Data Mining Assignment 3 - Part 2

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Step 1: Write the probability density function of the Poisson distribution.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Step 2: Write the likelihood function.

This is simply the product of the PDF for the observed values $x_1, \ldots, x_n$.

$$L(\lambda; x_1, ..., x_n) = \prod_{j=1}^{n} \frac{\lambda^{x_j} e^{-\lambda}}{x_j!}$$

Step 3: Simplify the calculations.

We can do that by calculating the natural log of the likelihood function:

- $l(\lambda; x_1, ..., x_n) = ln\left(\prod_{j=1}^{n} \frac{\lambda^{x_j} e^{-\lambda}}{x_j!}\right)$
- $l(\lambda; x_1, ..., x_n) = \sum_{j=1}^{n} ln\left(\frac{\lambda^{x_j} e^{-\lambda}}{x_j!}\right)$
- $l(\lambda; x_1, ..., x_n) = \sum_{j=1}^{n} [ln(\lambda^{x_j}) + ln(e^{-\lambda}) - ln(x_j!)]$
- $l(\lambda; x_1, ..., x_n) = \sum_{j=1}^{n} [x_j ln(\lambda) - \lambda - ln(x_j!)]$
- $l(\lambda; x_1, ..., x_n) = -n\lambda + ln(\lambda) \sum_{j=1}^{n} x_j - \sum_{j=1}^{n} ln(x_j!)$

Step 4: Calculate the derivative of the natural log likelihood function with respect to λ.

Next, we can calculate the derivative of the natural log likelihood function with respect to the parameter λ:

- $\frac{d}{d\lambda} l(\lambda; x_1, ..., x_n) = \frac{d}{dy}\left(-n\lambda + ln(\lambda)\sum_{j=1}^{n} x_j - \sum_{j=1}^{n} ln(x_j!)\right)$
- $\frac{d}{d\lambda} l(\lambda; x_1, ..., x_n) = -n + \frac{1}{\lambda}\sum_{j=1}^{n} x_j$

Step 5: Set the derivative equal to zero and solve for λ.

- $-n + \frac{1}{\lambda}\sum_{j=1}^{n} x_j = 0$
- $\lambda = \frac{1}{n}\sum_{j=1}^{n} x_j$

Thus, the MLE turns out to be:

$$\lambda = \frac{1}{n}\sum_{j=1}^{n} x_j$$

This is equivalent to the **sample mean** of the *n* observations in the sample.