Q2 B]

Deriving procedure to perform Lukas Kanade

We know $u(x,y) = a_1 x + b_1 y + c_1$

$v(x,y) = a_2 x + b_2 y + c_2$ ... i.e motion is affine

We assume the brightness constancy as well,

$$I(x+u, y+v+t+1) = I(x,y,t)$$

where $I(x,y,t)$ is intensity of pixel at time t

We approximate L.h.s of brightness constancy eqn using 1^{st} order Taylor series expansion.

$$I(x+u, y+v, t+1) \approx I(x,y,t) + I_x u + I_y v + I_t = 0$$

Substituting the affine motion model,

$$I_x * (a_1 x + b_1 y + c_1) + I_y * (a_2 x + b_2 y + c_2) + I_t = 0$$

To estimate the parameters $\{a_1, a_2, b_1, b_2, c_1, c_2$ we use the least squares minimization approach.

$$\therefore E = \sum_{x,y=0,0}^{x,y=m,n} \left[I_x(a_1 x + b_1 y + c_1) + I_y(a_2 x + b_2 y + c_2) + I_t \right]^2$$

To minimize this function E we take

$$\frac{\partial E}{\partial a_1}, \frac{\partial E}{\partial b_1}, \frac{\partial E}{\partial c_1}, \frac{\partial E}{d_1}, \frac{dE}{b_2}, \frac{dE}{c_2} = 0$$

This results in system of 6 linear eqn with 6 unknowns.

In matrix form,

$$A * p = b$$

$A \Rightarrow 6 \times 6$

$p \Rightarrow 6 \times 1$ parameters

$b \Rightarrow$ spatial/temporal gradients

Then, get p if A is invertible and $\therefore$ motion parameters

Upate the motion vectors by the parameters

$$u(x,y) = a_1 x + b_1 y + c_1$$

$$v(x,y) = a_2 x + b_2 y + c_2$$

Iterate this process again and again until a convergence criteria is met.