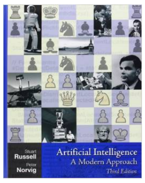
Deadline: 2015-09-21 23:59
Recitation (øvingsforelesning): 2015-09-10 18:15, S6

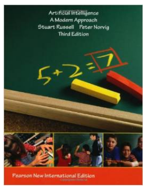
Propositional and Predicate Logics

Purpose: Gain experience with propositional and prepositional (first-order) logic by solving many small problems.

Note: The textbook, *Artificial Intelligence: A Modern Approach, 3rd edition*, exists in two versions:



"Blue version"



"Green version"

All references to chapters and figures in the textbook are given as 1.2.3 / 2.3.4, for the blue and the green version respectively.

For each question with more than 3 parts (e.g. question 4 in section 1), **you only need to deliver 3 of the parts** (any 3 parts). This also applies to exercises from the textbook. As proper preparation for the exam, we strongly advise that you eventually do all the parts. The questions in this homework are very typical of those that appear on exams.

1 Models and Entailment in Propositional Logic

1. Exercise 7.1 / 6.1 — "Suppose the agent has…"

   Build the complete model table and show both entailments using model checking.

2. Exercise 7.4 / 6.4, parts (c)–(l) — "Which of the following are correct?"

   Build the complete model tables to show whether the sentences are true or not (remember that you only need to do three of these parts – see the box near the top of the page!).

3. Consider a logical knowledge base with 100 variables, $A_1, A_2, \ldots, A_{100}$. This will have $Q = 2^{100}$ possible models. For each logical sentence below, give the number of models that satisfy it.

   Feel free to express your answer as a fraction of Q (without writing out the whole number $1267650600228229401496703205376 = 2^{100}$) or to use other symbols to represent large numbers.

   Example: The logical sentence A_1 will be satisfied by $\frac{1}{2}Q = \frac{1}{2}2^{100} = 2^{99}$ models.

   (a) $A_{24} \wedge A_{50}$

   (b) $\neg A_{38} \vee \neg A_{47}$

   (c) $(A_1 \vee \neg A_{57}) \wedge (A_1 \vee \neg A_{99})$

   (d) $A_{58} \Leftrightarrow A_{90}$

   (e) $A_1 \wedge \neg A_2 \wedge A_3 \wedge \neg A_4 \wedge A_5 \wedge \neg A_6$

(f) $(\neg A_1 \wedge \neg A_2 \wedge \ldots \wedge \neg A_{50}) \vee (A_{51} \wedge A_{52} \wedge \ldots \wedge A_{100})$

(g) $(A_1 \wedge A_2 \wedge A_3) \vee (A_4 \wedge A_5 \wedge A_6)$

2 Resolution in Propositional Logic

1. Convert each of the following sentences to Conjunctive Normal Form (CNF).
   (a) $A \wedge \neg B \wedge C$
   (b) $\neg A \vee \neg B \vee \neg C$
   (c) $(\neg A \wedge B) \Rightarrow C$
   (d) $(A \vee B) \Rightarrow \neg C$
   (e) $(A \wedge B) \Leftrightarrow (C \wedge D)$
2. Consider the following Knowledge Base (KB):
   - $(B \vee \neg C) \Rightarrow \neg A$
   - $(\neg A \vee D) \Rightarrow B$
   - $A \wedge C$

   Use resolution to show that $KB \models \neg D$.
3. Exercise 7.17 / 6.18 — "Consider the following sentence…"

3 Representations in First-Order Logic

1. Exercise 8.31 / 8.28 — "Consider a first-order logical knowledge base…"
2. Exercise 8.21 / 8.20 — "Arithmetic assertions can be written…"
3. Exercise 8.23 / 8.22 — "Write in first-order logic the assertion…"

   Feel free to use several logical sentences to express this one natural-language statement.
4. Translate into first-order logic the sentence "Everyone's DNA is unique and is derived from their parents' DNA." You must specify the precise intended meaning of your vocabulary terms. (*Hint:* Do not use the predicate $Unique(x)$, since uniqueness is not really a propery of an object in itself!)

4 Resolution in First-Order Logic

1. Find the unifier (θ) – if possible – for each pair of atomic sentences. $Ocean$, $Surrounds$ and $Includes$ are predicates, while $CapitalOf$ is a function that maps a country to the name of its capital.
   (a) $Ocean(x)$ … $Ocean(Atlantic)$ Answer: $\theta = \{x/Atlantic\}$
   (b) $Surrounds(Pacific, Hawaii)$ … $Surrounds(x, Hawaii)$
   (c) $Surrounds(x, Iceland)$ … $Surrounds(Atlantic, y)$
   (d) $Surrounds(x, Iceland)$ … $Surrounds(Atlantic, x)$

(e) $Includes(x, y)$ ... $Includes(Iceland, Reykjavik)$

(f) $CapitalOf(x)$ ... $CapitalOf(Iceland)$

(g) $Includes(Iceland, y)$ ... $Includes(x, CapitalOf(x))$

(h) $Surrounds(Atlantic, x)$ ... $Includes(y, Iceland)$

Do **one of the next two** resolution proofs as part of your assignment. Save the other as preparation for the exam.

2. Use resolution to prove Green(Linn) given the information below. You must first convert each sentence into CNF. Feel free to show only the applications of the resolution rule that lead to the desired conclusion. For each application of the resolution rule, show the unification bindings, θ.

   - Hybrid(Prius)
   - Drives(Linn, Prius)
   - ∀x: Green(x) ↔ Bikes(x) ∨ [∃y: Drives(x, y) ∧ Hybrid(y)]

3. Follow the same procedure as above to prove that the Tigers can win a championship: CWC (Tigers), when given the following facts:

   - ∀x: [PC(x) ∨ [∃y: Player(y, x) ∧ Hero(y)]] → CWC(x)
     If a team is a previous champion (PC) or has a player on it that is a hero, then they Can Win a Championship (CWC).
   - ∀x: Hero(x) ↔ [∃y, z: Player(x, y) ∧ PC(z) ∧ Beat(y, z)]
     A player is a hero if and only if he has played on a team that has beaten a previous champion.
   - PC(Bears)
     The Bears are previous champions.
   - Player(Jack, Tigers)
     Jack plays for the Tigers.
   - Beat(Tigers, Bears)
     The Tigers have beaten the Bears.