Relational algebra exercises #2

DBM1

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Operator	**Notation**	**Meaning**
Union	$r \cup s$	$\{t \mid t \in r \vee t \in s\}$
Intersection	$r \cap s$	$\{t \mid t \in r \wedge t \in s\}$
Difference	$r \setminus s$	$\{t \mid t \in r \wedge t \notin s\}$
Projection	$\pi_{A_1,\ldots,A_n}(r)$	$\{t\vert_{A_1,\ldots,A_n} \mid t \in r\}$
Selection	$\sigma_C(r)$	$\{t \mid t \in r \wedge C(t)\}$
Renaming	$\rho_\gamma(r)$	$\{t.\gamma^{-1} \mid t \in r\}$
Cartesian product	$r \times s$	$\{t.\gamma_R \cup t'.\gamma_S \mid t \in r \wedge t' \in s\}$
Natural join	$r \bowtie s$	$\{t \cup t' \mid t \in r \wedge t' \in s\}$
Theta join	$r \bowtie_\theta s$	$\{t \mid t \in (r \times s) \wedge \theta(t)\}$
Division	$r \div s$	$\{t \mid \forall t' \in s, t \cup t' \in r\}$

Table 1: Relational algebra cheat sheet

Exercise 1

Consider the following database schema:

- Visits(Drinker, Bar)
- Likes(Drinker, Beer)
- Serves(Bar, Beer)

and the following set of constraints:

- $\pi_{Drinker}(Visits) = \pi_{Drinker}(Likes)$
- $\pi_{Bar}(Serves) = \pi_{Bar}(Visits)$
- $\pi_{Beer}(Likes) = \pi_{Beer}(Serves)$

Formulate the following queries in relational algebra. Give the inline and the tree representations of each resulting expression.

1. The drinkers that visit a bar that serves a beer that they like.
2. All the drinkers with the beers they do not like.
3. The drinkers that like all beers in all bars that they visit.

4. The pairs of beers that are not served in a common bar.
5. The pairs of beers that are served in two different bars.

Exercise 2

Let R and S be two relation schemas defined over the same set of attributes, i.e. $\Omega_R = \Omega_S$. Let $X \subseteq \Omega_R$, $A \in \Omega_R \setminus X, c \in dom(A)$ and F a formula over R.

Give 2 boundaries to each of the following expressions.

1. $|\sigma_F(R)|$
2. $|R \cup S|$
3. $|R \setminus S|$
4. $|\pi_A(R)|$
5. $|R \bowtie S|$

Exercise 3

Let R_1, R_2, S be three relation schemas with $\Omega_{R_1} = \Omega_{R_2} = \{A, B, C\}$ and $\Omega_S = \{C\}$, and $F = (B = C)$ a formula.

For each of the following relation algebra equivalences, give a proof it holds.

1. $\pi_A(R_1 \cup R_2) = \pi_A(R_1) \cup \pi_A(R_2)$
2. $\sigma_F(R_1 \cup R_2) = \sigma_F(R_1) \cup \sigma_F(R_2)$
3. $\sigma_F(R_1 \setminus R_2) = \sigma_F(R_1) \setminus \sigma_F(R_2)$
4. $\sigma_F(R_2 \bowtie S) = \sigma_F(R_2) \bowtie \rho_{C \to B}(S)$
   For each of the following relation algebra equivalences, give a a counter-example showing it does not hold.
5. $\pi_A(R_1 \setminus R_2) = \pi_A(R_1) \setminus \pi_A(R_2)$
6. $\pi_A(R_1 \cap R_2) = \pi_A(R_1) \cap \pi_A(R_2)$
7. $(R_1 \div S) \times S = R_1$
8. $(R_1 \cup R_2) \div S = (R_1 \div S) \cup (R_2 \div S)$