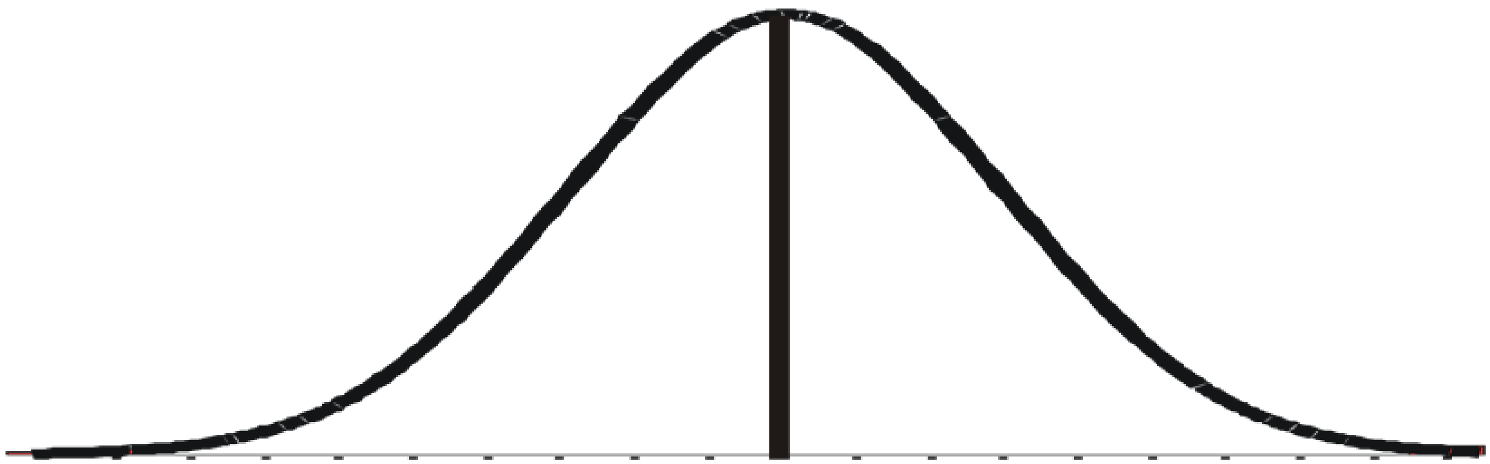
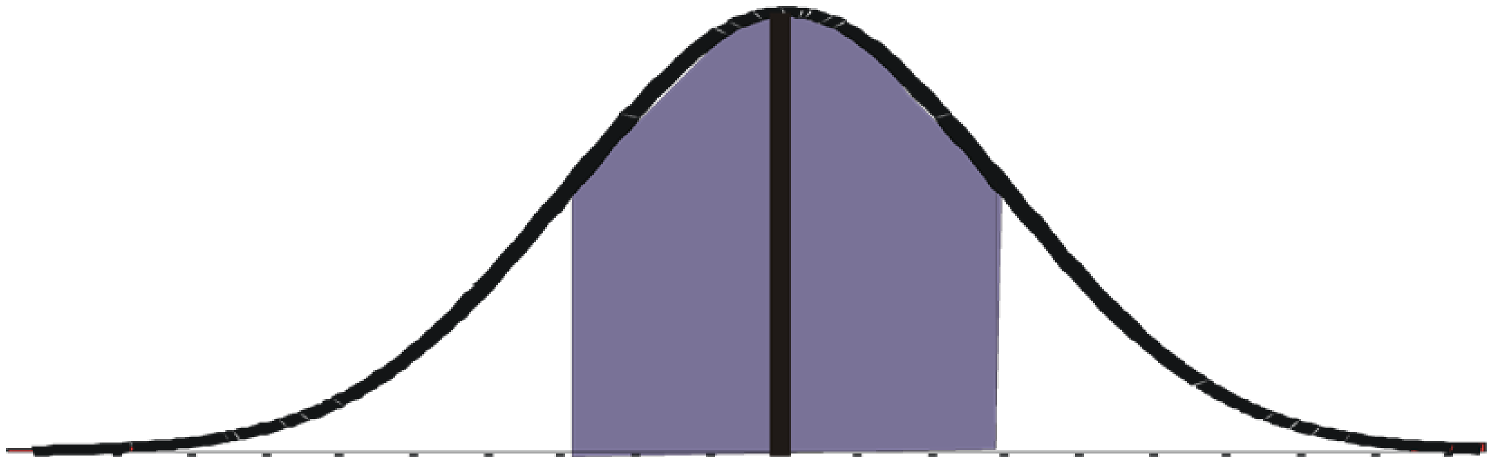
z Scores & the Normal Curve Model

The normal distribution and standard deviations



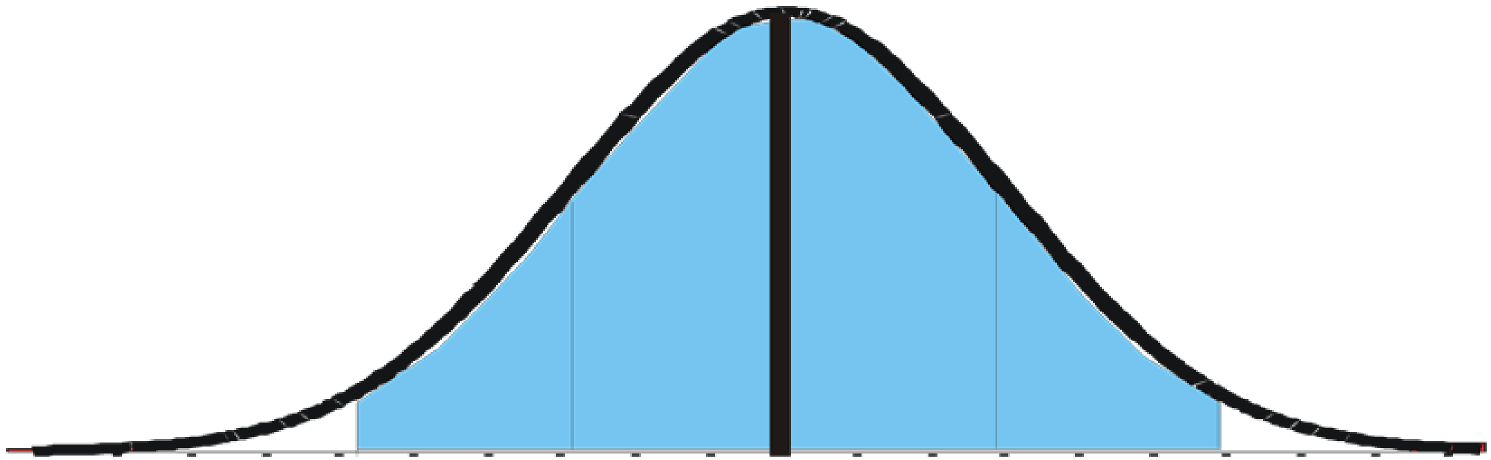
The normal distribution and standard deviations



In a normal distribution:

Approximately **68%** of scores will fall within **one** standard deviation of the mean

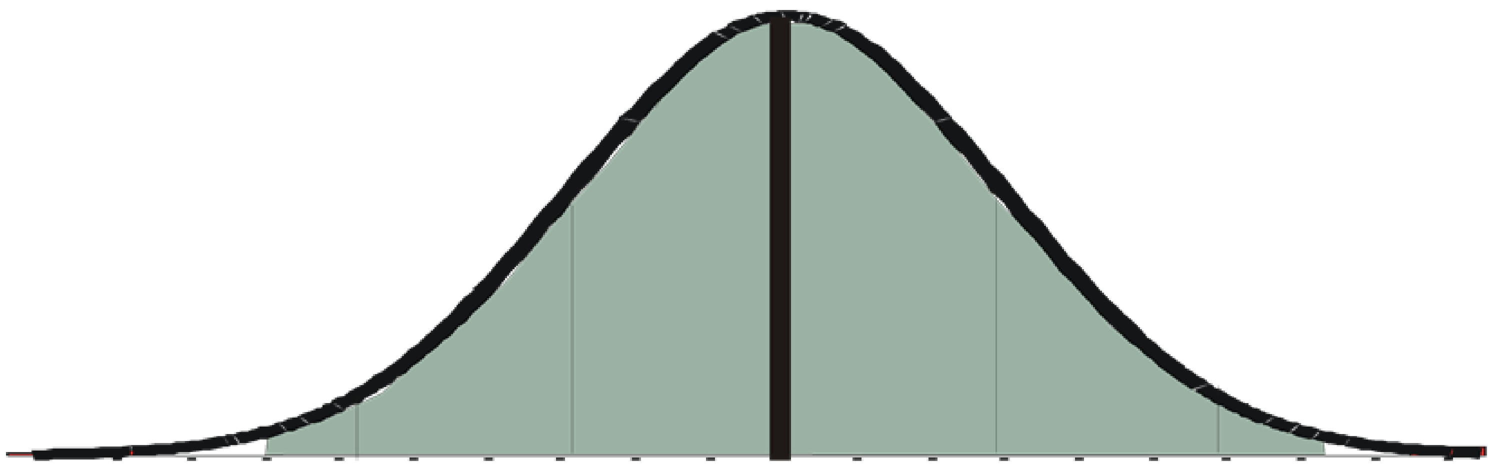
The normal distribution and standard deviations



In a normal distribution:

Approximately **95%** of scores will fall within **two** standard deviations of the mean

The normal distribution and standard deviations

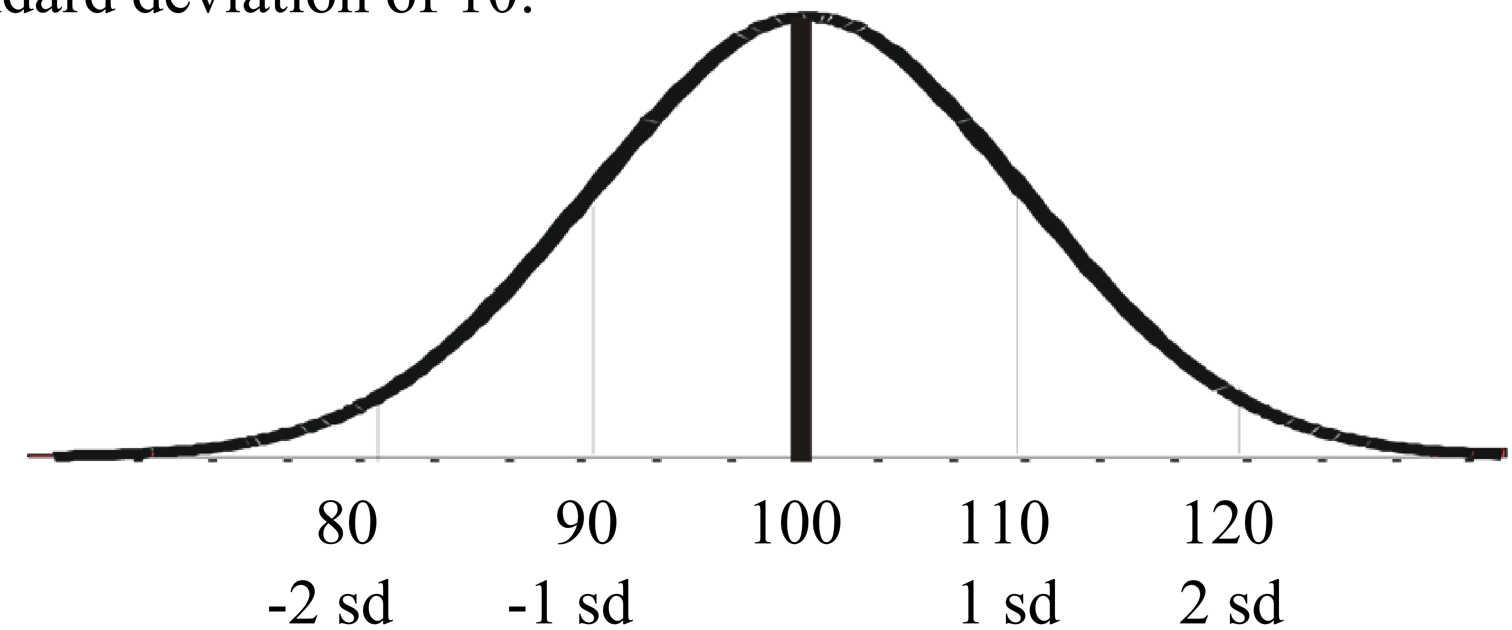


In a normal distribution:

Approximately **99%** of scores will fall within **three** standard deviations of the mean

Using standard deviation units to describe individual scores

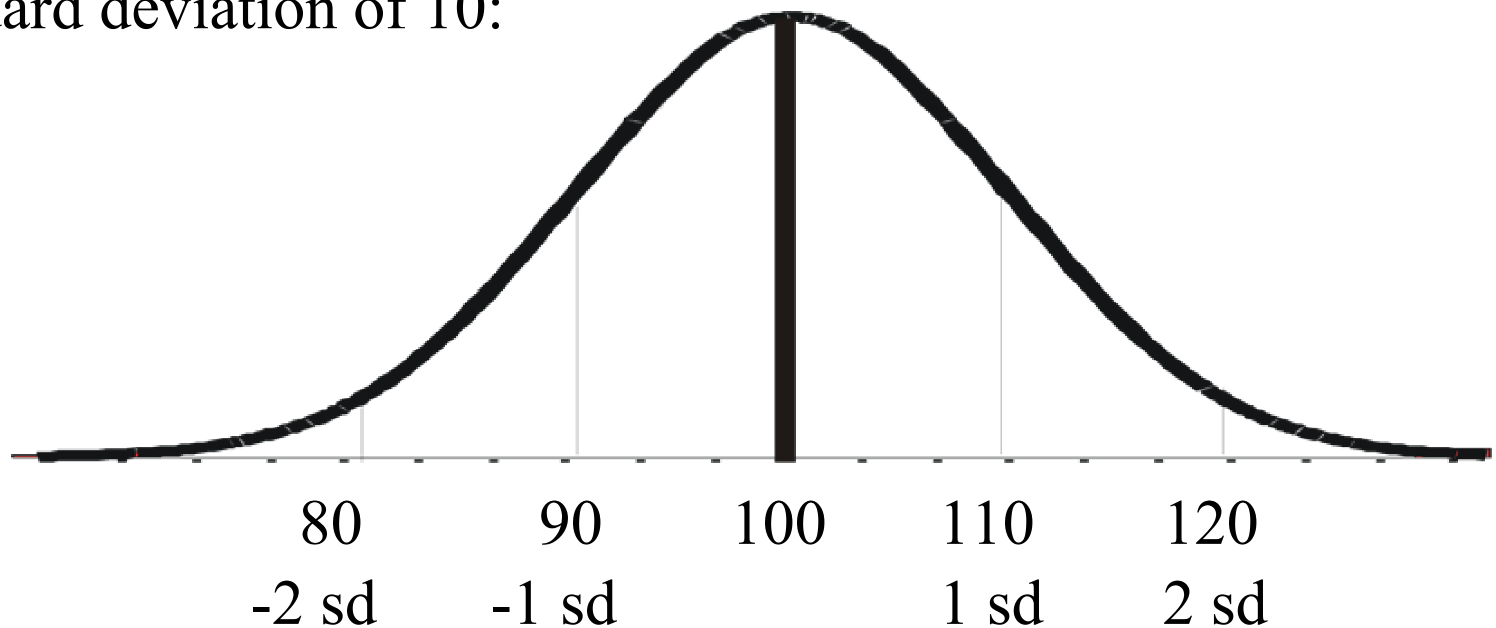
Here is a distribution with a mean of 100 and and standard deviation of 10:



What score is one sd below the mean?	90
What score is two sd above the mean?	120

Using standard deviation units to describe individual scores

Here is a distribution with a mean of 100 and and standard deviation of 10:



How many standard deviations below the mean is a score of 90?	1
How many standard deviations above the mean is a score of 120?	2

Z scores

What is a z-score?

A z score is a raw score expressed in standard deviation units.

z scores are sometimes called standard scores

Here is the formula for a z score:

$$z = \frac{X - \overline{X}}{S}$$

Computational Formula

$$z = \frac{X - \overline{X}}{S}$$

- Score minus the mean divided by the standard deviation
- Different formula for the population

Using z scores to compare two raw scores from different distributions

You score 80/100 on a statistics test and your friend also scores 80/100 on their test in another section. Hey congratulations you friend says—we are both doing equally well in statistics. What do you need to know if the two scores are equivalent?

the mean?

What if the mean of both tests was 75?

You also need to know the standard deviation

What would you say about the two test scores if the *S* in your class was 5 and the *S* in your friends class is 10?

Calculating z scores

What is the z score for your test: raw score = 80; mean = 75, S = 5?

$$z = \frac{X - \overline{X}}{S} \qquad z = \frac{80 - 75}{5} = 1$$

What is the z score of your friend's test: raw score = 80; mean = 75, S = 10?

$$z = \frac{X - \overline{X}}{S} \qquad z = \frac{80 - 75}{10} = .5$$

Who do you think did better on their test? Why do you think this?

Why z-scores?

- Transforming scores in order to make comparisons, especially when using different scales
- Gives information about the relative standing of a score in relation to the characteristics of the sample or population
  - Location relative to mean
  - Relative frequency and percentile

What does it tell us?

- z-score describes the location of the raw score in terms of distance from the mean, measured in standard deviations
- Gives us information about the location of that score relative to the “average” deviation of all scores

Fun facts about z scores

- Any distribution of raw scores can be converted to a distribution of z scores

the mean of a distribution has a z score of _____?

zero

positive z scores represent raw scores that are ____________ (above or below) the mean?

above

negative z scores represent raw scores that are ____________ (above or below) the mean?

below

Computing Raw Score when Know z-score

- $X = (z)\ (S_X) + \underline{M}$

Z-score Distribution

- Mean of zero
  - Zero distance from the mean
- Standard deviation of 1
- The z-score has two parts:
  - The number
  - The sign
- Negative z-scores aren’t bad
- Z-score distribution always has same shape as raw score

Uses of the z-score

- Comparing scores from different distributions
- Interpreting individual scores
- Describing and interpreting sample means

Comparing Different Variables

- Standardizes different scores
- Example in text:
  - Statistics versus English test performance
  - Can plot different distributions on same graph
  - increased height reflects larger N

The Standard Normal Curve

- Theoretically perfect normal curve
- Use to determine the relative frequency of z-scores and raw scores
- Proportion of the area under the curve is the relative frequency of the z-score

Thank You