FOURIER ANALYSIS (751799001, 701866001, 114-1) - HOMEWORK 2

Return by September 24, 2025 (Wednesday) 23:59

Total marks: 50

Special requirement. All homework must be prepared by using LaTeX.

Exercise 1 (10 points)**.** Let $f(x) = x^2$ for $x \in (-\pi, \pi)$. Compute its Fourier series, which in fact converges pointwisely on $(-\pi, \pi)$ and find the sum of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$.

Exercise 2 (10 points)**.** Put $x = \frac{\pi}{4}$ in the Fourier sine series of the constant function $f(x) = 1$ for $x \in (0, \pi)$, which in fact converges pointwisely on $(0, \pi)$, to compute the sum

$$\left(1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + \cdots\right) + \left(\frac{1}{3} - \frac{1}{7} + \frac{1}{11} - \frac{1}{15} + \cdots\right) = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \cdots .$$

(Note: The left-hand-side cannot be arbitrarily rearranged because they are only conditionally, not absolutely, convergent.)

Exercise 3 (10 points)**.** Compute the Fourier series of $|\sin x|$ in the interval $(-\pi, \pi)$, which in fact converges pointwisely on $(-\pi, \pi)$. Use it to find the sums

$$\sum_{k=1}^{\infty} \frac{1}{4k^2 - 1} \quad \text{and} \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{4k^2 - 1}.$$

Exercise 4 (10 points)**.** Compute the Fourier series of e^x on $(-\pi, \pi)$.

Exercise 5 (10 points)**.** Show how the Fourier series on $(-\ell, \ell)$ can be derived from the series on $(-\pi, \pi)$ by changing variables $y = \frac{\pi}{\ell} x$.