APPROXIMATION AND ESTIMATION

Learning outcomes

At the end of this chapter, Learners will:

- Express numbers to given significant figure.
- Estimate measures or quantities
- Round-off numbers to specified degree of accuracy.
- Express numbers in scientific notation or standard form.
- Approximate numbers in scientific notation to given degree of accuracy
- Apply the concept of approximation in real life

CONCISE INFORMATION

1. You should know these terms:

- ‘Standard form’ is also known as Scientific notation
- Significant means important
- Approximate/estimate figures are ***not*** exact but just nearer to the exact figures (usually used in science and mathematics).

2. Significant figures

(a) Significant figures of large numbers (from 1 and above).

Rules about Zero

When counting the number of significant figures

- Zeros at the end of non-zero numbers are not significant e.g., in the number 87 500, the two zeros are not significant or important.
- Zeros in between non-zero numbers are significant. E.g. In the number 702 091, the two zeros are significant.

Example 1

How many significant figures have the following numbers?

(i) $\underline{4}\ \underline{8}\ \underline{7}\ \underline{2}\ \underline{8}$, has 5 significant figures

(ii) $\underline{7}\ \underline{0}\ \underline{2}\ \underline{0}\ \underline{9}\ \underline{1}$, has 6 significant figures

(iii) $\underline{8}\ \underline{7}\ \underline{5}\ 0\ 0$, has 3 significant figures

(b) Significant figures of small numbers (*ie numbers below 1*)

Rules about zero

When asked about the number of significant figures, know that;

(i) Zero before non-zero numbers are not important (significant). E.g. in the number 0.0034, the 3 zeros in front are not significant.

(ii) Zeros at the end of non-zeros numbers are significant, e.g. in the number 0.1460, the zero at the end (ie after 6) is significant, but the zero at the beginning is not significant.

(iii) Zeros in between non zero numbers are significant e.g. in the number 0.02030, the zero between 2 and 3 and one at the end are significant.

Example 2

How many significant figures have the following numbers?

(i) $0.\ \underline{2}\ \underline{5}\ \underline{3}$, has 3 significant figures

(ii) $0.\ 0\ 0\ \underline{3}\ \underline{4}$, has 2 significant figures

(iii) $0.\ \underline{1}\ \underline{4}\ \underline{6}\ \underline{0}$, has 4 significant figures

(iv) $0.\ 0\ \underline{2}\ \underline{0}\ \underline{3}\ \underline{0}$, has 4 significant figures

3. Standard form

(a) Standard form of large numbers (*numbers from 1 and above*).

Example 3

(i) $47\,1\ 85 = 4.7185 \times 10^4$

(ii) $2\ 56.\ 1\ 8 = 2.5618 \times 10^2$ (*Start counting from the decimal point from **right** to the **left** as shown*).

(b) Standard form of small numbers (*i.e. numbers less than 1*).

Example 4

Write the following numbers in standard form

(i) $0.000436 = 4.36 \times 10^{-4}$

(ii) $0.05020 = 5.02 \times 10^{-2}$ (*Start counting form the decimal point from* ***left*** *to the* ***right***).

Mixed Examples

1. Write 47568 in standard form correct to 2 significant figures (*Sig Fig*).

Solution

$47568 = \mathbf{\underline{4}.\underline{7}568} \times \mathbf{10^4}$ ------- *Standard Form*

To write it to 2 Sig Fig, we will consider the two underlined numbers.

If the next number after the last significant figure is equal to or more than five, the last significant figure increases by 1. i.e. 7 will increase by 1 and become 8, since the next number is 5.

Therefore, we have;

$\mathbf{4.8 \times 10^4}$ in standard form, correct to 2 significant figures.

Alternatively,

We first write the given correct to 2 significant figures then express the resultant value in scientific notation.

47 568 correct to 2 significant figures is 48 000

Expressing 48 000 in scientific notation, we have

$\mathbf{4.8 \times 10^4}$ in standard form, correct to 2 significant figures.

Note: When writing 47568 *correct to* 2 *significant figures, do not forget to replace the digits* 5, 6 *and* 8 *with zeroes.* (*The wrong answer is when you write 47568 as 48*)

2. Write the product of 0.123 and 0.47 in standard form correct to 2 significant figures.

Solution

Product: $0.123 \times 0.47 = 0.05781$

$0.05781 = 5.781 \times 10^{-2}$ in standard form

$= 5.8 \times 10^{-2}$ in standard form correct to 2 sig fig

Therefore 0.05781 in standard form correct to two sig fig $= \mathbf{5.8 \times 10^{-2}}$

Alternatively,

$$0.123 \times 0.47 = \frac{123}{1000} \times \frac{47}{100}$$

$$= \frac{5781}{100000}$$

$$= 0.05781$$
$$= 5.781 \times 10^{-2}$$
$$= \mathbf{5.8 \times 10^{-2}}$$