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KCET-2017-2nd May
Questions – Mathematics

1. If A and B are finite sets and $A \subset B$, then

(A) $n(A \cup B) = n(A)$

(B) $n(A \cap B) = n(B)$

(C) $n(A \cup B) = n(B)$

(D) $n(A \cap B) = \phi$

2. The value of $\cos^2 45° + \sin^2 15°$ is

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{\sqrt{3}}{4}$

(C) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(D) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$

3. $3+5+7+\cdots$ to n term is

(A) $n(n+2)$

(B) $n(n-2)$

(C) n^2

(D) $(n+1)^2$

4. If $\left(\dfrac{1+i}{1-i}\right)^m = 1$ then the least positive integral value of m is,

(A) 2

(B) 3

(C) 4

(D) 1

5. If $\left|x-2\right| \le 1$, then

(A) $x \in [1,3]$

(B) $x \in (1,3)$

(C) $x \in [-1,3)$

(D) $x \in (-1,3)$

6. If ${}^{n}C_{12} = {}^{n}C_{8}$, then n is equal to,

(A) 26

(B) 12

(C) 6

(D) 20

7. The total number of terms in the expression of $(x+\alpha)^{47} - (x-\alpha)^{47}$ after simplification is

(A) 24

(B) 47

(C) 48

(D) 96

8. Equation of line passing through the point $(1,2)$ and perpendicular to the line $y = 3x - 1$ is

(A) $x + 3y - 7 = 0$

(B) $x + 3y + 7 = 0$

(C) $x + 3y = 0$

(D) $x - 3y = 0$

9. The eccentricity of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ is

(A) $\frac{2\sqrt{5}}{6}$

(B) $\frac{2\sqrt{5}}{4}$

(C) $\frac{2\sqrt{13}}{6}$

(D) $\frac{2\sqrt{13}}{4}$

10. The perpendicular distance of the point $P(6,7,8)$ from XY-plane is

(A) 8

(B) 7

(C) 6

(D) 5

11. The value of $\lim_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta}$ is

(A) 4/9

(B) 9/4

(C) 9/3

(D) 3/4

12. The contrapositive statement of the statement ‘If x is a prime number then x is odd” is

(A) If x is not a prime number then x is not odd

(B) If x is a prime number then x is not odd

(C) If x is not a prime number then x is odd

(D) If x is not odd, then x is not a prime number

13. If the coefficient of variation is 60 and the standard deviation is 24, then the arithmetic mean is

(A) 40

(B) 7/20

(C) 20/7

(D) $1/40$

14. The range of the function $f(x)=\sqrt{9-x^2}$

(A) $(0,3)$

(B) $[0,3]$

(C) $(0,3]$

(D) $[0,3)$

15. Let $f:R\to R$ be defined by $f(x)=x^4$ then

(A) f is one-one and onto

(B) f may be one-one and onto

(C) f is one-one not onto

(D) f is neither one-one nor onto.

16. The range of $\sec^{-1} x$ is,

(A) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(B) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(C) $[0, \pi]$

(D). $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

17. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$ then $\cot^{-1} x + \cot^{-1} y$ is equal to

(A) π

(B) $\frac{\pi}{5}$

(C) $\frac{2\pi}{5}$

(D) $\frac{3\pi}{5}$

18. If $f(x) = 8x^3,\ g(x) = x^{\frac{1}{3}}$ then $fog(x)$ is

(A) $8x$

(B) $8^3 x$

(C) $(8x)^{1/3}$

(D) $8x^3$

19. If $A = \frac{1}{\pi}\begin{vmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{vmatrix}$ and

$B = \begin{vmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{vmatrix}$ then A-B is equal to

(A) I

(B) 0

(C) $2I$

(D) $\frac{1}{2}I$

20. If a matrix is both symmetric and skew symmetric. then

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is scalar matrix

(D) A is square matrix

21. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ then the value of x and y are

(A) $x = 3, \ y = 3$

(B) $x = -3, \ y = 3$

(C) $x = 3, \ y = -3$

(D) $x = -3, \ y = -3$

22. Binary operation * on $R-\{-1\}$ defined by $a*b=\frac{a}{b+1}$ is

(A) * is associative and commutative

(B) * is associative but not commutative

(C) * is neither associative nor commutative

(D) * is commutative but not associative

23. If $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix}=\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ then x is equal to

(A) 2

(B) 4

(C) 8

(D) $\pm 2\sqrt{2}$

24. If A is square matrix of order 3×3, then $|KA|$ is equal to

(A) $K|A|$

(B) $K^2|A|$

(C) $K^3|A|$

(D) $3K|A|$

25. The area of triangle with vertices $(K,0),(4,0),(0,2)$ is 4 square units, then the value of K is

(A) 0 or 8

(B) 0 or -8

(C) 0

(D) 8

26. Let $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$ then

(A) $\Delta_1 = -\Delta$

(B) $\Delta_1 = \Delta$

(C) $\Delta_1 \neq \Delta$

(D) $\Delta_1 = 2\Delta$

27. If $f(x) = \begin{cases} Kx^2 & x \le 2 \\ 3 & x > 2 \end{cases}$ is continuous at $x = 2$, then the value of K is

(A) 3

(B) 4

(C) $3/4$

(D) $4/3$

28. The value of C in Mean value theorem for the function $f(x) = x^2$ in $[2,4]$ is

(A) 3

(B) 2

(C) 4

(D) 7/2

29. The point on the curve $y^2 = x$ where the tangent makes an angle of $\pi/4$ with X- axis is

(A) $\left(\frac{1}{2},\frac{1}{4}\right)$

(B) $\left(\frac{1}{4},\frac{1}{2}\right)$

(C) $\left(4,2\right)$

(D) $\left(1,1\right)$

30. The function $f\left(x\right)=x^2+2x-5$ is strictly increasing in the interval

A) $\left(-1,\infty\right)$

(B) $(-\infty, -1)$

(C) $[-1, \infty)$

(D) $(-\infty, -1]$

31. The rate of change of volume of sphere with respect to its surface area when the radius is 4 cm is

A) $4\ \mathrm{cm^3/cm^2}$

(B) $2\ \mathrm{cm^3/cm^2}$

(C) $6\ \mathrm{cm^3/cm^2}$

(D) $8\ \mathrm{cm^3/cm^2}$

32. If $y = \tan^{-1}\left(\dfrac{\sin x + \cos x}{\cos x - \sin x}\right)$ then $\dfrac{dy}{dx}$ is equal to

(A) $1/2$

(B) $\pi/4$

(C) 0

(D) 1

33. $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$ then $\frac{dy}{dx}$ is equal to

(A) $\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

(B) $\begin{vmatrix} l & m & n \\ f'(x) & g'(x) & h'(x) \\ a & b & c \end{vmatrix}$

(C) $\begin{vmatrix} f'(x) & l & a \\ g'(x) & m & b \\ h'(x) & n & c \end{vmatrix}$

(D) $\begin{vmatrix} l & m & n \\ a & b & c \\ f'(x) & g'(x) & h'(x) \end{vmatrix}$

34. If $\sin x = \dfrac{2t}{1+t^2}$, $\tan y = \dfrac{2t}{1-t^2}$ then $\dfrac{dy}{dx}$ is equal to

(A) 1

(B) 0

(C) -1

(D) 2

35. The derivative of $\cos^{-1}\left(2x^2-1\right)$ w.r.t $\cos^{-1} x$ is

(A) 2

(B) $\dfrac{-1}{2\sqrt{1-x^2}}$

(C) $\dfrac{2}{x}$

(D) $1-x^2$

36. If $y=\log(\log x)$ then $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{-(1+\log x)}{(x\log x)^2}$

(B) $\frac{-(1+\log x)}{x^2\log x}$

(C) $\frac{(1+\log x)}{(x\log x)^2}$

(D) $\frac{(1+\log x)}{x^2\log x}$

37. $\int\frac{x+3e^x}{(x+4)^2}dx$ is equal to

(A) $\frac{1}{(x+4)^2}+C$

(B) $\frac{e^x}{(x+4)^2}+C$

(C) $\dfrac{e^x}{(x+4)}+C$

(D) $\dfrac{e^x}{(x+3)}+C$

38. $\int \dfrac{\cos 2x-\cos 2\theta}{\cos x-\cos\theta}dx$ is equal to

(A) $2(\sin x+x\cos\theta)+C$

(B) $2(\sin x-x\cos\theta)+C$

(C) $2(\sin x+2x\cos\theta)+C$

(D) $2(\sin x-2x\cos\theta)+C$

39. $\int \sqrt{x^2+2x+5}dx$ is equal to

(A) $\dfrac{1}{2}(x+1)\sqrt{x^2+2x+5}+2\log\left|x+1+\sqrt{x^2+2x+5}\right|+C$

(B) $(x+1)\sqrt{x^2+2x+5}+2\log\left|x+1+\sqrt{x^2+2x+5}\right|+C$

(C) $\left(x+1\right)\sqrt{x^2+2x+5}-2\log\left|x+1+\sqrt{x^2+2x+5}\right|+\text{C}$

(D) $\left(x+1\right)\sqrt{x^2+2x+5}+\frac{1}{2}\log\left|x+1+\sqrt{x^2+2x+5}\right|+\text{C}$

40. $\int_0^{\pi/2}\frac{\tan^7 x}{\cot^7 x+\tan^7 x}\mathrm{d}x$ is equal to

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{3}$

41. $\int_{-5}^{5}\left|x+2\right|\mathrm{d}x$ is equal to

(A) 29

(B) 28

(C) 27

(D) 30

42. $\int_{-\pi/2}^{\pi/2} \frac{dx}{e^{\sin x}+1}$ is equal to

(A) 0

(B) 1

(C) $-\frac{\pi}{2}$

(D) $-\frac{\pi}{2}$

43. $\int_{0}^{\pi/2} \frac{1}{a^2 \cdot \sin^2 x + b^2 \cdot \cos^2 x} dx$ is equal to

(A) $\frac{\pi a}{4b}$

(B) $\frac{\pi a}{2b}$

(C) $\frac{\pi b}{4a}$

(D) $\frac{\pi}{2ab}$

44. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is

(A) $\frac{32}{3}$ sq. units

(B) $\frac{256}{3}$ sq. units

(C) $\frac{64}{3}$ sq. units

(D) $\frac{128}{3}$ sq. units

45. Area of the region bounded by the curve $y = \cos x.\ x = 0$ and $x = \pi$ is

(A) 2 sq. units

(B) 4 sq. units

(C) 3 sq. units

(D) 1 sq. units

46. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2}$ is

(A) 1

(B) 2

(C) 3

(D) 4

47. General solution of the differential equation $\frac{dy}{dx}+y=1\ (y\neq 1)$ is

(A) $\log\left|\frac{1}{1-y}\right|=x+C$

(B) $\log|1-y|=x+C$

(C) $\log|1+y|=x+C$

(D) $\log\left|\frac{1}{1-y}\right|=-x+C$

48. The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is $(x\neq 0)$

(A) x^2

(B) $\log|x|$

(C) $e^{\log x}$

(D) x

49. If $\vec{\mathbf{a}} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\hat{\mathbf{b}} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal, then value of λ is

(A) 0

(B) 1

(C) $\frac{3}{2}$

(D) $-\frac{5}{2}$

50. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is equal to

(A) 1

(B) 3

(C) $-\frac{3}{2}$

(D) $\frac{3}{2}$

51. If $\vec{a}$ & $\vec{b}$ are unit vectors, then angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a}-\vec{b}$ to be unit vector is

(A) $30°$

(B) $45°$

(C) $60°$

(D) $90°$

52. Reflexion of the point (α,β,γ) in XY plane is

(A) $(\alpha,\beta,0)$

(B) $(0,0,\gamma)$

(C) $(-\alpha,-\beta,\gamma)$

(D) $(\alpha,\beta,-\gamma)$

53. The plane $2x-3y+6z-11=0$ makes an angle $\sin^{-1}(\alpha)$ with X axis. The value of α is

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{\sqrt{2}}{3}$

(C) $\frac{2}{7}$

(D) $\frac{3}{7}$

54. The distance of the point $(-2,4,-5)$ from the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is

(A) $\frac{\sqrt{37}}{10}$

(B) $\sqrt{\frac{37}{10}}$

(C) $\frac{37}{\sqrt{10}}$

(D) $\frac{37}{10}$

55. A box has 100 pens of which 10 are defective. The probability that out of a sample of 5 pens drawn one by one with replacement and atmost one is defective is

(A) $\frac{9}{10}$

(B) $\frac{1}{2}\left(\frac{9}{10}\right)^4$

(C) $\left(\frac{9}{10}\right)^5+\frac{1}{2}\left(\frac{9}{10}\right)^4$

(D) $\frac{1}{2}\left(\frac{9}{10}\right)^5$

56. Two events A and B will be independent if

(A) A and B are mutually exclusive

(B) $P(A' \cap B') = (1 - P(A))(1 - P(B))$

(C) $P(A) = P(B)$

(D) $P(A) + P(B) = 1$

57. The probability distribution of X is

X	0	1	2	3
$P(X)$	0.3	k	2k	2k

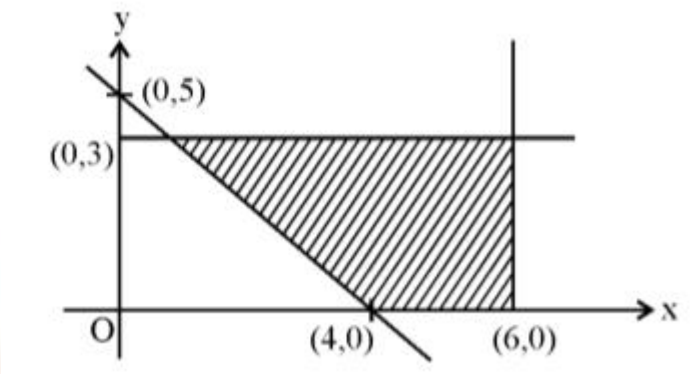
(A) 0.14

(B) 0.3

(C) 0.7

(D) 1

58. The shaded region in the figure is the solution set of the inequations



(A) $5x+4y\geq 20,\ x\leq 6,\ y\geq 3,\ x\geq 0, y\geq 0$

(B) $5x+4y\leq 20,\ x\leq 6,\ y\leq 3,\ x\geq 0, y\geq 0$

(C) $5x+4y\geq 20,\ x\leq 6,\ y\leq 3,\ x\geq 0, y\geq 0$

(D) $5x+4y\geq 20,\ x\leq 6,\ y\leq 3,\ x\geq 0, y\geq 0$

59. If an LPP admits optimal solution at two consecutive vertices of a feasible region, then

(A) the required optimal solution is at the midpoint of the line joining two points.

(B) the optimal solution occurs at every point on the line joining these two points

(C) the LPP under consideration is not solvable

(D) the LPP under consideration must be reconstructed

60. $\int_{0.2}^{3.5} [x] dx$ is equal to

(A) 4

(B) 4.5

(C) 3.5

(D) 3

KCET-2017-2nd May
Answer keys

1	C	16	D	31	B	46	A
2	B	17	B	32	D	47	A
3	A	18	A	33	C	48	A
4	C	19	A	34	A	49	D
5	A	20	B	35	A	50	C
6	D	21	A	36	A	51	A
7	A	22	C	37	C	52	D
8	A	23	D	38	A	53	C
9	A	24	C	39	A	54	B
10	A	25	A	40	B	55	C
11	A	26	B	41	A	56	A
12	D	27	C	42	D	57	A
13	A	28	A	43	D	58	C
14	B	29	B	44	B	59	B
15	D	30	C	45	A	60	B

KCET-2017-2nd May
Solutions – Mathematics

1. Consider the function $A \subset B$.

$$A \cap B = A$$
$$n(A \cap B) = n(A)$$

Also,

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= n(A) + n(B) - n(A) \\ &= n(B) \end{aligned}$$

2. Consider the expression.

$$\cos^2 45° + \sin^2 15°$$

Simplify the above expression as follows,

$$\cos^2 45° + \sin^2 15° = \cos(30° + 15°)\cos(60° - 15°)$$

$$= \cos 60° \cos 30°$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{4}$$

3. Consider the A.P

$$3 + 5 + 7 + \cdots + n$$

The sum of the A.P is calculated as,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2(3) + (n-1)2\right]$$

$$= \frac{n}{2}[6 + 2n - 2]$$

$$= n(n+2)$$

4. Consider the equation.

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

Simplify the above equation as follows,

$$\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^m = 1$$

$$\left(\frac{1+2i-1}{1-i^2}\right)^m = 1$$

$$\left(\frac{2i}{1+1}\right)^m = 1$$

$$\left(\frac{2i}{2}\right)^m = 1$$

Solve further,

$$\left(\frac{2i}{2}\right)^m = 1$$

$$i^m = 1$$

$$i^m = 1^4$$

$$m = 4$$

5. Consider the function.

$|x-2| \leq 1$

The above inequation is solved as,

$$\begin{aligned} -1 &\leq x-2 \leq 1 \\ -1+2 &\leq x-2+2 \leq 1+2 \\ 1 &\leq x \leq 3 \end{aligned}$$

Thus, $x \in [1,3]$.

6. It is known that if,

$^{n}C_{x} = {}^{n}C_{y}$

Then,

$x+y=n$

Now,

$^{n}C_{12} = {}^{n}C_{8}$

So,

$$\begin{aligned} 12+8 &= n \\ 20 &= n \\ n &= 20 \end{aligned}$$

7. Consider the expansion $(x+\alpha)^{47}-(x-\alpha)^{47}$

Here, $n=47$ which is odd.

The total number of term in the expansion is,

$$\begin{aligned}\text{Total}&=\frac{n+1}{2}\\&=\frac{47+1}{2}\\&=\frac{48}{2}\\&=24\end{aligned}$$

8. Consider the equation of line,

$y=3x-1$

So the slope is $m=3$

The slope of line perpendicular to the given line $m_1=-\frac{1}{3}$

The equation of required line is calculated as,

$$y-2=-\frac{1}{3}(x-1)$$

$$3y-6=-x+1$$

$$x+3y-7=0$$

9. Consider the ellipse,

$$\frac{x^2}{36}+\frac{y^2}{16}=1$$

The equation can be written as,

$$\frac{x^2}{6^2}+\frac{y^2}{4^2}=1$$

So,

$$a=6$$
$$b=4$$

The eccentricity is calculated as,

$$e=\sqrt{1-\frac{b^2}{a^2}}$$

$$=\sqrt{1-\frac{4^2}{6^2}}$$

$$=\sqrt{1-\frac{16}{36}}$$

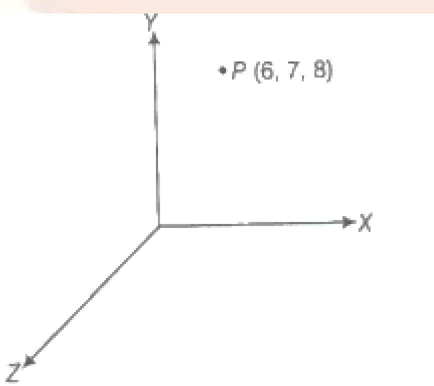
$$=\sqrt{\frac{20}{36}}$$

Solve further,

$$e=\frac{2\sqrt{5}}{6}$$

10. The given point is $P(6,7,8)$.

Consider the figure



Distance of any point from XY-plane is $|z|$

Therefore,

$$\begin{aligned} D &= |z| \\ &= |8| \\ &= 8 \end{aligned}$$

11. Consider the limit $\lim\limits_{\theta \to 0} \dfrac{1-\cos 4\theta}{1-\cos 6\theta}$.

The above expression is solved as,

$$\begin{aligned} \lim_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta} &= \lim_{\theta \to 0} \frac{2\sin^2 2\theta}{2\sin^2 3\theta} \\ &= \lim_{\theta \to 0} \frac{\sin^2 2\theta}{4\theta^2} \cdot \frac{1}{\left(\frac{\sin 3\theta}{3\theta}\right)^2} \cdot \left(\frac{4}{9}\right) \\ &= \frac{4}{9} \end{aligned}$$

12. The contrapositive statement of the statement ‘If x is a prime number then x is odd” is “ If x is not odd then x is not a prime number.

13. The coefficient of variation is $\mathrm{CV}=60$ and the standard deviation is $\sigma=24$.

The mean is calculated as,

$$\begin{aligned}
CV &= \frac{\sigma}{\overline{X}}\times 100 \\
60 &= \frac{24}{\overline{X}}\times 100 \\
\overline{X} &= \frac{24}{60}\times 100 \\
&= 40
\end{aligned}$$

14. Consider the function.

$$f\left(x\right)=\sqrt{9-x^{2}}$$

Let ,

$$y=\sqrt{9-x^2}$$
$$y^2=9-x^2$$
$$x^2=9-y^2$$
$$x=\sqrt{9-y^2}$$

Now,

$$9-y^2\geq 0$$
$$(3+y)(3-y)\geq 0$$

So,

$$f(x)\in[0,3]$$

15. Consider the function $f(x)=x^4$.

Let $f(x_1)=x_1^4$ and $f(x_2)=x_2^4$

So,

$$f(x_1)=f(x_2)$$
$$x_1^4=x_2^4$$
$$x_1=\pm x_2$$

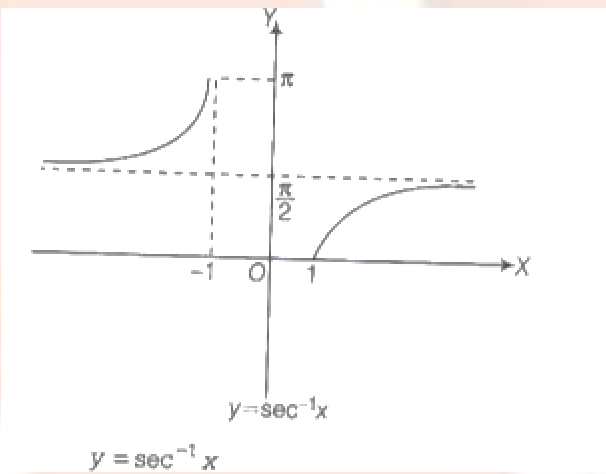
So $f\left(x\right)$ is not one-one.

The co-domain of $f\left(x\right)$ is R and the range is $f\left(x\right)\in\left[0,\infty\right)$.

Since co-domain is not equal to the range of the function.

Thus, the function is not onto.

16. Consider the graph of $\sec^{-1}x$.



The range is $y\in\left[0,\pi\right]-\left\{\frac{\pi}{2}\right\}$.

17. Consider the function.

$$\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$$

Simplify the above equation as follows,

$$\frac{\pi}{2}-\cot^{-1}x+\frac{\pi}{2}-\cot^{-1}y=\frac{4\pi}{5}$$

$$\cot^{-1}x+\cot^{-1}y=\pi-\frac{4\pi}{5}$$

$$\cot^{-1}x+\cot^{-1}y=\frac{\pi}{5}$$

18. Consider the function.

$$f(x)=8x^3$$

And,

$$g(x)=x^{\frac{1}{3}}$$

Now,

$$fog(x) = f\left(g(x)\right)$$
$$= f\left(x^{1/3}\right)$$
$$= 8\left(x^{1/3}\right)^3$$
$$= 8x$$

19. Consider the matrix,

$$A = \frac{1}{\pi}\begin{vmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{vmatrix}$$

Simplify the determinant.

$$A = \frac{1}{\pi}\left\lfloor \sin^{-1}(\pi x)\cot^{-1}(\pi x) - \tan^{-1}\left(\frac{x}{\pi}\right)\sin^{-1}\left(\frac{x}{\pi}\right)\right\rfloor$$

And,

$$B = \begin{vmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{vmatrix}$$

Simplify the determinant.

$$B=\left\lfloor \cos^{-1}(\pi x)\tan^{-1}(\pi x)-\sin^{-1}\left(\frac{x}{\pi}\right)\tan^{-1}\left(\frac{x}{\pi}\right)\right\rfloor$$

The value of $A-B$ is calculated as,

$$A-B=\begin{pmatrix} \dfrac{1}{\pi}\left\lfloor \sin^{-1}(\pi x)\cot^{-1}(\pi x)-\tan^{-1}\left(\dfrac{x}{\pi}\right)\sin^{-1}\left(\dfrac{x}{\pi}\right)\right\rfloor \\ -\left[\cos^{-1}(\pi x)\tan^{-1}(\pi x)-\sin^{-1}\left(\dfrac{x}{\pi}\right)\tan^{-1}\left(\dfrac{x}{\pi}\right)\right] \end{pmatrix}$$

$$=\mathrm{I}$$

20 Let A be a matrix that is both symmetric and skew symmetric.

Therefore,

$A'=A$

And,

$A'=-A$

This case is only possible when A is a zero matrix.

21. Consider the given condition.

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix}+\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

The above matrix is solved as,

$$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

So,

$$\begin{aligned} 2+y&=5 \\ y&=5-2 \\ y&=3 \end{aligned}$$

And,

$$\begin{aligned} 2x+2&=8 \\ 2x&=6 \\ x&=\frac{6}{2} \\ x&=3 \end{aligned}$$

22. The binary operation * is defined as,

$$a*b=\frac{a}{b+1} \qquad \ldots\ldots \text{(I)}$$

So,

$$b*a=\frac{b}{a+1}$$

Thus,

$$a*b \neq b*a$$

Hence, the operation is not commutative.

Now,

$$a*(b*c)=a*\left(\frac{b}{c+1}\right)$$

$$=\frac{a}{\left(\frac{b}{c+1}\right)+1}$$

$$=\frac{a(c+1)}{b+c+1}$$

And,

$$\left(a*b\right)*c=\frac{a}{b+1}*c$$

$$=\frac{\frac{a}{b+1}}{c+1}$$

$$=\frac{a}{\left(b+1\right)\left(c+1\right)}$$

So,

$$a*\left(b*c\right)\neq\left(a*b\right)*c$$

The operation is not associative.

23. The given condition is,

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix}=\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

The value of x is calculated as,

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

$$3 - x^2 = 3 - 8$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

24. For a $n \times n$ order matrix of degree 2,

$$|KA| = K^n |A|$$

Here n is the order of the matrix.

Since the matrix id of the 3×3.

$$|KA| = K^3 |A|$$

25. The vertices of the triangle are $(K,0),(4,0),(0,2)$.

The area of the triangle is $4\,\text{sq units}$.

The area of the triangle is calculated as,

$$A=\frac{1}{2}\left|K\left(0-2\right)+4\left(2-0\right)+0\left(0-0\right)\right|$$

$$4=\frac{1}{2}\left|-2K+8\right|$$

$$\left|-2K+8\right|=8$$

$$-2K+8=\pm 8$$

Solve further,

$$-2K=8-8$$
$$K=0$$

And,

$$-2K=-8-8$$
$$K=\frac{-16}{-2}$$
$$=8$$

26. Consider the matrices.

$$\Delta=\begin{vmatrix} Ax & x^2 & 1 \\ Bx & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$$

And,

$$\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$$

Apply $C_1 \to xC_1$, $C_2 \to yC_2$ and $C_3 \to zC_3$.

$$\Delta_1 = \frac{1}{xyz}\begin{vmatrix} Ax & By & Cz \\ x^2 & y^2 & z^2 \\ xyz & xyz & xyz \end{vmatrix}$$

$$= \frac{xyz}{xyz}\begin{vmatrix} A & B & C \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} A & B & C \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

Since $|A'| = |A|$

So,

$$\Delta_1 = \begin{vmatrix} A & B & C \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} Ax & x^2 & 1 \\ Bx & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$$

$$= \Delta$$

27. Consider the piece wise function.

$$f\left(x\right) = \begin{cases} kx^2 & x \le 2 \\ 3 & x > 2 \end{cases}$$

The function is continuous at $x = 2$. So,

$$\lim_{x \to 2} kx^2 = \lim_{x \to 2} 3$$

$$k\left(2\right)^2 = 3$$

$$4k = 3$$

$$k = \frac{3}{4}$$

28. The mean value theorem is given as,

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

Now,

$$f(x)=x^2$$

$$f'(x)=2x$$

Therefore,

$$2c=\frac{f(4)-f(2)}{4-2}$$
$$2c=\frac{16-4}{2}$$
$$c=\frac{12}{4}$$
$$c=3$$

29. Consider the curve $y^2=x$.

Differentiate with respect to x.

$$2y\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=\frac{1}{2y}$$

The slope of the tangent is $\frac{1}{2y}$.

Also the slope of the tangent is $\tan\frac{\pi}{4}$. So,

$$\frac{1}{2y}=\tan\frac{\pi}{4}$$

$$\frac{1}{2y}=1$$

$$2y=1$$

$$y=\frac{1}{2}$$

Thus,

$$\left(\frac{1}{2}\right)^2=x$$

$$\frac{1}{4}=x$$

$$x=\frac{1}{4}$$

The required point is $\left(\frac{1}{4},\frac{1}{2}\right)$.

30. Consider the function $f\left(x\right)=x^2+2x-5$.

Differentiate with respect to x.

$$\begin{aligned} f'\left(x\right)&=2x+2 \\ &=2\left(x+1\right) \end{aligned}$$

Now,

$$\begin{aligned} f'\left(x\right)&>0 \\ 2\left(x+1\right)&>0 \\ x+1&>0 \\ x&>-1 \end{aligned}$$

The function is increasing in $\left(-1,\infty\right]$.

31. Let the radius of the sphere be a.

The volume of the sphere is,

$V = \frac{4}{3}\pi a^3$

Differentiate with respect to a.

$$\frac{dV}{da} = \frac{4}{3}\pi\left(3a^2\right)$$
$$= 4\pi a^2$$

The surface area of the sphere is,

$$S = 4\pi\left(a^2\right)$$

Differentiate with respect to a.

$$\frac{dS}{da} = 4\pi\left(2a\right)$$
$$= 8\pi a$$

So,

$$\frac{dV}{dS} = \frac{4\pi a^2}{8\pi a}$$
$$= \frac{a}{2}$$
$$= \frac{4}{2}$$
$$= 2\ \text{cm}^3/\text{cm}^2$$

32. Consider the expression.

$$y=\tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$$

Simply the above expression as follows,

$$y=\tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$$
$$=\tan^{-1}\left(\frac{\tan \pi/4+\tan x}{1-\tan \pi/4 \tan x}\right)$$
$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+x\right)\right)$$
$$=\left(\frac{\pi}{4}+x\right)$$

Differentiate with respect to x.

$$\frac{dy}{dx}=1$$

33. Consider the matrix.

$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

Differentiate.

$$\frac{dy}{dx}=\left[\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}+\begin{vmatrix} f(x) & g(x) & h(x) \\ \frac{d}{dx}l & \frac{d}{dx}m & \frac{d}{dx}n \\ a & b & c \end{vmatrix}+\begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ \frac{d}{dx}a & \frac{d}{dx}b & \frac{d}{dx}c \end{vmatrix}\right]$$

$$=\left[\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}+\begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}+\begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}\right]$$

$$=\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

And,

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ a & b & c \\ l & m & n \end{vmatrix} = \begin{vmatrix} f'(x) & l & a \\ g'(x) & m & b \\ h'(x) & n & c \end{vmatrix}$$

34. The given functions are,

$$\sin x = \frac{2t}{1+t^2}$$

So,

$$x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$$

$$= 2\tan^{-1} t \qquad \text{...... (I)}$$

And,

$$\tan y = \frac{2t}{1-t^2}$$

So,

$$y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

$$= 2\tan^{-1} t \qquad \text{..... (II)}$$

From equation (I) and (II),

$$y = x$$

$$\frac{dy}{dx} = 1$$

35. Consider the expression.

$$\cos^{-1}\left(2x^2 - 1\right)$$

Let $u = \cos^{-1}\left(2x^2 - 1\right)$ and $v = \cos^{-1} x$.

Substitute $x = \cos\theta$.

$$\begin{aligned} u &= \cos^{-1}\left(2\cos^2\theta - 1\right) \\ &= \cos^{-1}\left(\cos 2\theta\right) \\ &= 2\theta \\ &= 2\cos^{-1} x \end{aligned}$$

Calculate $\frac{du}{dv}$.

$$\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

$$=\frac{\frac{d}{dx}2\cos^{-1}x}{\frac{d}{dv}\cos^{-1}x}$$

$$=\frac{-\frac{2}{\sqrt{1-x^2}}}{\frac{-1}{\sqrt{1-x^2}}}$$

$$=2$$

36. Consider the equation.

$$y=\log\left(\log x\right)$$

Differentiate with respect to x.

$$\frac{dy}{dx}=\frac{1}{x}\left(\frac{1}{\log x}\right)$$

$$=\left(x\log x\right)^{-1}$$

Differentiate with respect to x.

$$\frac{d^2y}{dx^2} = -(x\log x)^{-1-1}\left[1\log x + x\left(\frac{1}{x}\right)\right]$$

$$= -(x\log x)^{-2}(\log x + 1)$$

$$= \frac{-(\log x + 1)}{(x\log x)^2}$$

37. Consider the integral,

$$I = \int \frac{x+3e^x}{(x+4)^2}dx$$

The above integral is solved as,

$$I = \int \frac{(x+3)e^x}{(x+4)^2}dx$$

$$= \int e^x\left[\frac{x+4-1}{(x+4)^2}\right]dx$$

$$= \int e^x\left[\frac{1}{x+4} - \frac{1}{(x+4)^2}\right]dx$$

$$= \int e^x\left[\frac{1}{x+4}\right]dx + \int e^x\left[-\frac{1}{(x+4)^2}\right]dx$$

Solve further,

$$I=\frac{e^x}{x+4}-\int e^x\left[\frac{-1}{\left(x+4\right)^2}\right]dx+\int e^x\left[-\frac{1}{\left(x+4\right)^2}\right]dx$$

$$=\frac{e^x}{x+4}+C$$

38. Consider the integral.

$$I=\int\frac{\cos 2x-\cos 2\theta}{\cos x-\cos\theta}dx$$

The integral is solved as,

$$I=\int\frac{\left(2\cos^2 x-1\right)-\left(2\cos^2\theta-1\right)}{\cos x-\cos\theta}dx$$

$$=2\int\frac{\left(\cos^2 x\right)-\left(\cos^2\theta\right)}{\cos x-\cos\theta}dx$$

$$=2\int\frac{\left(\cos x-\cos\theta\right)\left(\cos x+\cos\theta\right)}{\cos x-\cos\theta}dx$$

$$=2\int\left(\cos x+\cos\theta\right)dx$$

Solve further,

$$I = 2\left[\sin x + x\cos\theta\right] + C$$

39. Consider the integral,

$$I = \int \sqrt{x^2 + 2x + 5}\,dx$$

The integral is solved as,

$$\begin{aligned}
I &= \int \sqrt{\left(x+1\right)^2 + 4}\,dx \\
&= \int \sqrt{\left(x+1\right)^2 + 2^2}\,dx \\
&= \left(\begin{array}{l} \dfrac{x+1}{2}\sqrt{\left(x+1\right)^2 + 2^2} + \\ \dfrac{\left(2\right)^2}{2}\log\left|x+1+\sqrt{\left(x+1\right)^2 + 2^2}\right| + C \end{array}\right) \\
&= \left(\begin{array}{l} \dfrac{x+1}{2}\sqrt{x^2 + 2x + 5} + \\ 2\log\left|x+1+\sqrt{x^2 + 2x + 5}\right| + C \end{array}\right)
\end{aligned}$$

40. Consider the integral.

$$I=\int_0^{\pi/2}\frac{\tan^7 x}{\cot^7 x+\tan^7 x}dx \qquad \ldots\ldots \text{(I)}$$

Simplify the above equation.

$$I=\int_0^{\pi/2}\frac{\tan^7(\pi/2-x)}{\cot^7(\pi/2-x)+\tan^7(\pi/2-x)}dx$$

$$=\int_0^{\pi/2}\frac{\cot^7(x)}{\tan^7(x)+\cot^7(x)}dx \qquad \ldots\ldots \text{(II)}$$

Add equation (I) and (II).

$$2I=\int_0^{\pi/2}\frac{\tan^7(x)+\cot^7 x}{\cot^7 x+\tan^7 x}dx$$

$$2I=\int_0^{\pi/2}1dx$$

$$2I=\frac{\pi}{2}$$

$$I=\frac{\pi}{4}$$

41. Consider the integral $\int_{-5}^{5}|x+2|\,dx$

Solve the integral as follows,

$$I=-\int_{-5}^{-2}(x+2)\,dx+\int_{-2}^{5}(x+2)\,dx$$

$$=-\left[\frac{x^2}{2}+2x\right]_{-5}^{2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{5}$$

$$=\left[\begin{array}{l}-\left[\left(\frac{4}{2}-4\right)-\left(\frac{25}{2}-10\right)\right]\\+\left[\left(\frac{25}{2}+10\right)-\left(\frac{4}{2}-4\right)\right]\end{array}\right]$$

$$=29$$

42. Consider the integral $\int_{-\pi/2}^{\pi/2}\frac{dx}{\mathrm{e}^{\sin x}+1}$

Let $I=\int_{-\pi/2}^{\pi/2}\frac{dx}{\mathrm{e}^{\sin x}+1}$ …… (I)

Solve the integral as follows,

$$I=\int_{-\pi/2}^{\pi/2}\frac{dx}{\mathrm{e}^{\sin x}+1}$$

$$=\int_{-\pi/2}^{\pi/2}\frac{dx}{\mathrm{e}^{-\sin x}+1}$$

$$=\int_{-\pi/2}^{\pi/2}\frac{e^{\sin x}dx}{\mathrm{e}^{-\sin x}+1} \quad \text{…… (II)}$$

Add equation (I) and (II).

$$2I=\int_{-\pi/2}^{\pi/2} 1dx$$

$$2I=\left[x\right]_{-\pi/2}^{\pi/2}$$

$$2I=\pi$$

$$I=\frac{\pi}{2}$$

43. Consider the integral.

$$I=\int_0^{\pi/2}\frac{1}{a^2\cdot\sin^2 x+b^2\cdot\cos^2 x}dx$$

The above integral is solved as,

$$I=\int_0^{\pi/2}\frac{1}{a^2\cdot\sin^2 x+b^2\cdot\cos^2 x}dx$$

$$I=\int_0^{\pi/2}\frac{\sec^2 x}{a^2\cdot\tan^2 x+b^2}dx$$

Let $\tan x=t$.

$$\sec^2 xdx=dt$$

The limit change from 0 to infinity.

Now,

$$I=\int_0^{\infty}\frac{1}{a^2\cdot t^2+b^2}dt$$

$$=\frac{1}{a^2}\int_0^{\infty}\frac{1}{\cdot t^2+\frac{b^2}{a^2}}dt$$

$$=\frac{1}{a^2}\frac{1}{\frac{b}{a}}\left[\tan^{-1}\frac{t}{\left(\frac{b}{a}\right)}\right]_0^{\infty}$$

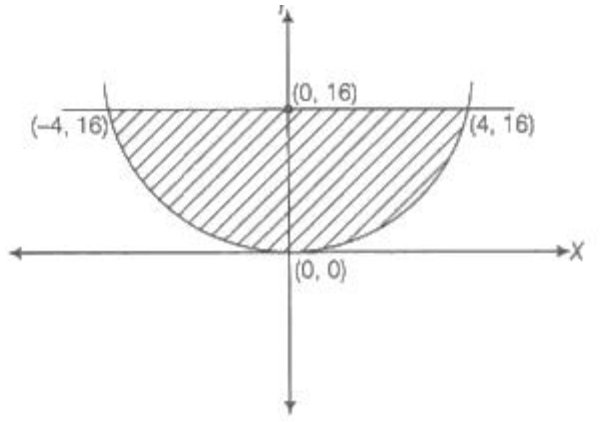
$$=\frac{1}{ab}\left[\tan^{-1}\infty-\tan^{-1}0\right]$$

Solve further,

$$I=\frac{1}{ab}\frac{\pi}{2}$$

$$=\frac{\pi}{2ab}$$

44. Consider the figure showing the area bounded by the curve $y=x^2$ and the line $y=16$.



The required area is calculated as,

$$A = 2\int_0^{16} x dy$$

$$= 2\int_0^{16} y^{\frac{1}{2}} dy$$

$$= 2\left[\frac{y^{3/2}}{3/2}\right]_0^{16}$$

$$= 2\left[\frac{16^{3/2}}{3/2}\right]$$

Solve further,

$$A = \frac{4}{3}(64)$$

$$= \frac{256}{3} \text{ sq units}$$

45. Consider the curve $y = x$.

The required area is calculated as,

$$A = \int_0^{\pi} |\cos x| dx$$
$$= 2\int_0^{\pi/2} \cos x dx$$
$$= 2[\sin x]_0^{\pi/2}$$
$$= 2 \text{ sq units}$$

46. Consider the differential equation.

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2}$$

The power of the term $\frac{d^2y}{dx^2}$ is 1.

Therefore, the degree of the given differential equation is 1.

47. Consider the equation.

$$\frac{dy}{dx}+y=1$$

The equation is solved as follows,

$$\frac{dy}{dx}=1-y$$

$$\frac{dy}{1-y}=dx$$

Integrate both side of the equation.

$$\int\frac{dy}{1-y}=\int dx$$

$$-\log|1-y|=x+\text{C}$$

$$\log\left|\frac{1}{1-y}\right|=x+\text{C}$$

48. Consider the differential equation.

$$x\frac{dy}{dx}+2y=x^2$$

The equation can be rewritten as,

$$\frac{dy}{dx}+\frac{2}{x}y=x$$

The integrating factor is calculated as,

$$\begin{aligned}\mathrm{IF}&=e^{\int\frac{2}{x}dx}\\&=e^{2\log x}\\&=e^{\log x^2}\\&=x^2\end{aligned}$$

49. Consider the vectors.

$\mathbf{a}=2\mathrm{i}+\lambda\mathrm{j}+\mathrm{k}$

And,

$\mathbf{b}=\mathrm{i}+2\mathrm{j}+3\mathrm{k}$

Since,

$\mathbf{a}\cdot\mathbf{b}=0$

Therefore,

$$\left(2i+\lambda j+k\right)\left(i+2j+3k\right)=0$$

$$2+2\lambda+3=0$$

$$2\lambda=-5$$

$$\lambda=\frac{-5}{2}$$

50. The three unit vectors are $\mathbf{a},\mathbf{b},\mathbf{c}$ and $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$.

Calculate the value of $\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}$.

$$\mathbf{a}\cdot\left(\mathbf{a}+\mathbf{b}+\mathbf{c}\right)=0$$

$$\mathbf{a}\cdot\mathbf{a}+\mathbf{b}\cdot\mathbf{a}+\mathbf{c}\cdot\mathbf{a}=0$$

$$\mathbf{b}\cdot\mathbf{a}+\mathbf{c}\cdot\mathbf{a}=-\left|\mathbf{a}\right|^2$$

$$\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}=-1$$

Similarly,

$$\mathbf{b}\cdot\mathbf{a}+\mathbf{b}\cdot\mathbf{c}=\mathbf{-1}$$

And,

$$\mathbf{c}\cdot\mathbf{a}+\mathbf{c}\cdot\mathbf{b}=-1$$

On addition,

$$2(\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a})=-3$$

$$\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}=\frac{-3}{2}$$

51. Consider the unit vectors $\mathbf{a}$, $\mathbf{b}$ and $\sqrt{3}\,\mathbf{a}-\mathbf{b}$

So,

$$\left|\sqrt{3}\mathbf{a}-\mathbf{b}\right|=1$$

$$\left(\sqrt{3}\mathbf{a}-\mathbf{b}\right)\cdot\left(\sqrt{3}\mathbf{a}-\mathbf{b}\right)=1$$

$$3|\mathbf{a}|^2+|\mathbf{b}|^2-2\sqrt{3}\mathbf{a}\cdot\mathbf{b}=1$$

$$3(1)^2+(1)^2-2\sqrt{3}\,|\mathbf{a}||\mathbf{b}|\cos\theta=1$$

Solve further,

$$4-2\sqrt{3}\cos\theta=1$$

$$2\sqrt{3}\cos\theta=3$$

$$\cos\theta=\frac{\sqrt{3}}{2}$$

$$\theta=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Solve further,

$\theta = \frac{\pi}{6}$

$= 30°$

52. The reflection of any point (x, y, z) in XY plane is $(x, y, -z)$.

Therefore, the reflection of (α, β, γ) in XY plane is $(\alpha, \beta, -\gamma)$

53. Consider the plane $2x - 3y + 6z - 11 = 0$.

The angle made with the X axis is $\sin^{-1}(\alpha)$.

The direction ratios of the normal plane is $\langle 2, -3, 5 \rangle$

And, the direction ratios of X axis is $\langle 1, 0, 0 \rangle$.

Therefore,

$$\sin\theta = \frac{2\times1+(-3)\times0+6\times0}{\sqrt{2^2+(-3)^2+(6)^2}\cdot\sqrt{(1)^2+0^2+0^2}}$$

$$\sin\theta = \frac{2}{7}$$

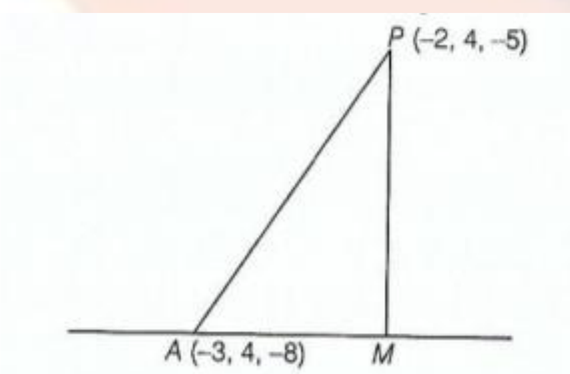
$$\theta = \sin^{-1}\left(\frac{2}{7}\right)$$

So,

$$\sin^{-1}(\alpha) = \sin^{-1}\left(\frac{2}{7}\right)$$

$$\alpha = \frac{2}{7}$$

54. Consider the figure depicting a line that passes through $A(-3,4,-8)$ and is parallel to the vector $\mathbf{b} = 3\mathrm{i}+5\mathrm{j}+6\mathrm{k}$.



Let M be the foot of the perpendicular from $P\left(-2,4,-5\right)$.

Then,

$$\mathbf{AP}=-\mathrm{i}-3\mathrm{k}$$

So,

$$\begin{aligned}\left|\mathbf{AP}\right|&=\sqrt{1+9}\\&=\sqrt{10}\end{aligned}$$

Therefore, AM is equal to the projection of **AP** on **b.**

$$\begin{aligned}AM&=\left|\frac{\mathbf{AP}\cdot\mathbf{b}}{\mathbf{b}}\right|\\&=\left|\frac{\left(-\mathrm{i}-3\mathrm{k}\right)\cdot\left(3\mathrm{i}+5\mathrm{j}+6\mathrm{k}\right)}{\left(3\mathrm{i}+5\mathrm{j}+6\mathrm{k}\right)}\right|\\&=\left|\frac{-3-18}{\sqrt{9+25+36}}\right|\\&=\frac{21}{\sqrt{70}}\end{aligned}$$

Therefore,

$$PM = \sqrt{AP^2 - AM^2}$$

$$= \sqrt{10 - \frac{441}{70}}$$

$$= \sqrt{\frac{259}{70}}$$

$$= \sqrt{\frac{37}{10}}$$

55. The probability of the defective ball is,

$$p = \frac{10}{100}$$

$$= \frac{1}{10}$$

The probability that the ball is not defective is,

$$q = 1 - \frac{1}{10}$$

$$= \frac{9}{10}$$

The probability the out of a sample of 5 pens drawn one by one without replacement and almost one is defective is calculated as,

$$P = {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5 + {}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^5 + 5\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^5 + \left(\frac{1}{2}\right)\left(\frac{9}{10}\right)^4$$

56. Two events are said to be independent if,

$$P(A' \cap B') = P(A') \cdot P(B')$$

Also if A and B are independent then A' and B' are also independent.

Therefore,

$$P(A' \cap B') = P(A') \cdot P(B')$$
$$= (1 - P(A)) \cdot (1 - P(B))$$

If A and B are mutually exclusive then,

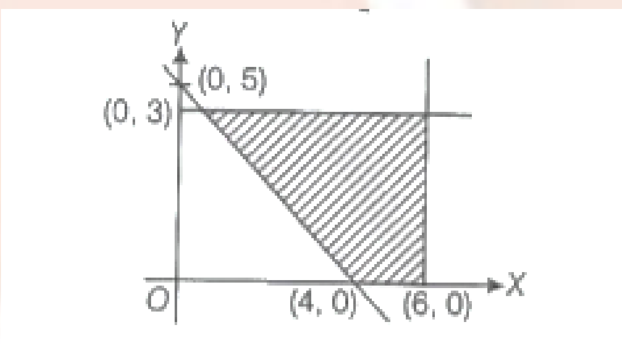
$$P(A \cap B) = 0$$

57. The sum of all probabilities is always equal to 1.

So,

$$0.3 + K + 2K + 2K = 1$$
$$5K = 0.7$$
$$K = \frac{0.7}{5}$$
$$= 0.14$$

58. Consider the figure.



The equation of line passing through point $(0,5)$ and $(4,0)$ is,

$5x + 4y = 20$

The shaded region lies above the line $5x + 4y = 20$. So the inequation is,

$5x + 4y \geq 20$

The shaded region lies below the line $y = 3$. So the inequation is,

$y \leq 3$

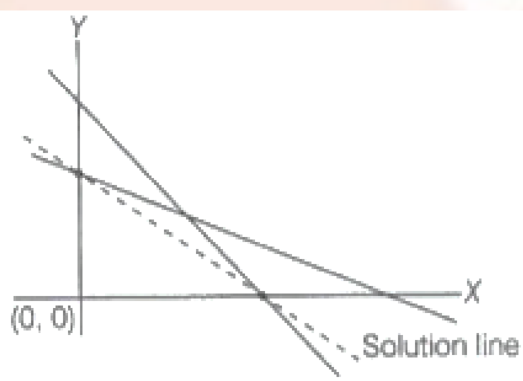
The shaded region lies left the line $x = 6$. So the inequation is,

$x \leq 6$

Also the shaded region lies in the first quadrant.

$x \geq 0$
$y \geq 0$

59. When a LPP admits an optimal solution at two consecutive vertices of a feasible region, then the optimal solution occurs at every point on the line joining these two points.



60. Consider the integral.

$$I=\int_{0.2}^{3.5}[x]dx$$

The above integral is solved as,

$$\begin{aligned}I&=\int_{0.2}^{1}[x]dx+\int_{1}^{2}[x]dx+\int_{2}^{3}[x]dx+\int_{3}^{3.5}[x]dx\\&=\int_{0.2}^{1}0dx+\int_{1}^{2}1dx+\int_{2}^{3}2dx+\int_{3}^{3.5}3dx\\&=0+1[x]_1^2+2[x]_2^3+3[x]_3^{3.5}\\&=(2-1)+2(3-2)+3(3.5-3)\end{aligned}$$

Solve further,

$$\begin{aligned}I&=(2-1)+2(3-2)+3(3.5-3)\\&=1+2+1.5\\&=4.5\end{aligned}$$