Learning outcomes

At the end of this chapter, Learners will:

- Describe different types of relationships
- Map functions to show relationship
- Identify mappings from arrow diagrams.
- Find the range of the function when domain is given
- Find a function given a set of ordered pairs.
- Draw graphs of linear functions
- Solve problems involving functions

CONCISE INFORMATION

(a) Relations and mappings

Relation

This is a rule which associates the elements for two sets: the *domain* (the set containing of objects) and *co-domain* (the set containing of images).

There are different types of relations: *one-to-many*, *many-to-one*, *many-to-many* and *one-to-one*

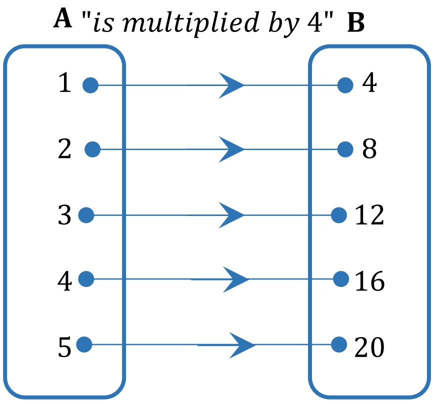
These relations are represented in four different ways. (a) Sets of ordered pairs (b) Arrow diagrams (c) Cartesian graph (d) Formula.

Mapping

This is a relation which is either one-to-one or many-to-one and every element of the domain has an image.

Example 1

(i) What relation is illustrated in the diagram below from set A to set B?

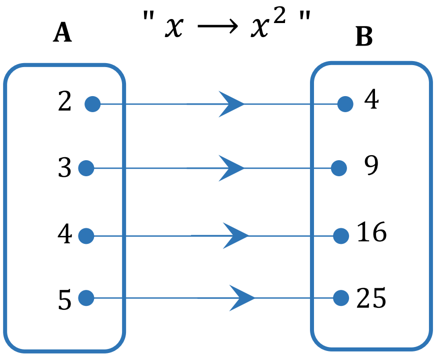


$\boldsymbol{NB}$: *Critically analyze how members in set A are related to elements in set B.*

∴ The relation is "is multiplied by 4"

(ii) If x is a variable on set $A = \{2, 3, 4, 5\}$, show the mapping of $x \longrightarrow x^2$ from set A to set B using arrow diagram.

Solutions



This shows that the variable x is mapped on x^2 in set B.

(b) Functions

A function is a 'mapping' where there is only one image for each member of the domain. A function is usually denoted by letter '***f***'

The following are the common ways of expressing a function:

(i) Set builder notation: f = {(x, y): y = f(x)}

(ii) Sets of ordered pairs

(iii) Arrow diagram

(iv) Formula: f(x) = y

(v) Functional notation: f: $x \longrightarrow y$

Example 2

(i) If $f(x) = 2x^2$ and $x \epsilon R$, find $f(-3)$

Solution

Clue: *You have to substitute x with the given value* -3

$f(x) \ = \ 2x^2$

$f(-3) \ = \ 2(-3)^2$

NB: *(-3) is the input. 18 is the output.*

$f(-3) \ = \ 2(-3 \times -3)$.

$f(-3) = 2 \times 9$

$\therefore \boldsymbol{f(-3) \ = \ 18}$

(ii) Given that $f(x) = 1 - x + x^2$, find $f(4)$.

Solution

$f(x) = 1 - x + x^2$

$f(4) = 1 - 4 + 4^2$

$f(4) = 1 - 4 + 16$

NB: *4 is the input. 13 is the output.*

$\therefore \boldsymbol{f(4) \ = \ 13}$

(iii) Given that $S(t) = \frac{2t+4}{t}$, find $S(3)$.

Solution

Note that '*f*' can be replaced by any letter, in this case '*S*'

$S(t) = \frac{2t+4}{t}$

$S(3) = \frac{2(3)+4}{3}$

$S(3) = \frac{6+4}{3}$

$\therefore \boldsymbol{S(3) = \frac{10}{3}}$

The range of a function

The *range* is the set of elements of *images* while the elements of the *domain* are called *objects*. On the Cartesian (XOY) plane the *x* values form objects while the corresponding values of *y* are the images.

Example

Given the function $f(x) = 3 - 2x$, x represents objects and that $f(x)$ represents images. Therefore given that for $x = -1$, the corresponding value is $f(-1) = 3 - 2(-1) = 5$. In this case, the object (input) of -1 has a corresponding image is 5.

Example

The domain of the function $f: x \longrightarrow 2x + 3$ is given as D = {– 2, – 1, 0, 1}. Find the range of the function.

Solution

Substitute each of the values in D into the function to find the corresponding images.

x	– 2	– 1	0	1
$2x + 3$	**– 1**	**1**	**3**	**5**

Therefor the range of the function is **{– 1, 0, 3, 5}**

Note: *the stated elements of the domain and the range can be expressed as sets of ordered pairs*

We can equally find the domain of a given function when we know the range.

Example

Find the value of x for which $f(x) = -1$ given that $f(x) = 3 - 2x$.

Solution

For $f(x) = 3 - 2x$ and $f(x) = -1$

$3 - 2x = -1$

$-2x = -4$

$x = 2$

In the above example $x = 2$ is the object for the image $f(x) = -1$. This process is then used to find as many objects (domain) for the given images (range).

Finding a function when given sets of ordered pairs

When given sets of ordered pairs to find a function we can either use

(i) the concept of simultaneous equations on linear functions
(ii) apply the basic operations of arithmetic on the objects(values of x) to find the corresponding values of y. This method is unreliable on linear function whose coefficient of x is not 1.

A linear function is a function in the form of $f(x) = ax + b$ (or $y = ax + b$), where a and b are constants.

Given a set of ordered pairs, (– 2, – 7), (– 1, – 5), (0, – 3) and (1, – 1), we choose any two ordered pairs and substitute the corresponding values of x and y in the equation $y = ax + b$ and solve for a and b.

Let us use the pairs (– 2, – 7) and (– 1, – 5).

We have $-7 = -2a + b$ and $-5 = -a + b$. Solving these two equations simultaneously gives that $a = 2$ and $b = -3$ so that the equation $y = ax + b$ now becomes $\boldsymbol{y = 2x - 3}$.

This can be used in case the function is given as an arrow diagram.

Graphs of linear functions

A linear function is in the form of $f(x) = ax + b$ (or $y = ax + b$), where a and b are constants.
To draw the graph of a linear function,

(i) Plot at least two ordered pairs of coordinates
(ii) Draw a line passing through these coordinates

The graph of a linear function is a straight line.

Example

Draw the graph of the function $y = 2x - 4$ for the domain $-1 \leq x \leq 3, x \in \boldsymbol{R}$.

Solution

Using the given domain, develop a table of values of x with the corresponding values of y.

x	-1	0	1	2	3
y	- 6	- 4	- 2	0	2

Plot these ordered pairs on the Cartesian plane.

Draw a line passing through these points.