

# Fractions Intro

## PhET Sim Design Document

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### Public URL:

[https://docs.google.com/document/d/1Q77\\_mqmngoJASfTpggtKS2xBEkRcdRMTSMUraNanuxg/edit?hl=en\\_US](https://docs.google.com/document/d/1Q77_mqmngoJASfTpggtKS2xBEkRcdRMTSMUraNanuxg/edit?hl=en_US)

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## Audience

This simulation is targeted at helping middle school and younger age students who are working on understanding fractions and equivalence between fractions, but mainly 5th grade and higher. This sim should follow naturally into “Fraction Operations”

## Learning Goals

The basic learning goals include building a flexible conceptual framework of fractions through the use of “manipulatives” and extending this understanding to arithmetic operations with fractions. Specifically:

- Recognize a fraction in multiple representations (pie pieces, rectangular blocks)
- Understand a fraction “ $a/b$ ” as “ $a$  parts of size  $1/b$ ”
- Recognize equivalent fractions, especially mixed number fractions, such as  $5/4 = 1\frac{1}{4}$

## Standards

- **Math TEKS 4.2** Use concrete objects and pictorial models to generate equivalent fractions. Model fraction quantities greater than one using concrete objects and pictorial models. Compare and order fractions using concrete objects and pictorial models
- **CCSS 4.NF** Extend understanding of fraction equivalence and ordering. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

## Mockup

### Intro Tab

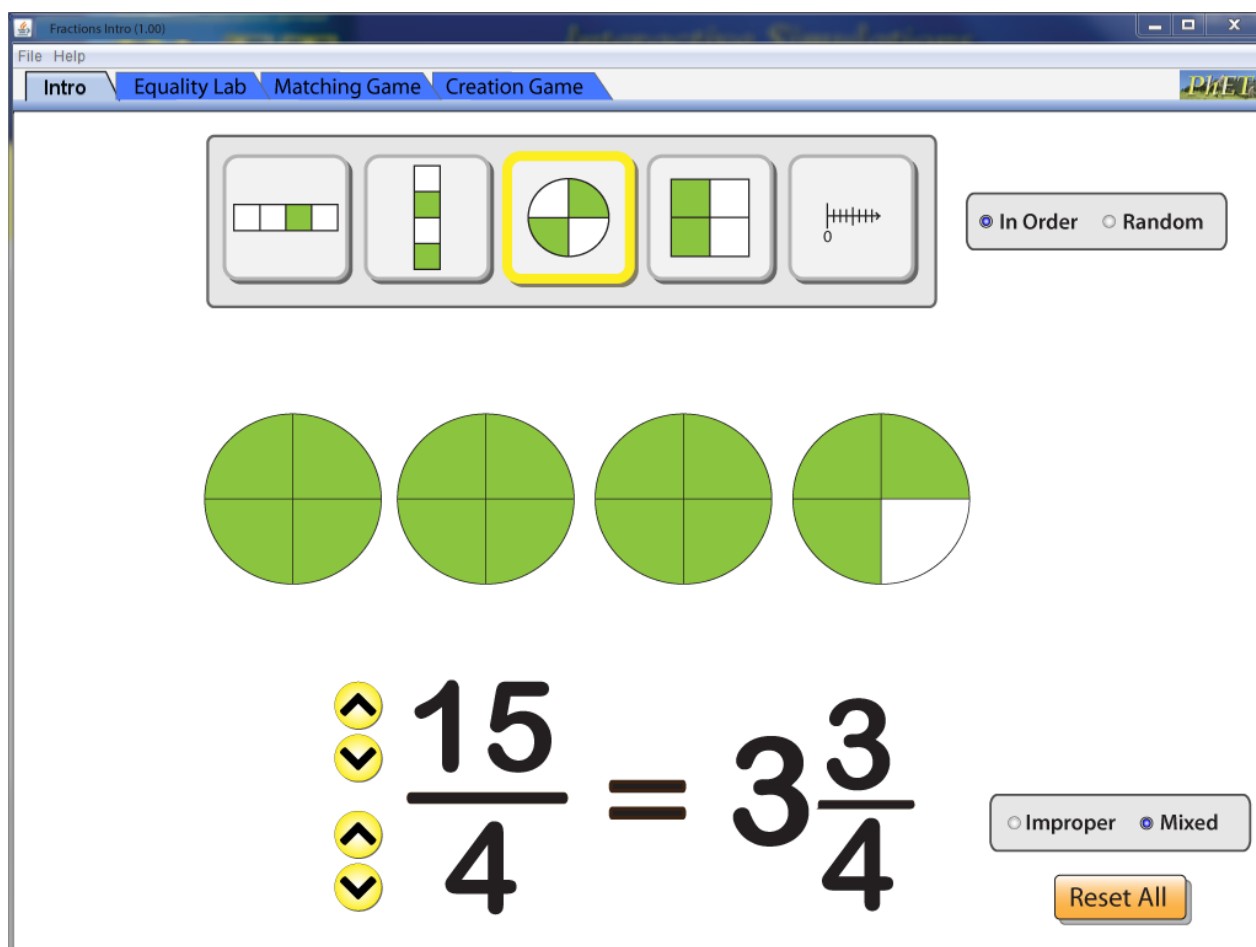
The intro tab will have many representations (virtual manipulatives) from which to choose including:

- 2d abstract representations such as horizontal rectangles, vertical rectangles, divided circles, divided squares, and a number line.
- 2d “real world” representations, such as slices of pizza and glasses of water
- Possibly 3d representations such as cake

The representation and the fraction will always be linked, and the numerator and denominator can be changed by the use of large spinners. The representations will not scale in size as more are added, so only 6 “wholes” will fit in the play area. So the numerator and denominator will have to be simultaneously constrained so that no more

than 6 wholes appear on the screen. The denominator's largest possible value will be “8”.

In addition, there will be the option to display the fraction as a mixed number and a reduced fraction as well as the option to randomly fill the virtual manipulatives, by means of taking virtual manipulative pieces from a bucket. It will be interesting to see if this ability to toggle has analogies to the ideas of “physically distributed learning” that have been observed by researchers.



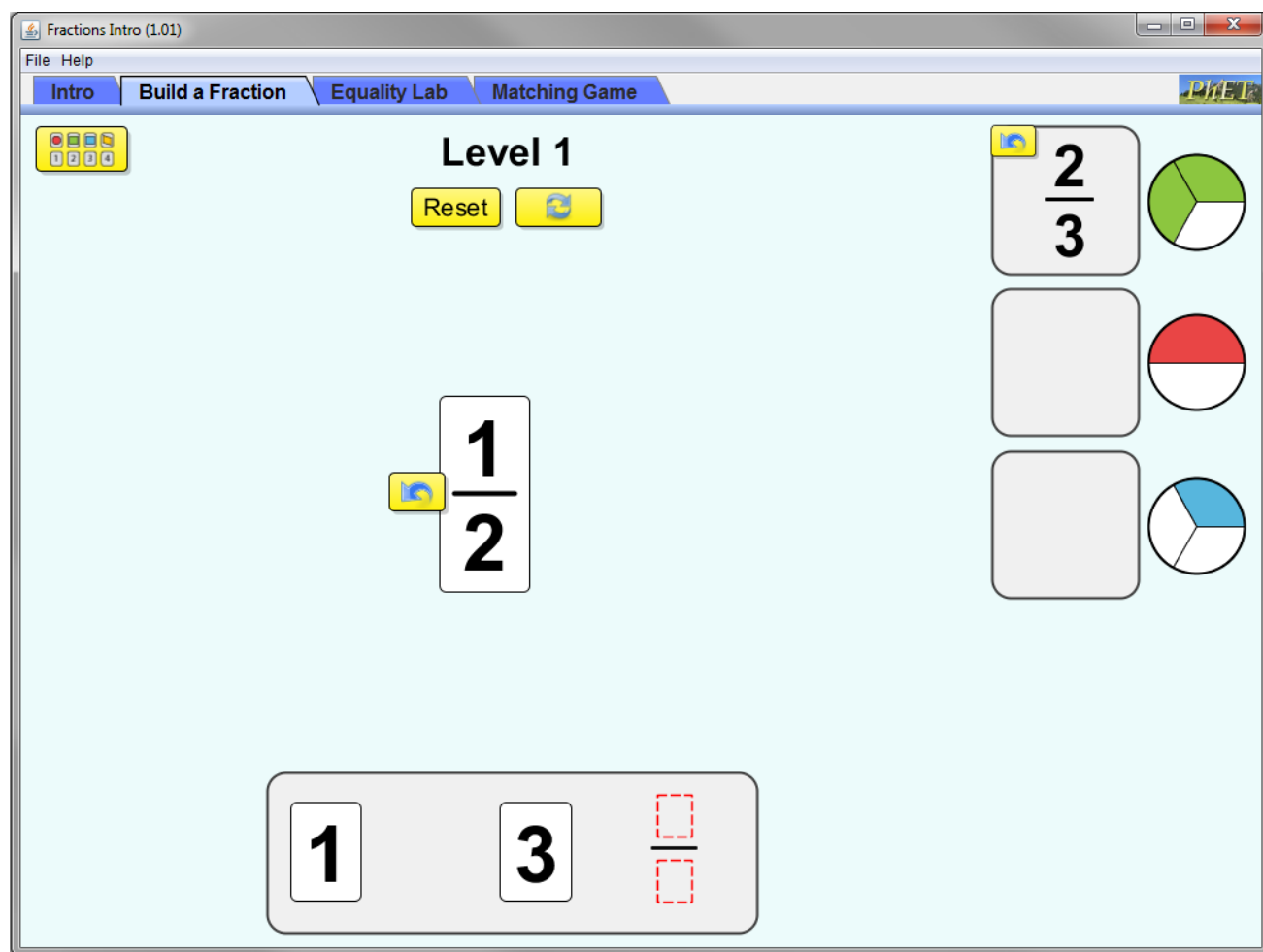
## Build a Fraction Tab

The Build a Fraction tab will also be made into a “standalone” sim called “Build a Fraction”, but this sim will include a tab that covers mixed numbers.

Two styles of challenges will be present in the build a fraction game tab. This tab will be an “implicit game” in the same spirit as “build a molecule”, so a goal is implied but there is no explicit scoring.

In one style of challenge the pictorial representations will be shown and the symbolic fraction will need to be created, in the second style, a symbolic fraction will be shown and an appropriate pictorial representation will need to be created from a pile of virtual manipulative pieces.

### “Numbers” Challenges



#### Level 1:

- fractions are  $\{1/2, 1/3, 2/3\}$
- if refresh button is pressed, colors and numbers are shuffled
- always circles
- just enough cards to complete targets

#### Level 2:

- Distribution of fractions ranging from  $1/2$  to  $4/5$ . As in the numerator could be 1, 2, 3, or 4 and the denominator could be 2, 3, 4, or 5 with the stipulation that the fraction is always less than 1.
- circles or rectangles, but all targets one shape
- just enough cards to complete targets

#### Level 3:

- All targets "six flowers"
- Range  $1/6$  to  $5/6$
- cards available to allow multiple solutions. For instance,  $2/6$ , could be represented as  $1/3$

#### Level 4:

- All triangles seems good,
- numerator and denominator able to range from 1-9
- just enough cards to complete targets

#### Level 5:

- numerator able to range from 1-9, and denominator able to range from 1-9, with the number less than 1
- all representations possible (circle, "9 and 4 square", bars, triangles, 6 flower, perhaps regular polygons), I
- all cards available to fulfill challenges in the most straightforward way, for instance a  $4/5$  representation has a 4 and a 5 available.
- just enough cards to complete targets

#### Level 6:

- 4 targets from this level forward
- Same as level 5, but now random fill is possible
- card constraints at this point, so at least one of the representations only has cards available to match it with a "nonobvious fraction". For instance if  $3/9$  appears, and  $5/9$  appears, we have 1(5) and 1(9), but not 2(9), so that

1/3 would need to be used to match.

#### Level 7:

- Top two representations are equivalent, and bottom 2 representations are equivalent but still numbers less than 1
- A built in check to draw a different fraction for the top 2 and the bottom 2
- Possible fractions sets from which to draw 2 each  $\{1/2, 2/4, 3/6\}$  ,  $\{1/3, 2/6, 3/9\}$ ,  $\{2/3, 4/6, 3/9\}$ ,  $\{1/4, 2/8\}$ ,  $\{3/4, 6/8\}$
- The representations are both be equal, for instance, 2 pies divided the same, and two bars divided the same, so that the learning goal is focused on the same exact picture can be represented by 2 different fractions. Always displaying the simplified fraction as the picture.
- Cards constrained, so for instance if  $\{1/2, 3/6\}$  is drawn for the top pair and  $\{3/4, 6/8\}$  drawn for the bottom, we would have 1(1), 1(2), 2(3), 1(4), 2(6), 1(8)

#### Level 8:

- Introduce double representations at this level (numbers greater than 1)
- 8 cards, 4 each of 2 numbers
- randomly choose from  $\{2/3, 3/2, 2/2, 3/3\}$ ,  $\{2/4, 4/2, 2/2, 4/4\}$ ,  $\{3/4, 4/3, 3/3, 4/4\}$ ,  $\{3/5, 5/3, 3/3, 5/5\}$ ,  $\{3/6, 6/3, 3/3, 6/6\}$

#### Level 9:

- Representations both less than 1 and greater than 1
- All representations possible
- No card constraints (as in straightforward matching of number and picture possible)

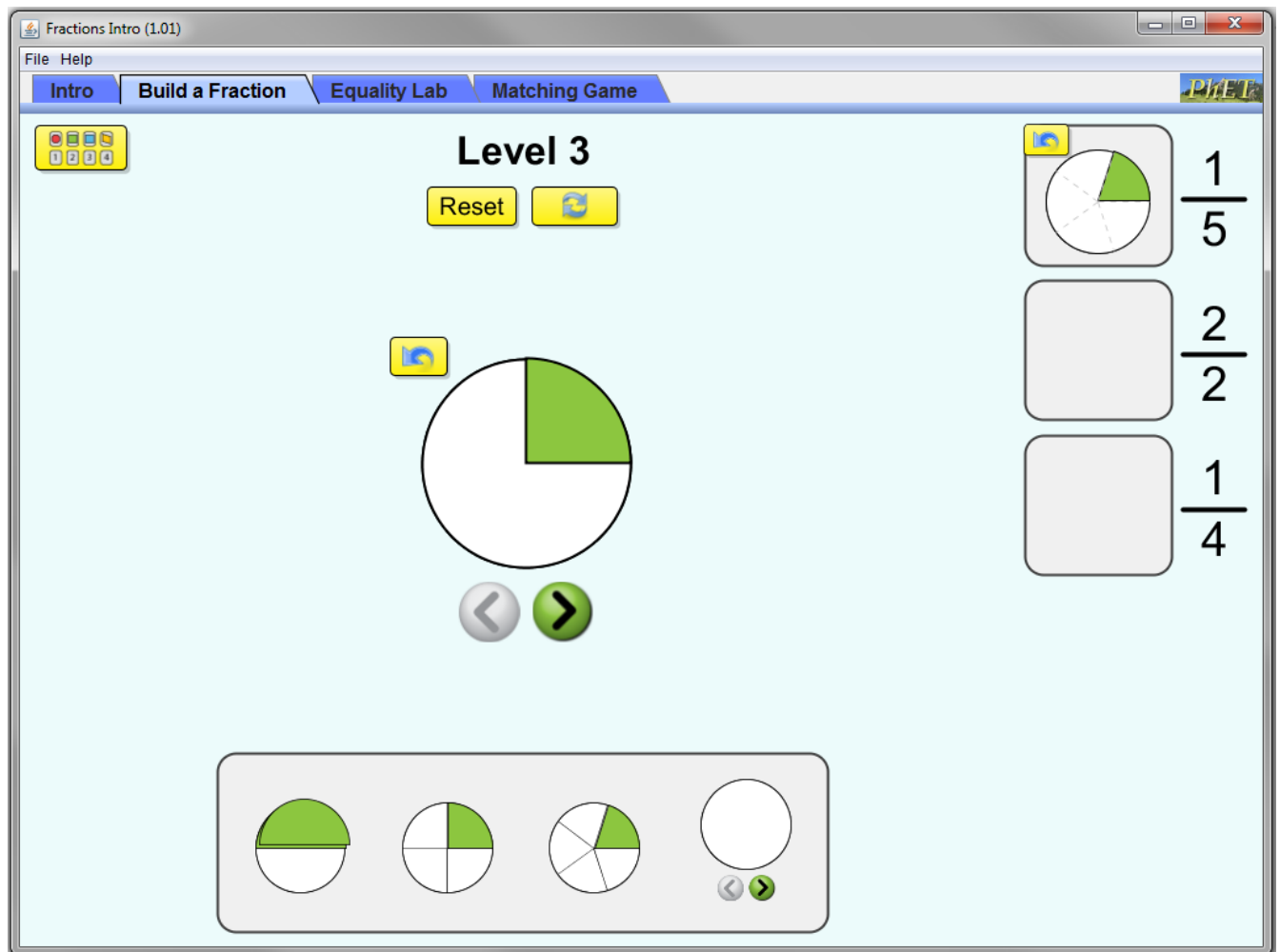
#### Level 10:

- Same as level 9 but with card constraints
- One or two representations use a prime number scale factor for each to generate the cards, for instance if one of the representations was 4/3, we use the scale factor (3/3), and we would need a 12 and a 9 card.

#### Summary Table

Level	Number of Targets	Target Shape	Simplification required?	Target Fill	Range (N) of Target	Range of Target Denominator (D)
1	3	circles	no	ordered	$0 < N < 1$	$D \in \{2, 3\}$
2	3	circles or rectangles	no	ordered	$0 < N < 1$	$D \in \{2, 3, 4, 5\}$
3	3	“six flowers”	yes	ordered	$0 < N < 1$	$D \in \{2, 3, 6\}$
4	3	triangles	no	ordered	$0 < N \leq 1$	$D \in \{2, 3, 4, 5, 6, 7, 8, 9\}$
5	3	varies	no	ordered	$0 < N \leq 1$	$D \in \{2, 3, 4, 5, 6, 7, 8, 9\}$
6	4	varies	yes	ordered	$0 < N \leq 1$	$D \in \{2, 3, 4, 5, 6, 7, 8, 9\}$
7	4	varies	yes	ordered	$0 < N < 1$	$D \in \{2, 3, 4, 6, 8, 9\}$
8	4	varies	yes	ordered	$0 < N < 2$	$D \in \{2, 3, 4, 5, 6\}$
9	4	varies	yes	random	$0 < N < 2$	$D \in \{2, 3, 4, 5, 6, 7, 8, 9\}$
10	4	varies	yes	random	$0 < N < 2$	varies

“Shapes” Challenge



Level 1:

- Two "draws", one target should be from the set  $\{1/1, 2/2, 3/3\}$  and the second draw for the next two targets from the set  $\{1/2, 1/3, 2/3\}$

Level 2:

- Choosing from a distribution of fractions ranging from  $1/2$  to  $4/5$ . The numerator can be 1, 2, 3, or 4 and the denominator could be 2, 3, 4, or 5 with the stipulation that the fraction is always less than 1.
- No "wholes" in the shapes piles.
- 2 possible ways to make at least one of the targets



### Level 3:

- Like level 2, but now fractions ranging from  $1/1$  to  $6/6$ , and with "whole" pieces available.
- Number of pieces of each fraction allowing for multiple solutions

### Level 4:

- All 3 targets the same, 2 possible target values  $\{1/2, 1/1\}$ .
- No "whole" pieces available
- constrain one of the targets so that two different sizes must be used.

### Level 5:

- numerator able to range from 1-8, and denominator able to range from 1-8, with the number less than or equal to 1
- all pieces available to fulfill targets in the most straightforward way (so for instance if  $3/8$  appears there will be  $3 \frac{1}{8}$  pieces)

### Level 6:

- switch to 4 targets for this level
- all targets are made from only 2 stacks of pieces
- So for instance we give a stack of thirds and a stack of halves, and  $\{2/3, 2/4, 5/6, 1/1\}$  are the target fractions, but we constrain the pieces so that some fractions must be made in "interesting" ways.  $2/3$  could just be made with 2 third pieces, but  $5/6$  would need to be made of a  $1/2$  and a  $1/3$ .
- It seems the sets that would work well for pieces would be,  $\{1/2, 1/3\}$ ,  $\{1/2, 1/4\}$ ,  $\{1/3, 1/4\}$ ,  $\{1/2, 1/6\}$ ,  $\{1/3, 1/6\}$ ,  $\{1/4, 1/8\}$ ,  $\{1/2, 1/8\}$
- the constraint should be such that only enough pieces exist to complete the targets

### Level 7:

- Top two targets, and bottom 2 targets are equivalent but still numbers less than 1
- A built in check to draw a different fraction for the top 2 and the bottom 2
- Possible fractions sets from which to draw 2 each  $\{1/2, 1/3, 2/3, 1/4, 3/4, 5/6, 3/8, 5/8\}$
- Shape pieces constrained so that for instance if  $1/2$  and  $1/2$  appears for the top targets, a  $1/2$  piece might be available but the other one

will need to be made with a  $\frac{1}{4}$  and  $\frac{1}{4}$ , or a  $\frac{1}{3}$  and a  $\frac{1}{6}$  or such.

Level 8 :

- Introduce numbers larger than 1 at this level
- On this level at least 2 numbers larger than 1 as targets
- Enough pieces available to match targets in "obvious ways"...so if  $\frac{5}{4}$  is a target a whole piece is available and a  $\frac{1}{4}$  piece available
- for numbers larger than 1, uses only  $\frac{1}{2}$ 's or  $\frac{1}{4}$ 's on this level

Level 9

--Same as level 8 but now some targets only allow "non-obvious" matches with pieces. For instance, if the target is greater than one, no "wholes" should be available. So if  $\frac{5}{4}$  is a target it would need to be built from something like 2 half pieces and a quarter piece

Level 10

- Same as level 7 but now all targets are greater than one.
- Still top two targets same, and bottom two targets the same
- No whole pieces available, and targets must be built in interesting ways. We could say something like the target must be built from 3 or more pieces as a way to constrain the pieces given. So for instance something like  $\frac{4}{3}$  would have to be built by something like  $1(\text{half}) + 2(\text{quarters}) + (\frac{1}{3})$

Summary Table

## Build a Fraction Sim (with mixed numbers)

This standalone sim will have the same interface as the Build a Fraction tab, but will include a second tab that covers mixed numbers.

On the mixed numbers interface: for the shapes challenges, targets will be numbers less than one or mixed numbers, for the numbers challenges, the user will be constrained to only put together cards to make a mixed number. For instance, they will not be able to put cards together in the fractional portion of the mixed number to form an improper fraction. So if for instance, a 5 has been placed in the numerator, only a card value greater than 5

will be allowed in the denominator, any other value will fly back to the floating panel much the way atoms in BAM are repelled when not placed in the correct order for bonding.

The levels for the mixed numbers challenges are described below.

Mixed numbers are written as  $\{x:x/x\}$ , so  $\{1:2/3\}$  should be read as “one and two thirds”

### “Numbers” Challenges

#### Level 1:

- Circles as targets
- $\{1:1/2, 2:1/2, 3:1/4\}$  as the challenges
- just enough cards to complete targets
- As before, refreshing will randomly reorder, recolor

#### Level 2:

- Circles or Rectangles as targets, but all targets are the same shape
- 1, 2, or 3, as whole number
- Fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4\}$

#### Level 3:

- All targets shaped like “six flowers”
- 1, 2, or 3, as whole number
- Fractional portion from the set  $\{1/2, 1/3, 2/3, 1/6, 5/6\}$
- So, if a “six flower” is showing  $3/6$ , we will want a 1 and 2 card in the deck

#### Level 4:

- All triangles
- 1, 2, or 3, as whole number
- Fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4, 1/9, 2/9, 4/9, 5/9, 7/9, 8/9\}$

#### Level 5:

- All representations possible, but each target is only one type of representation
- 1, 2, or 3, as whole number
- Fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, 5/6, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7, 1/8, 3/8, 5/8, 7/8, 1/9, 2/9, 4/9, 5/9, 7/9, 8/9\}$
- 2 of the representations match cards exactly, 1 of the representations requires simplifying to a solution

#### Level 6:

- Same as level 5 (now with 4 targets)
- Random fill now possible, so for instance  $\{2:1/4\}$  could be represented by 2 full circles with a

partially filled circle in between them. As in, we do not need to strictly fill from left to right.

-- 2 of the representations require simplifying

Level 7:

--Top two representations are equivalent in magnitude, and bottom 2 representations are equivalent in magnitude

-- For instance if the top two representations are  $\{1:1/2\}$ , the first representation could be a full circle and a half circle divided in halves, and the second circle could be a full circle and a half circle divide in fourths.

Level 8:

--Same as level 6

--All 4 representations require simplifying

Level 9:

-- All representations, random fill, and simplifying possible

-- Now representations within the targets can have different divisions, do this for 2 of the targets

--So, for instance if  $\{1:3/4\}$  is being represented by circles, the first circle could be divided in  $1/4$ 's and the second circle divided in  $1/8$ 's, with pieces randomly distributed between the two circles.

Level 10:

--Same as level 9, but now all 4 targets can have different internal divisions in representations.

### "Shapes" Challenges

Level 1:

--  $\{1:1/2, 2:1/2, 2:1/4\}$  as the targets

-- Wholes,  $1/2$ 's, and  $1/4$ 's to complete targets

-- as before refreshing will randomly reorder targets, and choose between circles/rectangles

-- a few extra pieces to allow multiple pathways to a solution (for instance, 2 halves that could form a whole)

Level 2:

-- Targets with 1 or 2 as whole number, fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4\}$

-- Wholes,  $1/2$ 's,  $1/3$ 's, and  $1/4$ 's

-- a few extra pieces to allow multiple pathways to a solution

Level 3:

-- All targets 1, 2, or 3, as whole number, fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4, 1/6, 5/6\}$

-- a few extra pieces to allow multiple pathways to a solution

Level 4:

- All targets the same
- 1, 2, or 3, as whole number, fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4\}$
- Pieces constrained so only enough pieces to complete targets.
- Force some wholes to be built from fractional portions. So if all targets were  $\{1:1/2\}$ , only 1 or 2 whole pieces would be available

#### Level 5:

- Targets with 1, 2, or 3, as whole number, fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, 5/6, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7, 1/8, 3/8, 5/8, 7/8\}$
- A few more cards than needed, but at least one target must be constructed with "nontrivial" pieces. For instance  $\{1:1/3\}$  only have two  $1/6$  pieces available for building

#### Level 6:

- Targets with 1, 2, or 3, as whole number, fractional portion from the set  $\{1/2, 1/3, 2/3, 1/6, 5/6\}$  or  $\{1/2, 1/4, 3/4, 1/8, 3/8, 5/8, 7/8\}$
- Pieces will be wholes, and either  $\{1/2$ 's and  $1/6$ 's $\}$  or  $\{1/2$ 's and  $1/8$ 's $\}$
- Only enough pieces to fulfill targets. Pieces chosen to minimize small pieces, so for instance if  $5/8$  is a fractional portion it will be built with a  $1/2$  and a  $1/8$  piece.

#### Level 7:

- Top two targets are the same, bottom two targets are the same
- Targets with 1, 2, or 3, as whole number, fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4, 1/6, 5/6, 1/8, 3/8, 5/8, 7/8\}$
- Only enough pieces to fulfill targets. One of each of the top and bottom targets require "nontrivial" pieces to build the solution.

#### Level 8:

- Targets with 1, 2, or 3, as whole number, fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, 5/6\}$
- Only enough pieces to fulfill targets
- At least 2 targets require "nontrivial" pieces

#### Level 9:

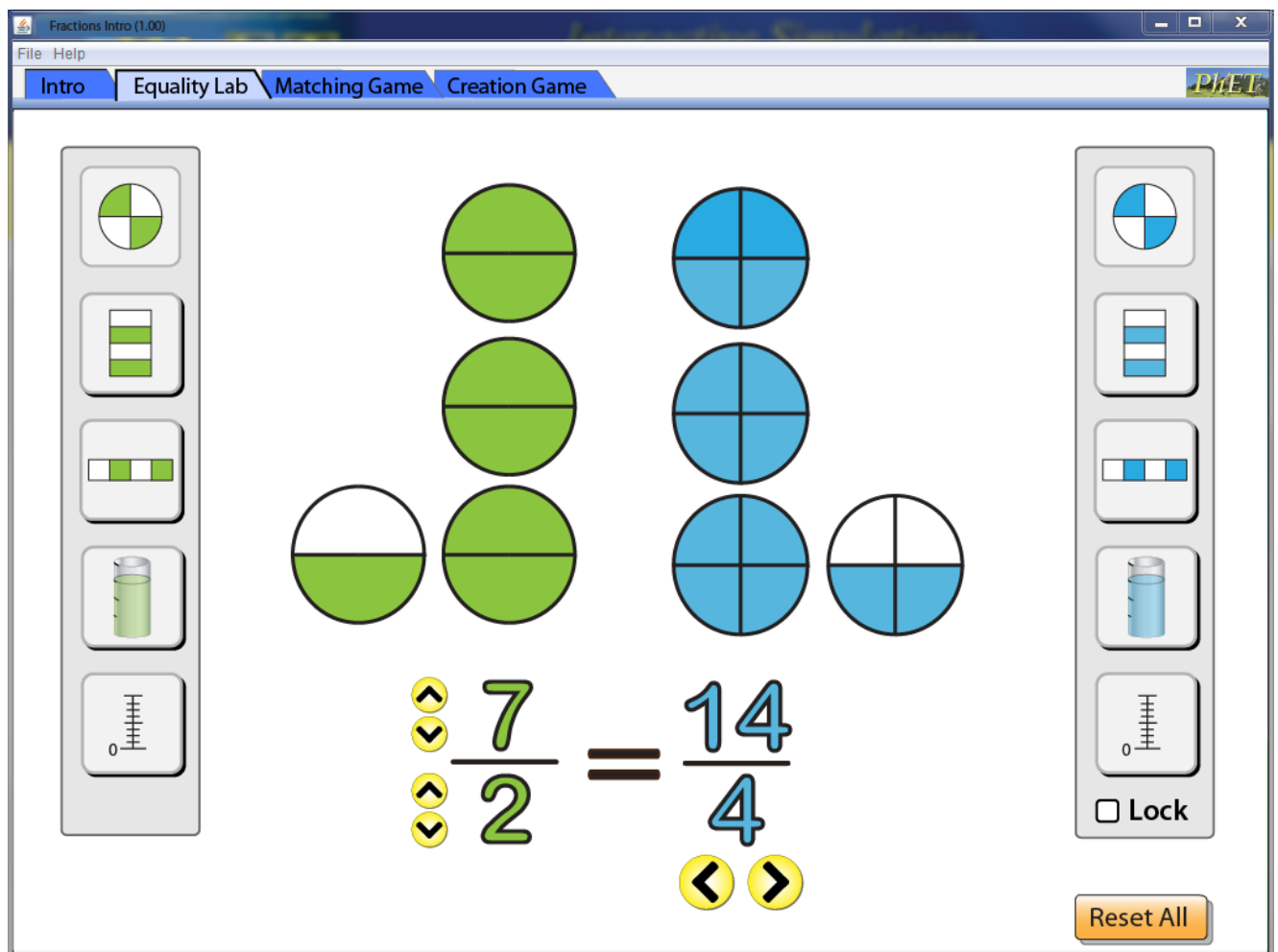
- Same as level 8, but now all 4 targets must be built with some "nontrivial pieces"

#### Level 10:

- Same as level 9, but fractional portion from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4, 1/6, 5/6, 1/8, 3/8, 5/8, 7/8\}$

## Equality Lab Tab

The main learning goal of this tab is to understand the equality of simplifiable fractions (such as  $\frac{1}{2}$  and  $\frac{2}{4}$ ). The sim will open with a default of each numerical representation being  $\frac{1}{1}$  and as the “main” fraction is changed the equivalent fraction will change accordingly. If the spinners on the equivalent fraction are changed the main fraction will remain the same but the equivalent fraction will move up or down to the next equivalent fraction. Initially the sim will open with both the kit on the left and the right “locked” so that the same representation appears on each side. The student will also be able to change the rightmost representation to a number line.



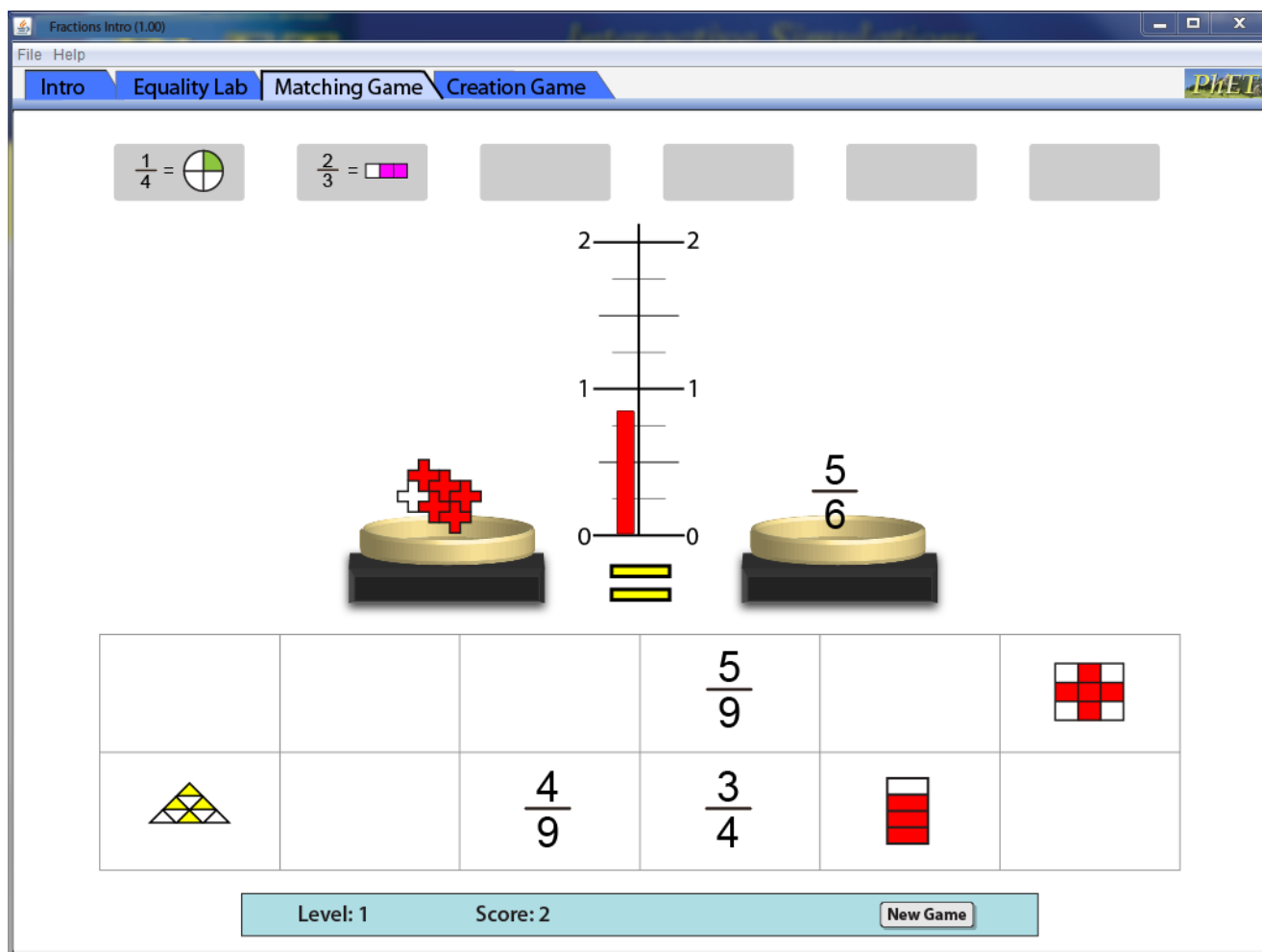
Although the vertical alignment of this tab is different from the horizontal layout of the intro tab, the developer (SR) and AP both felt the vertical alignment allowed for a much cleaner design, and would still be easy for students to figure out. Interviews and feedback will help to resolve this question.

## Match Game Tab

The match game will also be a “standalone sim” called “Fraction Matcher”. The standalone sim will include a tab for “mixed numbers”, but the match game tab will only include improper fractions when the number is greater than one.

The match game will include many abstract shapes and levels. The levels are described below. Each fraction and representation can be placed on the measuring devices, if the fraction and representation match a point will be scored, and the fraction and matching representation will “fly” to the upper left, and remain displayed as a matched pair. In the game students will have the ability to match numbers to representations, numbers to numbers, and representations to representations.

Scoring will follow our most “common” paradigm, with 2 points for a correct answer on the first try, and 1 point for a correct answer on the second try. After the second try, if the answer is still not correct, the student will be awarded no points, and a button will appear that when pressed will display the correct answer. After an answer has been displayed the student will be able to continue trying to make matches.



### Matching Game (first tab of Fraction Matcher)

#### Level 1:

- Only “exact” matches will be present. So for instance if there is a  $\frac{3}{6}$  and a pie with 6 divisions and 3 shaded slices, there will not be a  $\frac{1}{2}$  present. In other words, the numerical representation on this level will exactly match the virtual manipulative.
- Only numbers/representations  $\leq 1$  possible
- Only circles and rectangles will be used on this level
- Fractions from the set  $\{\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, 1/1\}$

#### Level 2:

- Unsimplified fractions possible on this level. So, for instance  $\frac{3}{6}$  and  $\frac{1}{2}$  could both



be present. Or a virtual representation of  $\frac{3}{6}$  could have the numerical of  $\frac{1}{2}$  be its only possible match

- Still only numbers/representations  $\leq 1$  possible
- More variety of shapes introduced (regular polygons, stacked triangles, interestingly divided rectangular shapes, etc.)
- Denominators  $\{2,3,4,6\}$  used

Level 3:

- Numbers/representations greater than “1” introduced, with a max value of “2”
- Numbers larger than one left in “improper” form.
- Variety of shapes again increased
- Denominator can now range up to “8”
- Unsimplified fractions present

Level 4:

- Denominator can now range to “9”

Level 5:

- Scale factors for unsimplified fraction are  $\frac{2}{2}$  and  $\frac{3}{3}$ , so denominator can range up to 27

Level 6:

- Scale factors for unsimplified fractions are  $\frac{4}{4}$  and  $\frac{5}{5}$ , so denominator can range up to 45
- For numbers great than one, the virtual representations can be filled in “random order”, in earlier levels the first whole is always completely filled.

Level 7:

- Scale factors for unsimplified fractions are  $\frac{6}{6}$  and  $\frac{7}{7}$ , so denominator can range up to 63
- Random fill possible

Level 8:

- Scale factors for unsimplified fractions are  $\frac{2}{2}$ ,  $\frac{3}{3}$ ,  $\frac{4}{4}$ ,  $\frac{5}{5}$ ,  $\frac{6}{6}$  and  $\frac{7}{7}$ ,  $\frac{8}{8}$ ,  $\frac{9}{9}$  so denominator can range up to 81
- Random fill possible

## **Fraction Matcher Mixed Numbers Tab**

Level 1:

- Only “exact” matches will be present. So for instance if there is a  $\frac{3}{6}$  and a pie with 6 divisions and 3 shaded slices, there will not be a  $\frac{1}{2}$  present . In other words, the

numerical representation on this level will exactly match the virtual manipulative.

- Only numbers/representations  $\leq 1$  possible
- Only circles and rectangles will be used on this level
- Fractions from the set  $\{1/2, 1/3, 2/3, 1/4, 3/4, 1/1\}$  or a mixed number with these fraction such a “1 and  $1/2$ ”
- Numerator in mixed number fraction always “1”

Level 2:

- Unsimplified fractions possible on this level. So, for instance  $3/6$  and  $1/2$  could both be present. Or a virtual representation of  $3/6$  could have the numerical of  $1/2$  be its only possible match
- More variety of shapes introduced (regular polygons, stacked triangles, interestingly divided rectangular shapes, etc.)
- Denominators  $\{2,3,4,5, 6\}$  used
- Numerator in mixed number fraction always “1”

Level 3:

- Variety of shapes again increased
- Denominator can now range up to “8”
- Unsimplified fractions present (but only for numbers less than “1”)
- Numerator in mixed number can range up to 8

Level 4:

- Denominator can now range to “9”

Level 5:

- Scale factors for unsimplified fraction are  $2/2$  and  $3/3$ , so denominator can range up to 27

Level 6:

- Scale factors for unsimplified fractions are  $4/4$  and  $5/5$ , so denominator can range up to 45
- The virtual representations can be filled in “random order”, in earlier levels the pieces are “sequentially” filled into the representation

Level 7:

- Scale factors for unsimplified fractions are  $3/3$ ,  $6/6$ , and  $7/7$ , so denominator can range up to 63
- Random fill possible

Level 8:

- Scale factors for unsimplified fractions are  $3/3, 4/4, 5/5, 6/6$  and  $7/7, 8/8, 9/9$  so denominator can range up to 81

Random fill possible

## Discussion

Several sims will cover fraction topics: Fractions Intro, Fraction Relationships (comparing fractions), Fraction Operations, and Fair fractions

### Fractions Intro:

4 tabs: Intro, Build a Fraction, Equality Lab, Match Game,

### Fraction Relationships

This sim will cover the relationship of fractions to decimals and percents, and possibly comparing sizes of different fractions. Ideally this sim can be leveraged off work done on Fractions Intro, but development will wait until Fractions Intro has been refined through the interview process

### Fraction Operations:

Development will wait until refinement of Fractions Intro through interview process

## Online Resources

### **Conceptua Math (decent manipulatives examples)**

<http://www.conceptuamath.com/fractions.html>

(username: Ariel Paul, password: phet1977)

### **Melvin's Make a Match (game for equivalent fractions/multiple representations)**

<http://pbskids.org/cyberchase/games/equivalentfractions/>

### **Slice and Clone Perceptual Learning Tools**

[http://illusory.psych.ucla.edu/PLM/articles/SnC\\_PLM\\_desc\\_results.pdf](http://illusory.psych.ucla.edu/PLM/articles/SnC_PLM_desc_results.pdf)

### **The National Library of Virtual Manipulatives**

<http://nlvm.usu.edu/>

## **National Council Of Teachers of Mathematics**

<http://illuminations.nctm.org/Activities.aspx?grade=all&srchstr=fraction>

## **Interactivate**

<http://www.shodor.org/interactivate/activities/>

## **Thinklets**

<http://www.fi.uu.nl/thinklets/>

## **References**

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# Interview and Classroom Feedback

Several short interviews have been done on the Intro tab as of Jan 2012:

These were short (5 minute) interviews to test interface usage of the *Intro* tab. Students used the up/down controls for numbers without issue, as well as the Reduced and Mixed check boxes.

They selected the cake first, then used other objects. These interviews demonstrated that the controls were intuitive for middle school students, and that students were comfortable switching between objects and relating them to symbolic representations.

## Notes:

### Physics meeting 9-13-2012

- Change free play to “fraction lab” and make cards only go to 8
- People happy with graphical changes to BAMF

### Physics meeting 9-6-2012

- Distinguishing the containers for BAMF people seem happy
- “Do not enter” cue for mixed fractions
- Free play tab, cards 1-9, fraction and mixed fraction skeleton (unlimited), 2 cards in each pile, but “infinite” stack, shape stacks ( $1 - \frac{1}{8}$ )

### Physics meeting 8-30-2012

- Keep multiple containers possible for BAMG
- Find ways to shrink representations to fit better
- Try to have representation variety for shapes targets

### Physics meeting 8-16-2012

- Take out Geometry and Area on sim meta data
- Level 4 matching game, at least one pair of shape and number, and number and shape

unreduced. As in if  $\frac{2}{3}$  the number,  $\frac{4}{6}$  should be the shape and vice versa

- Make better icon for home button
- Trish will write tips after AP gives her level descriptions for BAF

### **Physics meeting 7-19-2012**

Move Smiley face in Fraction matcher to be more in the negative space

Keep in mind for interviews if students find the second level selection screen

Change container division buttons to green

People happy “reset/refresh” under level label

Try no flashing, see if interviews reveal major student difficult. Could always cue in other ways, such as a shell that looks like a filled fraction.

### **Physics meeting 6-27-2012**

KP suggests Fraction Matcher 2 tabs one with mixed numbers one without

KP agrees that dynamic number line not so useful for the Fraction matcher, but good in equality lab and might be a generally useful tool in future math sims.

Suggestion for sound option for Build a fraction. Sound is default off (mainly Trish's suggestion that sound can be distracting)

For mixed fractions fraction matcher tab, level icons will show pictures greater than 1

Up/down buttons on container in Build a fraction, change to left/right which are side by side underneath container

For mixed numbers on Build a fraction do numbers less than or equal to 3

Just add circle representations to Build a fraction

For Build a Fraction “shapes” mode we will have the toggling of divisions in containers as an option that is default off.

## **General Updates up to 6-26-2012**

Fraction Matcher is now a break out sim with 8 levels that includes mixed numbers in levels 7 and 8.

Build a Fraction tab will also be a break out sim, but is still under development, and the break out sim will have 2 tabs, the second tab including mixed numbers.

For Build A Fraction, in the “shapes” mode, when pieces are placed in a container, they will sequentially fill. We realized it would be problematic to have pieces of different fractional sizes to be randomly placed in an open container.

KP requested a “dynamic” number line for the Equality Lab tab, SR has implemented this feature, but we will likely keep a static number line in the Match game.

The match game now has a levels interface as the start screen, and we are trying to make the sim as tablet friendly as possible for possible porting to the iPad

## **Physics Meeting 4-12-2012**

KP requested a more traditionally scored game for the “Match Game”

For the game:

- have to have item on both test trays before hitting "check answer"
- put them on, then push "check answer". Then shows the “<,=, or >” and number line
- +2 if correct first try, then try again option, +1 if correct 2nd try, then show correct answer, the correct match flies to be where the last item manipulated was, and the incorrect item flies back to the match possibilities
- got it right => "next" button appears (saves saves match), plays a ding sound, shows a smile face and a +2. Show running score on the left hand side.
- got it wrong => "try again" (moves last one back to initial cells)

AP: how many levels to go?

maybe go to level 6, then repeat 6 indefinitely

-at higher levels, make tougher improper fractions, for instance have half a pie matching “27/54”...go up to 64 for the denominator

- ability to click an item to have it fly to the first open platform. clicking when in the pan would send it back, dragging the items will still work

For equality lab tab,

-keep the representations locked with the option of comparing to a number line. For right hand side, have "same as left" and representation vs "number line"

## **Physics Meeting 2-16-2012**

### **Review of Schwartz Feedback:**

- 1) Icon on bucket --- shows the piece within a circle
- 2) Issue with bucket filling with stuff - after you change denominator  
Always pieces in the buckets ...
- 3) Putting a "1/3" next to the icon.
- 4) Is there a way to do drag and drop with the numberline?  
Keep number line as is
- 5) Start simulation off with just one OR start with numerator/denominator locked  
OR another tab  
Number of objects ...

## **Physics Meeting 2-2-2012**

### **Review of Schwartz Feedback:**

- 1) Better connect numerator and denominator to their meaning
- 2) Issue of  $\frac{1}{2}$  of everything vs  $\frac{6}{2}$ .
- 3) Removing Reduced and Mixed --- might be too much for this tab
- 4) Equity Tab --- Issues with scale (mixing magnitude and fraction)  
Numberline equality tester
- 5) Should we include inequality in this first sim
- 6) A "fair sharing" tab --- How do I divide a group.  
Are there sims out there
- 7) Was initially confused that the fraction wasn't "operating" on the cake.
- 8) More context

### **Ideas:**

Bucket full of pieces == a your piece mode where image on bucket changes size.  
Image of "Whole" --- the thing without any slices, totally full.  
Make images at the top is solids --- Number line == 0 to 1  
"# of Pieces" == Numerator --- into to bucket or out of bucket  
Grey out the arrows during the time of animation  
No reduced and mixed  
Keep pieces going to slots closest to where they were



Animate the numerator --- into and out of bucket

**Later:**

Animate denominator ...

Animate change in size

Animate repositioning when going from  $\frac{3}{3}$  to  $\frac{3}{2}$

**Schwartz Group meeting 2-1-2012**

**Fraction:**

What is  $\frac{1}{3}$  of 6?

Fair sharing, and partitive model?

How long kids will explore?

More context --- more real world connections, kind of boring, math for the sake of math, felt it was passive, may or maybe not confused by ghost pies, confusing of how you should read it.

DS: Balance scale --- is a no-no;  $\frac{3}{6}$  or  $\frac{1}{2}$ . Cause confusions about absolute scale versus relative... Conflation of magnitude sense of fraction. What is the property being balanced ... if it is the ratio, then the total number of equivalents.

What the whole is, is taken for granted here?

$\frac{8}{7}$  to  $\frac{8}{6}$  end up with more cake...

These things are reflecting each other ...

We aren't having them do operations of the cake? That the cake is a fixed quantity ... we made Fraction represents the cake.

What kids are suppose to learn ...

-  $\frac{2}{5}$  looks like this.

- more pieces for more numerator

- get more places when they do the denominator

We want them to get this qualitative relationship

Need to separate out --- idea of "less cake" and "more partitions".

Like the number line ... all kind of the same thing?

Physical manipulatives?

Right now is through teaching is partitioning.

3 models:

Partitive Whole

Numberline

Fair share / Set quantity --- Grouping model. All these little tiles does this

mean.

$$26/8=13/4:$$

Reduced on

Grouping

Scattered beans ... clicke on 8 beans ... click on 4, and they get colored.

Same thing as  $\frac{1}{2}$ . Same thing as 2 beans where 1 is colored.

Give me  $\frac{1}{4}$  of 8 ... make it into 4 separate groups and pick one for them. 4 represents number of groups I have, and answer is number of items in a group.

Both magnitude and fraction --- this isn't working well.

Have a set model ... have kids define what a whole is....

What is the whole?

Grouping --- you want them to conserve the cake.

Fractions --- they do it as part of the "unit" ... they never see a fraction as

Ariel --- could highlight in red, the  $\frac{13}{4}$ .

Magnitude perspective --- does equality lab flow nicely from this? (This is what we do).

Probably want to introduce greater than, less than. --- would stick it in here.

Knowing whether  $\frac{4}{7}$  and

Or forget reduced and mixed, and put in equivalence and bigger/smaller.

Moving towards reduced.

Making links between the numbers and the pieces ... can get lost when playing it around.

Perhaps making animated response --- to draw attention to the cause of the division.

Paper folding

Cake/Pizza

Rods ???

Measurement --- cooking?

Carpentry ---- building?, pipes?,

Money --- usually used for decimals?

Make wrong fraction something bad happens, right fraction something good happens.

Build up cake....

What the unit is?

Somebody didn't get their fair share?

Aswaldo --- car game learning about  $\frac{1}{4}$  of 6 .... how to compute this?

Ideas for tips

Mostly what each level challenge provides.