Advanced Chemical Engineering Thermodynamics (CBE 60553)

Homework 2 **Due September 11, 2017**

Solve each problem on separate sheets of paper, and clearly indicate the problem number and your name on each. Carefully and neatly document your answers. You may use a mathematical solver like Matlab or Mathematica. Use plotting software for all plots.

1 The fundamental equation knows all, sees all

A system is described by the fundamental equation:

$$U = \left(\frac{v_0\theta}{R^2}\right)\frac{S^3}{NV} \tag{1}$$

where R, θ, and v_0 are all positive constants.

1. Find the three equations of state of the system
2. Convince yourself (and me) that the three equations are first-order homogeneous and that $T(S, V, N)$ is intrinsically positive.
3. Plot the "adiabats" (loci of constant entropy) in the $P - v$ plane.
4. Find the "mechanical equation of state" $P = P(T, v)$ of the system and plot the corresponding isotherms (loci of constant temperature) in the $P - V$ plane.

2 But it reveals itself through its equations of state

A particular system is described by the thermal and mechanical equations of state below, where A is a positive constant.

$$T = \frac{3As^2}{v} \qquad P = \frac{As^3}{v^2}$$

1. Find the corresponding chemical potential $\mu(s, v)$.
2. Find the corresponding fundamental equation $u(s, v)$.

3 Relax that constraint

Equations of state of two systems are given below. System 1 contains 2 moles and is at 250 K. System 2 contains 3 moles and is 350 K. Suppose the two systems are connected by a diathermal wall. What are the energies and temperatures of the two systems after equilibrium is established?

$$U^{(1)} = \frac{3}{2}Rn^{(1)}T^{(1)} \qquad U^{(2)} = \frac{5}{2}Rn^{(2)}T^{(2)}$$

4 Work with an ideal gas

When an ideal monatomic gas is placed inside a piston and compressed quasi-statically, the temperature and volume are observed to vary according to

$$T = \left(\frac{V}{V_0}\right)^{\eta} T_0$$

where η is a constant.

1. Calculate the work w done on the gas when one mole is compressed from V_0 to $V_1 < V_0$.

2. Calculate the corresponding change in energy ΔU of the gas.

3. Calculate the corresponding heat transfered to the gas q by energy balance.

4. Calculate the heat transfered to the gas q by integrating $\bar{d}q = TdS$.

5. Calculate the value of η for which $q = 0$.

5 Maximum work, minimum work

One mole of a monatomic ideal gas is contained in a cylinder of volume 10^{-3} m^3 at 400 K. The gas is to be brought to a final state of twice the volume and the same temperature.

1. What is the maximum work that can be delivered to a reversible work source, assuming a 300 K thermal reservoir is available?

2. One way to extract this work would be to expand the gas adiabatically to 300 K, compress it isothermally at 300 K, and finally compress it adiabatically to the final state. Find the work and heat transfers in each step.

3. What is the minimum work necessary to return the gas to its initial state, assuming the same 300 K reservoir is available?

