Problem Set 4: Demand Estimation
450-1

This problem set is designed to help you to understand the nuts and bolts of discrete choice demand models and imperfect competition. In this exercise, we will use simulated fake data so that you know the true distributions and then we will try alternative approaches to estimation. Please read the full problem set before starting, as it may save some heart-ache. **The problem set is quite long and you should start working on this as soon as possible!** Please turn in your answers in a typed-up pdf, and add your code as an appendix in a separate pdf.

1 Market Simulation

1.1 Model

1.1.1 Demand:

Consumers' preference for product j and market m is assumed to take the following form:

$$\begin{aligned} U_{ijm} &= X_{jm}\beta - \alpha_i p_{jm} + \xi_{jm} + \epsilon_{ijm} \\ \alpha_i &= \alpha + \sigma_\alpha v_{ip} \\ U_{i0} &= 0 \end{aligned} \tag{1}$$

where the product characteristics are iid with distributions:

- $X_{jm} = (X_{1jm}, X_{2jm}, X_{3jm})$, with $X_{1jm} = 1$ (a constant).
- $X_2 \sim U[0,1]$ and $X_3 \sim N(0,1)$
- $\xi_{jm} \sim N(0,1)$.

Similarly, the consumer taste shocks are iid with distributions

- $\nu_{ip} \sim LN(0,1)$, where LN is the lognormal distribution
- ϵ_{ijm} is drawn from type I extreme value distribution.

1.1.2 Supply:

The marginal cost of producing product j in market m is given by:

$$MC_{jm} = \gamma_0 + \gamma_1 W_j + \gamma_2 Z_{jm} + \eta_{jm} \tag{2}$$

where $W_j \sim N(0,1)$, $Z_{jm} \sim N(0,1)$ and $\eta_{jm} \sim N(0,1)$. All products are produced by single-product firms. The markets are regional, while the firms are national. Therefore, W_j is a common cost shifter for firm j across all markets.

1.1.3 Parameters:

In the remainder of this problem set, you will estimate the demand parameters $\theta = \{\beta, \alpha, \sigma_\alpha\}$, and the supply parameters γ.

Let the true parameter values be:

- $\beta = (5, 1, 1)$
- $\alpha = 1$ and $\sigma_\alpha = 1$
- $(\gamma_0, \gamma_1, \gamma_2) = (2, 1, 1)$.

Provided in matlab files are the simulated markets, prices and market shares for two simulations (generated using the true parameters): 100 markets and 3 products, 100 markets and 5 products.

For each of these two simulations, compare the distribution of prices, profits and consumer surplus. For consumer surplus, simulate draws of consumers from the true distribution and calculate their optimal purchasing decision and welfare

2 BLP and Hausman Instruments

Unless specified, use the dataset with $J = 3$ and $M = 100$ for the following exercises:

1. Consider the following set of moment conditions: $E[\xi|X] = 0$ and $E[\xi|p] = 0$.

   (a) Using the $(J, M) = (3, 200)$ dataset, compute the values of $E[\xi_{jm} X_{jm}]$, $E[\xi_{jm} p_{jm}]$ and $E[\xi_{jm} \bar{p}_{jm}]$, where $\bar{p}_{jm}$ is the average price of products in the other markets.

(b) Which of these moment conditions is valid? Which of them are relevant? Why?

(c) Can you use both BLP and Hausman instruments in this setting? Why? Why not?

2. Estimate θ a.la. BLP, but using *demand-side moments only* i.e. $E[\xi|X] = 0$ and writing the problem as an MPEC

   (a) Write down the BLP moments.

   (b) Construct your objective function.

   (c) Construct the constraints function.

   (d) Construct the gradient and Hessian.

   (e) Try to estimate θ and the standard errors. Report the estimates, bias and standard errors for each parameter.

   Note: You should start the optimization routine at several different starting values, and ensure that you are confident about your results. Comment on which parameters appear to be most stable across runs?

   (f) Compute the price elasticity of demand at equilibrium prices, profits and consumer surplus at the estimated parameters. Compare with the true values.

   (g) Repeat the estimation for $M = 10$. How do the estimates, standard errors and stability of the optimization routine change?

3. Estimate θ a.la. BLP, but assuming incorrectly that $E[\xi|p] = 0$ within each market. Compare the parameter estimates to the true values and the ones obtained using BLP instruments. Comment.

3 Adding Supply-side Instruments

Unless specified, use the dataset with $J = 3$ and $M = 100$ for the following exercises:

1. Estimate θ assuming $E[\xi|X, W] = 0$.

   (a) Write down the BLP moments, as well as a moment with the cost shifter W.

(b) Estimate θ and the standard errors. Report the estimates, bias and standard errors for each parameter. Compute elasticity of demand at equilibrium prices, profits and consumer surplus at the estimated parameters. Compare with the true values.

(c) Repeat the estimation for $M = 10$. How do the estimates, standard errors and stability of the optimization routine change?

(d) Compare the answers obtained here with the true values and the estimates using the BLP instruments alone.

2. Estimate θ and γ jointly, assuming that $E[\xi, \eta | X, W] = 0$.

(a) Write down marginal costs under the three pricing assumptions: 1) perfect competition, 2) perfect collusion, and 3) oligopoly (correct model).

(b) Using your most preferred estimates so far, compute the marginal costs. Comment on why you prefer these estimates. Compare the marginal costs to the true marginal costs in the data.

(c) Estimate the demand and supply parameters under these three assumptions. Comment on the estimates, standard errors, and demand elasticities at observed prices comparing them to previous estimates.

4 Bonus: Merger exercise

1. Pick a set of parameter estimates that you trust the most for the following exercises. Why do you prefer these over the others?

2. Suppose firm 1 and firm 2 plan to merge.

(a) Write down the merged firm's pricing problem.

(b) Predict the new set of prices using *estimated* parameters. How does markup change?

(c) Compare consumer surplus, prices and profits.