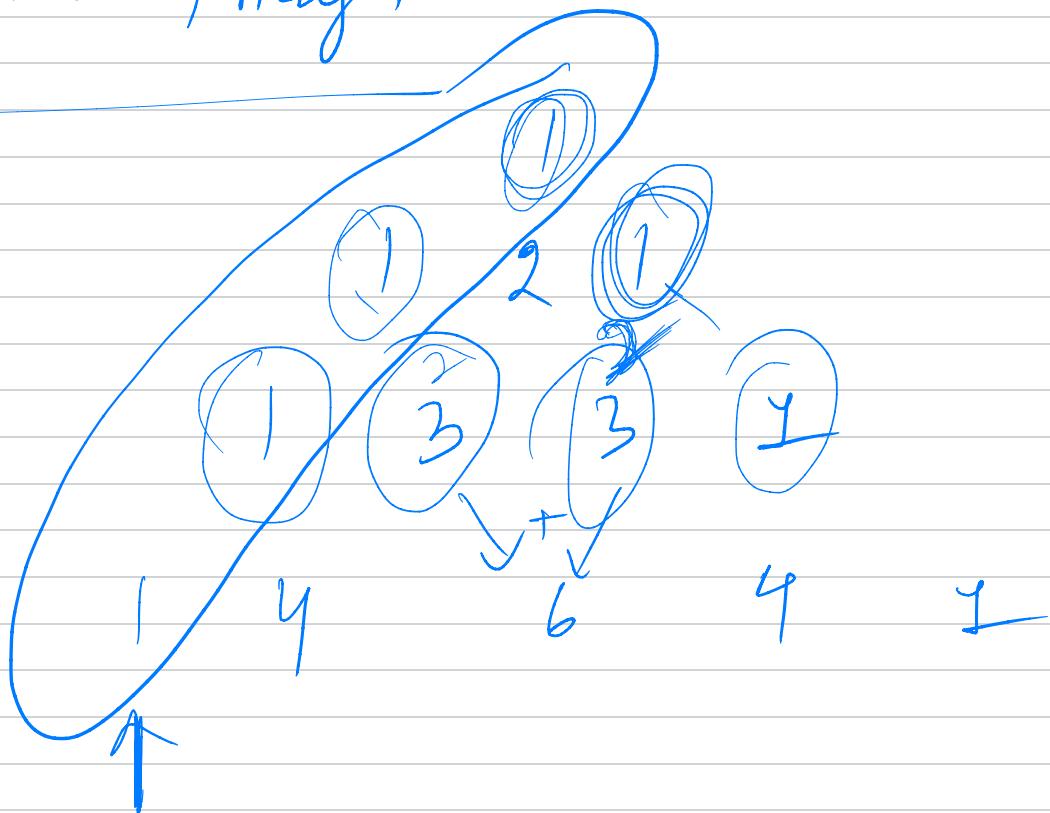
$y_2 = g(x_2)$

wanted work in sequential

$y_3 = \text{LOOKUP } (g, x_1).$

get()

PASCAL'S Triangle



Java Streams

① `FOR(S : STUDENTS)`

`PRINTS S`

`STUDENTS. STREAM(). (s → PRINTS)`

②

`FOR(S : STUDENTS)`

`if (s is ACTIVE)`

`ACTIVE. APP(s)`

`FOR(A : ACTIVE)`

`AGGREGATE = A. AGG ;`

`Avg = AGGREGATE / ACTIVE. SIZE()`

USING STREAMS

STUDENTS - STREAM().

FILTER($s \rightarrow s \text{ IS ACTIVE})$.

MAP($s \rightarrow s.\text{AGE}$).

TAKEAWAYS

AVERAGE();

→ IT'S EASY TO CONVERT SEQUENTIAL STREAM TO PARALLEL

→ USE PARALLEL STREAM instead
OF STREAM

→ AVG & FOR EACH ARE TERMINAL
OPERATIONS

→ FILTER & MAP ARE INTERMEDIATE
OPERATIONS

DATA RACES &

2 TYPES DETERMINISM

FUNCTIONAL

DET

Same input will
always lead to
Same output

FUTURE
ASYNCS

Read

STRUCTURAL
DETERMINISM

SUM1 = SUM OF
LOWER HALF

SUM2 = SUM

OF UPPER HALF.

SUM = SUM1 +
SUM2

SAME INPUT lead to
SAME COMPUTATIONAL

GRAPH (Structure of parallel
program)

BUT IN PARALLEL
PROG

(DATA RACE)
Read & Write

Waste of
Read & Write & WRITE in parallel.

We are
interested in
parallel program
that are both
structurally &
functionally
plus no
delta race
but not
always!
Benign Non
Determinism
(Next Page)

→ (vice versa if code is having DRF it is f₂S Deterministic)

DRF → Data Race Free

→ (can be achieved when we achieve functional and deterministic)

→ FUNCTIONAL + STRUCTURAL Deterministic

BENIGN NON DETERMINISM
Cases where we get diff output but they are all accepted

eg Search → PARALLEL LOOP

FOR [i : [0 : M-N]]

FOUND = TRUE

ASYNC FOR [j : [0 : N-i]] {

If (TEXT(i+j) != PAI[i])

FOUND = FALSE

If (FOUND = TRUE) Index = i;

If Multiple both have

same string then

any index is the right

Example:

Made by Pushpa Sood
https://www.pushpasood.com

PARALLEL LOOPS

When the tasks ~~set~~ in being performed inside the loop is independent of each other we can easily use parallelism to compute them independently.

for {
 ~~if~~

~~if~~ p == head ; p != null ;
 ~~p = p - Next~~)

{

~~async p - compute~~

y

y

This will be
converted to
a parallel form

Any for all

forall (i : [0 - n - 1])

y

a[i] = b[i] + c[i])

A better way is by using the JAVA API's parallel stream

$a = \text{IntStream.rangeClosed}(0, n-1)$.
parallel.

- to Array ($i \rightarrow b[i] + c[i]$),

Matrix multiplication This is used to measure speed of fast computation in the world (benchmarking)

$$C \leftarrow A \times B$$

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$C[i][j] = \sum_{k=0}^{n-1} A[i][k] * B[k][j]$$

~~FOR ALL~~ → Parallelism

~~FOR~~ $[i, j] : [0 : N-1, 0 : N-1]$

Can we run this in parallel

$C[i][j] = 0$ No! This will lead to data race

~~FOR~~ $(k : [0 : N-1]) \{$

$C[i][j] += A[i][k] *$

$B[k][j];$

y

y

It will remove
seq

Barriers in parallel loop

OUTPUT

This order will be retained

FORALL ($I : [0 : N-1]$)

Hellow TWO
Hellow zero
BYE TWO

$S = \text{LOOKUP}(I);$

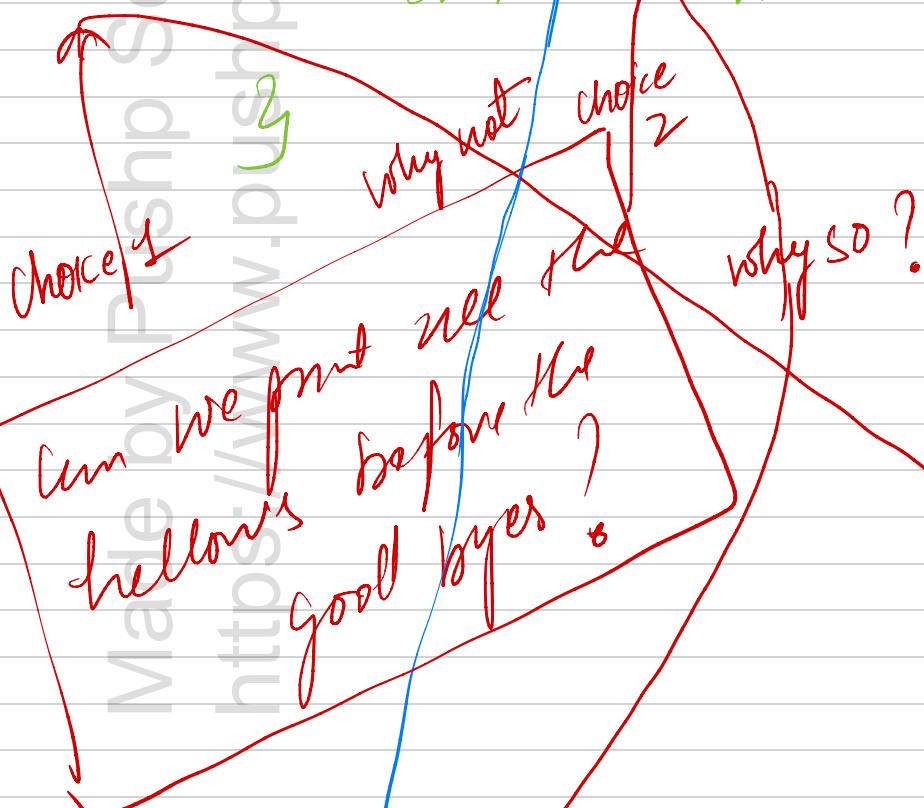
FRONT = "Hellow" + S

Hellow ON

FORALL ~~BARRIER~~

FRONT = "BYE" + S

Different combination



Output with barrier

Hellow 2

b 0 D

n 1

< BARRIER >

good bye 1

n 2

n 0

Computationally

which can be very

FORALL for

if we have LOOKUP
we have smaller coverage

which is very

$$x_0 = 0 \quad x_N = 2$$

$$x_i^0 = \left(x_{i-1} + x_{i+1} \right) / 2$$

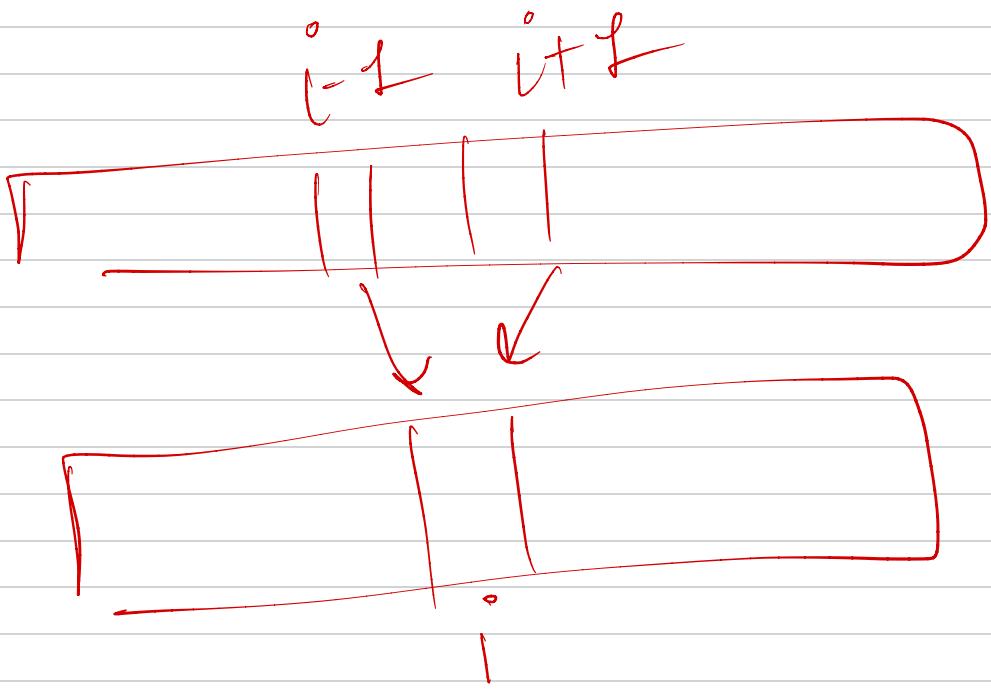
$$0 < i < N$$

$$N = 10$$

①

0	0.1	0.2	...	0.8	0.9	1
---	-----	-----	-----	-----	-----	---

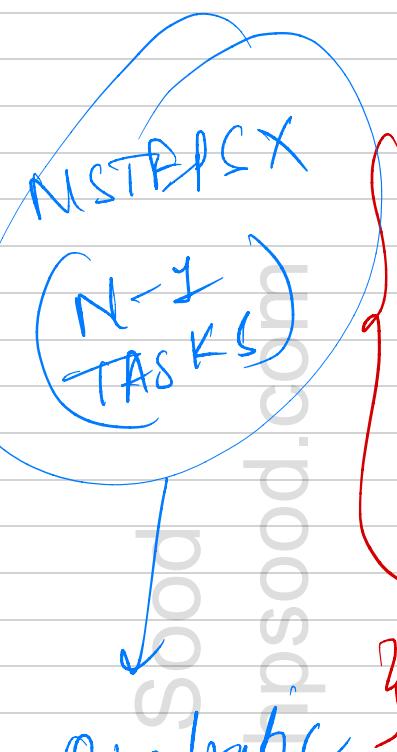
Made by Pushpendra Sood
 old X
 New X
<https://www.pushpendra.com>



ONE DIMENSIONAL ITERATIVE AVERAGING
 APPROACH

sequential **FOR [ITER : [i : N STEPS]] {**

*No of time we want
to repeat*



FOR ALL [I : [i : N-1]] {

$$\text{NEWX}[i] = \text{AVG} (\text{OLDX}[i-1], \\ \text{OLDX}[i+1])$$

}

SWAP POINTERS

FOR ALL [I : [i : N-1]] { BARRIER

$(N-1)$ TASKS

$(N-1) \times N$ STEPS

FOR [ITER : [i : N STEPS]] {

$$\text{NEWX}[i] = \text{AVG} (\text{OLDX}[i-1],$$

SWAP POINTERS

}

FOR ALL $(I : [0 : N-1])$

Too Many !
N TASKS

$$A[I] = B[I] + C[I]$$

Solution

FOR ALL $(g : [0 : Ng-1])$

No of groups
Total task will be Ng

How to determine?

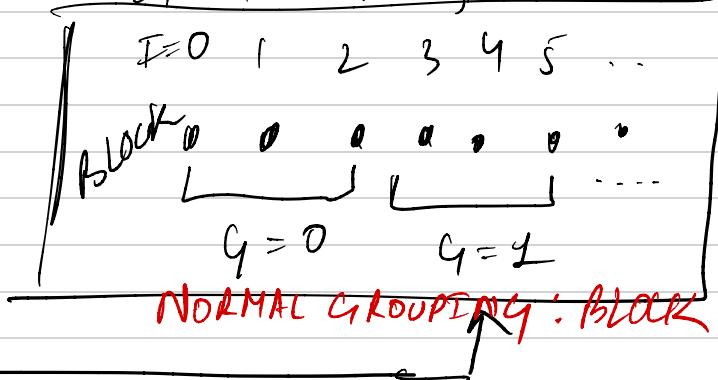
for $i : \boxed{\text{MyGroup}}(g)$

Not always

$Ng, [0 : N-1]$ create these many tasks

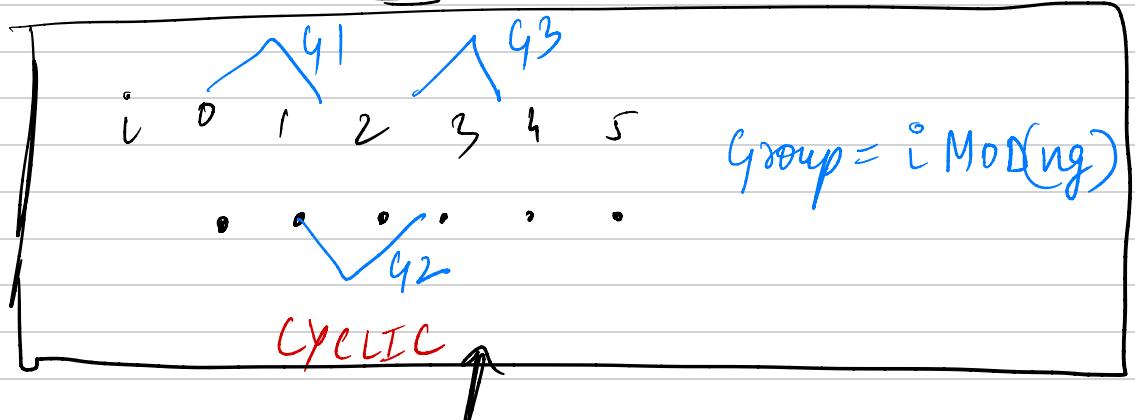
$$A[I] = B[I] + C[I]$$

CHUNKS OF EQUAL SIZE



fix to too many task
Is chunking vs iteration grouping

①



②

Made by PushupSood
<https://www.pushupsood.com>

Split phase barriers with JAVA PHASERS

CPLF SPAN = 200

FORALL { i : [1:N] } S ~ 100 unit of time

Do local work ARRIVE
print "HELLO" + i;
My id = LOOKUP(i);
AWAIT ADVANCE NEXT
print "GOODBYE" + myid;

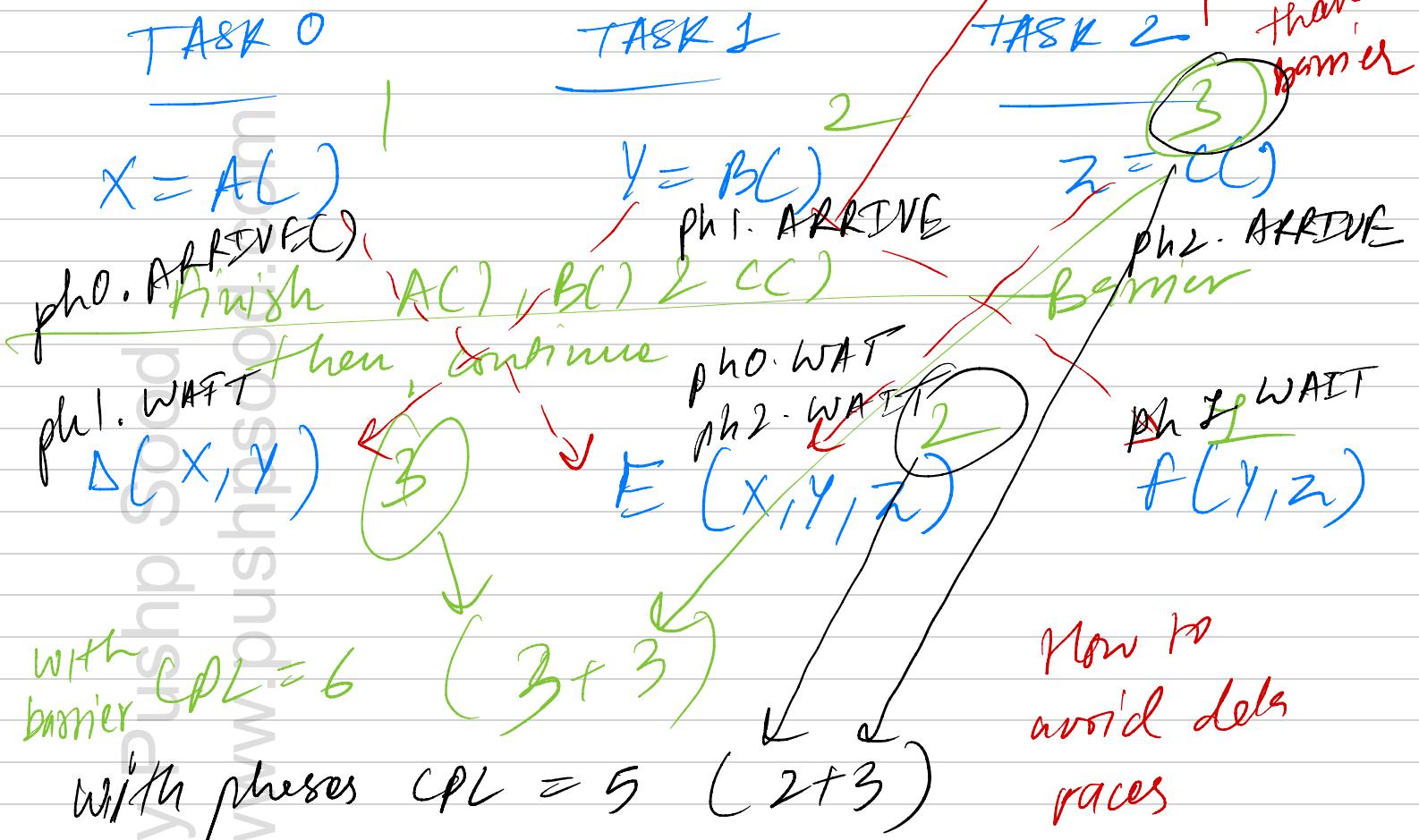
W/ PHASER CPLF = 200
SPAN

- ARRIVE AND AWAIT ADVANCE
it is actually a barrier

- ARRIVE
LOCAL WORK
AWAIT ADVANCE

Not associated
or can be executed

POINT - TO - POINT SYNC



1-D Iterative Averaging with Processors

$\text{ph}[i] = \text{new phaver}[N+2];$
FORALL ($i : [0 : N-1]$) {

FOR (ITER: $[0 : \text{NSTEP}-1]$) {

$$\text{NEWX}[i] = \text{AVG}(\text{OLDX}[i-1])$$

$\text{ph}[i].\text{ARRIVE}();$

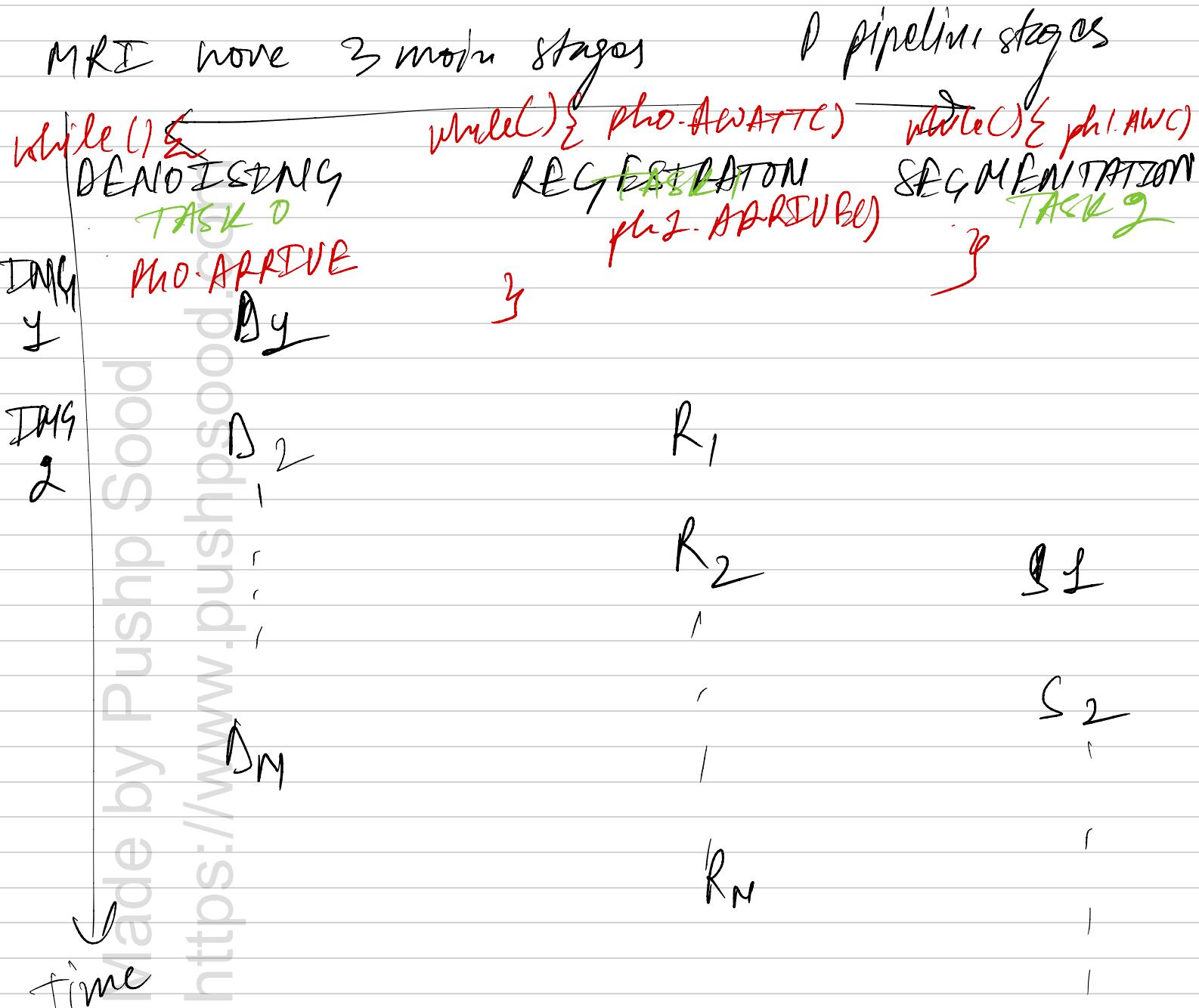
$\text{ph}[i-1].\text{WAIT}(); \text{OLDX}[i+1]);$

$\text{ph}[i+1].\text{WAIT}();$ Barrier

SWAP NEWX, OLDX

fixed
earlier

Pipelining

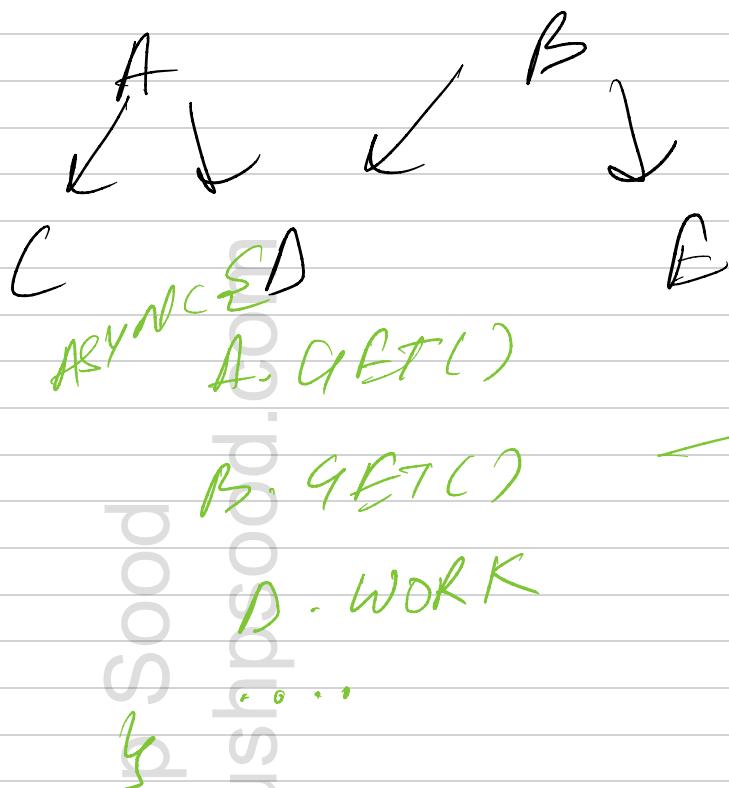


$$WORK = N + P$$

$$CPL = N + P - 1$$

$$\text{PARALLELISM} = \frac{N+P}{(N+P-1)} \approx P$$

DATA Flow



possibility 1

works but implicit dependency!

ASYNC {
 A. WORK ; A.PUT() }
 ASYNC { b. WORK ; b.PUT() } explicit
 ASYNC AWAIT (A) {
 C. WORK } precondition
 ASYNC AWAIT (A, B) {
 D. WORK }
 ASYNC AWAIT (B) {
 E. WORK }
}

possibility 2