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A New Slip Theory of Plasticity[1]

A slip theory is presented for the calculation of the incremental stress-strain relations of a face-centered cubic polycrystal such as aluminum and its alloys, from its experimental uniaxial curves. This theory satisfies the conditions of equilibrium, condition of continuity of displacement as well as the slip characteristics of the component crystals. Numerical calculations based on this theory give results which are in good agreement with known experimental data on thin wall cylinders subject to different ratios of incremental axial and torsional loadings after being compressed beyond the elastic range.

Introduction

Plastic stress-strain relation of polycrystalline metal is of basic importance for the inelastic analysis of metallic solids and structures. It is the aim of the present study to develop a physically more rigorous theory of plasticity for metals.

Tests [1-4][2] have shown that under stress, single crystals slide along certain crystal directions on certain crystal planes. These planes correspond to the dislocation glide planes and these directions correspond to Burger's vectors of dislocation [5-6]. Also, slip depends on the resolved shear stress (the shear stress along the direction of slip on the slip plane) and is independent of the normal pressure on the plane. This coincides with dislocation theory where the force to move a dislocation line is directly proportional to the shear stress on the slip plane along the Burger's vector. The quantitive relations between plastic strain and dislocation movement have been elegantly shown by Mura [7] and Kroner [8] and other distinguished investigators.

The main difference between a single crystal and a polycrystal is the presence of grain boundaries in polycrystals. The grain boundary has been estimated to be only a few atoms thick [9, 10]. Hence for the present study, grain boundaries are considered as surfaces of zero thickness between two crystals of different orientations. Polycrystalline metals undergo considerable plastic deformation before cracks occur. Hence, the actual slip distribution in the aggregate must satisfy, throughout the aggregate, the condition of continuity of displacement, the condition of equilibrium and the stress-strain relationship of the individual crystals. A method satisfying all these three conditions has been developed by Lin, Uchiyama, and Martin [11] in 1961 to calculate the stress field in metals at the very initial stage of plastic deformation. This method has been much further developed by Lin and Ito [13 15] to calculate the slip fields of polycrystalline aggregates from the resolved shear stress versus the sum of slip characteristics of single h.c.p. (hexagonal close-packed) and f.c.c. (face-centered cubic) crystals.

This method is modified further in the present study and hence is reviewed here for reference.

Method of Satisfying Both Equilibrium and Compatibility Conditions

Referring to a set of rectangular coordinates (x_i) where $i = 1, 2, 3$, the strain component e_{ij} consists of elastic and plastic parts

$$e_{ij} = e_{ij}{}^E + e_{ij}{}^P \tag{1}$$

where the superscript E and P denote elastic and plastic, respectively. Neglecting the anisotropy of elastic constants, the stress-strain relationship may be represented by

$$\tau_{ij} = \delta_{ij}\lambda e_{kk}{}^E + 2\mu e_{ij}{}^E = \delta_{ij}\lambda(e_{kk} - e_{kk}{}^P) + 2\mu(e_{ij} - e_{ij}{}^P) \tag{2}$$

where δ_{ij} is the Kronecker delta, λ and μ are Lame's constants, and the repeated indices denote summation. The equation of equilibrium within the body is

$$\tau_{ij,j} + F_i = 0 \tag{3}$$

where the subscript j after the comma denotes partial differentiation with respect to x_j; and F_i denotes the i-component of the body force. The equilibrium condition on the boundary with normal ν is

$$\overset{\nu}{S}_i = \tau_{ij}\nu_j \tag{4}$$

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[2] Numbers in brackets designate References at end of paper.

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where S_j is the i-component of the boundary force and ν_j is the cosine of the angle between the normal ν and x_j-axis. Substituting (2) into (3) and (4), we obtain

$$\delta_{ij}\lambda e_{kk,j} + 2\mu e_{ij,j} - (\delta_{ij}\lambda e_{kk,j}{}^{P} + 2\mu e_{ij,j}{}^{P}) + F_i = 0 \qquad (5)$$

$$\overset{\nu}{S}_i = \nu_j[\delta_{ij}\lambda e_{kk} + 2\mu e_{ij} - (\delta_{ij}\lambda e_{kk}{}^{P} + 2\mu e_{ij}{}^{P})]. \qquad (6)$$

It is seen that the parenthesis terms in (5) and (6) are equivalent to F_i and S_i, respectively, in causing the strain field e_{ij}, and are denoted by $\bar{F}_i$ and $\bar{S}_i$. These are called the equivalent body and surface forces, respectively. If we let $e_{kk}{}^{P}$ be thermal volumetric strain $3\alpha T$, where α is the coefficient of thermal expansion and T is the temperature, equations (5) and (6) give the well-known Duhamel's analogy [16] between temperature gradient and applied body force in an elastic medium.

Plastic strain caused by slip produces no volumetric strain. Hence $\bar{F}_i$ and $\bar{S}_i$ reduce, respectively, to,

$$\bar{F}_i = -2\mu e_{ij,j}{}^{P}; \qquad \overset{\nu}{\bar{S}}_i = 2\mu e_{ij}{}^{P}\nu_j \qquad (7)$$

Consider a point x' in the aggregate sliding in one or more slip systems. Let (y_i) be a set of rectangular coordinates with y_1 along the normal to the slip plane and y_2 along the slip direction of a slip system at the source point x', where slip has taken place. Denoting the plastic shear strain caused by slip in the nth slip system by $[\gamma_{y(1)y(2)}(x')]_n$, the equivalent body force components along y_1 and y_2-directions due to $\gamma_{y(1)y(2)}(x')$ in the nth slip system is

$$[\bar{F}_{y(1)}(x')]_n = -2\mu\left[\frac{\partial\gamma_{y(1)y(2)}(x')}{\partial y_2}\right]_n; \; [\bar{F}_{y(2)}(x')]_n = -2\mu\left[\frac{\partial\gamma_{y(1)y(2)}(x')}{\partial y_1}\right]_n$$

If slip has occurred in N slip systems at this point x', the equivalent body force along the x_k reference axis is given as

$$\bar{F}_k(x') = -2\mu\sum_{n=1}^{N}\left[\frac{\partial\gamma_{y(1)y(2)}(x')}{\partial x_l'}L_{kl}(x')\right]_n \qquad (8)$$

where

$$L_{kl} = \left(\frac{\partial x_l'}{\partial y_2}\frac{\partial x_k'}{\partial y_1} + \frac{\partial x_l'}{\partial y_1}\frac{\partial x_k'}{\partial y_2}\right).$$

Consider a fine grained metal. Each individual crystal is much smaller than the aggregate. To calculate the stress field caused by slip in a crystal at the interior of the aggregate, the equivalent body force due to plastic strain gradients can be considered to be acting in an infinite medium. The anisotropy of elastic constants of single crystals vary from one metal to another. This anisotropy of aluminum is small [10]. Since this study is mainly for aluminum and its alloys, this anisotropy is neglected. The displacement field in an infinite isotropic elastic medium caused by body force $F_k(x')$ acting in a volume V' has been given by Kelvin [16, 17]. From this displacement field the stress field is obtained [11]. Let $\tau_{ij}{}^{S}(x)$ be the stress at x caused by the equivalent body force $\bar{F}(x')$. From Kelvin's solution, we have

$$\tau_{ij}{}^{S}(x) = \int_{v'}\phi_{ijk}(x,x')\bar{F}_k(x')dv' \qquad (9)$$

where

$$\phi_{ijk}(x,x') = \frac{-6\mu A(x_i - x_i')(x_j - x_j')(x_k - x_k')}{r^5} + \frac{2\mu^2 A}{(\lambda + \mu)r^3}[\delta_{ij}(x_k - x_k') - \delta_{ik}(x_j - x_j') - \delta_{jk}(x_i - x_i')]$$

and

$$r^2 = (x_i - x_i')(x_i - x_i').$$

Substituting (8) into (9) we have

$$\tau_{ij}{}^{S}(x) = -2\mu\int_{v'}\phi_{ijk}(x,x')\sum_{n=1}^{N}\left[\frac{\partial\gamma_{y(1)y(2)}(x')}{\partial x_l'}L_{kl}(x')\right]_n dv' \qquad (10)$$

Let (z_i) be a set of rectangular coordinates with z_1 normal to the slip plane and z_2 along the slip direction of a slip system q at a field point x. The resolved shear stress in this slip system at this point caused by slip throughout the aggregate is obtained by transforming the stress $\tau_{ij}{}^{S}(x)$ from x-axes to z-axes.

$$[\tau_{z(1)z(2)}{}^{S}(x)]_q = \left[\frac{\partial x_i\partial x_j}{\partial z_1\partial z_2}\right]_q\tau_{ij}{}^{S}(x) \qquad (11)$$

where the subscript q outside the brackets refers to the z-axes of the qth slip system at point x. For N slip systems at the point x, the plastic strain

$$[e_{z(1)z(2)}{}^{P}(x)]_q$$

due to slip in these N systems referring to y-axes is

$$[e_{z(1)z(2)}{}^{P}(x)]_q = \sum_{n=1}^{N}\left[\frac{\partial y_1}{\partial z_1}\frac{\partial y_2}{\partial z_2} + \frac{\partial y_2}{\partial z_1}\frac{\partial y_1}{\partial z_2}\right]_{qn}[\gamma_{y(1)y(2)}(x)]_n \qquad (12)$$

where the subscripts q and n outside the brackets refer to the z-axes of the qth slip system and to the y-axes of the nth slip system at point x, respectively. Let the resolved shear stress in the qth slip system at point x caused by the homogeneous stress field $\tau_{ij}{}^{0}$ be denoted by $[\tau^{0}{}_{z(1)z(2)}(x)]$. The resolved shear stress in the qth slip system at point x becomes [12, 14]

$$[\tau_{z(1)z(2)}(x)]_q = [\tau_{z(1)z(2)}{}^{0}(x)]_q + [\tau_{z(1)z(2)}{}^{S}(x)]_q - 2\mu[e_{z(1)z(2)}{}^{P}(x)]_q \quad (13)$$

The critical shear stress $\tau_c(x)$ is taken to be the same in all slip systems at point x and is assumed to depend on the sum of slip in all slip systems, at point x. This corresponds to isotropic strain-hardening. The critical stress is, thus, written as

$$\tau_c(x) = f\left(\sum_{n=1}^{N}\int\left|\left[d\gamma_{y(1)y(2)}(x)\right]_n\right|\right) \qquad (14)$$

where f is a function, and the argument is the sum of the absolute values of the incremental slip that has occurred during the strain history of point x. For a slip system m to be active, the resolved shear stress must be equal to the critical shear stress, i.e.,

$$\left[\tau_{z(1)z(2)}{}^{0}(x)\right]_m + \left[\tau_{z(1)z(2)}{}^{S}(x)\right]_m - 2\mu\left[e_{z(1)z(2)}{}^{P}(x)\right]_m = f\left(\sum_{n=1}^{N}\int\left|\left[d\gamma_{y(1)y(2)}(x)\right]_n\right|\right) \quad (15)$$

Writing (15) in incremental form, we have

$$\left[\Delta\tau_{z(1)z(2)}{}^{0}(x)\right]_m + \left[\Delta\tau_{z(1)z(2)}{}^{S}(x)\right]_m - 2\mu\left[\Delta e_{z(1)z(2)}{}^{P}(x)\right]_m = f'\left(\sum_{n=1}^{N}\int\left|\left[d\gamma_{y(1)y(2)}(x)\right]\right|_n\right)\sum_{p=1}^{P}\left|\left[\Delta\gamma_{y(1)y(2)}(x)\right]_p\right| \quad (16)$$

where f' denotes differentiation of f with respect to its argument, P denotes the number of currently active slip systems at a point during an increment of the homogeneous stress $\Delta\tau_{ij}{}^{0}$. At every point where slip is taking place, there are P unknown $\Delta\gamma_{y(1)y(2)}$ and P equations given by (16). Hence, the $\Delta\gamma_{y(1)y(2)}$ at all sliding points may be determined by (16). This method satisfies the conditions of continuity of displacement, condition of equilibrium, and the stress-strain relationship of the component crystals. This method has been used to calculate the initial incremental plastic stress-strain relationship of pure zinc and aluminum polycrystalline aggregates under different combined loadings from a given single crystal resolved shear stress versus sum of slip curve [12–15].

Present Polycrystal Model

Single crystal test data are available for pure aluminum but not for aluminum alloys. However, aluminum alloys are much more commonly used than pure aluminum for engineering structures. Besides, the size of single crystals used in single crystal

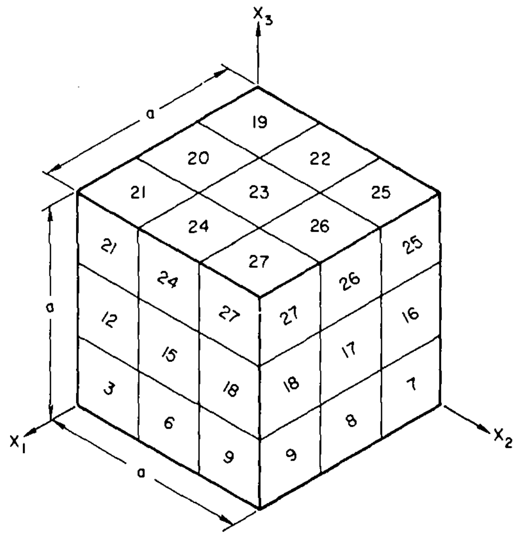

Fig. 1 Basic block of 27 cube-shaped crystals denoting the crystal numbers

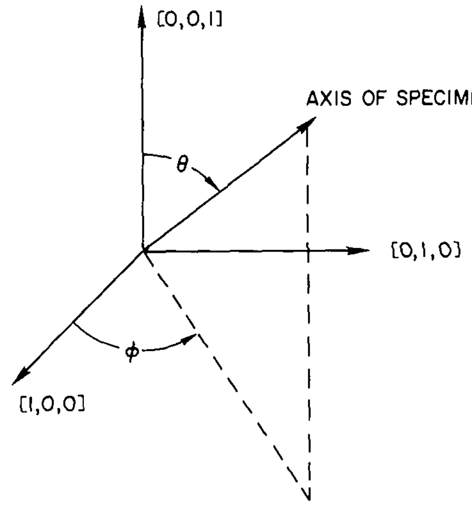

Fig. 2 Specimen axis relative to the cubic axes of an f.c.c. crystal

tests is much larger than that of the grains in polycrystals and it is known that the stress-strain curve of crystals varies with grain size [18–20]. Hence the stress-strain relationship obtained from single crystal test may not be the same as that of the component crystal in a fine grained polycrystal. In order to overcome this difficulty, the component crystal stress-strain relationship is here derived from the experimental tensile stress-strain curve of the polycrystal. This approach is similar to the derivations of the characteristic shear function from the tensile stress-strain curve in the development of the first slip theory of plasticity by Batdorf and Budiansky [21]. Based on this derived component crystal stress-strain relationship, the incremental stress-strain relations of the polycrystal under arbitrary combined loadings are calculated.

In the present study, a large three-dimensional region in the infinite medium is considered to be entirely filled with innumerous identical basic cubic blocks of 27 cube-shaped f.c.c. crystals (see Fig. 1) having different orientations. Each f.c.c. crystal has four slip planes, on each of which, there are three slip directions. The orientation of the crystal is defined by the angles θ and ϕ (Fig. 2), which the specimen axis (x_1, x_2, x_3) make with the crystal axes. The orientations of the 27 crystals were chosen such that the orientations of the specimen axis x_1 relative to the 27 sets of crystal axes on a standard stereographic projection of a cubic crystal [10] are quite uniformly distributed in a unit stereographic triangle while the specimen axes x_2 and x_3 relative to these 27 sets of crystal axes are only approximately uniformly distributed in two other stereographic triangles. The orientations of these 27 crystals defined by their crystal axes related to the three specimen axes as shown in Fig. 2 are tabulated in Table 1.

The grain size is very small as compared to the large three-dimensional region which is filled with these basic cube-shaped blocks of crystals. The average incremental stress versus average incremental plastic strain of the interior center basic block is considered to represent the incremental stress-strain of the polycrystal. The size of the total region may be considered to be infinite as compared to the size of one block. Hence to calculate the stress field in this center block, the values of $\Delta\gamma_{y(1)y(2)}$ and $\partial\Delta\gamma_{y(1)y(2)}/\partial x'_l$ at (x'_1, x'_2, x'_3) are taken to be the same at any point defined by $(x'_1 - m_1a,\ x'_2 - m_2a,\ x'_3 - m_3a)$ where m_1, m_2, and m_3 are any positive or negative integers and "a" is the linear dimension of the basic cube-shaped block given in Fig. 1. The integral in (10) over a three-dimensional infinite region reduces to an integral over one basic block with $\phi_{ijk}(x,x')$ expressed as $\phi_{ijk}(x_1, x_2, x_3, x'_1 - m_1a, x'_2 - m_2a, x'_3 - m_3a)$ with m_1, m_2, m_3 summed over all positive and negative integers. Those blocks were grouped according to their distances from the center block, the contributions to the stress field in the center block due to successive equidistant groups were evaluated. The sum of these contributions from all groups expressed as an infinite series was found to converge rapidly. A more detailed discussion of this procedure is given by Ito [22].

Table 1 Crystal orientation and arrangement

Crystal No.*	Crystal Angles† in degrees	SPECIMEN AXIS x_1	x_2	x_3
1	θ	61.5	75.4	84.9
	φ	41.5	33.4	32.3
2	θ	71.5	76.9	85.7
	φ	26.5	22.1	22.5
3	θ	76.5	80.6	66.8
	φ	21.5	36.2	32.2
4	θ	76.5	85.8	62.5
	φ	31.5	30.9	38.7
5	θ	86.5	81.4	81.7
	φ	11.5	27.3	26.1
6	θ	61.5	76.4	87.0
	φ	36.5	25.5	34.9
7	θ	71.5	87.1	81.6
	φ	36.5	35.5	16.8
8	θ	81.5	66.6	76.0
	φ	36.5	32.8	21.0
9	θ	76.5	87.7	86.2
	φ	26.5	25.9	13.2
10	θ	81.5	75.4	61.7
	φ	31.5	40.1	41.8
11	θ	71.5	59.4	80.4
	φ	41.5	37.4	36.6
12	θ	86.5	85.5	77.8
	φ	36.5	11.9	37.3
13	θ	81.5	77.4	70.8
	φ	21.5	36.2	31.8
14	θ	71.5	60.6	87.8
	φ	31.5	42.3	35.7
15	θ	86.5	81.7	75.5
	φ	16.5	41.1	43.3
16	θ	86.5	89.2	84.8
	φ	41.5	6.2	41.8

*See Figure 1

†See Figure 2

Table 1 (Continued)

Crystal No.*	Crystal Angles† in degrees	SPECIMEN AXIS x_1	x_2	x_3
17	θ	76.5	87.4	72.0
	φ	41.5	22.6	44.2
18	θ	81.5	71.7	69.9
	φ	26.5	32.4	25.5
19	θ	86.5	72.1	64.8
	φ	31.5	24.8	33.6
20	θ	81.5	68.3	81.6
	φ	41.5	44.9	22.0
21	θ	86.5	85.5	73.1
	φ	26.5	16.6	27.6
22	θ	86.5	81.0	70.4
	φ	21.5	32.9	36.1
23	θ	81.5	85.7	72.0
	φ	16.5	40.8	42.2
24	θ	66.5	71.9	76.2
	φ	31.5	37.3	32.3
25	θ	66.5	82.9	82.7
	φ	36.5	23.7	33.3
26	θ	76.5	66.9	86.1
	φ	36.5	42.4	26.9
27	θ	66.5	64.7	80.1
	φ	41.5	38.7	42.8

*See Figure 1

†See Figure 2

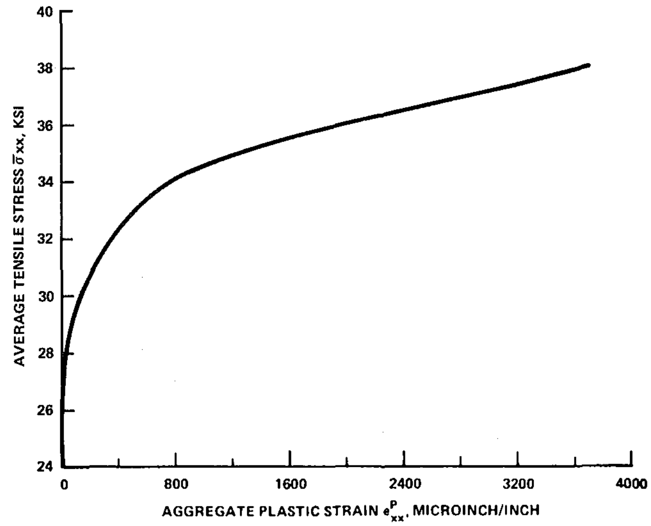

Fig. 3 Polycrystal aluminum stress-plastic strain curve in uniaxial tension

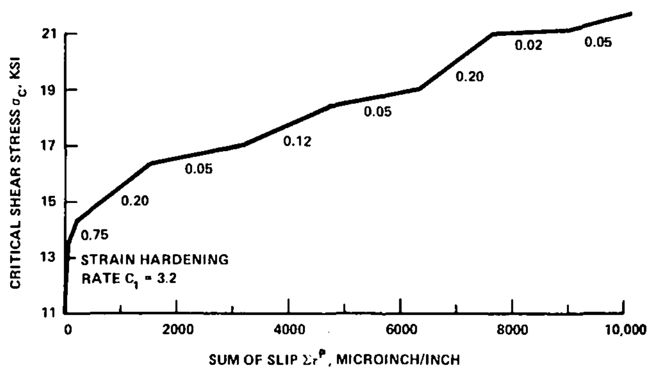

Fig. 4 The component aluminum single crystal critical shear stress—sum of slip curve

The method may be applied to any number of sliding points considered, but the computation increases rapidly with the number of these points. Therefore, in the present study, one point located at the center of each of the 27 crystals is considered and it is assumed that the stress and the plastic strain calculated at this point represents the average plastic strain in the crystal (see Ito [22]).

With $\tau_{ij}^S(x)$ calculated by (10), the stress field is readily obtained by

$$\tau_{ij}(x) = \tau_{ij}^0 + \tau_{ij}^S(x) - 2\mu e_{ij}^P(x) \tag{17}$$

which follows (2). The average stress $\bar{\tau}_{ij}$ over the 27 basic crystals is readily obtained by taking the average stress of these 27 crystals

$$\bar{\tau}_{ij} = \tau_{ij}^0 + \bar{\tau}_{ij}^S - 2\mu\bar{e}_{ij}^P \tag{18}$$

where the bar on the top denotes the average values.

Numerical Calculations. Polycrystalline aluminum alloy 14ST4 is considered. Young's modulus is taken to be 10^7 psi (1 psi = 0.703 gm/mm^2) and Poisson's ratio, 0.3. The tensile stress-strain curve has been carefully obtained by Budiansky, et al. [23] and is reproduced in Fig. 3.

The average polycrystal stress $\bar{\tau}_{ij}$ is approximately equal to τ_{ij}^0 when the average plastic strain $\bar{e}_{ij}^P$ is small [14]. When the polycrystal is loaded far beyond the elastic range, $\Delta\tau_{ij}$ may differ considerably from $\Delta\tau_{ij}^0$. In order to calculate the slip distribution in the polycrystal under tensile loadings, i.e., all $\Delta\bar{\tau}_{ij}$ except $\Delta\bar{\tau}_{11}$ vanishes, trial values of $\Delta\tau_{ij}$ are prescribed to obtain $\Delta\bar{\tau}_{ij}$ close to a purely tensile state. The incremental plastic strain $\Delta\bar{e}_{ij}^P$ is obtained by averaging the plastic strain of the 27 crystals [24].

The tangent modulus, the slope of the tensile stress-strain curve of the polycrystal $\Delta\bar{\tau}_{11}/\Delta\bar{e}_{11}^P$, depends on the rate of hardening f' of the component crystals. Different trial values of this rate were prescribed to approximate the tangent modulus of the experimental tensile stress-strain curve. In this way, the critical shear stress τ_c versus the sum of slip curve of the component crystal was obtained and approximately represented by segments of straight lines where the subsequent segment was determined for the next loading increment. The calculated component crystal critical shear stress versus the sum of slip is shown in Fig. 4.

The incremental slip must take place in the direction of the resolved shear stress, i.e., the work done due to slip in a slip system must be positive. Hence

$$\left[\Delta e_{z(1)z(2)}{}^P(x)\right]$$

in an active slip system not only has to satisfy (16), but also must be along the same direction as the resolved shear stress

$$\left[\tau_{z(1)z(2)}(x)\right]_m.$$

Let those slip systems having the resolved shear stress reaching the critical shear stress be referred to as the potentially active slip systems. If (16) is applied to all those potentially active slip systems of all the grid points, the incremental slip of these slip systems by solving (16) may not all be in the direction of the resolved shear stress. Hence the number of active slip systems may be less than that of the potentially active ones. At the same time, all the resolved shear stresses in the nonactive slip systems should not exceed its corresponding critical shear stress.

When the loading is far beyond the elastic range, as in the present study, the number of potentially active slip systems in all

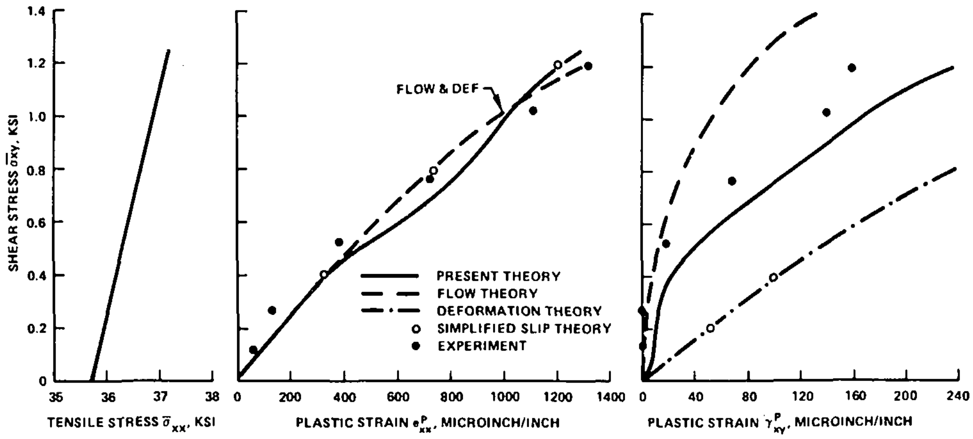

Fig. 5 Loading path and plastic strain for $\Delta\bar\sigma_{xx}/\Delta\bar\sigma_{xy}$ = 1.180

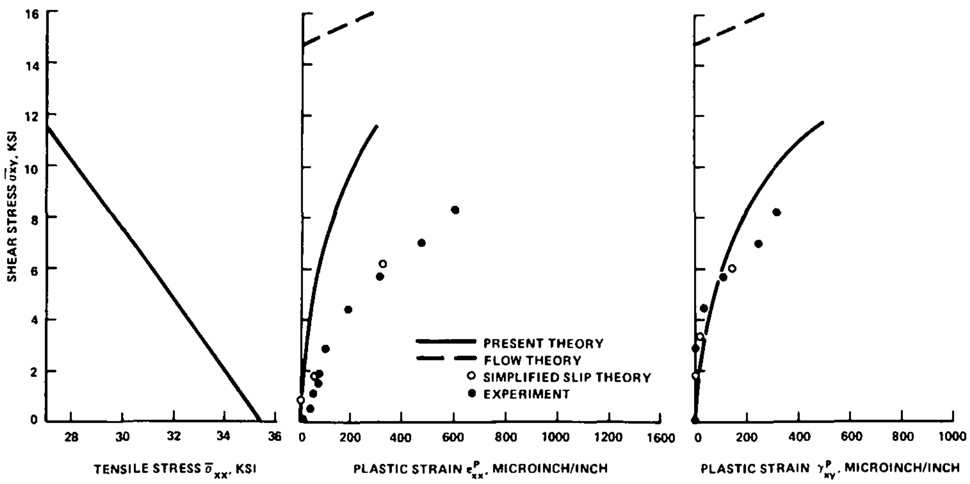

Fig. 6 Loading path and plastic strain for $\Delta\bar\sigma_{xx}/\Delta\bar\sigma_{xy}$ = −0.656

crystals is large and the determination of the set of active slip systems may not be an easy task. Hence, in the present calculation, the following relaxation procedure is used.

A load increment $\Delta\tau_{ij}^0$ is prescribed without allowing any slip to occur and the excess resolved shear stress, i.e., $|\tau_{z(1)z(2)}| - \tau_c$, in all potentially slip systems, is calculated. The slip system with maximum excess resolved shear stress is allowed to slide to relax the excess resolved shear stress. Slip in this slip system changes the resolved shear stresses in all slip systems in all crystals. Then again, the slip system with the maximum excess resolved shear stress is allowed to slide and relax the stress. The process is repeated until the maximum excess resolved shear stress is sufficiently small. This relaxation method has been found to be more efficient than to directly solve the simultaneous equations for each trial combination of active slip systems out of the potentially active ones.

Budiansky, et al. [23], have experimentally obtained incremental stress-strain data of 14S-T4 Al Al under different ratios of incremental compressive and torsional loadings after the material was stressed beyond yielding in compression. The incremental plastic strains under these loadings were calculated from the present theory. The calculated results based on the present model are compared with the experimental data and other plasticity theories as shown in Figs. 5–7.

Discussion and Conclusion

1 A method is given to calculate the incremental stress-strain relation of f.c.c. polycrystals under arbitrary loadings from the experimental tensile stress-strain curve of the metal. This method does not depend on the single crystal experimental stress-strain curve and hence is applicable to many alloys, of which single crystal stress-strain curve is not available.

2 This theory is physically rigorous; it satisfies the condition of continuity of displacement, the condition of equilibrium, the slip characteristics of the component crystals.

3 The stress-strain curves calculated by the present theory, the simplified slip theory of Batdorf and Budiansky [21] and the plastic flow theory are shown with experimental data for loading paths as shown in Figs. 5–7. For the case $\Delta\bar\sigma_{xx}/\Delta\bar\tau_{xy} = 1.18$ (Fig. 5) all these theories seem to agree well with the experimental plastic axial strain e_{xx}^P, however, the present theory agrees with the experimental shear-strain γ_{xy}^P a little better than the plastic flow theory and much better than the plastic deformation theory and the simplified slip theory. For the case $\overline{\Delta\sigma}_{xx}/\overline{\Delta\tau}_{xy} = -0.656$ (Fig. 6) both the present theory and the simplified slip theory give much better agreement with experimental result than the plastic flow theory for both e_{xx}^P and γ_{xy}^P and the simplified slip theory gives better agreement with the experimental values of e_{xx}^P than the present theory. For the case $\overline{\Delta\tau}_{xx}/\overline{\Delta\tau}_{xy} = 0.052$ (Fig. 7) the present theory and the plastic flow theory give much better agreement with experimental data for plastic shear strain γ_{xy}^P than the simplified slip theory and the plastic deformation theory. It is seen from the foregoing that, as a whole, the present theory gives better representation of the experimental data than the other theories.

The accuracy of the calculated results may be improved by

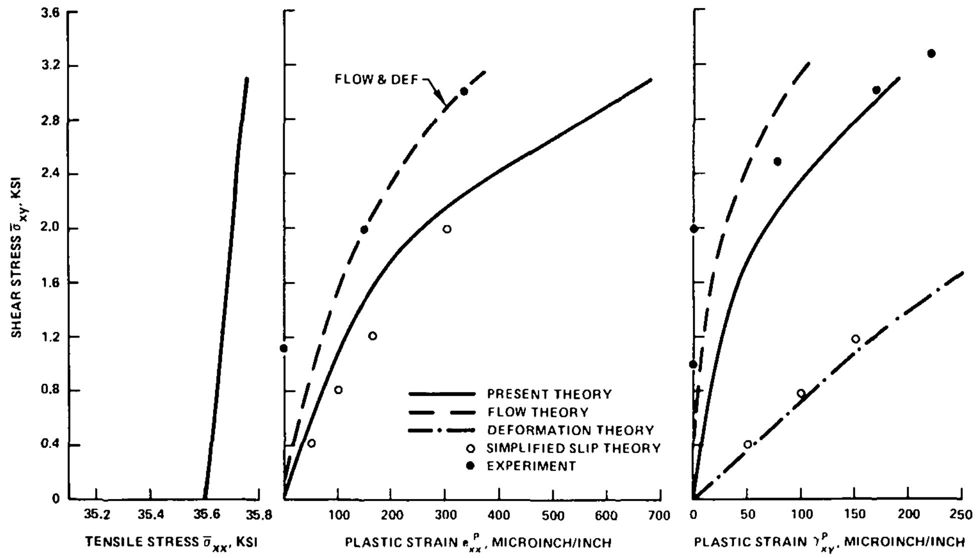

Fig. 7 Loading path and plastic strain for $\Delta\bar{\sigma}_{xx}/\Delta\bar{\sigma}_{xy} = 0.052$

considering more crystal orientations and more grid points in each crystal. But this will increase greatly the amount of computation required.

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