第二章：深度学习反向传播
——优化论五部曲

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回归与分类

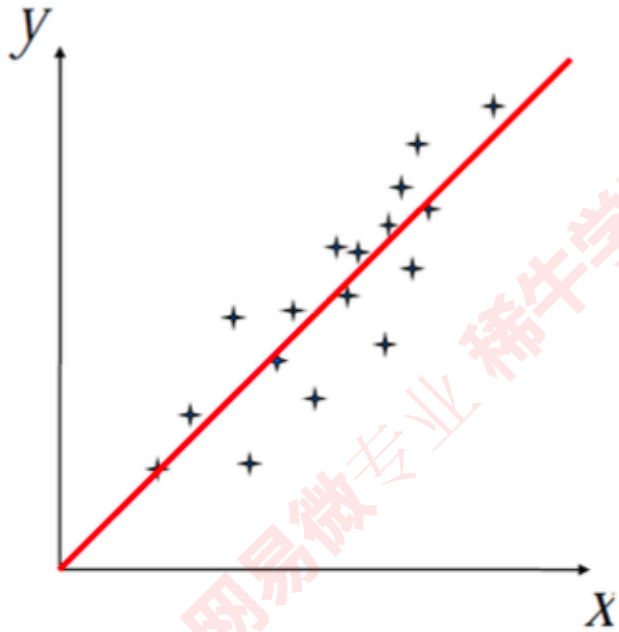
神经网络

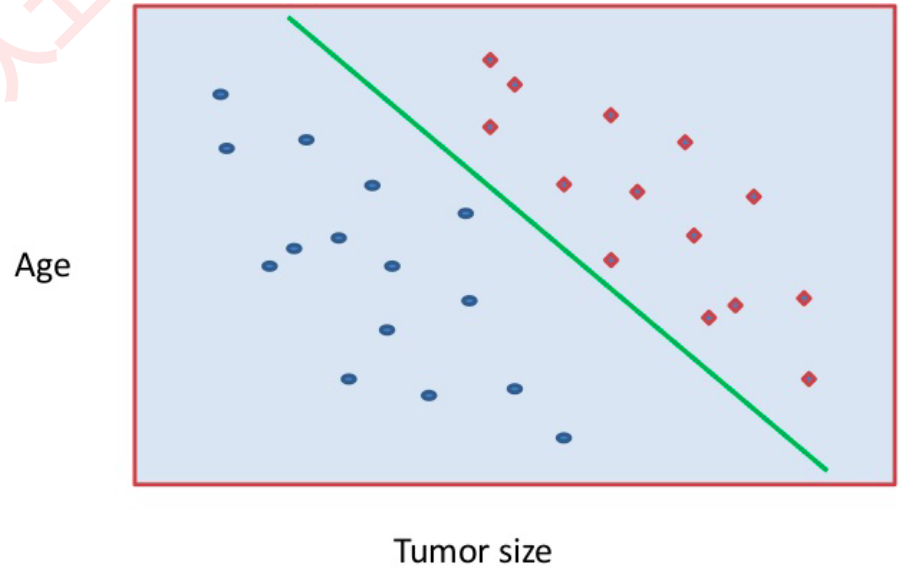
BP 算法

计算图

回归 VS 分类

- 给定训练样本集 $\{(\mathbf{x}_1, y_1), \cdots, (\mathbf{x}_N, y_N)\}, \mathbf{x}_i \in \mathbb{R}^d$
  - 回归：预测的是连续值
  - 分类：预测的是离散值
    - 二分类：$y_i \in \{0, 1\}$





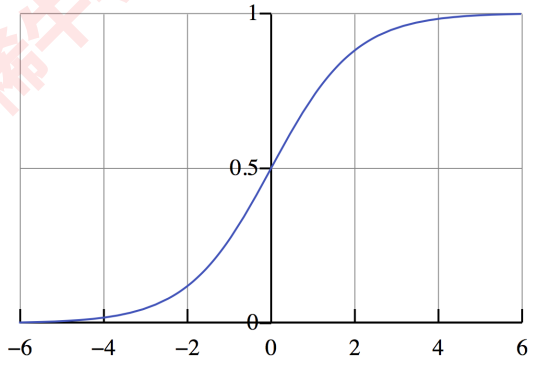
Sigmoid 函数

- $y = \mathbf{w}^T\mathbf{x} + b$
- 非线性映射 1

$$z = \begin{cases} 0 & y < 0 \\ 0.5 & y = 0 \\ 1 & y > 0 \end{cases}$$

- 非线性映射 2

$$z = \frac{1}{1 + e^{-y}}$$



S 函数求导 (课堂手推)

- $\sigma(x) = \frac{1}{1+e^{-x}}$
- $\frac{1}{dx}\sigma(x) = \sigma(x) - \sigma(x)^2$

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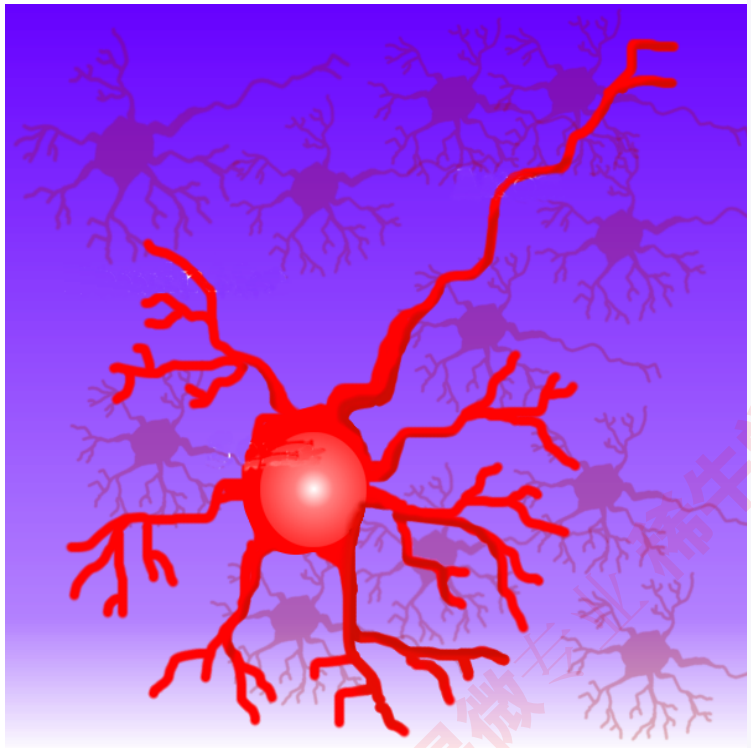
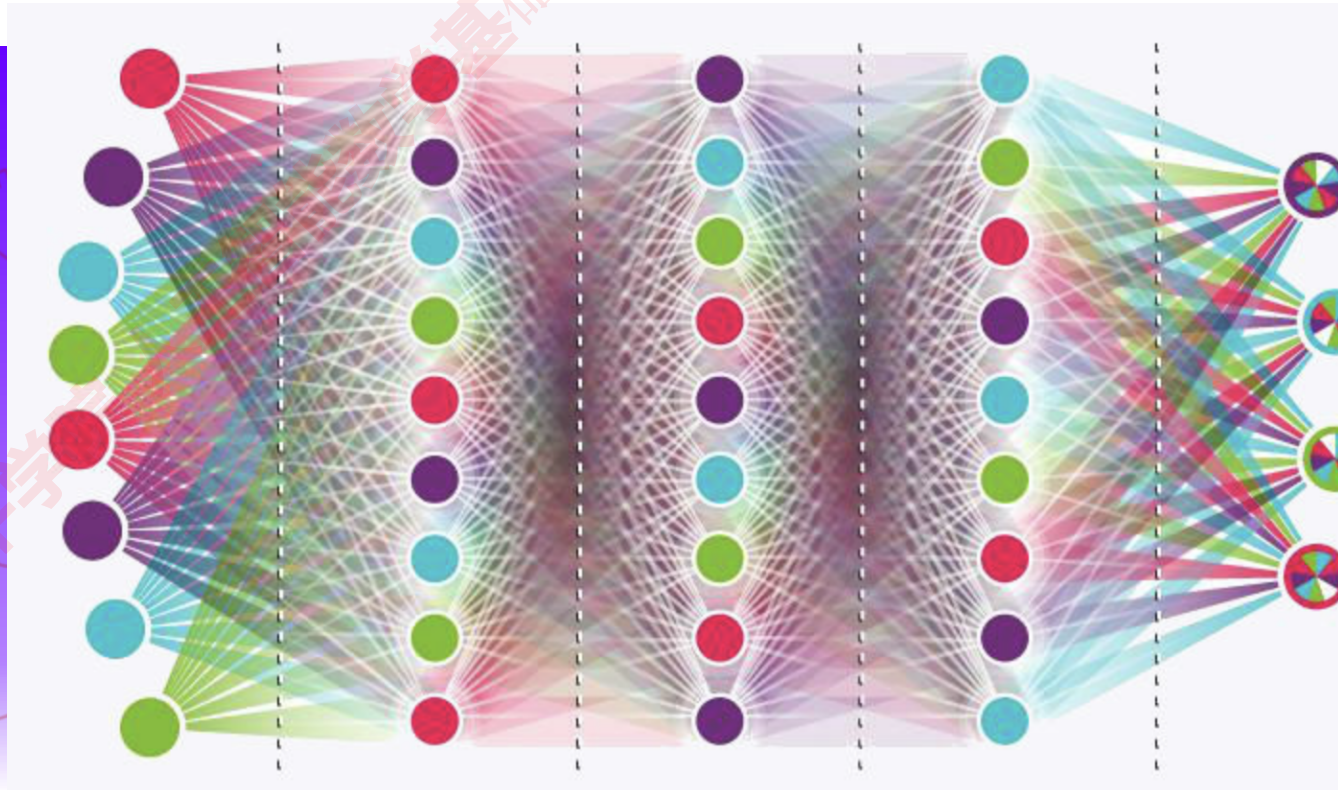
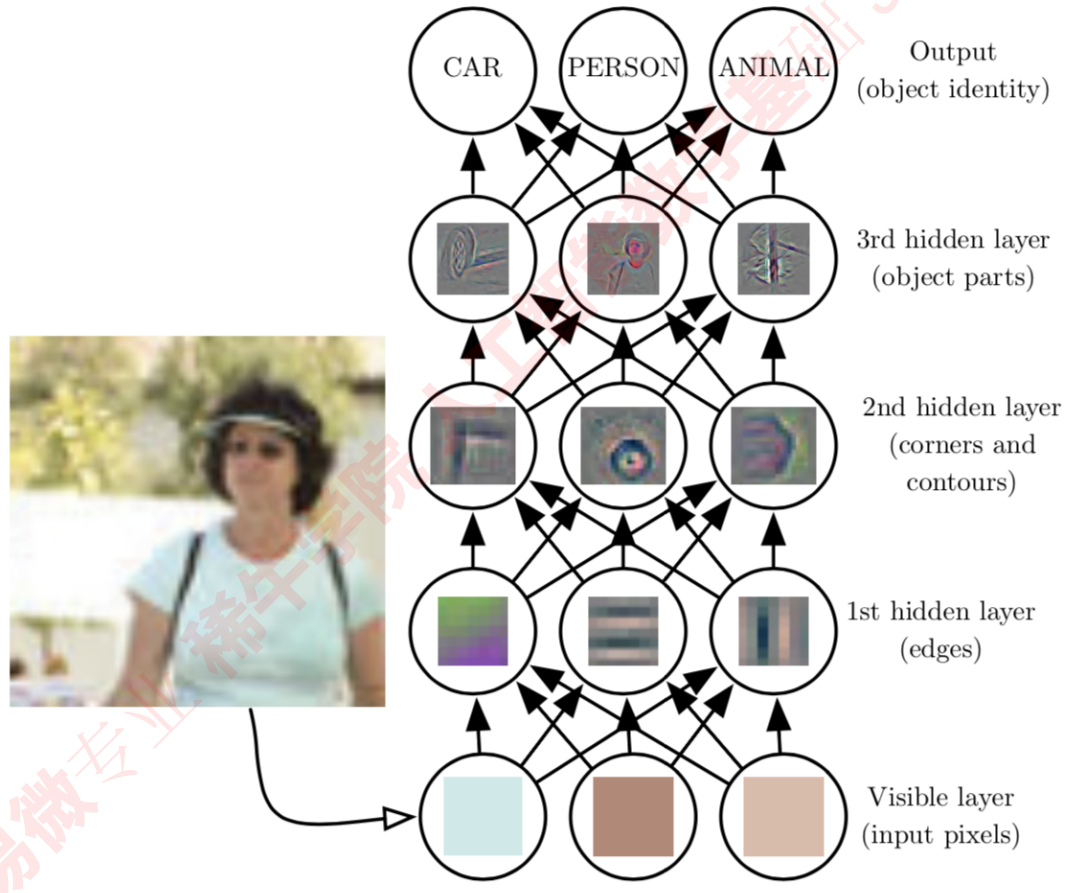
架构



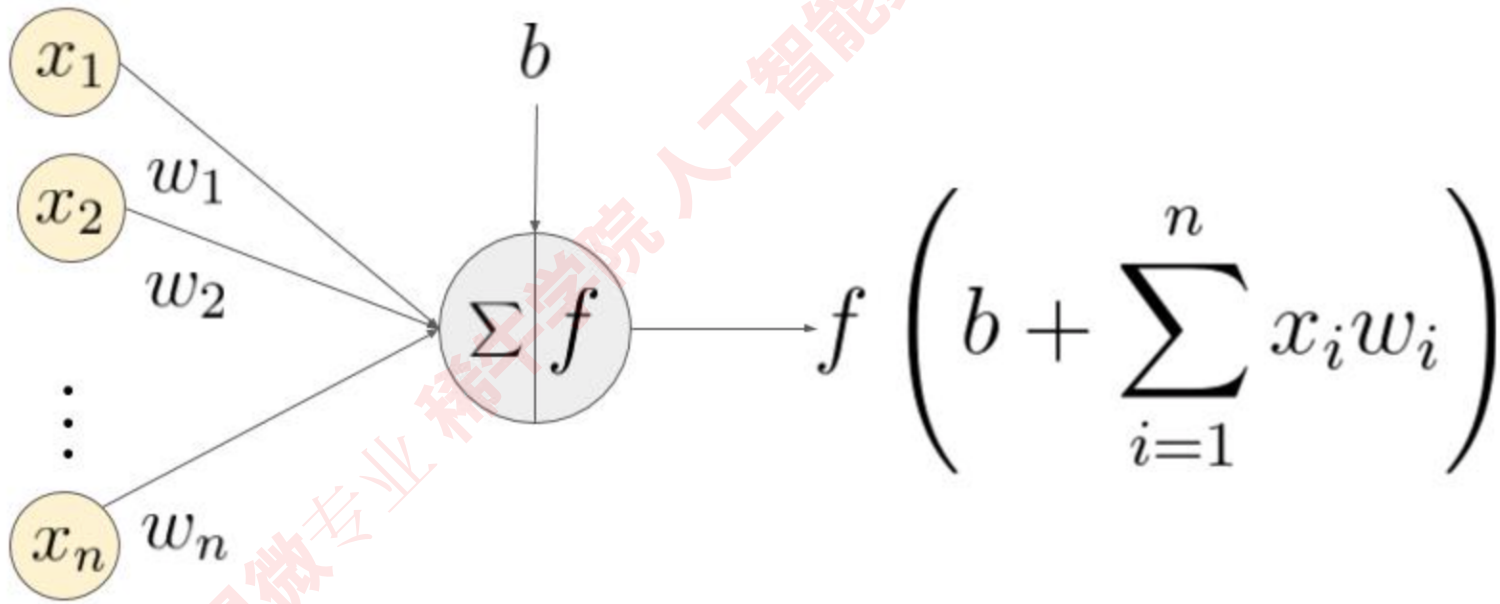


功能 (DL 例子)

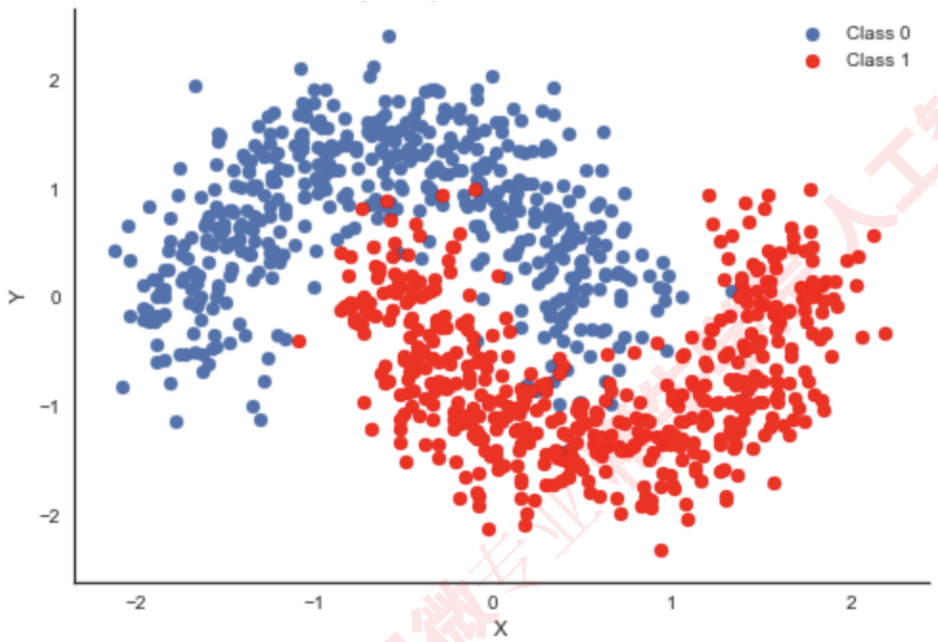


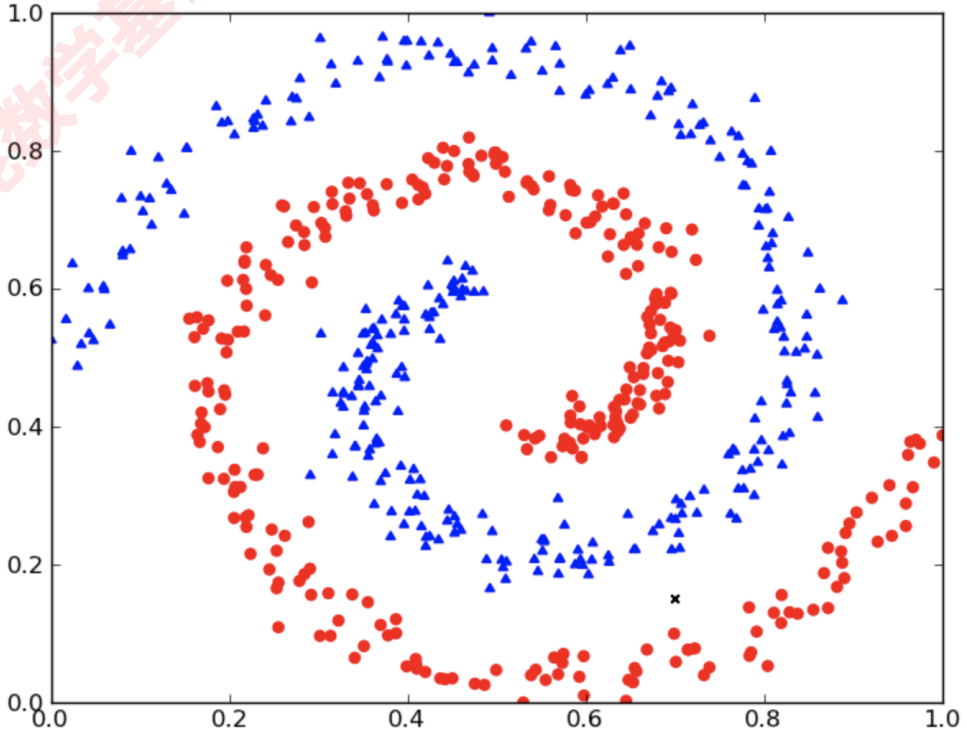
神经元

- 激活函数
- 相互嵌套

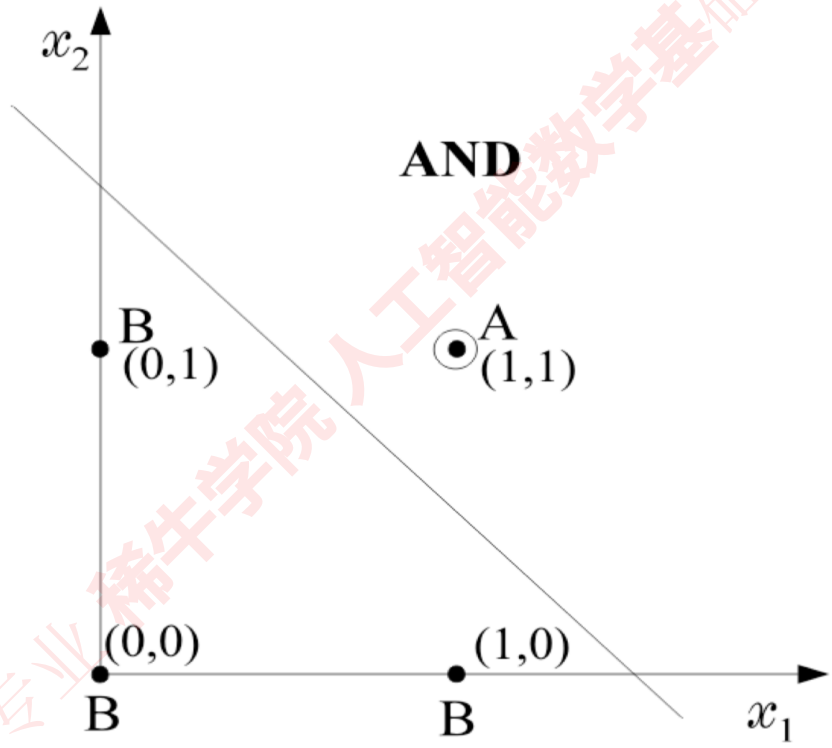


切分原理

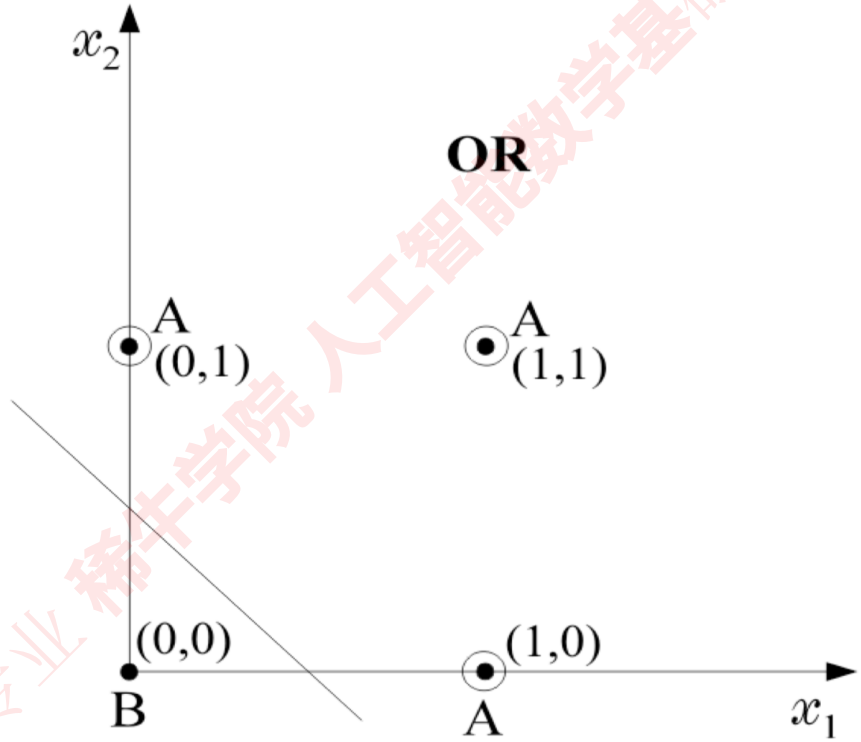




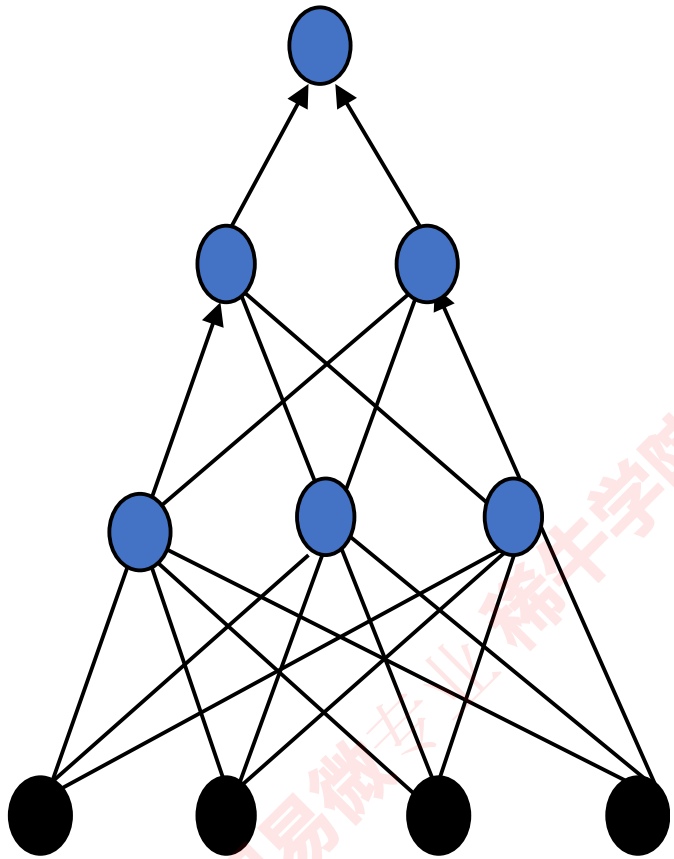
逻辑与

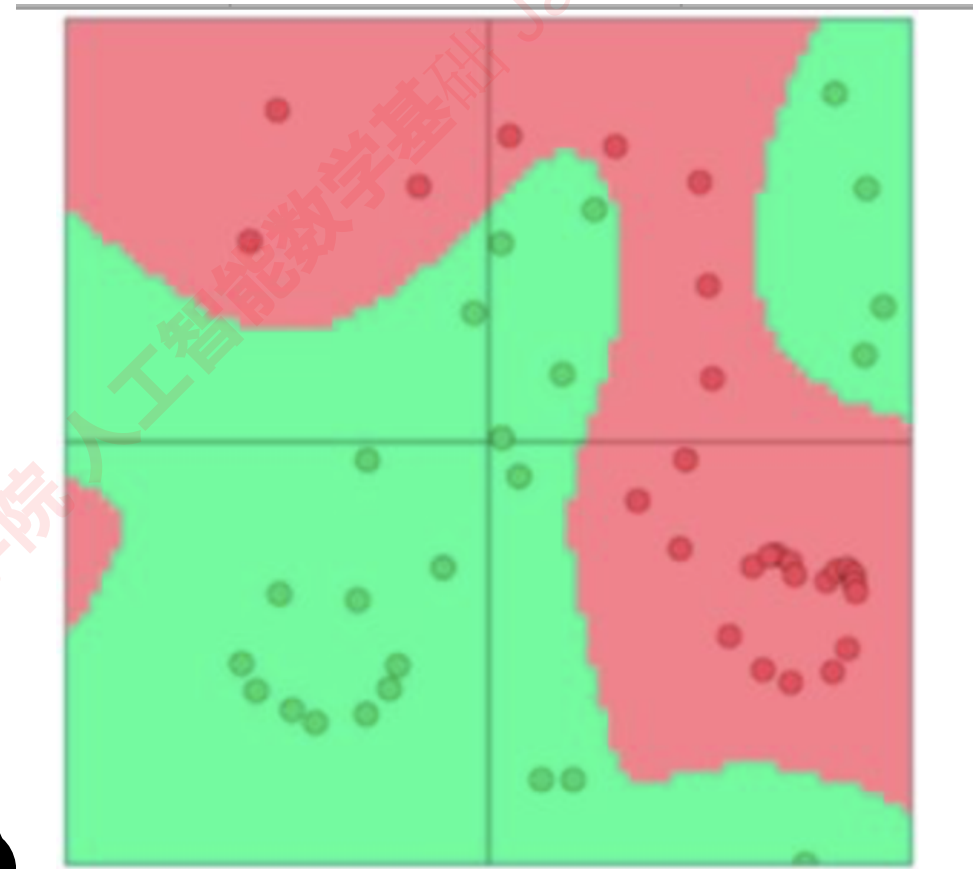


逻辑或

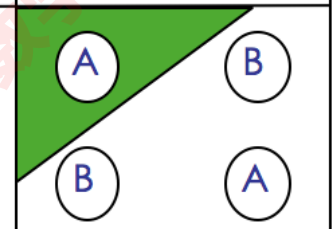
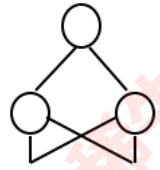
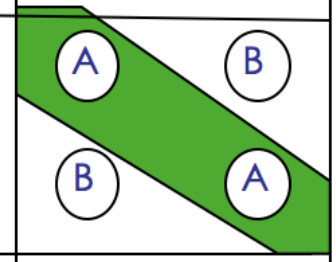
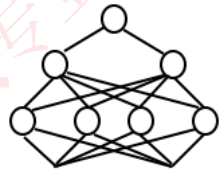
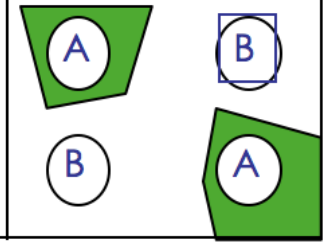


空间切分





切分原理

Structure	Types of Decision Regions	Exclusive-OR Problem
Single-Layer 	Half Plane Bounded By Hyper plane	
Two-Layer 	Convex Open Or Closed Regions	
Three-Layer 	Arbitrary (Complexity Limited by No. of Nodes)	

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(Back Propagation, BP) 算法 (手推 1)

(Back Propagation, BP) 算法 (手推 2)

(Back Propagation, BP) 算法 (手推 3)

BP 算法数学描述总结

符号定义

- x_j^l: 层 l 下的节点 j 的输入
- $w_{i,j}^l$: 从层 $l-1$ 中的节点 i 到层 l 中的节点 j 这一段的权重
- $\sigma(x) = \frac{1}{1+e^{-x}}$
- θ_j^l : 层 l 下的节点 j 的偏置
- O_j^l: 层 l 下的节点 j 的输出
- t_j: 目标值
- $E = \frac{1}{2}\sum_{k \in K}(O_k - t_k)^2$

BP(输出层)

- 目标是计算：$\frac{\partial E}{\partial w_{j,k}}$
- 具体计算

$$
\begin{aligned}
\frac{\partial E}{\partial w_{j,k}} &= \frac{\partial}{\partial O_k}\frac{1}{2}\sum_{k\in K}\left(O_k - t_k\right)^2 \times \frac{\partial}{\partial x_k}O_k \times \frac{\partial}{\partial w_{j,k}}x_k \\
&= \left(O_k - t_k\right) \times \frac{\partial}{\partial x_k}\sigma\left(x_k\right) \times O_j \\
&= \left(O_k - t_k\right) \times \sigma\left(x_k\right)\left(1 - \sigma\left(x_k\right)\right) \times O_j \\
&= \underbrace{\left(O_k - t_k\right) O_k \left(1 - O_k\right)}_{\delta_k} O_j
\end{aligned}
$$

BP(隐层)

- 目标是计算：$\frac{\partial E}{\partial w_{i,j}}$
- 具体计算

$$
\begin{aligned}
\frac{\partial E}{\partial w_{i,j}} &= O_j\left(1-O_j\right) O_i \sum_{k \in K}\left(O_k-t_k\right) O_k\left(1-O_k\right) w_{j,k} \\
&= O_i O_j\left(1-O_j\right) \sum_{k \in K} \delta_k w_{j,k} \\
&= O_i \delta_j
\end{aligned}
$$

BP 总结

- 对于输出层节点 $k \in K$

$$\frac{\partial E}{\partial w_{j,k}} = O_j \delta_k$$

其中 $\delta_k = (O_k - t_k)\, O_k\, (1 - O_k)$

- 对于隐层节点 $j \in J$

$$\frac{\partial E}{\partial w_{i,j}} = O_i \delta_j$$

其中 $\delta_j = O_j\, (1 - O_j) \sum_{k \in K} \delta_k w_{j,k}$

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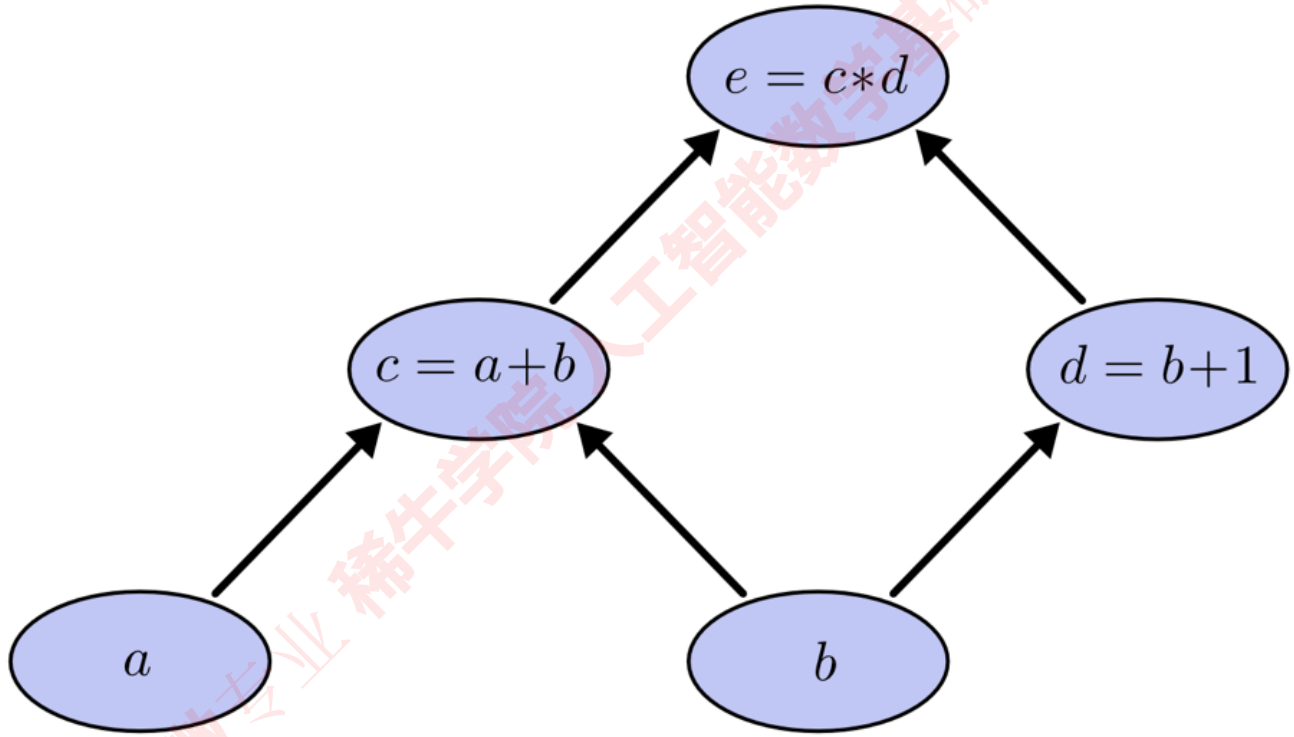
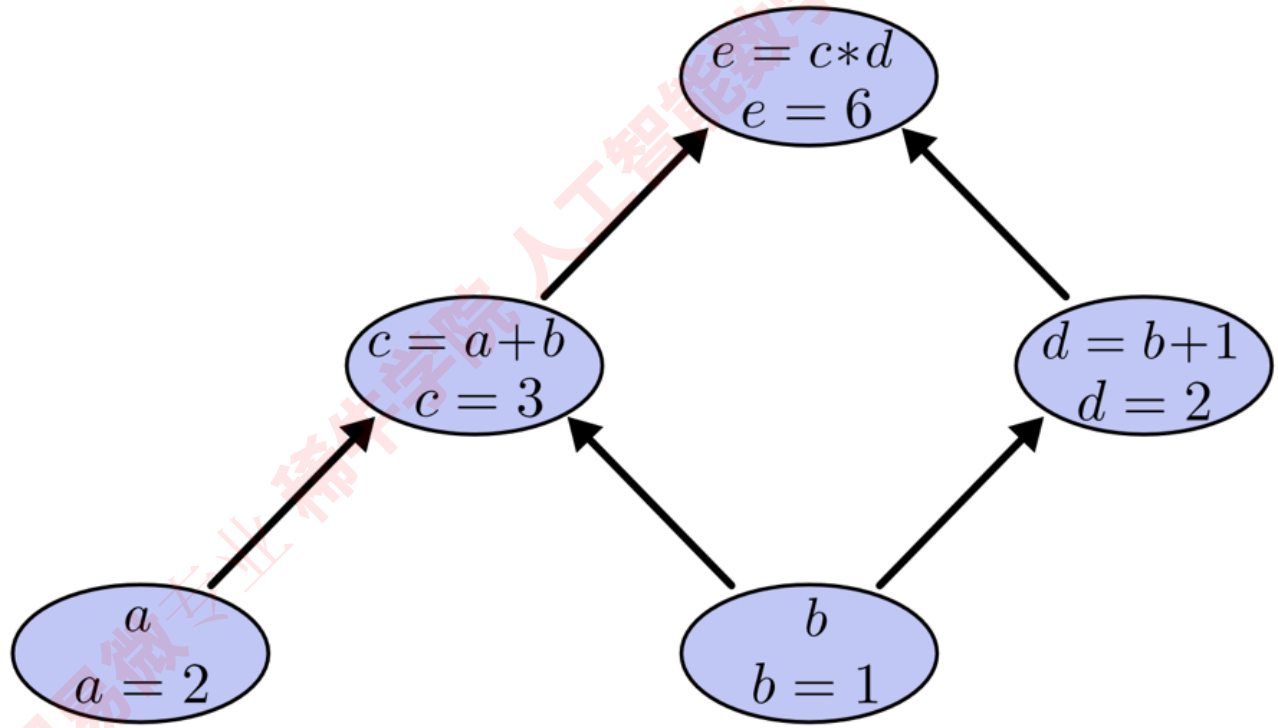
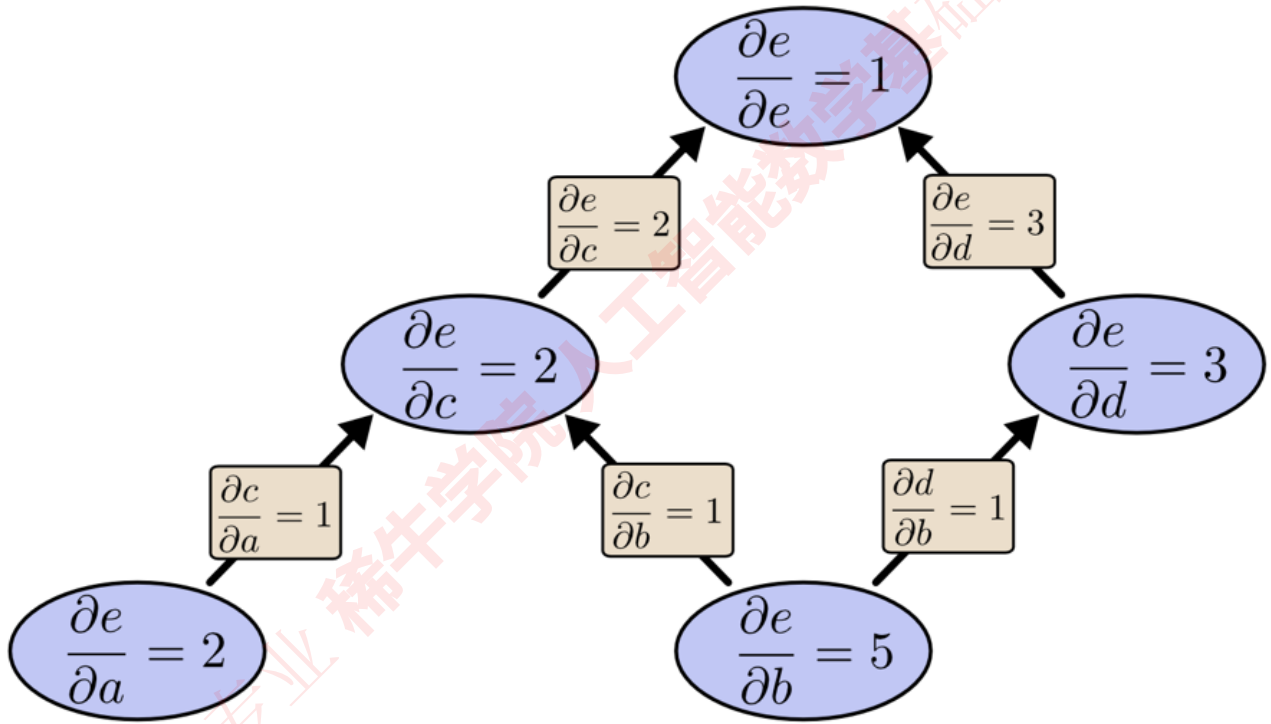


计算图

$$\frac{\partial e}{\partial a} = \frac{\partial e}{\partial c} \times \frac{\partial c}{\partial a}$$
$$\frac{\partial e}{\partial b} = \frac{\partial e}{\partial c} \times \frac{\partial c}{\partial b} + \frac{\partial e}{\partial d} \times \frac{\partial d}{\partial b}$$



BP



BP 案例演示

本章总结

本章参考资料

- Bishop, Christopher M. Pattern recognition and machine learning. springer, 2006.

- Goodfellow, Ian, Yoshua Bengio, and Aaron Courville. Deep learning. MIT press, 2016.

- Calculus on Computational Graphs: Backpropagation