Interception Calculations

1 Basic Trigonometry Identities

1.1 Spherical Coordinates

Cartesian $\longleftrightarrow$ Spherical

$\mathbf{x} = \rho\ \mathbf{cos(}\theta\mathbf{)}\ \mathbf{sin(}\phi\mathbf{)}$

$\mathbf{y} = \rho\ \mathbf{sin(}\theta\mathbf{)}\ \mathbf{sin(}\phi\mathbf{)}$

$\mathbf{z} = \rho\ \mathbf{cos(}\phi\mathbf{)}$

$\rho = \sqrt{x^2 + y^2 + z^2}$

$\theta = \mathbf{arctan(y\ /\ x)}$

$\phi = \mathbf{arctan(}\sqrt{x^2 + y^2}/z)$

2 Derivatives of Trigonometric Functions

2.1 Two-Dimensional

$\frac{d\theta}{dt} = \frac{-y}{x^2+y^2}\ \frac{dx}{dt} + \frac{x}{x^2+y^2}\ \frac{dy}{dt}$

2.2 Spherical

$\frac{d\rho}{dt} = \frac{x}{\sqrt{x^2+y^2+z^2}}\ \frac{dx}{dt} + \frac{y}{\sqrt{x^2+y^2+z^2}}\frac{dy}{dt} + \frac{z}{\sqrt{x^2+y^2+z^2}}\frac{dz}{dt}$

$\frac{d\phi}{dt} = \frac{zx}{\sqrt{x^2+y^2}(x^2+z^2+y^2)}\ \frac{dx}{dt} + \frac{zy}{\sqrt{x^2+y^2}(x^2+z^2+y^2)}\ \frac{dy}{dt}$ - $\frac{\sqrt{y^2+x^2}}{z^2+y^2+x^2}\ \frac{dz}{dt}$

$\frac{d\theta}{dt} = \frac{-y}{x^2+y^2}\ \frac{dx}{dt} + \frac{x}{x^2+y^2}\ \frac{dy}{dt}$

3 Simplified Ballistics Calculations

3.1 Data Collection

The image capture, Sonar, or Radar needs two separate locations to get a triangulation of the target position and needs to capture twice in order to determine the vector.

3.2 Problem

Assuming that drag is negligible and that gravity isn't significant is short time frame.

Target is initially found at (ρ, ϕ, θ) has a trajectory with assumed Constant Velocity at and is moving ($\frac{d\rho}{dt}$, $\frac{d\phi}{dt}$, $\frac{d\theta}{dt}$) which is converted to ($\frac{dX}{dt}$, $\frac{dY}{dt}$, $\frac{dZ}{dt}$) .

Our launch system is located at Cartesian Coordinates (0,0,0). Our launch projectile has a maximum velocity $\frac{d\rho}{dt}$ = M = $\{R > 0\}$.

3.3 Solution of three equations

X Component: $\mathbf{X}_{mi} + \ \rho_m cos(\theta)sin(\phi)\ \Delta T_{int} = \mathbf{X}_{ti} + \frac{dX_t}{dt} \quad \Delta T_{int}$

Y Component: $\mathbf{Y}_{mi} + \ \rho_m sin(\theta)sin(\phi)\ \Delta T_{int} = Y_{ti} + \frac{dY_t}{dt} \quad \Delta T_{int}$

Z Component: $\mathbf{Z}_{mi} + \ \rho_m cos(\phi) \ \ \Delta T_{int} = Z_{ti} + \frac{dZ_t}{dt} \quad \Delta T_{int}$

Using an initial guess of the target's position to get an early solution, Scipy.Optimize.fsolve converges on a solution $[\Delta T, \theta, \phi]$