CHAPTER 4 RATIO AND PROPORTION

Learning outcomes
At the end of this chapter, Learners will:
- Solve problems that involve ratio and proportion

CONCISE KNOWLEDGE

Ratio
A ratio is the relationship (or comparison) between two numbers of the same unit. The relationship can be written using a full colon (:), a fraction or the comparative word 'to'.

Equivalent ratios
A ratio can be changed by multiplying (dividing) the comparable quantities by the same multiplier (or factor).

When simplifying ratio of measures, express the measures in the same unit.

Example
Express 2hrs: 45mins as a ratio in its lowest term

Solution

(i) First change 2hrs to minutes i.e.
$2 \times 60 = 120\ minute$
Then $\frac{120}{15} : \frac{45}{15}$
2hrs : 24minutes = 8: 3

OR
(ii) Express minutes into hours
$2hrs:\ 45\ minutes$
$2hrs : \frac{3}{4} hrs$
$2 \times 4 : \frac{3}{4} \times 4\ (multiply\ both\ sides\ by\ 4)$
$\mathbf{8 : 3}$

Dividing in a given ratio
A ratio can be used to divide a given quantity or measure.

Given a value ***P*** to be shared in the ratio $a: b$ the portion,

(i) for a is given by $\frac{a}{a+b} \times \boldsymbol{P}$
(ii) for b is given by $\frac{b}{a+b} \times \boldsymbol{P}$

This concept can be extended to a ratio of three numbers.

Given a value $\boldsymbol{P}$ to be shared in the ratio $a: b: c$ the portion,

(i) for a is given by $\frac{a}{a+b+c} \times \boldsymbol{P}$

(ii) for b is given by $\frac{b}{a+b+c} \times \boldsymbol{P}$

(iii) for c is given by $\frac{c}{a+b+c} \times \boldsymbol{P}$

PROPORTION

When given a problem on direct or inverse proportion you can either use unitary or proportion method

Direct Proportion

Example

If 3 books cost K45.00, how much will 8 books cost?

Unitary method

3 books cost K45.00

1 book will cost $= \text{K}\ \frac{45}{3}$

$\therefore$ 8 Books will cost $= \frac{45}{3} \times 8$

$= \boldsymbol{K120.00}$

Proportion method

Let x be the cost of 8 books

3 books K45 (*cross multiply*)

8 books x

$3x = 45 \times 8$

$\frac{3x}{3} = \frac{45 \times 8}{3}$

$\boldsymbol{x = K120.00}$

Inverse Proportion

Example 1

There is enough food for 9 pupils to last 8 days. How many days will the food last if there are only 6 pupils?

Solution

Unitary method

9 pupils, the food lasts 8 days

1 pupil, the food will last

$= 9 \times 8$ days [more]

6 pupils, the food will last $= \frac{9 \times 8}{6}$

= 12 days

Proportion method

Let x be the days the food will last with 6 pupils

9 pupils : 8days

6 pupils : x

$\frac{9}{6} = \frac{x}{8}$ $\left(inverse\ \frac{8}{x}\ to\ \frac{x}{8}\right)$ *and cross multiply*

$\frac{6x}{6} = \frac{9 \times 8}{6}$

$\boldsymbol{x = 12days}$

Read and understand the question i.e. after finding the answer, go back and read the question again.

Example 2

16 workers take 6 days to dig a well. How many more days will be needed to dig the same well if 4 of the workers fell sick?

Solution

Unitary method

16 workers dig in 6 days

1 worker will dig in (16 × 6) days [***More***]

(If 4 workers are sick, 12 will remain

$i.e\ 16 - 4 = 12$)

$\therefore$ 12 workers will dig in $= \frac{16 \times 6}{12}$

= 8 days

How many more days? 8 – 6 = 2 more days

Proportion method

16 : 6

12 : x

$\frac{16}{12} = \frac{x}{6}$…..*cross multiply*

$\frac{12x}{12} = \frac{16 \times 6}{12}$

$x = 8\ days$

8 – 6 = 2 more days

Note;

(i) This is an inverse proportion

(ii) You are **not** calculating for the 4 workers but (16 - 4) workers.

(iii) The question is asking for how many more days not how many days.

Representative Fraction (RF)

A ratio of distance on the map to the distance on the ground.

This is given by $\frac{\text{distance on the map}}{\text{distance on the ground}}$

It is usually in the form of 1: x or $\frac{1}{x}$ where 1 unit on the map is equivalent to x units on the ground.

Rate

This is a ratio of two measurements with different units of measurements.

Speed is an example of the comparison of distance against time. S = D/T. The denominator quantity must be unitary.

A car that takes 4 hours to cover a distance of 300km is said be moved at the speed of $\frac{300km}{4hrs} =$ 75km/hr (read 75 km per hour).