**Topics: Normal distribution, Functions of Random Variables**

1. 1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

**Ans**- We have a normal distribution with *μ* = 45 minutes and *σ* = 8 minutes. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X d" 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X d" 50).

Z = (X - *μ*)/ *σ* = (X - 45)/8.0 Thus the question can be answered by using the normal table to find

Pr(X d" 50) = Pr(Z d" (50 - 45)/8.0) = Pr(Z d" 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* = 6. For each statement below, please specify True/False. If false, briefly explain why.

A. More employees at the processing center are older than 44 than between 38 and 44.

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans** - We have a normal distribution with *μ* = 38 and *σ* = 6. Let X be the number of employees. So according to question

a) Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X d" 44).

Z = (X - *μ*)/ *σ* = (X - 38)/6

Thus, the question can be answered by using the normal table to find

Pr(X d" 44) = Pr(Z d" (44 - 38)/6) = Pr(Z d" 1)=84.1345%

Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore the statement that More employees at the processing center are older than 44 than between 38 and 44 is TRUE.

b) Probability of employees less than age of 30 = Pr(X<30).

Z = (X - *μ*)/ *σ* = (30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X d" 30) = Pr(Z d" (30 - 38)/6) = Pr(Z d" -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees).

Therefore, the statement B of the question is also TRUE.

3. If X1 ~ N (? , ? ^2) and X2 ~ N (? , ? ^2) are iid normal random variables, then what is the difference between 2 X1 and X1 + X2? Discuss both their distributions and parameters.

**Ans**- As we know that if X <" N (?1, ? 1^2), and Y <" N(?2, ? 2^2) are two independent random variables then X + Y <" N(?1 + ?2, ? 1^2 +? 2^2), and X " Y <" N(?1 " ?2, ? 1^2 +? 2^2) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z <" N(a?1 + b?2, a^2? 1^2 + b^2? 2^2 ).

Therefore in the question

2X1~ N(2 u,4 ? ^2) and

X1+X2 ~ N(? + ?, ? ^2 + ? ^2 ) ~ N(2 u, 2? ^2 )

2X1-(X1+X2) = N( 4?,6 ? ^2)

4. Let X ~ N(100, 202) , Find two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

**Ans**. Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So, the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z= (X-?) / ?

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \*? + ? = X

Z (-0.005) \*20+100 = -(-2.57) \*20+100 = 151.4

Z (+0.005) \*20+100 = (-2.57) \*20+100 = 48.6

So, option D is correct.