FOURIER ANALYSIS (751799001, 701866001, 114-1) - HOMEWORK 5

Return by October 15, 2025 (Wednesday) 23:59

Total marks: 50

Special requirement. All homework must be prepared by using LaTeX.

Exercise 1 (10 points)**.** Prove the symmetry properties and Parseval's identity for Schwartz class $\mathscr{S}(\mathbb{R}^n)$ described in Proposition 2.3.8.

Exercise 2 (10 points)**.** Prove the basic properties of Fourier transform on Schwartz class $\mathscr{S}(\mathbb{R}^n)$ described in Proposition 2.4.10.

Exercise 3 (10 points)**.** Prove the basic properties of Fourier transform on tempered distribution $\mathscr{S}'(\mathbb{R}^n)$ described in Proposition 2.5.5.

Exercise 4 (10 points)**.** Let $n = 1$, and we consider the Heaviside function

$$H(x) = \begin{cases} 1 & , x > 0, \\ 0 & , x \leq 0, \end{cases}$$

for all $x \in \mathbb{R}$. Compute the Fourier transform $\hat{H}$ of H in $\mathscr{S}'(\mathbb{R}^n)$.

Exercise 5 (10 points)**.** Let $n = 1$, let $\omega \in \mathbb{R}$ and let $f(x) := \sin(\omega x)$ for all $x \in \mathbb{R}$. Compute the Fourier transform $\hat{f}$ of f in $\mathscr{S}'(\mathbb{R}^n)$.