Hydrogen Refueling Infrastructure in Scotland

Pannathorn Naksung, Puttiwat Wanna

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1 Introduction

The Scottish government is planning a hydrogen refueling infrastructure in Scotland.

1.1 Facilities

The facilities have three main parts and have associated candidate locations, so they can not be moved.

1. **Refueling Station -** There are 25 candidate sites, each with its own associated demand, which has to be satisfied by its own attached production facility or centralized facility.
2. **Production Facility -** There are two types of production facilities - centralized facilities with high capacity and localized facilities attached to the refueling station. Localized facilities can be separated into two types - large and small.
3. **Transportation -** If hydrogen is produced by the centralized production facility, it needs to be transported to the station. Note that each tube trailer can only transport to one station at a time and needs to come back to the facility it left from before transporting to another station.

1.2 Data Used

The following data were used to find a solution.

1. **Demand -** The demand of each station in kg.

c1	c2	c3	c4	c5	c6	c7	c8	c9
5997.927	6653.629	5154.747	4761.837	187.836	772.174	4404.681	4371.787	1940.248
c10	c11	c12	c13	c14	c15	c16	c17	c18
5344.083	6030.954	7237.152	3133.319	4103.515	8245.546	5912.590	7094.504	5154.764
	c19	c20	c21	c22	c23	c24	c25	
	8317.610	7234.712	8675.991	2113.757	3055.495	4810.460	4986.685	

Table 1: Hydrogen demands for each station in kg

2. **Facility Opening Cost -** There is an associated cost for each facility opened shown in table 4 below.
3. **Production Requirements -** Centralized facilities and each type of localized facility have their own production requirement and limits.

Infrastructure	Opening Cost (£)
Centralized	25,000.00
Large Localized	6000.00
Small Localized	3500.00
Refueling Station	50,000.00

Table 2: Facility opening cost

Production Facility	Minimum (kg)	Maximum (kg)
Centralized	0.00	200,000.00
Large Localized	100.00	9500.00
Small Localized	100.00	5000.00

Table 3: Production requirements for facilities

4. **Production Cost -** Each production facility has a different production cost per kg of hydrogen.
5. **Transportation Cost -** Each tube trailer used has a transportation cost determining whether the tube is empty or not. An empty tube has a 75% discount on the transportation cost.

Production Facility	Production Cost (£ per kg)
Centralized	2.00
Localized	6.00

Table 4: Production cost for facilities

Tube Condition	Transportation Cost (£ per km)
Empty	10.00
Not Empty	40.00

Table 5: Tube trailers transportation cost

6. **Tube Trailers Capacity and Limitations -** Each tube trailer used to transport hydrogen from centralized facilities has a capacity of 1000.00 kg. Each station can receive up to 7 tube trailers.
7. **Station Covering -** If two (or more) stations are within a radius of 20 km of each other, one station can be opened and cover all of the other stations' demands, meaning that the demands can be combined into one station.

8. **Distances between Each Station and Each Centralized Facility** - The distance between each station and each of the centralized facilities in km. was given.

9. **Distances between Stations** - The distance between each station in km. was also given.

2 Methodology

2.1 Objective: what to optimize

The objective is to minimize all four associated costs and to optimize the infrastructure as much as possible. The costs are:

1. **Station Opening Cost** - Every refueling station has a fixed opening cost. The only aspect of this that can be reduced is the number of stations that open by adding a covering constraint, which will be explained in the constraints section. The station opening cost v could be written as

   $$v = \sum_1^i (s_i \times \lambda)$$

   where s_i tells whether a station i is open and λ is the opening cost. s_i is 1 if the station is open and 0 otherwise.

2. **Centralized Facility Opening Cost** - Each centralized facility can either be open or not. If it is open, then there is an associated cost, which sums up to be the opening cost for centralized facilities. The opening cost for centralized facilities w could be expressed as

   $$w = \sum_1^i (f_i \times \mu)$$

   where f_i tells whether a facility i is open and μ is the opening cost. f_i is 1 if the facility is open and 0 otherwise.

3. **Localized Facility Opening Cost** - There are two types of localized facilities - large and small. Each station can open only one type of localized facility or neither. The cost of opening a localized facility is simply calculated by the total number of localized facilities opened.

   $$x = \sum_1^i \sum_1^j (l_{i,j} \times \pi_j)$$

   where $l_{i,j}$ tells whether a station i opens a localized facility type j and π_j is the cost for opening a localized facility type j. $l_{i,j}$ is 1 when a station i opens a localized facility type j, otherwise, 0.

4. **Hydrogen Production Cost** - The production cost includes the cost for producing hydrogen at a centralized facility and a localized facility. The production cost y can be calculated by

   $$y = \sum_1^i (\sum_1^j p_{i,j} \times \phi^c + \sum_1^k q_{i,k} \times \phi^l)$$

   where $p_{i,j}$ is the amount of hydrogen produced from a facility j for a station i, $q_{i,k}$ is the amount of hydrogen produced on a localized facility type k for a station i, ϕ^c and ϕ^l are the cost per kg for producing hydrogen at a centralized facility and a localized facility respectively.

5. **Hydrogen Transportation Cost** - Hydrogen produced from a centralized facility needs to be transported using tube trailers. The transportation cost, z, includes the number of tube trailers used and the distance. This can be represented by

   $$z = \sum_1^i \sum_1^j (a_{i,j} \times \delta_{i,j} \times (2 - \psi) * \tau)$$

   where $a_{i,j}$ is a number of tubes used for a facility j to deliver to a station i, $\delta_{i,j}$ is the distance from a facility j to a station i, ψ is the discount for an empty tube transportation, and τ is the transportation cost per km per tube.

   One note on $(2 - \psi)$ is that a trailer needs to go both ways to the station and return. Therefore, on the first round, the cost with a non-empty tube would be just 1 and the return round is $1 - \psi)$, making both rounds combined as $1 + 1 - \psi = 2 - \psi$.

The objective, then, is to minimize all these objectives together, with constraints that will be described in the next section.

2.2 Constraints: what are the limiters

For convenient purposes, constraints will be divided into 4 sections, the refueling station, the production facilities, and the transportation

2.2.1 Facility Constraints

1. **Opening a Localized Facility** If the localized facility is built on a refueling station, there can be only one type of localized facility, large or small, on each refueling station. In addition, the station needs to be opened for the localized facility to be built. The constraint could be written as

$$\sum_1^j l_{i,j} \leq s_i \quad \forall i$$

Since these variables, $l_{i,j}$ and s_i, are binary, these constraints also apply.

$$l_{i,j} \in \{0,1\} \quad \text{and} \quad s_i \in \{0,1\} \quad \forall i \forall j$$

2. **Opening a Centralized Facility** - Three sites of centralized facilities are considered to be open. Each facility could be either open or not open. Hence, the constraint could be written as

$$f_i \in \{0,1\} \quad \forall i$$

2.2.2 Production Limit Constraints

1. **Centralized Facility Production Limit** - Each centralized production facility has a production limit, α. Since a facility can operate only if it is open, the constraint is

$$\sum_1^i p_{i,j} \leq f_i \times \alpha \quad \forall j$$

Moreover, production cannot be negative, and this constraint also applies.

$$p_{i,j} \geq 0 \quad \forall i \forall j$$

2. **Localized Facility Production Limit** - Each of the localized production facilities has its own production limit, β_j, and a minimum amount of hydrogen that is needed to be produced, γ_j. Since a facility can operate only if it is open, the constraint is

$$q_{i,j} \leq l_{i,j} \times \beta_j \quad \text{and} \quad q_{i,j} \geq l_{i,j} \times \gamma_j \quad \forall i \forall j$$

Additionally, the production cannot be negative, so this constraint also applies.

$$q_{i,j} \geq 0 \quad \forall i \forall j$$

2.2.3 Transportation Constraints

1. **Tube Transportation Cost** - For the centralized facility to send its hydrogen to a refueling station, tubes need to be used for transportation. Each tube has a capacity θ. A number of tubes need to cover the amount of hydrogen sent to a station. Due to the number of tubes being an integer, therefore,

$$a_{i,j} \times \theta \geq p_{i,j} \quad \text{and} \quad a_{i,j} \in \mathbb{Z}_0^+ \quad \forall i \forall j$$

2. **Maximum Tubes Per Site** - Only a number of tubes, σ, can be used to deliver hydrogen to each refueling station. Hence, the constraint is

$$\sum_1^j a_{i,j} \leq \sigma \quad \forall i$$

2.2.4 Station Constraints

1. **Station Covering** - Each station could be covered by a station other than itself if they are within a certain radius of each other. Since each station can be covered only once, the constraint could be written as

$$\sum_1^i (c_{i,j} \times z_{i,j}) = 1 \quad \forall j$$

where $c_{i,j}$ is whether a station i covers a station j and $z_{i,j}$, derived from a distance between stations, is whether a station i can cover a station j. $c_{i,j}$ and $z_{i,j}$ are 1 if a station i covers or can cover a station j and 0 otherwise, meaning

$$c_{i,j} \in \{0,1\} \quad \text{and} \quad z_{i,j} \in \{0,1\} \quad \forall i \forall j$$

Moreover, a station can only be covered by an open station. Another constraint is

$$c_{i,j} \leq s_i \quad \forall i \forall j$$

2. **Combined Demands** - The demand of each station needs to be met. However, the demand of a station can be satisfied by another station if it is covered by that station. Nevertheless, this combined demand cannot exceed the production sent to a station from each centralized facility combined with the production from the localized facility. This results in a constraint

$$\sum_1^j d_j \times c_{i,j} \leq \sum_1^k p_{i,k} + \sum_1^h q_{i,h} \quad \forall i$$

where d_j is the demand of a station j.

3 Results

The total cost of the infrastructure is as follows.

Source	Cost (£)
Station Opening Cost	1,050,000.00
Centralized Facility Opening Cost	75,000.00
Localized Facility Opening Cost	42,500.00
Production on Centralized Facility	154,853.03
Production on Localized Facility	289,616.92
Transportation Cost	222,575.35
Total	1,834,545.30

Table 6: Breakdown costs from each source

The following table shows the production from each facility to each station.

Station	Production from Centralized Facility 1 (kg)	Production from Centralized Facility 2 (kg)	Production from Centralized Facility 3 (kg)	Production from Large Localized Facility (kg)	Production from Small Localized Facility (kg)
station 1	0	5997.93	0	0	0
station 2	0	0	6653.63	0	0
station 3	5154.75	0	0	0	0
station 4	4761.84	0	0	0	0
station 5	960.01	0	0	0	0
station 6	0	0	0	0	0
station 7	0	7000.00	0	0	4639.39
station 8	0	0	4371.79	0	0
station 9	0	0	0	0	0
station 10	5344.08	0	0	0	0
station 11	0	0	0	6030.95	0
station 12	0	0	0	7237.15	0
station 13	0	0	0	0	3133.32
station 14	0	4103.52	0	0	0
station 15	0	6000.00	0	0	4185.79
station 16	1000.00	0	0	0	4912.59
station 17	0	0	0	0	0
station 18	0	0	5154.76	0	0
station 19	7000.00	0	0	0	1317.61
station 20	0	0	0	0	0
station 21	0	0	0	8675.99	0
station 22	2113.76	0	0	0	0
station 23	0	7000.00	0	0	3150.00
station 24	0	0	4810.46	0	0
station 25	0	0	0	0	4986.69
Total	26,334.43	30,101.44	20,990.64	21,944.10	26,325.39

Table 7: Production from each facility

Figures below show the location of open stations, the covering, and a facility that provides hydrogen for each station.

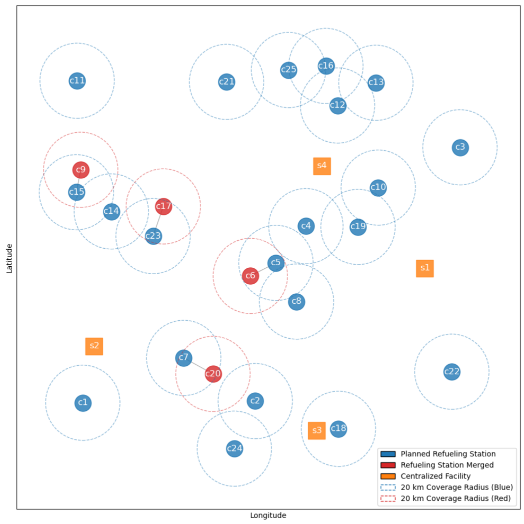

Figure 1: The planned station location and centralized production facility. Each circle determines the radius of whether the station can be merged.

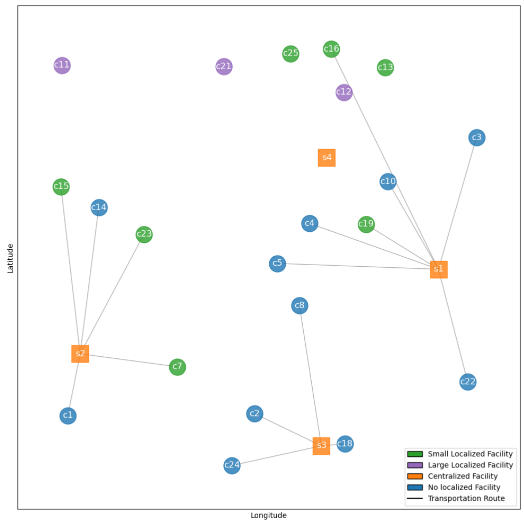

Figure 2: A graph showing open stations and facilities, including which station each facility sends the production to and where each large or small facility is located

4 Analysis

4.1 The Open Refueling Stations

The cost of opening refueling stations is the most significant source of the whole cost of the infrastructure. As seen in the results, all stations that can be combined will be combined. This is expected due to two reasons.

1. Every open station will go towards the total cost, so there is no incentive to not merge the stations if doable.
2. The production cost per kg of a centralized facility is so much cheaper than that of a localized one. Logically, if the total cost of a centralized facility (production, transportation, and shared opening cost) does not exceed that of the localized facility (production and opening cost), the refueling station will "choose" to supply from the centralized facility. Consequently, at refueling stations that can cover each other, the one that is the closest to the closest centralized facility will always be chosen to open to minimize the cost of transportation. In fact, even if the cost of the refueling station opening is not considered, the refueling station will still be merged. Note that this will only be true if the said station chooses its supply from a centralized station.

In short, if any stations are able to merge, it will do. This is a combination of the opening cost of a refueling station and an incentive for a refueling station to be as close to a centralized facility as possible.

4.2 The Production Facilities

The other part of the cost comes from the production cost and the transportation cost. Besides having lower production costs, the centralized facility has the advantage of sharing the opening costs with refueling stations, compared to the localized facility, which has to bear the opening cost alone (or with just a few merged stations). Looking at the production results, the general strategy can be succinctly summarized to:

4.2.1 If a refueling station is close to a centralized facility

If the cost of using a centralized facility does not exceed that of the localized facility, the station will "choose" to supply its hydrogen from the centralized facility until its demand or the transportation limit is reached and fulfill the remaining demands with an appropriate (cheapest) localized facility.

4.2.2 If a refueling station is far from a centralized facility

If the distance is too far, making the cost of using a centralized facility exceed the cost of using a small localized facility, a station will choose to use a small localized facility instead. If the demands exceed a small localized facility, three scenarios can happen.

1. If the cost of procuring the rest of the demands from a centralized facility is **cheaper** than upgrading from a small to a large localized facility, it will the rest of its supply from a centralized facility.
2. If the cost of supplying the remaining demands from a centralized facility **exceeds** the cost of upgrading from a small facility to a large one, the incentive then will be to upgrade the station to a large centralized facility.
3. If, instead, the demand still exceeds that of a large facility, then the station will be served both by a large localized facility and a centralized facility.

4.3 The Transportation

The transportation of hydrogen from the centralized station is a crucial part of the infrastructure. A limit on how much hydrogen can be transported to each station forced some high-demand stations to build their own localized facility as a top-up (some examples of this are stations 7, 19, and 23).

4.4 The Limit and the Scope of the Problems

The above analyses show some limitations that are worth exploring.

4.4.1 The Discrete Nature of the Transportation Cost

With more careful observation of the results, in the stations that are far from a centralized facility and are both served by a centralized facility and a small localized facility (stations 15 and 16), the demand will not be fully fulfilled by using a small localized facility up to its capacity, then fill the rest up with the centralized facility as expected. Instead, the amount served from the centralized facility will be a multiplier of the tube capacity. This is because the cost of transportation is not linear. For example, sending a non-empty tube will cost the same as sending a full tube, so the hydrogen will be sent at the maximum capacity of the tube. This means some of the results can not be interpolated linearly.

4.4.2 The Production Limit of the Centralized Facility

All previous analyses assumed that a centralized facility would not hit a production limit. This is indeed a good assumption since even combining all the demands together will not exceed the production limit of a single central facility. However, if there is an expansion of demands or a future expansion plan, the production limit may need to be considered.

5 Conclusion

The optimal supply chain design is presented in the results section. In total, the infrastructure will cost 1,834,545.30 £, with 21 refueling stations open. On the supply side, all 3 centralized facilities will be opened, with 3 large and 6 small localized stations needed. In general, because of the elevated cost of a station and the efficiency of the centralized facility, any stations that can be merged will be merged to the one that is the closest to a centralized facility. The station that is open will choose its supply location based on its proximity to the centralized location. If it is close, it will get its supply from the centralized facility. If not, it will get its supply by building an on-site localized facility. This is due to the cheaper cost of hydrogen production and the ability to share the cost of opening a centralized facility. This economy of scale effect is crucial to understanding the optimization of the supply chain. However, the effect is somewhat limited by the number of tubes that can be transported to a station. The following suggestion is based on using the economy of scale to an advantage and alleviating the transport limitation.

5.1 Suggestions

5.1.1 Lifting the transportation limit

The transportation limit costs the infrastructure 17,265.25 £. Lifting the limits from 7 tubes to 12 tubes or raising the max capacity of tubes to 1663 kg will make opening some of the localized facilities (stations 7, 19, 23) unnecessary, thus saving the cost if the improvement is cheaper than the cost saved. Note that this is just a cost save in a unit of time. In the long run, this will reduce the cost further.

5.1.2 Opening a new production facility

There are clusters of refueling stations (stations 12, 13, 16, 21, 25) that do not have a nearby centralized facility. Building a centralized facility closer to the said clusters can save some cost in the long run. For example, a facility in the middle of station 4 and station 12 (lat = 46.82, long = 29.20) will save 64,954.28 £ of infrastructure cost. Note that the provided new coordinate is in no way the most optimized place for putting a centralized facility but is one based on visual inspection.

5.1.3 Hydrogen pipeline

From the analysis section, it is clear that because of economies of scale, the centralized facility produces far cheaper hydrogen per unit than the localized one. Long-range transportation is the factor that incentivizes the opening of localized facilities. Consider building a hydrogen pipeline from a centralized facility to refueling stations. This will remove the constraint of transportation limits, making all stations use a centralized facility (which, in this case, is facility 1). Based on the calculation for the current infrastructure plan, this will save the cost by 508,153.29 £. Therefore, building the pipeline will result in an immediate return on investment if it can be built and maintained at a lower cost. Furthermore, in the long run, the pipeline will be more and more worthwhile. Building a pipeline will also reduce traffic congestion from tubes and reduce the carbon footprint from the transportation.

5.1.4 Covering Range Expansion

From the result, some stations are very close to each other but not within the range of the covering radius. One suggestion is to increase this covering radius so that more stations could be merged, thus reducing the station opening and transportation costs. One example is stations 16 and 25, which are 20.2 km apart, while the covering range is 20 km. If the covering range is increased to 21 km, the total cost will reduce to 1,785,547.35 £, saving 48,997.95 £. Therefore, it could reduce costs if the research and improvement that cost less than this amount is done to increase the covering range, even just 1 km, from 20 km to 21 km.

5.1.5 Find Routing Path

The routing problem research could be done to find a routing path for each tube trailer to be able to transport to more than one station on each run. This could reduce costs, but further data and analysis are needed to estimate the improvement precisely.

6 Appendix

6.1 Visualization of the distance between stations

The following graph visualizes the distance between each station and every station.

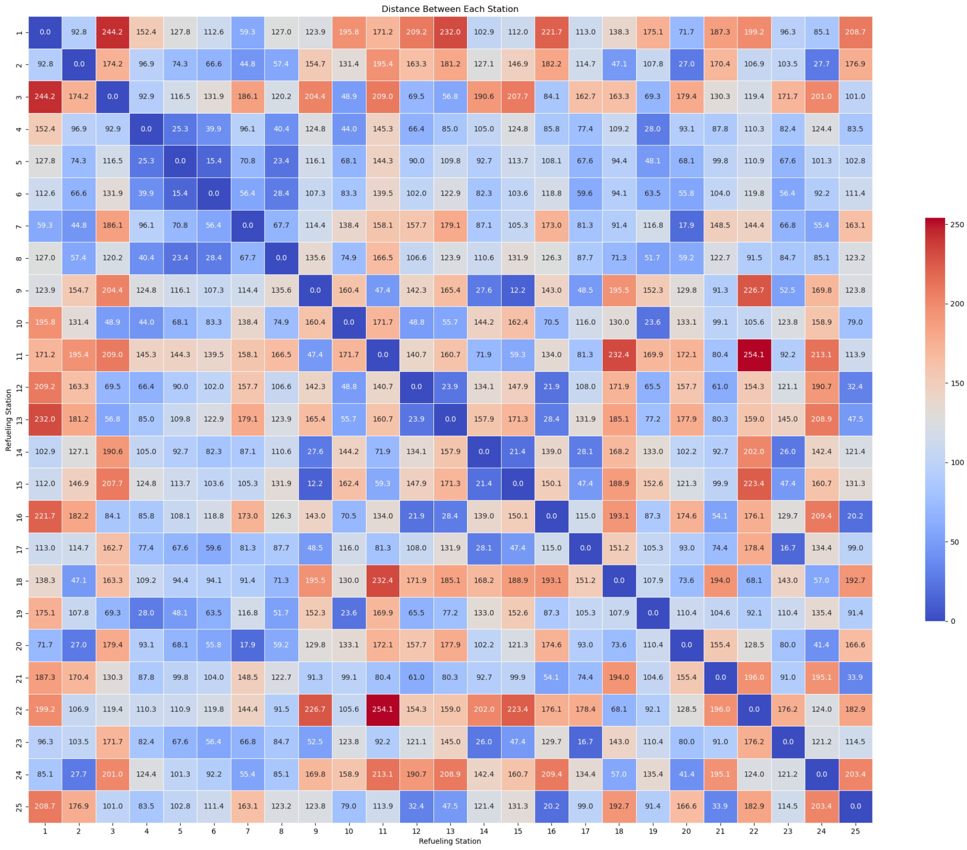

Figure 3: Heatmap of the distance between each refueling station.

Next is a graph that visualizes the distance between each station and every centralized production facility.

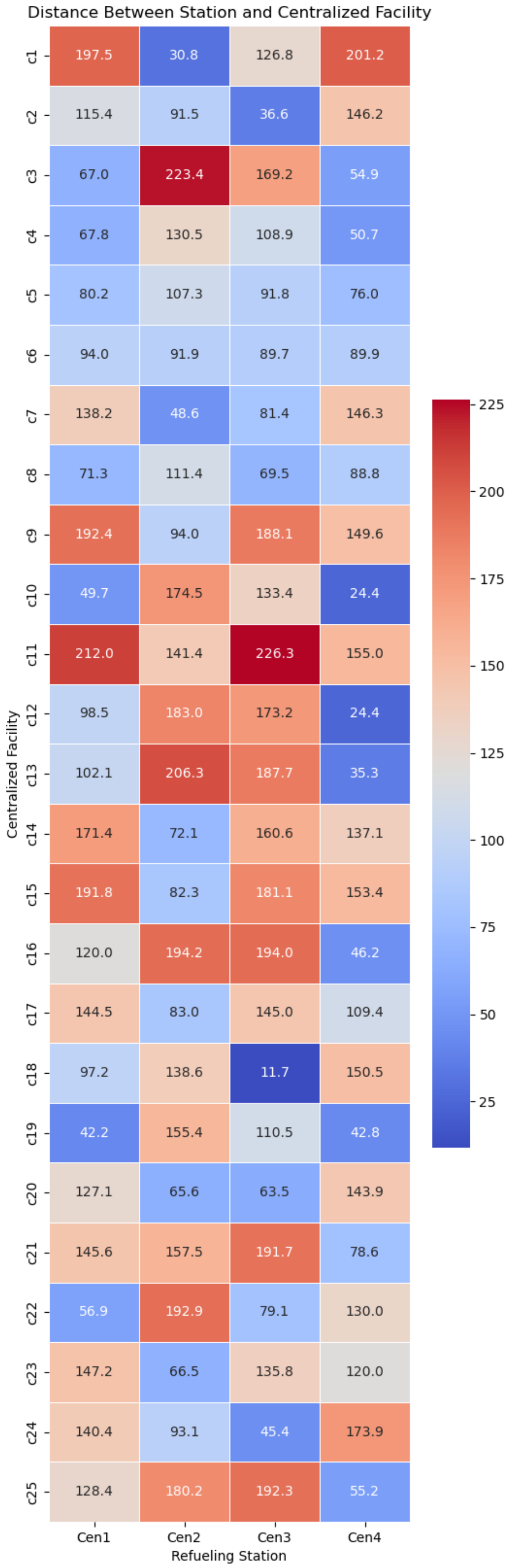

Figure 4: Heatmap of the distance between each refueling station and a centralized facility.

6.2 Visualization of decision boundary of a centralized refueling station

The decision boundary, or point of decision, was calculated by finding where it is more worthwhile to obtain supply from a centralized facility than a localized facility. Supply from a centralized facility has several advantages: it is cheaper to produce, and the opening cost can be shared between several stations. The disadvantage, however, is the transportation cost. The decision boundary is then calculated from said factors.

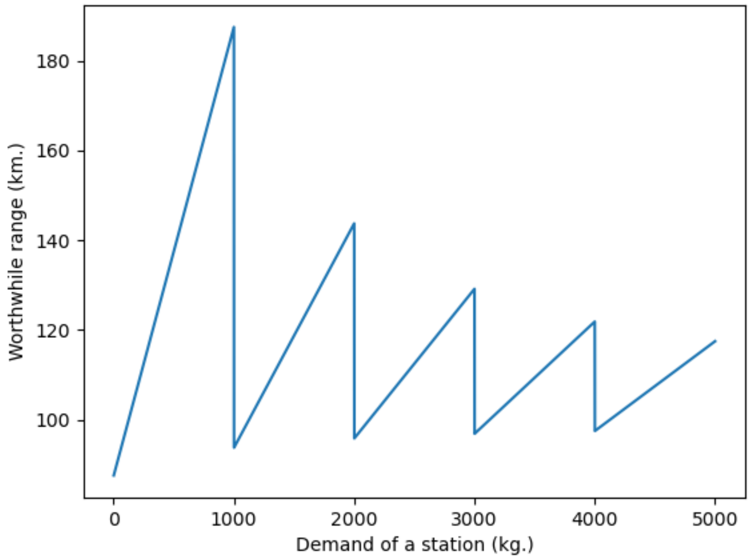

Figure 5: Worthwhile range in of each station that, if a station is located closer than worthwhile range, it will pick its supply from a centralized facility (This does not include the shared centralized facility opening cost), note that the high point of the graph is at a multiplier of the max capacity of tube = 1000.

6.3 Visualization of suggestions

6.3.1 Lifting the transportation limit

Lifting the transportation limits (this can be done by raising the maximum capacity or number of tubes that can be sent to a station) will make stations 7, 19, and 23 no longer need to open a localized facility.

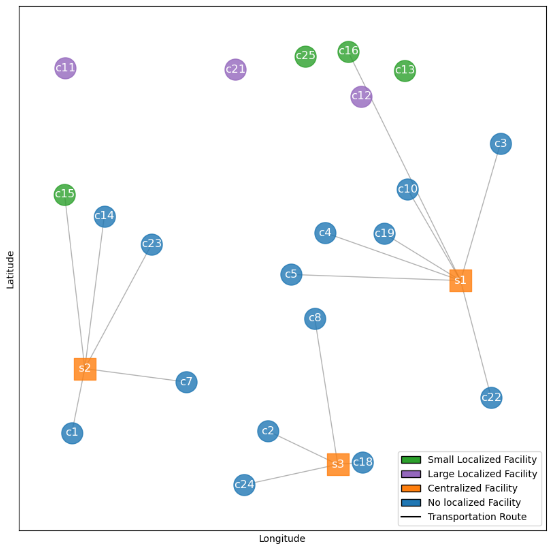

Figure 6: Infrastructure with the transportation lifted

6.3.2 Opening a new production facility

Opening a new refueling station at (lat = 46.82, long = 29.20) will make station 1 obsolete and reduce costs by making opening several localized facilities unnecessary.

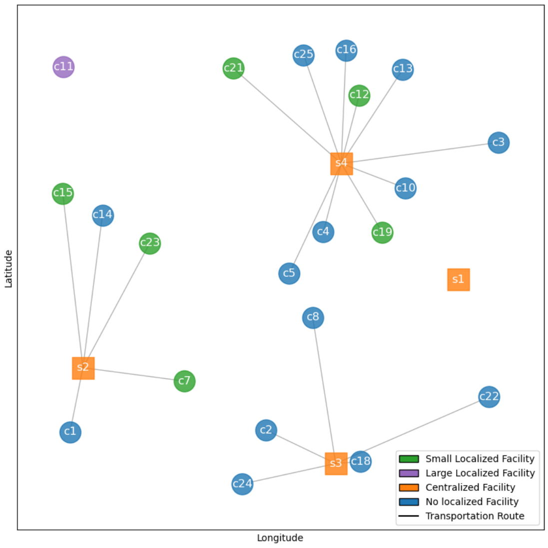

Figure 7: Infrastructure with the transportation lifted

6.3.3 Hydrogen pipeline

Implementing a hydrogen pipeline will effectively eliminate transportation costs and transportation limits. This calculated on the current infrastructure will only need a centralized facility. The facility that is chosen is Facility 1 because it is the facility that is, on average, the closest to every refueling station.

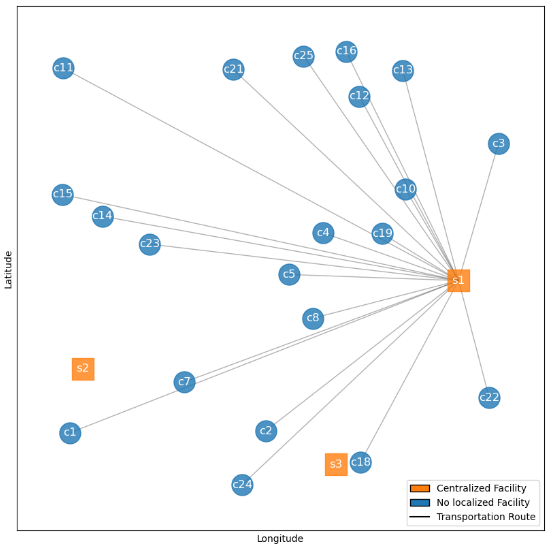

Figure 8: Infrastructure with the transportation lifted