

Solve each problem on separate sheets of paper, and clearly indicate the problem number and your name on each. Carefully and neatly document your answers. You may use a mathematical solver like Matlab or Mathematica. Use plotting software for all plots.

## 1 Partial molar concepts

Following is some data on the heat evolved when 1 mole of sulfuric acid ( $\text{H}_2\text{SO}_4$ ) is isothermally mixed with  $\text{H}_2\text{O}$  at 298 K.

$N_{\text{H}_2\text{O}}$ (mols)	0.25	1.0	1.5	2.33	4.0	5.44	9.0	10.1	19.0	20.0
$-\Delta H_{\text{mix}}$ (J)	8242	28200	34980	44690	54440	58370	62800	64850	70710	71970

1. Is this mixture ideal? Why?
2. Determine and plot the molar enthalpy of mixing as a function of mole fraction of  $\text{H}_2\text{SO}_4$ .
3. Estimate the heat evolved when 100 g of a 60%(w/w) sulfuric acid solution is mixed with 75 g of a 25%(w/w) sulfuric acid solution. *Hints:* What is the molar composition of the initial solutions? Of the final one?)
4. Estimate the partial molar enthalpies of  $\text{H}_2\text{O}$  and  $\text{H}_2\text{SO}_4$  in a 50%(w/w) solution.
5. The mixing enthalpy of a “regular” solution can be written as  $\chi_{12}x_1x_2$ . Fit the data to this model to estimate  $\chi_{12}$  and to estimate the partial molar enthalpies of  $\text{H}_2\text{O}$  and  $\text{H}_2\text{SO}_4$  in a 50%(w/w) solution.

## 2 Phase diagrams for liquids

Within the regular solution model, the free energy of mixing two liquids is given by

$$\Delta g_{\text{mix}} = RT \{x_A \ln x_A + x_B \ln x_B + \chi_{AB}x_Ax_B\}$$

1. Suppose  $\chi_{AB} = 5$  at 300 K for some mixture of liquids A and B. You prepare a mixture of 0.3 mol A and 0.7 mol B at this temperature. How many phases are present at equilibrium, what are their compositions, and how much of each phase (if more than one) is present?
2. What are the spinodal compositions at 300 K of the A/B mixture?
3. The binodal and spinodal curves meet at the critical point. The second and third derivatives of the free energy of mixing must vanish at this point. Find the critical composition and temperature of this mixture. Assume that  $\chi_{AB} \propto 1/T$ .

## 3 Funny phase diagrams

While  $\chi_{AB} \propto 1/T$  is the normal behavior, other dependencies are possible.

1. Construct a temperature vs. composition diagram for a system for which  $\chi_{AB}$  is a positive constant independent of temperature.
2. Construct a temperature vs. composition diagram for a system for which  $\chi_{AB} \propto T$ .

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**4 Two components, two phases, too much fun!**

At 300 K, the saturation pressure of A is ten times the saturation pressure of B. A and B mix ideally.

1. Write down an expression for the free energy of a two-component ideal liquid mixture as a function of pressure and composition,  $g^l(P, x_B)$ .
2. Write down an expression for the free energy of a two-component ideal gas mixture as a function of pressure and composition,  $g^v(P, y_B)$ .
3. Plot  $g^l$  and  $g^v$  vs composition at five pressure from  $P = P^{sat,B}$  to  $P = P^{sat,A}$ . Identify the important regions on each plot.

**5 Vapor-liquid equilibrium.**

The partial pressure of CS<sub>2</sub> above a CS<sub>2</sub>/dimethoxymethane (DMM) mixture at 35.2°C can be fit to the equation:

$$P_{\text{CS}_2} = x_{\text{CS}_2}(514.5 \text{ torr}) \exp(1.4967x_{\text{DMM}}^2 - 0.68175x_{\text{DMM}}^3)$$

1. Use the Gibbs-Duhem relation to determine the partial pressure of DMM as a function of composition. Assume the vapor is ideal.
2. Do CS<sub>2</sub> and DMM form a regular solution at these conditions? *Hint*: Determine the activities of each component and from these the excess free energy of mixing. Is it proportional to  $x(1-x)$ ?