KCET (MATHEMATICS)
SOLUTIONS AND KEY

Key:

1. d	2. b	3. a	4. a	5. c	6. b	7. a	8. b	9. a	10. b
11. a	12. d	13. d	14. d	15. b	16. d	17. a	18.c	19. b	20. a
21. c	22. a	23. b	24. b	25. a	26. d	27. d	28. d	29.wrong	30. c
31. b	32. b	33. b	34. a	35. d	36. b	37. a	38. c	39. b	40. c
41. c	42. c	43. a	44. a	45. b	46. a	47. c	48. b	49. b	50. a
51. c	52. c	53. b	54. b	55. a	56. a	57. c	58. d	59. c	60. a

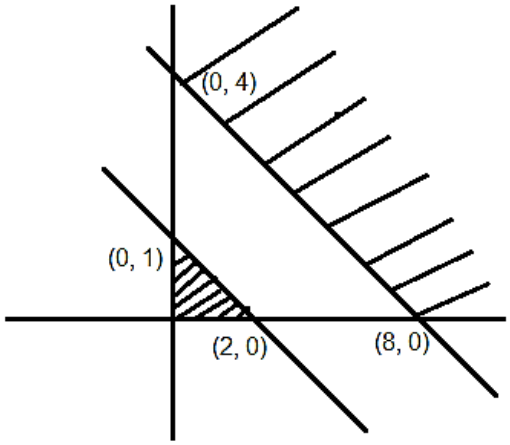
Solution:

1. **Sol: (d)**

$x + 2y \leq 2$

$x + 2y \geq 8$

$x, y \geq 0$



Has no feasible solution.

2. **Sol: (b)**

$M^2 + \sigma^2 = \sum x_i^2 \, P(x = x_i)$

$M = \sum x_i \; P(x = x_i) = \frac{5}{18} + \frac{21}{36} = \frac{6}{18} = \frac{1}{3}$

$\therefore \frac{1}{9} + \sigma^2 = \frac{5}{18} + \frac{4}{36} = \frac{7}{18}$

$\therefore \sigma^2 = \frac{7}{18} - \frac{2}{18} = \frac{5}{18}$

$\therefore$ S.D. = $\frac{1}{3}\sqrt{\frac{5}{2}}$

3. **Sol: (a)** P(both even) = $\frac{8}{17}\times\frac{7}{16} = \frac{7}{34}$

4. **Sol: (a)** P(3 are dead) = $\frac{4}{10}\times\frac{3}{9}\times\frac{2}{8} = \frac{1}{30}$

5. **Sol: (c)**

$x + 5 \leq -10$ (or) $x + 5 \geq 10$

$x \leq -15$ (or) $x \geq 5$

$x \in (-\infty,-15] \cup 45, \infty)$

6. **Sol: (b)**

${}^{n}C_2 = 45 \Rightarrow n(n-1) = 90 = 10 \times 9$

$\Rightarrow n = 10$

7. **Sol: (a)**

${}^{16}C_8$

8. **Sol: (b)**

$2^{2n} - 1 = 4^{2n} - 1 = (3 + 1)^n - 1$

$= 3m$

Divisible by 3

9. **Sol: (a)**

$3x - 4y + k = 0$; Point $(-2, 3)$

$-6 - 12 + k = 0$

$\Rightarrow k = 18$

$\therefore 3x - 4y + 18 = 0$

10. **Sol: (b)**

$\frac{1-i}{1+i} = -i \Rightarrow i^{96} = a + ib$

$\Rightarrow (i^4)^{24} = a + ib$

$\Rightarrow 1 + i(0) = a + ib$

11. **Sol: (a)**

Eccentricity = $\sqrt{2}$

$\therefore a = b$

12. **Sol: (d)**

$5! \times {}^{6}C_3 \times 3! = 120 \times 6 \times 5 \times 4$

$= 120 \times 120 = 14400$

13. **Sol: (d)**

Let 'd' be the common difference

$x^{-d} \cdot y^{2d} \cdot z^{-d} = (xz)^{-d} \cdot (y^2)^{-d}$

$= (xz)^{-d}(xz)^{d} = 1$

14. **Sol: (d)**

$\underset{x \to 0^-}{Lt} \frac{|x|}{x} = -1$; $\underset{x \to 0^+}{Lt} \frac{|x|}{x} = 1$

does not exist

15. **Sol: (b)**

$f(x) = x - \frac{1}{x} \Rightarrow f'(n) = 1 + \frac{1}{x^2}$

16. **Sol: (d)**

17. **Sol: (a)**

P(A) = 0.5 ; P(B) = 0.3

$\therefore P(\overline{A} \cap \overline{B}) = (1 - P(A))\,(1 - P(B))$

$= (0.5)\,(0.7) = 0.35$

$\simeq 0.4$

18. **Sol: (c)**

Sum = {8, 9, 10, 11, 12}

Prob = $\frac{15}{36} = \frac{5}{12}$

19. **Sol: (b)**

$P(A \cup B) = \frac{3}{5} + \frac{1}{5} - \frac{3}{25}$

$= \frac{20}{25} - \frac{3}{25} = \frac{17}{25} = \frac{68}{100}$

(exact answer is not given) = 0.68

20. **Sol: (a)**

$x < 0 : f(x) = -x + x = 0$

$\therefore fog(x) = 0$

21. **Sol: (c)**

total – bijections

$6^6 - 6!$

22. **Sol: (a)**

$f(-1) + f(2) + f(4)$

$= -3 + 4 + 8 = 9$

23. **Sol: (b)**

Use $\sin^{-1} x + \cos^{-1} x = \pi/2$

24. **Sol:(b)**

$$Let\ cso^{-1}\left(\frac{2}{\sqrt{5}}\right) = \theta$$
$$\Rightarrow \cos\theta = \frac{2}{\sqrt{5}}$$
$$\therefore \tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-cos\theta}{1+\cos\theta}} = \sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}}$$
$$= \sqrt{5} - 2$$

25. **Sol: (a)**

$$A^2 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = 2^2 A$$
$$A^3 = \begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix} = 2^4 A$$
$$\therefore K = 2^{n-1}$$

26. **Sol: (d)**

$$x + y = 2$$
$$-x + y = 4$$

Solving for x, y : $x = -1, y = 3$

27. **Sol: (d)**

$$A.A' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

28. **Sol: (d)**

Apply : $C_1 \to C_1 - C_2$

$$\therefore \begin{vmatrix} 4 & (5^x - 5^{-x})^2 & 1 \\ 4 & (6^x - 6^{-x})^2 & 1 \\ 4 & (7^x - 7^{-x})^2 & 1 \end{vmatrix} = 0$$

29. (wrong Question)

It should be given as:

$$\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$$

30.**Sol: (c)**

Area of the triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = k$

$$\therefore \begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2 = 16\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$$
$$= 16(4k^2) = 64k^2$$

31. **Sol: (b)**

$$|5A| = 5^3|A| = 125|A|$$

32. **Sol: (b)**

$$\lim_{x\to 0^-}\frac{\sqrt{1+Kx}-\sqrt{1-Kx}}{x}=\lim_{x\to 0^-}\frac{mKx}{x(\sqrt{1-Kx}+\sqrt{x}}$$

$$\lim_{x\to 0^+}\frac{2x+1}{x-1}=-1=K\ \ (after\ rationaltion)$$

$$\therefore K=-1$$

33. **Sol: (b)**

$$x=\frac{\cos y}{\cos(a+y)}$$

Diff. w.r.t y, we set

$$1=\frac{(-\cos(a+y)\sin y\sin(a+y)\frac{dy}{dx}}{\cos^2(a+y)}$$

$$\Rightarrow \frac{\cos^2(a+y)}{\sin(a+y-y)}=\frac{dy}{dx}$$

34. **Sol: (a)**

$$f(x)=|\cos x-\sin x|$$

$$f'(x)=-sin-\cos x$$

$$at\left(\frac{\pi}{6}\right)=\frac{-1}{2}-\frac{\sqrt{3}}{2}=-\frac{1}{2}(1+\sqrt{3})$$

35. **Sol: (d)**

$$y=\sqrt{x+y}\Rightarrow y^2=x+y$$

$$\Rightarrow 2y\frac{dy}{dx}=1+\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2y-1}$$

36. **Sol: (b)**

$$K=\lim_{x\to 1}\frac{\log_e^x}{x-1}=\lim_{x\to 1}\frac{1/x}{1}=1$$

37. **Sol: (a)**

$$V=x^3\ and\frac{dx}{x}x\ 100=3$$

$$dv=3x^2.dx$$

$$=3x^3.\frac{dx}{x}$$

$$=3.x^3\times\frac{3}{100}=0.09x^3$$

38. **Sol: (c)**

$$Let\ f=x^{-x}$$

$$\log f=-x\log x$$

$$\frac{1}{f}\frac{df}{dx}=-(1+\log x)$$

$$\therefore \frac{df}{dx} = -f(1 + \log_e^x \quad)$$

For eatrenum; $1 + \log_e^x \quad = 0$

$$\Rightarrow x = e^{-1} \Rightarrow \frac{1}{e}$$

$$Max = \left(\frac{1}{\frac{1}{e}}\right)^{1/e} = e^{1/e}$$

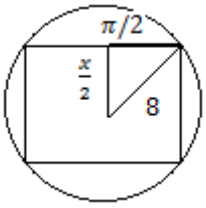
39. **Sol: (b)**

Stationary pt $\Rightarrow f'(x) = 0$

$$\Rightarrow x = \frac{1}{e}(like\ \theta(38);$$

40. **Sol: (c)**

Maximum area occused when it is a square



$$\therefore \frac{x^2}{4} + \frac{x^2}{4} = 64$$
$$\Rightarrow x^2 = 128$$

41. **Sol:(c)**

$$\int \frac{1}{1+e^x}dx = -\int \frac{-e^{-x}}{e^{-x}+1}dx$$
$$= -\ln|1 + e^{-x}| + c$$
$$= -\ln\left|\frac{e^x+1}{e^x}\right| + c$$
$$= \ln\left|\frac{e^x}{e^x+1}\right| + c$$

42. **Sol: (c)**

$$3 - 6x - 9x^2 = 4 - (3x+1)^2$$
$$\therefore \int \frac{1}{\sqrt{4-(3x+1)^2}}dx = \frac{1}{3}\sin^{-1}\left(\frac{3x+1}{2}\right) + C$$

43. **Sol: (a)**

$$\int e^{\sin x}(\sin x \cos x + \cos x)\,dx$$

We know $\int e^{g(x)}\big(g'(x)f(x) + f'(x)\big)dx = e^{g(x)}.f(x) + C$

$$= e^{\sin x}.\sin x + C$$

44. **Sol: (a)**

$$\int_{-2}^{2} |x\cos\pi x|\,dx = 2\int_{-2}^{2} |x\cos\pi x|\,dx$$

$$= 2 = \left[\int_{0}^{\frac{1}{2}} x\cos\pi x - \int_{1/2}^{3/2} (x\cos\pi x)dx + \int_{3/2}^{2} (x\cos\pi x)dx\right]$$

$$= 2\left[\frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{4}{2\pi} + \frac{3}{2\pi} + \frac{1}{\pi^2}\right]$$

$$= 2\left(\frac{8}{2\pi}\right) = \frac{8}{\pi}$$

45. **Sol: (b)**

$$\int_{0}^{1} \frac{dx}{e^x + e^{-1}} = \int_{0}^{1} \frac{e^x}{(e^x)^2 + 1}dx$$

$$= (\tan^{-1}(e^x))_0^1 = \tan^{-1}(e) - \frac{\pi}{4}$$

46. **Sol: (a)**

$$Let\ x = sin\theta$$

$$dx = cos\theta\ d\theta$$

$$\int_{0}^{\pi/6} \frac{\cos\theta\ d\theta}{(1+\sin^2\theta)cos\theta} = \int_{0}^{\pi/6} \frac{\sec^2\theta}{1+2\tan^2\theta} = d\theta$$

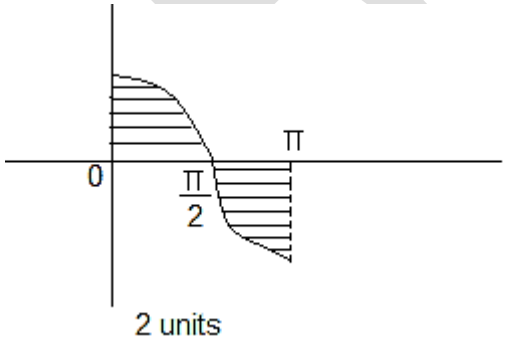
Multiply numeration and denomination by $\sec^2\theta$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\,tan\theta\right)_0^{\frac{\pi}{6}}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

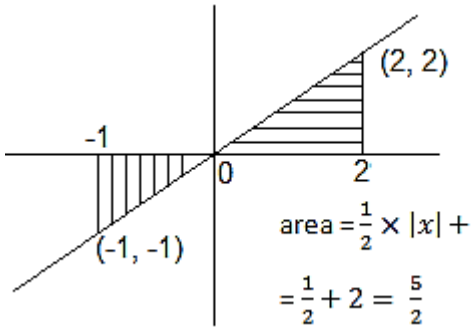
47. **Key: (c)**

Sol:



48. **Key: (b)**

Sol:



$\text{area} = \frac{1}{2} \times |x| + \frac{1}{2} \times 2 \times 2$

$= \frac{1}{2} + 2 = \frac{5}{2}$

49. **Key: (b)**

Sol: $\left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2$

Order = 2, degree = 3

50. **Key: (a)**

Sol: $x\frac{dy}{dx} = 3 + y$

$\Rightarrow \int \frac{dy}{3+y} = \int \frac{dx}{x}$

$\Rightarrow In\,(3 + y) = In\,(x) + inc$

$\Rightarrow 3 + y = cx$

Straight lines

51. **Key: (c)**

Sol: $\frac{dy}{dx} + y = \frac{1+y}{x}$

$\Rightarrow \frac{dy}{dx} + y\left(1 - \frac{1}{x}\right) = \frac{1}{x}$

I . F . $= e^{\int\left(1-\frac{1}{x}\right)dx} = e^x \cdot e^{-in\,x}$

$= \frac{e^x}{x}$

52. **Key: (c)**

Sol: $\left|\bar{a} \times \bar{b}\right|^2 + \left|\bar{a} \cdot \bar{b}\right|^2 = |\bar{a}|^2\left|\bar{b}\right|^2$

$\Rightarrow 144^9 = 16\left|\bar{b}\right|^2 \Rightarrow |\bar{b}| = 3$

53. **Key: (b)**

Sol: $\bar{a} \cdot \bar{b} = 0\ and\ |\bar{a}| = \left|\bar{b}\right| = 1$

$(3\bar{a} + 2\bar{b}) \cdot \left(5\bar{a} - 6\bar{b}\right)$

$= 15\,|\bar{a}|^2 - 18(0) + 10(0) - 12\left|\bar{b}\right|^2$

$= 3$

54. **Key: (b)**

Sol: $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$\Rightarrow a(b\,c-1)-1\,(c-1)+1\,(1-b)=0$

$\Rightarrow abc-(a+b+c)=-2$

55. **Key: (a)**

Sol: $\bar{a}\cdot\bar{b}=0 \Rightarrow \mu+\lambda-2=0$

$\Rightarrow \lambda+\mu=2$

and $1+\lambda^2+4=\mu^2+1+1$

$\therefore \lambda^2+3=\mu^2=(2-\lambda)^2$

$\Rightarrow \lambda^2+3=4+\lambda^2-4\lambda$

$\therefore \lambda=\frac{1}{4} \Rightarrow \mu=2-\frac{1}{4}=\frac{7}{4}$

56. **Key: (a)**

Sol: $\overline{PR}=\perp$ line

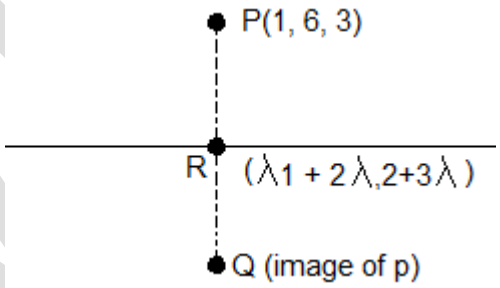
d.r's of $\overline{PR}=(\lambda-1,2\lambda-5,3\lambda-1)$

drs fo line : (1, 2, 3)

$\therefore \lambda-1+4\lambda-10+9\lambda-3=0$

$\Rightarrow 14\lambda=14 \Rightarrow \lambda=1$

$\therefore$ R (1, 3, 5)



$\therefore Q=(1,0,7)$

57. **Key: (c)**

Sol: $\frac{x}{1/2}=\frac{y}{1/3}=\frac{z}{-1}$; $\frac{x}{1/6}=\frac{y}{-1}=\frac{z}{-1/4}$

Here: $\frac{1}{12}-\frac{1}{3}+\frac{1}{4}=\frac{1-4+3}{12}=0$

Lines are perpendicular

58. **Key: (d)**

Sol: point (4, 2, k) satisfies the plans

$\therefore$ 2 (4) – 4(2) + k = 7

$\Rightarrow$ k = 7

59. **Key: (c)**

Sol: $xy + yz = 0 \Longrightarrow y(x + z) = 0$

Pain of perpendicular planes.

60. **Key: (a)**

Sol: $z = 3x + 9y$

$$z_{(5.5)} = 15 + 45 = 60$$
$$z_{(0,0)} = 0 + 90 = 90$$

Minimum value at (5.5)