CHAPTER 5 ALGEBRAIC EXPRESSIONS AND FORMULA

Learning outcomes

At the end of this chapter, Learners will:

- Formulate algebraic expressions
- Simplify algebraic expressions
- Apply the distributive law in simplifying algebraic expressions
- Evaluate algebraic expressions
- Solve simple equations using the additive and multiplicative inverse
- Construct formula from given statement.

CONCISE INFORMATION

Algebraic expressions

An algebraic expression is a sum (or difference) of two or more algebraic terms with at least one of the terms containing a variable.

Like terms have the same variable whose power (index) must be same. The terms $3x$ and $4x$ are *like* terms while the terms $3x$ and $4x^2$ are *unlike* terms. Two or more like terms can be added/subtracted based on the coefficients.

Simplifying algebraic expressions

(a) ***Simplification***; you should know how to simplify algebraic expressions by identifying like terms and carry out the operation.

E.g. Simplify

(i) 7a +3c – 4a – c *(collect the like terms)*

$= 7a – 4a + 3c - c$

$= \mathbf{3a + 2c}$.

(ii) $2(3x + 4y) + 3(x - y)$

$= 6x\ + 8y\ + 3x - 3y$ *(Expand the brackets)*

$= 6x + 3x + 8y - 3y$ *(Collect like terms)*

$= \mathbf{9x + 5y}$

(b) ***Factorization***; you should learn to get the common factor in each term and put it outside the bracket.

E.g. Factorise the following

(i) $4w - 4y$ (*the common factor is 4*)

$= \mathbf{4(w - y)}$

(ii) $9e + 6f - 3g$

$= (\underline{3} \times 3 \times e) + (\underline{3} \times 2 \times f) - (\underline{3} \times g)$ (*The common factor in each term is 3*)

$= \mathbf{3(3e + 2f - g)}$

The distributive law in simplifying algebraic expressions
The distributive law over addition or subtraction is also used in simplifying algebraic expressions.

Example
To simplify the expression $3(x - 2y)$, multiply each term inside the brackets by the 3 outside and then add the results where possible.
Thus $3(x - 2y) = 3 \times x + 3 \times -2y = \mathbf{3x - 6y}$
The process of multiplying each term inside the brackets by the number outside is called *expansion*.

Evaluating algebraic expressions (*substitution*)
This is when variables (letters) in an algebraic expression are replaced with numbers to determine the value of the expression.

Example
Find the value of $2y^2 - 3xy - x^2$ given that $x = -1$ and $y = 1$.
Solution
$2y^2 - 3xy - x^2 = 2 \times (1)^2 - 3 \times -1 \times 1 - (-1 \times -1)^2$
$= 2 \times 1 - 3 \times -1 - 1^2$
$= 2 + 3 - 1$
$= \mathbf{4}$

Solving simple equations using the additive and multiplicative inverse
An *additive inverse* of a number is another number such that the sum of the two numbers is zero. For example, the additive inverse of 3 is -3 and the additive inverse of -5 is 5.

A *multiplicative inverse* of a number is another number such that the product of the two numbers is one. For example, the multiplicative inverse of 3 is $\frac{1}{3}$ and the multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$.
Given a simple linear equation, use the concept of additive and /or multiplicative inverse to solve.

Example
Solve the equation $3x - 4 = 5$
Solution
The additive inverse of -4 is 4 and the multiplicative inverse 3 is $\frac{1}{3}$
$3x - 4 = 5$ ……….. *Add 4 on both sides*
$3x - 4 + 4 = 5 + 4$
$3x = 9$…………… *Multiply both sides by $\frac{1}{3}$ which is the multiplicative inverse of 3*
$\frac{1}{3} \times 3x = 9 \times \frac{1}{3}$

$\boldsymbol{x} = \mathbf{3}$

Construct formula from given statement
Given a mathematical problem that is not arithmetic or expressed as a word problem, we can convert such a problem to symbolic statement that can be solved algebraically as an equation. This involves representing the unknown quantity with a variable.

Example
An intercity bus was carrying 65 passengers whose combined mass was 4.03 tonnes. Each passenger carried an equal amount of luggage mass on the bus. If the total combined mass of the passengers and luggage was 7 930 kg, find the mass of the luggage carried by each passenger.
[Adapted from Monde M & Simutowe H(2014)]