

Notes for the Australian Mathematics Advanced Stage 6 Course

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Part I
Year 11

Chapter 1

Functions

1.1 Functions and Relations

A function takes in input and returns exactly one output, whereas a relation can return more than one. A function is a mapping between two sets.

1.1.1 Definition of a function

Functions can be thought of as a set of ordered pairs (x, y) .

1.1.2 Domain, range, independent and dependent variables

Definition 1.1.1 (Domain and range). The domain of a function is the set of all possible inputs values a function can have. The range of a function is the set of all possible outputs a function can have.

Definition 1.1.2 (Independent and dependent variables). A function $f(x)$ has x as the independent variable. The output (or $f(x)$ itself) is the dependent variable, as it is dependent on x .

Interval notation is used to describe unbroken portions of the real line.

Definition 1.1.3 (Interval notation). The *open* interval (a, b) represents the set of all real numbers between a and b but not including a or b . The *closed* interval $[a, b]$ contains all real numbers between a and b and includes the endpoints a and b . The *half-open* interval $[a, b)$ is the set of all real numbers between a and b but only includes the endpoint a . Similarly, the half-open interval $(a, b]$ only contains the endpoint b .

1.1.3 Graphs of functions

1.1.4 Types of functions and relations

Definition 1.1.4 (One-to-one). A *one-to-one* function takes in one input and returns exactly one output. It passes both horizontal and vertical line tests. For example,

$$f(x) = x$$

is a one-to-one function.

Definition 1.1.5 (Many-to-one). A *One-to-many* function has more than one input that produce the same output. It fails the horizontal line test but passes the vertical. An example is

$$f(x) = x^2$$

where both $x = -1$ and $x = 1$ give the output 1.

Definition 1.1.6 (One-to-many). A *one-to-many* relation has an input that outputs more than one number. It passes the horizontal line test but fails the vertical. An example relation is

$$y^2 = x$$

where $x = 1$ has the outputs -1 and $+1$.

Definition 1.1.7 (Many-to-many). A *many-to-many* relation has multiple input which output multiple of the same outputs. It fails both horizontal and vertical line tests. An example is the circle

$$x^2 + y^2 = 1$$

where $x = 1$ or $x = -1$ both produce outputs 1 and -1 .

1.1.5 Properties of functions

Definition 1.1.8 (Even function). A function f is even if it satisfies,

$$f(-x) = f(x).$$

The graph of f is symmetric about the y -axis.

Definition 1.1.9 (Odd function). A function f is odd if it satisfies,

$$f(-x) = -f(x).$$

The graph of f has point symmetry about the origin (if you spin the graph 180° about the origin, it's the same graph).

Definition 1.1.10 (Algebra of functions). Two functions f and g can be added ($f(x) + g(x)$), subtracted ($f(x) - g(x)$), multiplied ($f(x)g(x)$) together or divided ($f(x)/g(x)$ provided $g(x)$ is never 0), forming a new function. The domain of the new function is the intersection of the domain of f and the domain of g . The range is more difficult to find.

Definition 1.1.11 (Function composition). Function composition is another way to combine functions to form new functions. The composition of functions f and g is denoted as

$$f \circ g(x) = f(g(x)).$$

The domain of $f \circ g$ is the domain of g , whose outputs must also lie in the domain of f . The range of $f \circ g$ is all the outputs that from the range of g as input.

1.1.6 Solutions to functions

When we consider the equation

$$f(x) = 0,$$

we are solving for the x values that are sent to 0 by the function. Graphically, they are the x -intercepts. This is because we are finding all ordered pairs whose y (output) value is 0, corresponding to the x -intercepts.

Part II

Year 12

