Measurement Theory

Based on ideas from Houle et al 2011, Wolman2006 and Sarle 1997

(including many direct quotes from the papers, so beware!)

Houle D, Pélabon C, Wagner GP, Hansen TF. Measurement and meaning in biology. Q Rev Biol. 2011 Mar;86(1):3-34. Review. PubMed PMID: 21495498.

Readings for Tuesday (exploratory data analysis in R)

- Bolker Chapter 2
- Gelman and Hill Appendix B
- Dalgaard Chapter 10 (working with data).
- Zuur (Protocol for data exploration)
- Crawley chapters 3-4 (optional for more detail).

Readings for Effect sizes

- We may also discuss effect sizes, so you can start the readings for that (but you can wait until after the class on Tuesday).

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- Not true in F or K. That is the statement is only true in C, but not in F or K means that.

- If it is not true for all of these different scales of measurement, does it have any meaning?

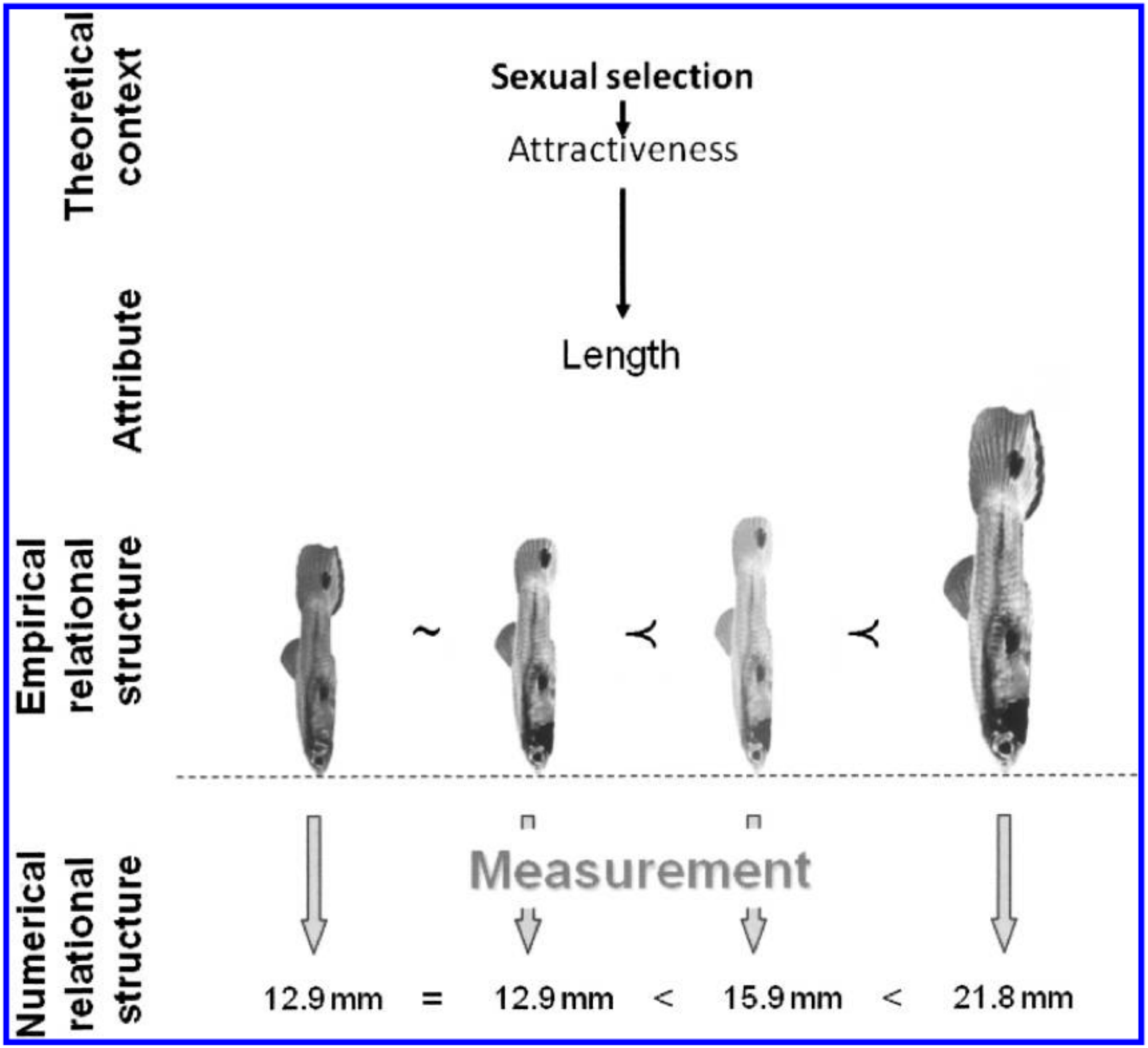
Measurement theory aims to relate what we measure to reality.

- We may be able to do statistics on a set of numbers, but if we do not understand the source of the numbers, it may be meaningless.

The basic premise of representational measurement theory is that measurement consists of the assignment of numbers to empirical entities.

If *E* represents the empirical entities (say a sample of fish), then a scale can be viewed as a function from E to the real numbers.

The fundamental requirement of representational measurement is that the process of assigning numbers to elements of E should preserve the empirical relations on E.


Theoretical context
Sexual selection
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12.9 mm
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- The key principle of measurement theory is that theoretical context, the rationale for collecting measurements, is essential to defining ***appropriate measurements*** and interpreting their values.
- Its goal is ensuring that inferences about measurements reflect the underlying reality we intend to represent.

When what is true about the relations of the numbers is true about the relations of the attributes, the conclusions we draw from the numbers are meaningful conclusions about nature.

(Houle et al 2011)

Measurement theory and statistics

- This has little to do with what “valid” statistical methods and inference can be applied.
- Instead it provides a framework to make sure that what we include in our model reflects reality.
- If we are interested in arbitrary scales for their own sake (the food critic example) then measurement theory does not matter so much.
- If we want to relate this scale to taste…. Then it matters a great deal.

What are permissible transformations?

- Permissible/admissible transformations are transformations of a scale of measurement that preserve the relevant relationships of the measurement process. (Sarle 1997).
- They transform one meaningful measurement scale into another meaningful measurement scale.
- Only these transformations maintain the underlying biological meaning of the attributes from our entities (usually an organism).
- Celsius to Fahrenheit

From Wolman 2006

Table 1. Admissible transformations of measurement scale, their associated scale types, and example scales for each scale type.

Admissible transformations	*Scale type*	*Examples*
$\phi(x) = x$ (identity)	absolute	relative frequency counting
$\phi(x) = \alpha x,\ \alpha > 0$	ratio	weight (mass), length, temperature (kelvins), time (duration)
$\phi(x) = \alpha x + \beta,\ \alpha > 0$	interval	temperature (Fahrenheit, etc.), time (calendar)
$x \geq y$ iff $\phi(x) \geq \phi(y)$ (monotone)	ordinal	Mohs hardness scale, rankings grades (school)
ϕ one-to-one (permutation)	nominal	naming classification (species)

What are the levels of measurement?

- Different "levels" of measurement involve different properties of the numbers that are measured (Sarle 1997).
- These properties are the relations and operations that are meaningful.
- Associated with each level is a set of permissible transformations.

What operations and relations should we consider? Concatenation.

- Are the combined lengths of fish A and B when lined up head to fin longer than fish C?
- When concatenation operations are possible, then we can construct a ***standard sequence*** for measurement (such as a ruler in this case).

Relation structures depending on paired comparisons

- No natural concatenation is possible.
- Can you say that the attractiveness of Brad Pitt + Matt Damon is greater than the attractiveness of Robert Pattinson?
- However under controlled conditions ordering may be possible for this.

What relations should we consider? Order

- Can order of attributes be established? Elephant is bigger than a cat which is bigger than a mouse.
- Wild-type is more functional than a hypomorph which is more functional than a null morph in terms of alleles.

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http://en.wikipedia.org/wiki/Level_of_measurement

Levels of measurement

- Nominal: Any numbers are just labels, they do not express any mathematical properties (i.e. no information of order between levels, or magnitude between levels).
  - Sex (M vs F). Species.
- Ordinal: Numbers indicate relative order, but the contain no information of the magnitude of difference.
  - Social rank in hyenas. Some description of allelic series.
- Interval: Numbers indicate order and magnitude, but there is no absolute zero.
- Ratio: Numbers indicate magnitude and order, with an absolute zero (age, height, length, # offspring, [P])

Nominal

- Two entities are assigned the same “value” or symbol if they have the same value of the attribute (examples).
- One-one transformations (equality) and many-one transformations are permissible. However the many-one loses information.
- Permissible statistics (mode, Chi-squared) (Stevens 1946, 1951)

Ordinal

- Entities are assigned numbers such that the ranking/ ordering of the numbers reflects the ordered relation for values of the attribute.
- If you have entities x & y with attribute values a(x) and a(y) with a(x) > a(y), then these are assigned measurement values m(x), m(y) such that m(x) > m(y).
- However concatenation (addition) may not be meaningful.
- Monotone increasing transformations are permitted (as long as order is maintained, any transformation is ok).
- Median and percentile are permissible.

Interval

- Entities are assigned measurement values such that differences between the numbers reflect differences of the attribute. That is you could fit them along a line.
- If m(x) – m(y) > m(u) – m(v) then

  a(x) – a(y) > a(u) – a(v)
- The origin and unit of measurement are considered arbitrary.
- Permissible stats: mean, standard deviation, Pearson correlation, regression, ANOVA.
- Affine (including linear) transformations are possible

    t(m) = c*m + d (c, d constants)

Log-Interval

- Things are assigned numbers such that ratios between the numbers reflect ratios of the attribute.
- If m(x) / m(y) > m(u) / m(v), then

  a(x) / a(y) > a(u) / a(v).
- Power transformations are permissible

  i.e. $t(m) = c*m^d$ (unit of measurement arbitrary).

Ratio

- Entities are assigned numbers such that differences and ratios between the numbers reflect differences and ratios of the attribute.
- Linear transformations are permitted

  t(m) = c*m. Only the unit of measurement is arbitrary (not the origin).

All statistics are permitted for interval including geometic and harmonic mean, coefficient of variation and logarithms.

Absolute

- Entities are assigned numbers such that all properties of the numbers reflect analogous properties of the attribute.
- No transformations permitted.

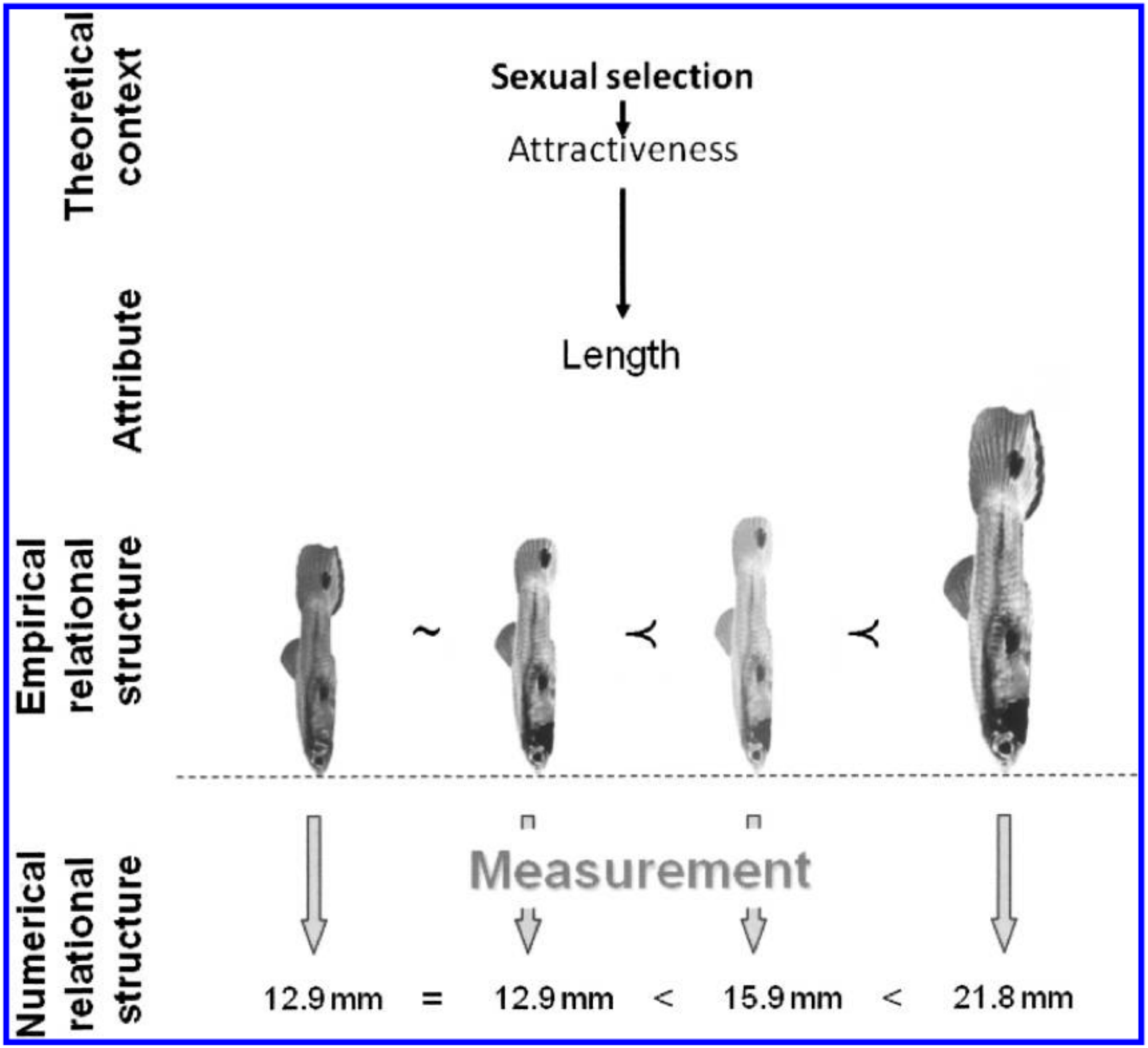
What scale is IQ?

TABLE 1

Classification of scale types (after Stevens 1946, 1959, 1968; Luce et al. 1990:113)

Scale type	Permissible transformations	Domain	Arbitrary parameters	Meaningful comparisons	Biological examples
Nominal	Any one-to-one mapping	Any set of symbols	Countable	Equivalence	Species, genes
Ordinal	Any monotonically increasing function	Ordered symbols	Countable	Order	Social dominance
Interval	$x \text{ -> } ax+b$	Real numbers	2	Order, differences	Dates, Malthusian fitness
Log-interval	$x \text{ -> } ax^b,\ a,\ b>0$	Positive real numbers	2	Order, ratios	Body size
Difference	$x \text{ -> } x+a$	Real numbers	1	Order, differences	Log-transformed ratio-scale variables
Ratio	$x \text{ -> } ax$	Positive real numbers	1	Order, ratios, differences	Length, mass, duration
Signed ratio*	$x \text{ -> } ax$	Real numbers	1	Order, ratios, differences	Signed asymmetry, intrinsic growth rate (r)
Absolute	None	Defined	0	Any	Probability

* Luce et al. (1990) defined this ratio scale but did not discuss or name it. Stevens did not consider this scale.



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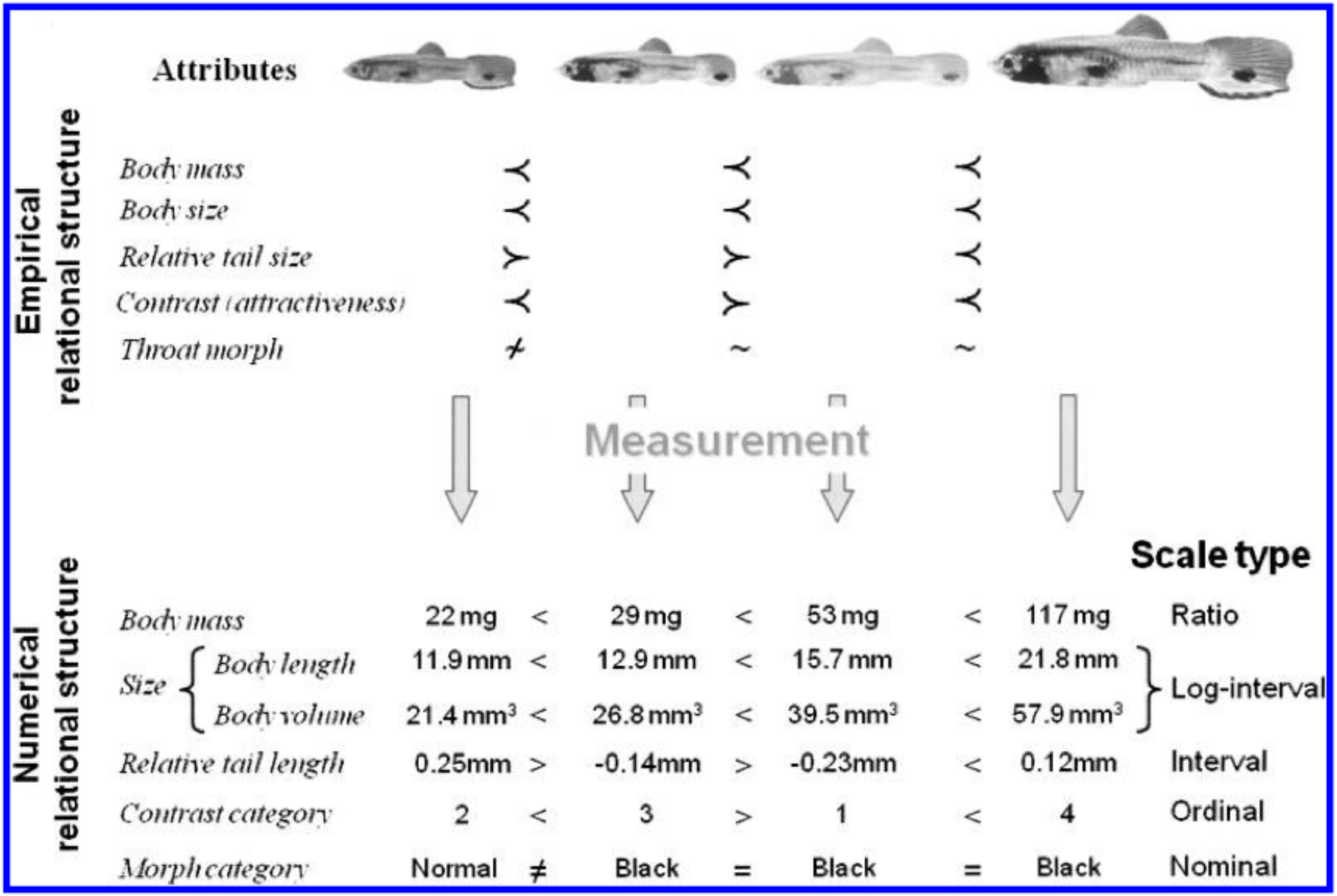

Attributes
Empirical relational structure
Body mass
Body size
Relative tail size
Contrast (attractiveness)
Throat morph
Measurement
Scale type
Numerical relational structure
Body mass
22 mg < 29 mg < 53 mg < 117 mg
Ratio
Size
Body length
11.9 mm < 12.9 mm < 15.7 mm < 21.8 mm
Body volume
21.4 mm³ < 26.8 mm³ < 39.5 mm³ < 57.9 mm³
Log-interval
Relative tail length
0.25mm > -0.14mm > -0.23mm < 0.12mm
Interval
Contrast category
2 < 3 > 1 < 4
Ordinal
Morph category
Normal ≠ Black = Black = Black
Nominal

Checklist from Houle et al 2011

- Keep theoretical context in mind.
- Honor your family of hypotheses.
- Make Meaningful definitions.
- Know what the numbers mean.
- Remember where the numbers come from.
- Respect scale type.
- Know the limits of your model.
- Never substitute a test for an estimate.
- Clothe estimates in the modest raiment of uncertainty.
- Never separate a number from its unit.