School of Mathematics
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THE UNIVERSITY of EDINBURGH

Risk and Logistics

Homework Assignment 2

March 7, 2025

Instructions

1. For this assignment, you will have to write a report and produce some code using Xpress or python.
2. You should attempt all questions.
3. The total marks for this assignment are: 35.
4. Explain in your report what you have been doing in a concise and reproducible way. Write full sentences, not just the final results. The style in which you present your answers is part of the marking.
5. Structure your code, include comments, and avoid unnecessary loops, conditions, and calculations. The style in which you write your code is also part of the marking.
6. **Use your imagination!** You don't have to do everything by the book. Moreover, it is okay to use simplifications and approximations, provided they are reasonable and well justified.

   If you have an idea for a new method or one that helps you to improve the efficiency or effectiveness (or both!) of an existing method, without deteriorating the other too much, then please go ahead. The questions are kept deliberately vague to allow you that freedom (it would be boring – both for you and me – if I'd tell you exactly what to do and what not).

   I am happy with any method that is reasonable and does the job, but I expect you to properly explain and motivate why you think this is a good idea - and, ideally, back it up with some empirical results.

   Marks will be awarded for creativity and clever ideas (to some extent, independent of whether or not they work), and the overall efficiency, i.e. runtime, and effectiveness, i.e. solution quality, of your heuristics.
7. The strict deadline for handing-in your assignment is **16:00 on Friday, 04 April 2025**.

   Please upload a PDF with your report on gradescope as assignment 2 and all your codes and data files on gradescope in assignment 2 code.

Setting the Scene

You have been approached by a surveying company who operates in Antarctica. This company has 1 072 surveying robots each positioned somewhere on the continent illustrated in Figure 1. These robots are all battery operated and equipped with drones which can fly to a charging station to charge. Each robot drone has a battery range and if the robot cannot make it to the charger before its range runs out then you can use a human operated vehicle to take the robot to a charger at an additional cost.

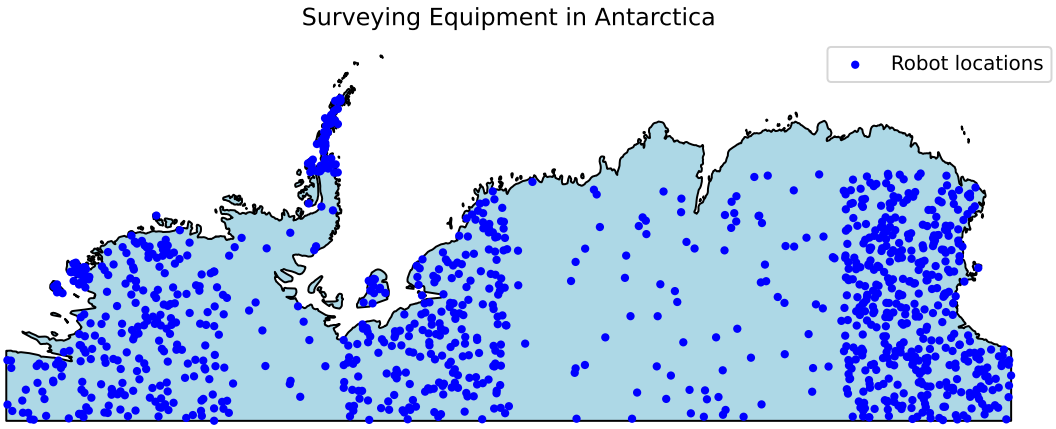

Figure 1: A graphical illustration of the robot locations.

The robots are all fitted with solar panels but given the variations in weather conditions and the uncertainty of how much battery is used for operation, the robots have a flying range which is stochastic in nature. Moreover, on any given day not every robot will require charging. More specifically, if r_i denotes the range of the robot then the probability that it will charge is $p_i = p(r_i) = \exp(-\lambda^2(r_i - r_{\min})^2)$ where λ is a parameter and $r_{\min}$ is the lowest possible range of the robot.

If you allocate a robot to a charger, you can allocate at most q robots to a charger, then it will always use the charger and it will recharge to the maximum possible range denoted by $r_{\max}$. The following costs are associated with the problem: $c_b, c_m, c_{\tilde{c}} \in \mathbb{R}$, c_h, where c_b is the annualised construction investment of a charging station (build cost £/station); c_m is the maintenance cost of a charger (£/charger); $c_{\tilde{c}}$ is the cost of charging (£/km) which is constant up to the full range of $r_{\max}$ miles; and c_h is the cost of moving the robot with insufficient range to a charger.

Your goal is to design the recharging infrastructure. More specifically, you must decide on where to build charging stations (they do not have to be build on land) and how many chargers to build at each charging station to ensure that all your robots can be charged. You can build at most m chargers at any charging station.

Data

I have uploaded on Learn

- *robot_ locations.csv*: A spreadsheet that contains a list of the 1 072 robot locations.
- *range.csv*: A spreadsheet containing the range of each robot in a deterministic setting.
- *range_ scenarios.csv*: A spreadsheet containing the ranges in 100 scenarios. If you need more then please let me know.

- *Data_ Xpress.txt*: A file that contains all of the above data in Xpress format.
- *Template.mos*: A Mosel template file that reads in all the data.

You are given the following parameter values.

- $m = 8$
- $q = 2$
- $c_b = 5000$
- $c_h = 1000$
- $c_m = 500$
- $c_c = 0.42$
- $\lambda = 0.012$
- $r_{\min} = 10$
- $r_{\max} = 175$

Questions

1. First consider the deterministic variant of the problem where every robot was the range in '*range.csv*'.
   (a) Formulate a MINLP which is suitable for solving this problem. Solve the MINLP for small problem instances i.e., use only a subset of the vehicles and present your results. **[8 marks]**
   (b) Develop and implement a construction heuristic that which is suitable for solving this problem. I leave the choice of heuristic to you.
   Describe your construction heuristic in the report.
   Run your heuristic on the instances as the previous question and get some performance guarantees. Then try to solve the full problem instance. **[5 marks]**
   (c) Develop and implement a local search heuristic that tries to improve your current plan. I leave it up to you how you define your neighbourhood(s), how you explore them, and how you check whether or not you found a better solution.
   Describe your improvement heuristic in the report.
   Run your heuristic on the instances as the previous question and get some performance guarantees. Then try to solve the full problem instance.
   Compare the final solutions with the starting solutions. **[12 marks]**
2. Solve the stochastic version of the problem. I will leave this completely up to you. **[10 marks]**

Notes

1. To use the square root of a decision variable in python use xp.sqrt(x).
2. To solve nonlinear problems in Xpress Mosel you will need the modules "mmnl" and "mmxnlp"
3. The same hints from the previous assignment are also valid here i.e., how to compute the MIP gap.