Fly Swatter Fire Control Computer Mathematics

1 Basic Trigonometry Identities

1.1 Spherical Coordinates

Cartesian $\longleftrightarrow$ Spherical

$\mathbf{x} = \rho\ \mathbf{cos}(\theta)\ \mathbf{sin}(\phi)$

$\mathbf{y} = \rho\ \mathbf{sin}(\theta)\ \mathbf{sin}(\phi)$

$\mathbf{z} = \rho\ \mathbf{cos}(\phi)$

$\rho = \sqrt{x^2 + y^2 + z^2}$

$\theta = \mathbf{arctan(y\ /\ x)}$

$\phi = \mathbf{arctan(}\sqrt{x^2 + y^2}/z)$

2 Derivatives of Trigonometric Functions

2.1 Two-Dimensional

$\frac{d\theta}{dt} = \frac{-y}{x^2+y^2}\ \frac{dx}{dt} + \frac{x}{x^2+y^2}\ \frac{dy}{dt}$

2.2 Spherical

$\frac{d\rho}{dt} = \frac{x}{\sqrt{x^2+y^2+z^2}}\ \frac{dx}{dt} + \frac{y}{\sqrt{x^2+y^2+z^2}}\frac{dy}{dt} + \frac{z}{\sqrt{x^2+y^2+z^2}}\frac{dz}{dt}$

$\frac{d\phi}{dt} = \frac{zx}{\sqrt{x^2+y^2}(x^2+z^2+y^2)}\ \frac{dx}{dt} + \frac{zy}{\sqrt{x^2+y^2}(x^2+z^2+y^2)}\ \frac{dy}{dt}$ - $\frac{\sqrt{y^2+x^2}}{z^2+y^2+x^2}\ \frac{dz}{dt}$

$\frac{d\theta}{dt} = \frac{-y}{x^2+y^2}\ \frac{dx}{dt} + \frac{x}{x^2+y^2}\ \frac{dy}{dt}$

3 Straight line Ballistics Calculations

3.1 Data Collection

The image capture, Sonar, or Radar needs two separate locations to get a triangulation of the target position and needs to capture twice in order to determine the vector.

3.2 Problem

Assuming that drag is negligible and that gravity isn't significant is short time frame.

Target is initially found at (ρ, ϕ, θ) has a trajectory with assumed Constant Velocity at and is moving ($\frac{d\rho}{dt}$, $\frac{d\phi}{dt}$, $\frac{d\theta}{dt}$) which is converted to ($\frac{dX}{dt}$, $\frac{dY}{dt}$, $\frac{dZ}{dt}$) using the derviatives of the trig functions shown above.

Our launch system is located at Cartesian Coordinates (0,0,0). Our launch projectile has a maximum velocity $\frac{d\rho}{dt}$ = M = $\{R > 0\}$.

3.3 Solution of three equations

X Component: $\mathbf{X}_{mi} + \ \rho_m cos(\theta) sin(\phi) \ \Delta T_{int} = \mathbf{X}_{ti} + \frac{dX_t}{dt} \quad \Delta T_{int}$

Y Component: $\mathbf{Y}_{mi} + \ \rho_m sin(\theta) sin(\phi) \ \Delta T_{int} = Y_{ti} + \frac{dY_t}{dt} \quad \Delta T_{int}$

Z Component: $\mathbf{Z}_{mi} + \ \rho_m cos(\phi) \ \ \Delta T_{int} = Z_{ti} + \frac{dZ_t}{dt} \quad \Delta T_{int}$

Using an initial guess of the target's position to get an early solution, Scipy.Optimize.fsolve converges on a solution $[\Delta T, \theta, \phi]$

4 Kinematics

4.1 Kinematic Equations

Each component X, Y, Z must be split and calculated separately.

Equation 1: $\Delta\mathbf{x} = \mathbf{v}\ \Delta\mathbf{t}$

Equation 2: $\mathbf{v}_f = v_i + \alpha\ \Delta\mathbf{t}$

Equation 3: $\mathbf{x}_f = x_i + v_i\Delta\mathbf{t}$ **+ (1/2)** $\alpha\ \Delta\mathbf{t}^2$

Equation 4: $\mathbf{x}_f = x_i + v\Delta\mathbf{t}$ **- (1/2)** $\alpha\ \Delta\mathbf{t}^2$

Equation 5: $\mathbf{v}_f^2 = v_i^2 + 2\alpha\ (\mathbf{x}_f - x_i)$

5 Ballistic Calculations using Kinematics

5.1 Calculation of Change in Position

Assuming gravitation acceleration is 9.81 $\frac{m}{s^2}$ **and is in the negative Z direction and air resistance is negligible.**

Δt is given as well an initial position and velocity in the Cartesian Plane.