VLSI DSP HW1

4107064136 林軒宇

```
clear;
clc;
%---------------------QR_cordic------------------%
row = 8;
col = 4;
A = random_matrix(row,col);
% Check if A has full column rank 4
while 1
    if rank(A) == min(row,col)
        break
    else
        A = random_matrix(row, col);
    end
end
%floating point of QR;
[Q,R] = QR(A);
check_Q_unit = Q * Q';
```

```matlab
%--Phase 1: Determine the required word length of matrix R elements first
R_sign = 1;
R_dec  = 2;
R_frac = 10;
R_len  = R_sign + R_dec + R_frac;
% Initialize Q_hat and R_hat matrices
Q_hat_inv = eye(row);
R_hat = sfi(A, R_len, R_frac);
% Eliminate A(q+1,p) by A(q,p)
for p_hat = 1:col
    for q_hat = (row-1):(-1):p_hat
        Q_hat_inv_single = eye(row, 'like', R_hat);
        rot_angle_hat = atan2(R_hat(q_hat+1,p_hat), R_hat(q_hat,p_hat));
        Q_hat_inv_single(q_hat  ,q_hat  ) =  cos(rot_angle_hat);
        Q_hat_inv_single(q_hat  ,q_hat+1) =  sin(rot_angle_hat);
        Q_hat_inv_single(q_hat+1,q_hat  ) = -sin(rot_angle_hat);
        Q_hat_inv_single(q_hat+1,q_hat+1) =  cos(rot_angle_hat);
        % inverse of Q matrix
        Q_hat_inv = Q_hat_inv_single * Q_hat_inv;
        Q_hat_inv = sfi(Q_hat_inv, R_len, R_frac);
        % A matrix after one rotation
        R_hat = Q_hat_inv_single * R_hat;
        R_hat = sfi(R_hat, R_len, R_frac);
    end
end
Q_hat = Q_hat_inv';
% Calculate the quantization error value by self-defined function
delta_1 = quantization_error(R, R_hat);

%-- Phase 2: Replace the trigonometric functions with the CORDIC scheme
%-- determine the required iteration number and the word length of the scaling factor K
iter_num = 7;
K_sign = 1;
K_dec  = 0;
K_frac = R_frac;
K_len  = K_sign + K_dec + K_frac;
```

```matlab
Z_sign = 1;
Z_dec  = 7;
Z_frac = 3;
Z_len  = Z_sign + Z_dec + Z_frac;


R_hat_cordic = sfi(A, R_len, R_frac);
% Eliminate A(q+1,p) by A(q,p)
for p_hat = 1:col
    for q_hat = (row-1):(-1):p_hat
        if R_hat_cordic(q_hat,p_hat) < 0
            for rev = p_hat:col
                R_hat_cordic(q_hat  ,rev) = -R_hat_cordic(q_hat  ,rev);
                R_hat_cordic(q_hat+1,rev) = -R_hat_cordic(q_hat+1,rev);
            end
        end
        x_v = R_hat_cordic(q_hat  ,p_hat); % initial input x of vectoring mode
        y_v = R_hat_cordic(q_hat+1,p_hat); % initial input y of vectoring mode

        z_v = 0;                           % initial input z of vectoring mode
        % call vectoring_mode
        [X_v, Y_v, Z_v, K_v] = vectoring_mode(x_v, y_v, z_v, iter_num, R_len, R_frac, K_len, K_frac, Z_len, Z_frac);
        R_hat_cordic(q_hat  ,p_hat) = X_v; % update R_hat_cordic after rotation
        R_hat_cordic(q_hat+1,p_hat) = Y_v;

        % computing rotation mode (col-p_hat) times: rotate the same angle as vectoring mode
        for rot = 1: (col-p_hat)
            x_r = R_hat_cordic(q_hat  ,p_hat+rot);   % initial input x of rotation mode
            y_r = R_hat_cordic(q_hat+1,p_hat+rot);   % initial input y of rotation mode
            z_r = -Z_v;                              % initial input z of rotation mode
            % call rotation_mode
            [X_r, Y_r, Z_r, K_r] = rotation_mode(x_r, y_r, z_r, iter_num, R_len, R_frac, K_len, K_frac, Z_len, Z_frac);
            R_hat_cordic(q_hat  ,p_hat+rot) = X_r;   % update R_hat_cordic after rotation
            R_hat_cordic(q_hat+1,p_hat+rot) = Y_r;
        end
    end
end
```

```matlab
%Determine the final quantization error value delta
delta_2 = quantization_error(R, R_hat_cordic);
output = [K_len,iter_num,delta_2];

display(A);
display(Q);
display(R);
formatSpec = 'Q x (Q)^-1:\n' ;
fprintf(formatSpec);
disp(check_Q_unit);
fprintf('word length, iteration number, delta');
disp(output);



function A1 =  random_matrix(row,col)
    A1 = randn([row col],'double');
    for i = 1:row
        for j = 1:col
            if abs(A1(i,j)) >1
                A1(i,j) = rand(1)*0.75 + 0.25;
            elseif abs(A1(i,j)) < 0.25
                A1(i,j) = rand(1)*0.75 + 0.25;
            elseif rank(A1) < 4
                A1(i,j) = rand(1)*0.75 + 0.25;
            end
        end
    end
end
```

```matlab
function alpha = angle_lut(i)
switch i
    case 0
        alpha = 45;
    case 1
        alpha = 26.6;
    case -1
        alpha = 14;
    case 3
        alpha = 7.1;
    case 4
        alpha = 3.6;
    case -3
        alpha = 1.8;
    case 6
        alpha = 0.9;
    case 7
        alpha = 0.4;
    case 8
        alpha = 0.2;
    case 9
        alpha = 0.1;
    otherwise
        alpha = 0;
end
```

```matlab
function [X, Y, Z, K] = vectoring_mode(x, y, z, iter_num, xy_len, xy_frac, k_len, k_frac, z_len, z_frac)
% Initialize X, Y, Z, K
X = x;
Y = y;
Z = z;
K = 1;

% Iterate iter_num times
for iter = 0:iter_num
    % decide the rotation direction
    % +1 means counterclockwise, -1 means clockwise
    d = -sign(X * Y);
    % store x(i) and y(i)
    X_temp = X;
    Y_temp = Y;
    X = X - d * bitsra(Y_temp, iter);
    % truncate X to signed fixed point each iteration
    X = sfi(X, xy_len, xy_frac);
    Y = Y + d * bitsra(X_temp, iter);
    % truncate Y to signed fixed point each iteration
    Y = sfi(Y, xy_len, xy_frac);
    alpha = angle_lut(iter);
    % truncate alpha to signed fixed point each iteration

    alpha_fix = sfi(alpha, z_len, z_frac);
    Z = Z - d * alpha_fix;
    % truncate Z to signed fixed point each iteration
    Z = sfi(Z, z_len, z_frac);
    K = K * cosd(alpha);
    % truncate K to fixed point each iteration
    K = sfi(K, k_len, k_frac);
end

% Multiply with the scaling factor K
X = X * K;
Y = Y * K;
% Truncate X Y to fixed point the same as input
X = sfi(X, xy_len, xy_frac);
Y = sfi(Y, xy_len, xy_frac);
```

```matlab
function [X, Y, Z, K] = rotation_mode(x, y, z, iter_num, xy_len, xy_frac, k_len, k_frac, z_len, z_frac)
% Initialize X, Y, Z, K
X = x;
Y = y;
Z = z;
K = 1;

% Iterate iter_num times
for iter = 0:iter_num
    % decide the rotation direction
    % +1 means counterclockwise, -1 means clockwise
    d = sign(Z);
    % store x(i) and y(i)
    X_temp = X;
    Y_temp = Y;
    X = X - d * bitsra(Y_temp, iter);
    % truncate X to fixed point each iteration
    X = sfi(X, xy_len, xy_frac);
    Y = Y + d * bitsra(X_temp, iter);
    % truncate Y to fixed point each iteration
    Y = sfi(Y, xy_len, xy_frac);
    alpha = angle_lut(iter);
    % truncate alpha to fixed point each iteration

    alpha_fix = sfi(alpha , z_len, z_frac);
    Z = Z - d * alpha_fix;
    % truncate Z to fixed point each iteration
    Z = sfi(Z, z_len, z_frac);
    K = K * cosd(alpha);
    % truncate K to fixed point each iteration
    K = sfi(K, k_len, k_frac);
end

% Multiply K by X at the final stage
X = X * K;
% Truncate X to fixed point the same as input
X = sfi(X, xy_len, xy_frac);
% Multiply K by Y at the final stage
Y = Y * K;
% Truncate Y to fixed point the same as input
Y = sfi(Y, xy_len, xy_frac);
```

```matlab
function [Q,R] = QR(A)
  [m,n] = size(A);
  R = A;
  Q = coder.nullcopy(repmat(A(:,1),1,m)); % Declare type and size of Q
  Q(:) = eye(m);                          % Initialize Q
  for j=1:n
    for i=j+1:m
      [R(j,j:end),R(i,j:end),Q(:,j),Q(:,i)] = ...
          givensrotation(R(j,j:end),R(i,j:end),Q(:,j),Q(:,i));
    end
  end
end

function [x,y,u,v] = givensrotation(x,y,u,v)
  a = x(1); b = y(1);
  if b==0
    % No rotation necessary.  c = 1; s = 0;
    return;
  else
    if abs(b) > abs(a)
      t = -a/b; s = 1/sqrt(1+t^2); c = s*t;
    else
      t = -b/a; c = 1/sqrt(1+t^2); s = c*t;
    end
  end
  x0 = x;              u0 = u;
  % x and y form R,   u and v form Q
  x(:) = c*x0 - s*y;  u(:) = c*u0 - s*v;
  y(:) = s*x0 + c*y;  v(:) = s*u0 + c*v;
end
```

```matlab
function delta = quantization_error(R, R_hat)
[row, col] = size(R);
r_diff = 0;
r_sum = 0;
R_hat_float = double(R_hat);
for i = 1:row
    for j = i:col
        r_diff = r_diff +( abs(R(i,j)) - abs(R_hat_float(i,j)) )^2;
        r_sum = r_sum + abs(R(i,j)) ^ 2;
    end
end
% Calculate the quantization error value
delta = sqrt(r_diff) / sqrt(r_sum);
```

Outcome:

A =

```
  -0.2682    0.6527    0.5409   -0.3633
  -0.4099   -0.7343    0.4869    0.8295
  -0.7113    0.5406    0.7723    0.3440
   0.3476    0.9758   -0.9896    0.6263
   0.3193    0.2559    0.5673   -0.2867
  -0.3983    0.2778    0.7417    0.7922
  -0.5435    0.6395    0.6484    0.4056
  -0.9119    0.3316    0.4489    0.7238
```

Q =

```
  -0.1786    0.3531   -0.2623   -0.3909   -0.7407   -0.0343   -0.2537   -0.0875
  -0.2730   -0.5119   -0.0321    0.5169   -0.3960   -0.4668   -0.0152   -0.1423
  -0.4738    0.2136   -0.1036   -0.1034    0.5103   -0.3700   -0.4135   -0.3744
   0.2315    0.6485    0.4624    0.4866   -0.1217   -0.2259   -0.0940   -0.0241
   0.2126    0.2068   -0.7045    0.1956    0.1389   -0.3522    0.0409    0.4821
  -0.2653    0.1045   -0.3008    0.5380         0    0.6848   -0.2565   -0.0637
  -0.3620    0.3007   -0.1383    0.0502         0         0    0.8296   -0.2622
  -0.6073    0.0544    0.3158   -0.0477         0         0         0    0.7254
```

R =

```
   1.5014   -0.3986   -1.4080   -1.0371
        0    1.6470   -0.1208    0.1115
        0         0   -1.2657    0.4587
        0    0.0000         0    1.1960
        0         0         0   -0.0000
   0.0000         0         0    0.0000
   0.0000         0         0         0
        0   -0.0000         0   -0.0000
```

Q x (Q)^-1:

```
   1.0000    0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000    0.0000
   0.0000    1.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
   0.0000    0.0000    1.0000   -0.0000   -0.0000    0.0000   -0.0000    0.0000
  -0.0000    0.0000   -0.0000    1.0000    0.0000    0.0000   -0.0000   -0.0000
  -0.0000    0.0000   -0.0000    0.0000    1.0000   -0.0000   -0.0000    0.0000
  -0.0000    0.0000    0.0000    0.0000   -0.0000    1.0000   -0.0000    0.0000
  -0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000    1.0000    0.0000
   0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000    0.0000    1.0000
```

word length, iteration number, delta   11.0000   7.0000   0.4269

A =

```
  0.3658   -0.4098    0.5360   -0.4494
 -0.9610   -0.5035    0.9724    0.3709
 -0.6537    0.8186    0.2570    0.9033
  0.5131    0.6103   -0.9742    0.8412
 -0.2710    0.7642    0.4706   -0.4147
 -0.9000    0.6480    0.5476    0.8743
 -0.2857    0.6981    0.5015   -0.3909
 -0.4624    0.4744    0.5894    0.4092
```

Q =

```
  0.2138    0.1639   -0.7171    0.5432    0.0645    0.0372   -0.0486   -0.3320
 -0.5617    0.5191    0.0342    0.1370   -0.4955   -0.3255    0.1753   -0.1135
 -0.3821   -0.3447    0.1395    0.2481    0.5291   -0.5459   -0.1080   -0.2541
  0.2999   -0.4823    0.3266    0.4825   -0.5299   -0.1054    0.1882   -0.1033
 -0.1584   -0.3982   -0.3730   -0.3611   -0.4355   -0.1591   -0.5668   -0.1030
 -0.5261   -0.1867    0.1191    0.2051         0    0.7471   -0.0943   -0.2551
 -0.1670   -0.3552   -0.3743   -0.3216         0         0    0.7679   -0.1129
 -0.2703   -0.1809   -0.2550    0.3406         0         0         0    0.8445
```

R =

```
  1.7107   -0.6413   -1.4277   -0.8369
 -0.0000   -1.6640    0.3995   -0.5316
       0   -0.0000   -1.0818    1.0365
       0         0         0    1.0308
 -0.0000         0   -0.0000    0.0000
       0    0.0000         0    0.0000
       0         0         0         0
  0.0000         0         0    0.0000
```

Q x (Q)^-1:

```
  1.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000    0.0000
 -0.0000    1.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000
  0.0000    0.0000    1.0000   -0.0000    0.0000   -0.0000    0.0000    0.0000
 -0.0000   -0.0000   -0.0000    1.0000   -0.0000   -0.0000    0.0000   -0.0000
  0.0000    0.0000    0.0000   -0.0000    1.0000   -0.0000    0.0000    0.0000
 -0.0000    0.0000   -0.0000   -0.0000   -0.0000    1.0000    0.0000   -0.0000
  0.0000   -0.0000    0.0000    0.0000    0.0000    0.0000    1.0000    0.0000
  0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000    1.0000
```

word length, iteration number, delta   11.0000   7.0000   0.5677

A =

0.9651	0.6557	0.4902	0.7598
-0.8658	-0.3862	0.7653	-0.8097
0.2774	0.5256	0.7783	0.8569
0.8115	0.3401	0.6438	0.4944
-0.2512	0.6598	0.5404	0.5492
0.5613	0.3856	0.4415	0.2654
0.9428	-0.3202	-0.7603	0.3083
-0.7745	0.8175	-0.6936	-0.9382

Q =

0.4663	-0.3600	0.1089	0.1776	-0.5767	-0.4815	0.0475	0.2070
-0.4183	0.1888	0.5509	0.4583	-0.1747	-0.1284	0.3837	-0.2856
0.1340	-0.3274	0.3278	-0.3653	-0.2542	0.3183	-0.2187	-0.6482
0.3921	-0.1624	0.2667	0.3521	0.6855	-0.2434	-0.2241	-0.2131
-0.1214	-0.4579	0.1860	-0.4742	0.3200	-0.1637	0.6014	0.1588
0.2712	-0.2122	0.1520	0.3577	0	0.7512	0.2471	0.3240
0.4555	0.2865	-0.4084	0.0200	0	0	0.5742	-0.4621
-0.3742	-0.6036	-0.5306	0.3844	0	0	0	-0.2575

R =

2.0696	0.2438	0.2326	1.4987
0	-1.5055	-0.5316	-0.4404
0	0	1.7480	0.5638
-0.0000	0	0	-0.8951
0.0000	0	0	-0.0000
0	0	0	0.0000
0	0	0	0
0.0000	0	-0.0000	0

Q x (Q)^-1:

1.0000	0.0000	-0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0000
0.0000	1.0000	0.0000	0.0000	-0.0000	0.0000	-0.0000	-0.0000
-0.0000	0.0000	1.0000	-0.0000	-0.0000	0.0000	-0.0000	0.0000
0.0000	0.0000	-0.0000	1.0000	0.0000	0.0000	0.0000	-0.0000
-0.0000	-0.0000	-0.0000	0.0000	1.0000	-0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	-0.0000	1.0000	0.0000	-0.0000
0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0000	1.0000

word length, iteration number, delta   11.0000   7.0000   0.4509

A =

0.7665	0.8328	-0.5138	0.7037
0.5404	-0.6946	0.7964	0.3566
0.2689	-0.4619	-0.6712	0.5658
0.3881	0.8836	0.7943	0.5278
0.8812	0.4359	0.7907	0.6098
0.8007	0.8967	0.2877	-0.6479
-0.3012	0.5047	0.6783	0.3826
-0.6987	-0.4009	0.3656	0.5510

Q =

0.4354	-0.2522	0.4254	0.4875	-0.4205	-0.3562	0.0722	0.1388
0.3070	0.6119	-0.4954	0.0164	-0.5087	-0.0122	0.1572	-0.0448
0.1527	0.3761	0.3511	0.4075	0.2919	0.6121	0.2933	-0.0006
0.2205	-0.4123	-0.3388	0.2462	-0.1807	0.5537	-0.4987	-0.1342
0.5006	0.0313	-0.3543	0.2079	0.6683	-0.3510	-0.0978	0.0172
0.4548	-0.2800	-0.0238	-0.5312	0	0.2616	0.3437	0.4954
-0.1711	-0.4135	-0.3526	0.2363	0	0	0.7125	-0.3345
-0.3969	0.0092	-0.2917	0.3927	0	0	0	0.7766

R =

1.7604	0.9725	0.3588	0.3451
-0.0000	-1.6230	-0.2960	0.0833
0	0	-1.7507	-0.3538
0	0	0	1.4871
0	0	0	0
-0.0000	0	0.0000	0
0	0	0	-0.0000
0	0	0	0

Q x (Q)^-1:

1.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000	-0.0000
-0.0000	1.0000	0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000
-0.0000	0.0000	1.0000	-0.0000	0.0000	0.0000	0.0000	-0.0000
0.0000	-0.0000	-0.0000	1.0000	0.0000	-0.0000	-0.0000	-0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	-0.0000	0.0000
0.0000	0.0000	0.0000	-0.0000	0.0000	1.0000	-0.0000	0.0000
0.0000	0.0000	0.0000	-0.0000	-0.0000	-0.0000	1.0000	0.0000
-0.0000	0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000	1.0000

word length, iteration number, delta   11.0000   7.0000   0.4216

A =

0.8516	0.9919	0.3002	-0.3523
0.9545	0.5530	0.2636	0.7629
0.6737	-0.4234	0.9109	-0.4807
-0.6691	0.3616	0.8378	0.8368
-0.4003	-0.3519	0.6506	0.9140
-0.6718	0.2695	0.9243	0.7195
0.5756	0.3534	0.4134	0.3866
-0.7781	0.4659	-0.8305	0.2814

Q =

-0.4213	0.6008	-0.0833	0.4277	0.4745	-0.1448	-0.1574	-0.0276
-0.4722	0.2818	-0.0630	-0.6525	-0.2846	0.1475	-0.4031	0.0517
-0.3333	-0.3714	-0.4293	0.3168	-0.1923	-0.0266	-0.1060	0.6461
0.3310	0.3276	-0.4742	0.0037	-0.4129	-0.6114	-0.0372	-0.1121
0.1980	-0.2023	-0.3682	-0.4678	0.6974	-0.2107	-0.1087	0.1517
0.3323	0.2633	-0.5202	0.0959	0	0.7337	-0.0161	-0.0422
-0.2848	0.1839	-0.1691	-0.2379	0	0	0.8877	0.1089
0.3849	0.4127	0.3811	-0.0794	0	0	0	0.7280

R =

-2.0215	-0.3196	-0.2786	0.6437
0	1.4268	0.0358	0.6478
0.0000	0	-1.9366	-0.8781
0	0	0	-1.2705
0.0000	0	0	0.0000
0.0000	0	0.0000	0
0	0	0	0
0	0	0	0

Q x (Q)^-1:

1.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0000	-0.0000
-0.0000	1.0000	-0.0000	0.0000	0.0000	-0.0000	0.0000	0.0000
0.0000	-0.0000	1.0000	0.0000	-0.0000	-0.0000	0.0000	0.0000
-0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	-0.0000
-0.0000	0.0000	-0.0000	0.0000	1.0000	0.0000	0.0000	-0.0000
-0.0000	-0.0000	-0.0000	0.0000	0.0000	1.0000	0.0000	-0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
-0.0000	0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0000	1.0000

word length, iteration number, delta   11.0000   7.0000   0.2984

A =

0.4817	0.7946	0.8372	0.7868
0.7703	0.8707	0.2917	0.2574
0.8824	-0.3818	0.9877	-0.5341
-0.7158	0.4289	0.3929	0.9417
-0.2805	-0.2991	0.8282	0.2820
0.5336	-0.8999	0.2798	0.7783
0.7971	0.6347	0.4182	-0.8976
0.4855	0.4518	-0.7745	-0.7923

Q =

-0.2631	0.3934	-0.4012	-0.3671	-0.1190	0.6648	-0.1115	0.1087
-0.4208	0.4005	-0.0379	-0.3270	0.3831	-0.5506	-0.2594	-0.1922
-0.4820	-0.3293	-0.3857	0.3466	-0.4116	-0.2021	-0.3969	0.1498
0.3910	0.3352	-0.3685	-0.1852	-0.4445	-0.4558	0.2530	0.3087
0.1532	-0.1356	-0.5178	0.2067	0.6871	0.0061	0.0512	0.4154
-0.2915	-0.5814	-0.0386	-0.6096	0	-0.0793	0.4277	0.1213
-0.4354	0.2623	-0.0997	0.4310	0	0	0.7164	-0.1808
-0.2652	0.1970	0.5230	0.0617	0	0	0	0.7833

R =

-1.8306	-0.4035	-0.5968	0.7277
0	1.7500	-0.0652	0.0219
0	0	-1.7591	-0.9673
-0.0000	0	0	-1.5844
0	0	0	0
0	0	0.0000	0.0000
-0.0000	0	0	0
0.0000	0	0.0000	0

Q x (Q)^-1:

1.0000	0.0000	0.0000	0.0000	-0.0000	0.0000	-0.0000	-0.0000
0.0000	1.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.0000	0.0000	1.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0000
0.0000	-0.0000	-0.0000	1.0000	0.0000	0.0000	-0.0000	0.0000
-0.0000	-0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	-0.0000
0.0000	-0.0000	-0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	1.0000	-0.0000
-0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	1.0000

word length, iteration number, delta   11.0000   7.0000   0.4623

A =

0.7907	0.3532	0.5425	0.9455
0.9381	0.7852	0.7138	0.5075
0.5531	0.9520	-0.8028	-0.5368
-0.9606	0.3436	0.7979	-0.3020
0.7349	-0.6155	0.8749	0.5487
0.7612	0.8124	0.8764	0.9149
0.4918	0.6642	0.9843	0.6620
0.4978	0.7146	0.8284	0.5205

Q =

-0.3794	0.0132	-0.0824	0.5141	0.4812	-0.5624	-0.1730	-0.0837
-0.4501	-0.2040	-0.0876	-0.3116	-0.6159	-0.3836	-0.2683	-0.2290
-0.2654	-0.4137	0.6129	-0.4273	0.4358	0.0649	0.0253	-0.0731
0.4610	-0.4796	-0.5256	-0.2748	0.2990	-0.1355	-0.2521	-0.1855
-0.3526	0.5785	-0.3751	-0.5160	0.3313	0.1406	-0.0532	-0.0407
-0.3653	-0.2707	-0.1966	0.3309	0	0.7030	-0.3251	-0.2128
-0.2360	-0.2585	-0.3166	0.0317	0	0	0.8517	-0.2256
-0.2389	-0.2870	-0.2323	-0.0603	0	0	0	0.8961

R =

-2.0840	-0.9889	-1.0050	-1.3921
0	-1.6668	-0.4123	0.0252
0	-0.0000	-2.0232	-1.0088
0	0.0000	0	0.6495
-0.0000	-0.0000	0	0
0	0.0000	0	0
-0.0000	0.0000	-0.0000	0
0	0	0	-0.0000

Q x (Q)^-1:

1.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000	-0.0000	0.0000
-0.0000	1.0000	-0.0000	0.0000	0.0000	-0.0000	-0.0000	0.0000
-0.0000	-0.0000	1.0000	-0.0000	0.0000	-0.0000	0.0000	0.0000
0.0000	0.0000	-0.0000	1.0000	0.0000	0.0000	-0.0000	-0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	-0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	-0.0000	0.0000
-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	-0.0000	-0.0000	0.0000	0.0000	1.0000

word length, iteration number, delta   11.0000   7.0000   0.5258

A =

-0.5000	0.9133	0.7906	0.2640
0.7165	-0.4732	0.7561	0.5066
0.5789	0.5784	-0.4049	0.3378
0.8610	0.8099	0.5279	0.5220
0.4936	0.7163	0.4347	0.3970
0.5070	0.5318	0.6599	-0.4828
-0.6327	0.4340	0.6714	0.5469
0.5486	0.5201	-0.3038	0.6134

Q =

0.2867	-0.6045	0.2718	0.0115	-0.1462	-0.3044	-0.5997	0.0684
-0.4108	0.3881	0.6395	0.3724	0.0054	-0.3448	-0.1125	0.0386
-0.3319	-0.2362	-0.4244	0.1177	-0.0637	-0.6394	0.1789	-0.4418
-0.4937	-0.3224	0.1356	0.0785	-0.5256	0.5300	-0.0191	-0.2651
-0.2830	-0.3291	0.0964	0.0579	0.8356	0.2336	-0.1026	-0.1887
-0.2907	-0.2213	0.2974	-0.7224	0	-0.2104	0.3464	0.3033
0.3628	-0.3519	0.3232	0.4186	0	0	0.6818	-0.0074
-0.3145	-0.2078	-0.3412	0.3751	0	0	0	0.7750

R =

-1.7441	-0.4990	-0.1859	-0.4688
0	-1.7476	-0.7213	-0.5548
0	0	1.5007	0.1853
0	-0.0000	-0.0000	1.1032
0	0	0	0
0	0	-0.0000	0
0	0	0	0
0	0	0.0000	0

Q x (Q)^-1:

1.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000
-0.0000	1.0000	0.0000	0.0000	0.0000	-0.0000	-0.0000	0.0000
-0.0000	0.0000	1.0000	0.0000	-0.0000	0.0000	-0.0000	-0.0000
-0.0000	0.0000	0.0000	1.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	0.0000	-0.0000	-0.0000	1.0000	-0.0000	0.0000	-0.0000
-0.0000	-0.0000	0.0000	0.0000	-0.0000	1.0000	-0.0000	0.0000
0.0000	-0.0000	-0.0000	-0.0000	0.0000	-0.0000	1.0000	0.0000
0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	1.0000

word length, iteration number, delta   11.0000   7.0000   0.4534

A =

```
 0.8503  -0.8665  -0.6187   0.4645
 0.4155  -0.4218   0.9345  -0.4491
-0.6548  -0.9427   0.6577   0.9493
-0.2963   0.9886   0.7868   0.7174
 0.8792  -0.9884   0.2874   0.5749
-0.9048   0.6030   0.6329   0.6705
-0.4042  -0.3744   0.4518   0.8118
-0.7258   0.6279  -0.5817   0.7351
```

Q =

```
 0.4424  -0.1867  -0.2824   0.4689  -0.6322   0.2155  -0.0588  -0.1421
 0.2162  -0.0904   0.5791  -0.2870  -0.3771   0.1653   0.0683   0.5943
-0.3407  -0.7521   0.1237   0.0374  -0.1164  -0.3418  -0.4131  -0.0242
-0.1542   0.4471   0.5143   0.5314  -0.1859  -0.4195  -0.0788  -0.1126
 0.4575  -0.2445   0.2340   0.4551   0.6403   0.1263  -0.1158   0.1726
-0.4708   0.0211   0.2463   0.1586        0   0.7858  -0.2013  -0.1845
-0.2103  -0.3488   0.1285   0.2168        0        0   0.8724  -0.0974
-0.3776   0.0972  -0.4161   0.3671        0        0        0   0.7349
```

R =

```
 1.9220  -1.2002  -0.4589  -0.8266
 0.0000   1.7970  -0.3829  -0.7774
      0        0   1.7252   0.1934
 0.0000        0        0   1.5773
      0  -0.0000   0.0000        0
 0.0000  -0.0000        0   0.0000
      0   0.0000   0.0000   0.0000
      0   0.0000   0.0000        0
```

Q x (Q)^-1:
```
 1.0000  -0.0000  -0.0000   0.0000   0.0000  -0.0000  -0.0000   0.0000
-0.0000   1.0000  -0.0000   0.0000  -0.0000  -0.0000  -0.0000  -0.0000
-0.0000  -0.0000   1.0000   0.0000   0.0000   0.0000  -0.0000  -0.0000
 0.0000   0.0000   0.0000   1.0000   0.0000   0.0000   0.0000  -0.0000
 0.0000  -0.0000   0.0000   0.0000   1.0000  -0.0000  -0.0000  -0.0000
-0.0000  -0.0000   0.0000   0.0000  -0.0000   1.0000  -0.0000  -0.0000
-0.0000  -0.0000  -0.0000   0.0000  -0.0000  -0.0000   1.0000  -0.0000
 0.0000  -0.0000  -0.0000  -0.0000  -0.0000  -0.0000  -0.0000   1.0000
```

word length, iteration number, delta   11.0000   7.0000   0.3646

A =

-0.8549	-0.9677	0.3210	0.9501
0.7513	0.4051	0.7404	-0.3646
0.3040	-0.3479	0.5550	0.7502
-0.2528	0.9503	0.8582	0.6134
0.8176	0.5628	0.8768	0.9788
0.6061	-0.5808	0.9910	-0.2962
0.5405	0.8751	-0.4149	0.5643
0.8981	0.5417	0.3585	0.5911

Q =

0.4515	0.3099	0.3663	-0.5076	0.3678	-0.2658	0.3007	0.1096
-0.3968	-0.0099	0.2334	0.3884	0.1678	-0.7652	0.1268	-0.0857
-0.1605	0.2976	0.1931	-0.3866	-0.7650	-0.2268	-0.1442	-0.2038
0.1335	-0.6369	0.6799	0.0771	-0.2383	0.1480	0.1483	0.0877
-0.4317	-0.0827	0.3079	-0.3534	0.4411	0.1994	-0.4688	-0.3647
-0.3200	0.5268	0.3284	0.2782	0	0.4803	0.4492	-0.0478
-0.2854	-0.3511	-0.3296	-0.3884	0	0	0.6551	-0.3266
-0.4743	-0.0457	-0.0242	-0.2879	0	0	0	0.8304

R =

-1.8937	-0.9786	-0.8707	-0.2341
0	-1.6971	0.2895	-0.3315
0	0	1.7045	0.8287
0	0	0	-1.6843
0	0.0000	0	0
0	0	0	0.0000
0	0	-0.0000	0
-0.0000	0	0	0

Q x (Q)^-1:

1.0000	-0.0000	0.0000	0.0000	0.0000	-0.0000	0.0000	0.0000
-0.0000	1.0000	0.0000	0.0000	-0.0000	0.0000	-0.0000	-0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	-0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	-0.0000	0.0000
0.0000	-0.0000	0.0000	0.0000	1.0000	-0.0000	-0.0000	-0.0000
-0.0000	0.0000	0.0000	0.0000	-0.0000	1.0000	0.0000	0.0000
0.0000	-0.0000	0.0000	-0.0000	-0.0000	0.0000	1.0000	0.0000
0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.0000	0.0000	1.0000

word length, iteration number, delta   11.0000   7.0000   0.4652