Normalized Wave Functions

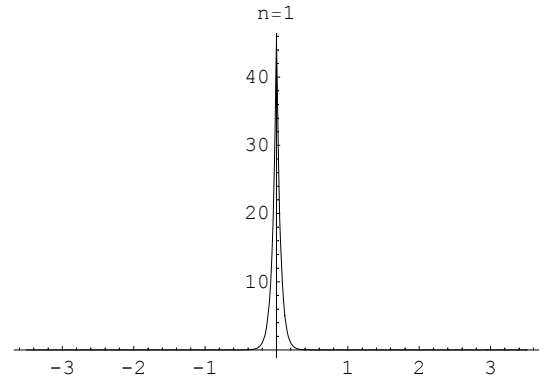
$$\psi[n_, l_, x_] := \frac{1}{\sqrt{4\,\pi}}\,\text{Abs}[x]^{l}\,\text{Exp}[-\text{Abs}[x]/(n\,a)]\sum_{j=0}^{n-l-1} b[j, n, l]\,\text{Abs}[x]^{j}$$

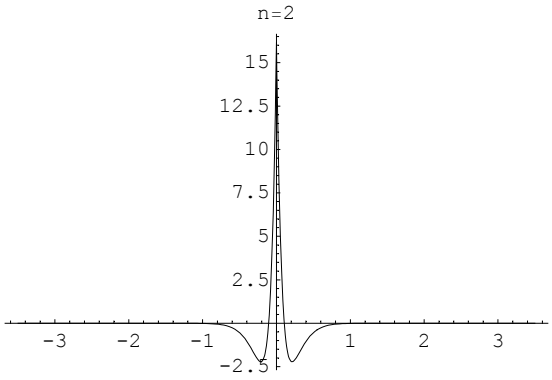
$$b[j_, n_, l_] := \text{If}\left[j == 0,\ 2\,(n\,a)^{-3/2},\ \frac{2}{n\,a}\,\frac{j+l-n}{j\,(j+2\,l+1)}\,b[j-1, n, l]\right]$$

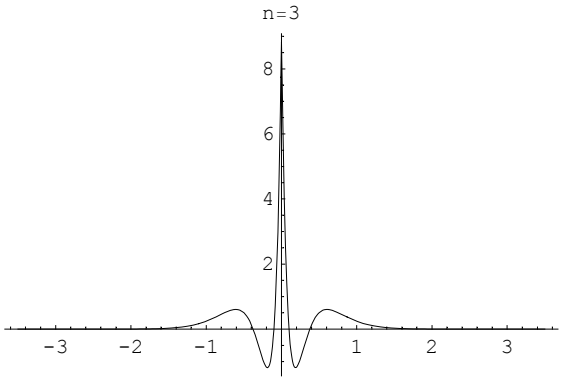
In[36]:=
```
hbar = 0.658;
ke2 = 1.44;
m = 5.68;
a = hbar^2 / (m ke2); E1 = -m ke2^2 / (2 hbar^2)
```

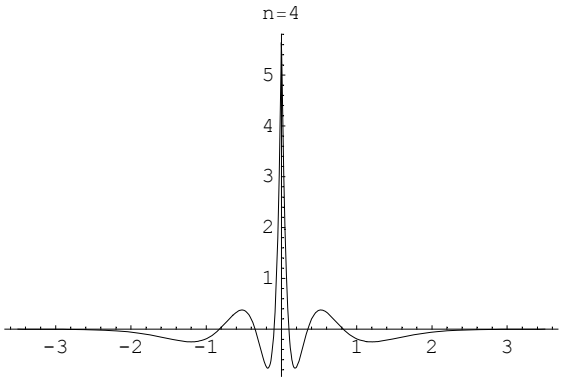
Out[39]= -13.6016

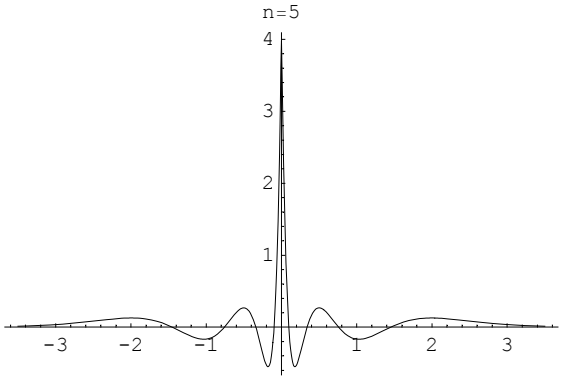
In[121]:=
```
Table[Plot[Evaluate[ψ[n, 0, x]], {x, -3.5, 3.5},
   PlotRange → All, PlotLabel → StringJoin["n=", ToString[n]]], {n, 10}];
```

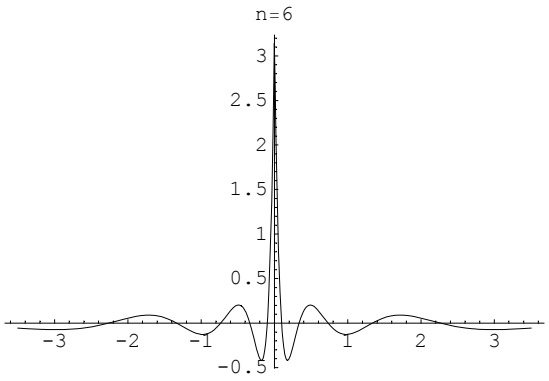


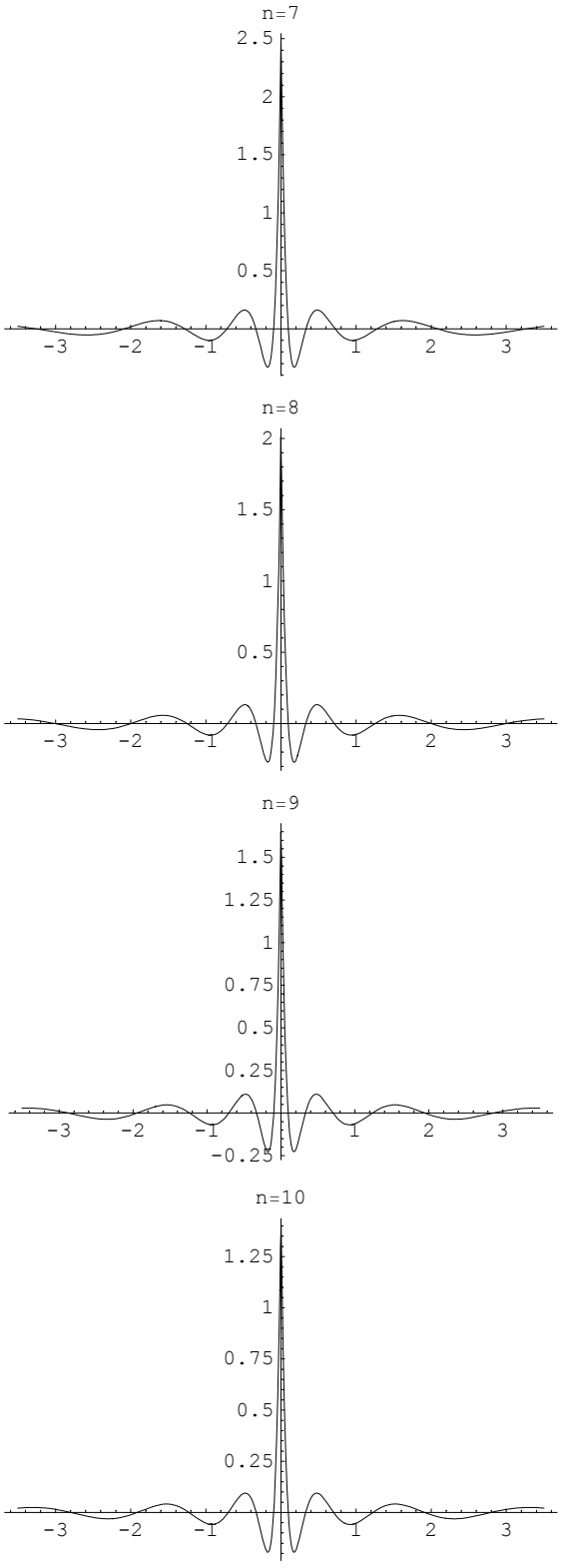



n=3
8
6
4
2
-3
-2
-1
1
2
3


n=4
5
4
3
2
1
-3
-2
-1
1
2
3

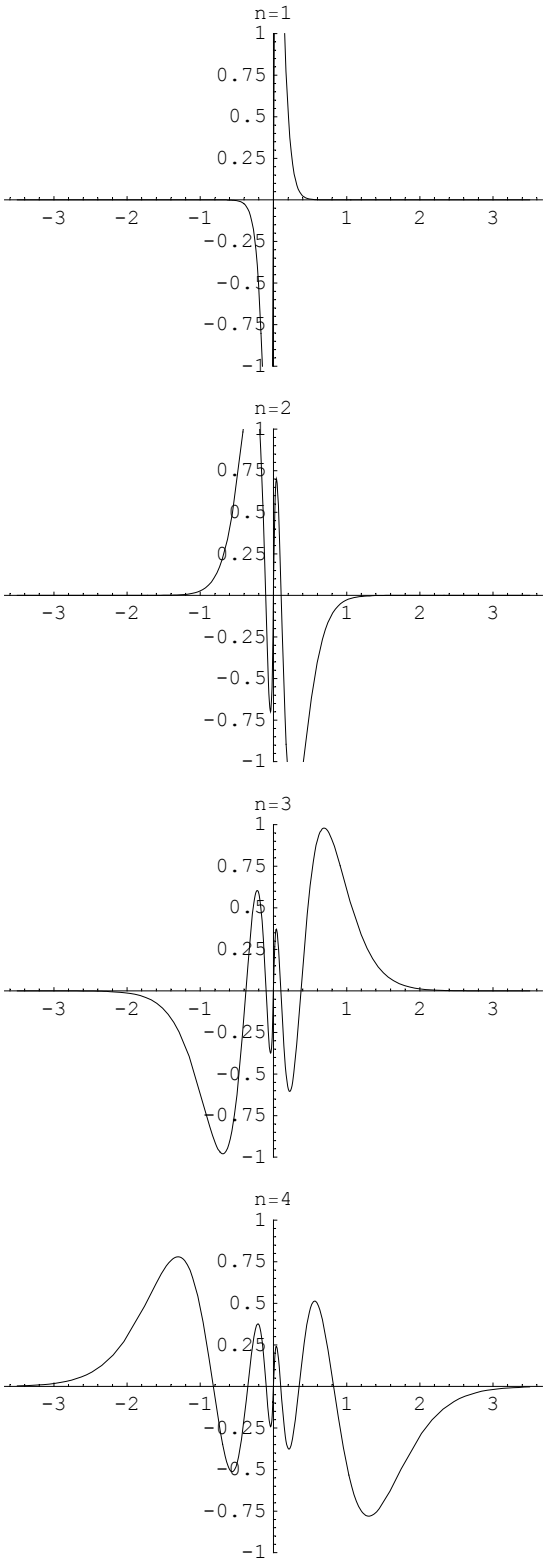

n=5
4
3
2
1
-3
-2
-1
1
2
3

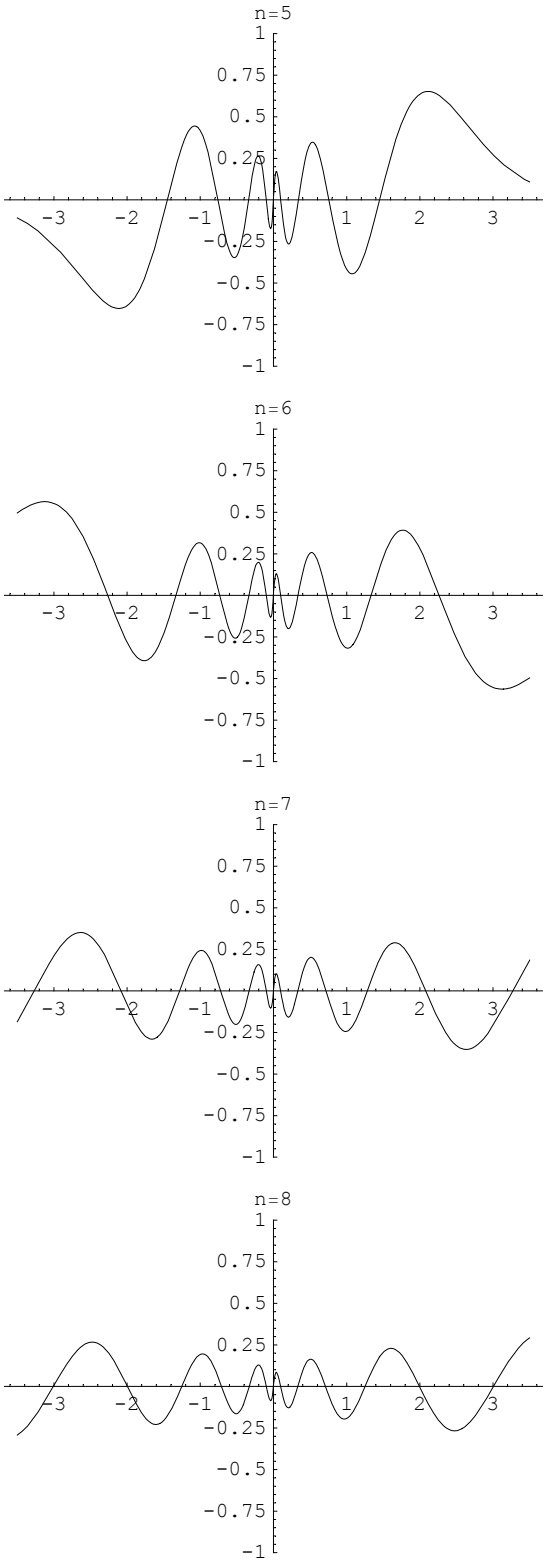

n=6
3
2.5
2
1.5
1
0.5
-0.5
-3
-2
-1
1
2
3

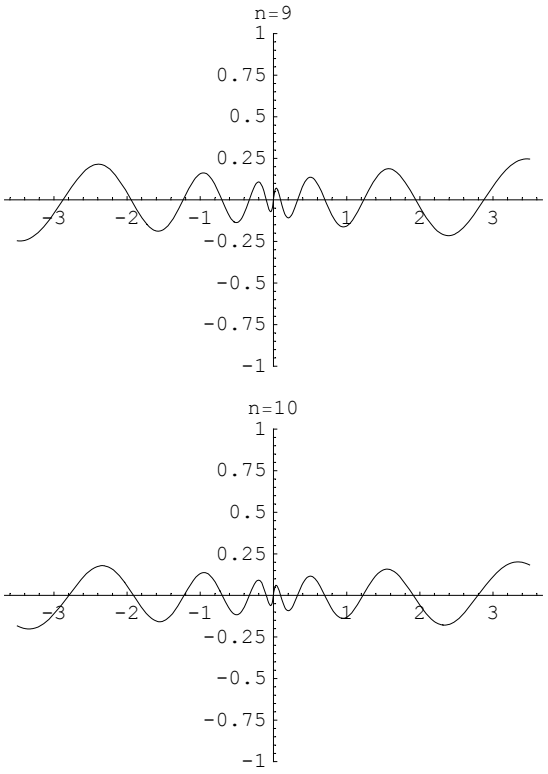


In[122]:=

```
Table[Plot[Evaluate[√(2 π) x ψ[n, 0, x]], {x, -3.5, 3.5},
   PlotRange → {-1, 1}, PlotLabel → StringJoin["n=", ToString[n]]], {n, 10}];
```

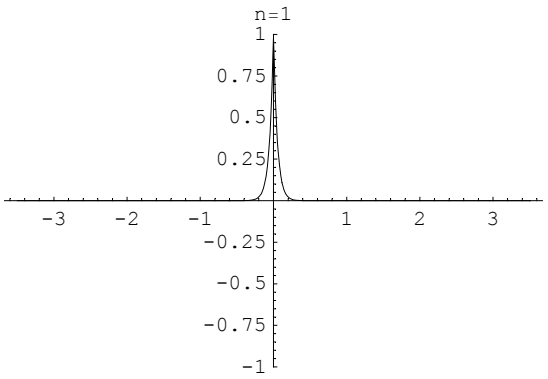

n=1
n=2
n=3
n=4

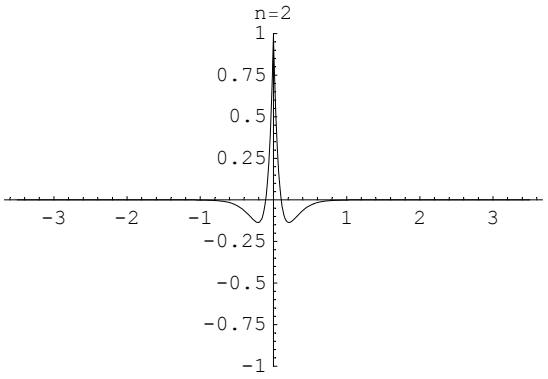

n=5
n=6
n=7
n=8
1
0.75
0.5
0.25
-0.25
-0.5
-0.75
-1
-3
-2
-1
1
2
3

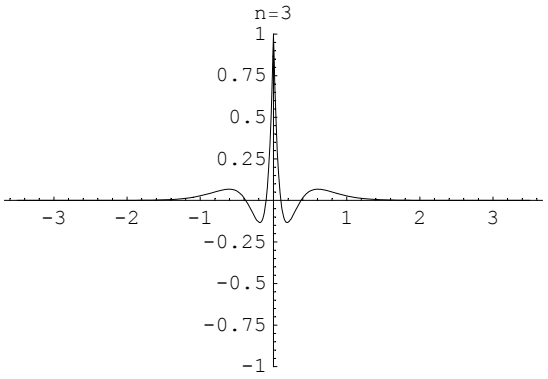


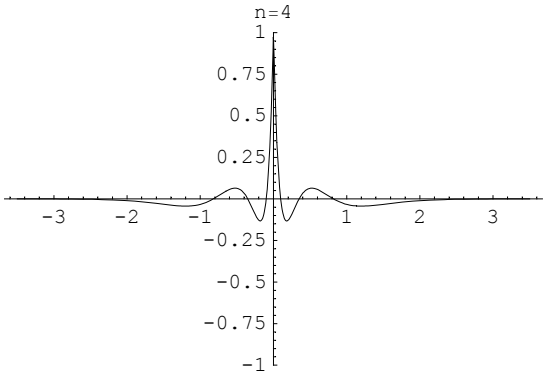
Scaled Wave Functions

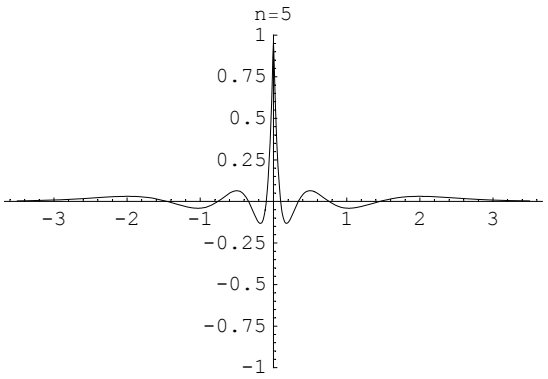
```
In[62]:= ψ3D[n_, x_] := √π (n a)^(3/2) ψ[n, 0, x]
         ψ1D[n_, x_] := x ψ[n, 0, x]

In[66]:= Table[Plot[Evaluate[ψ3D[n, x]], {x, -3.5, 3.5},
            PlotRange → {-1, 1}, PlotLabel → StringJoin["n=", ToString[n]]], {n, 10}];
```

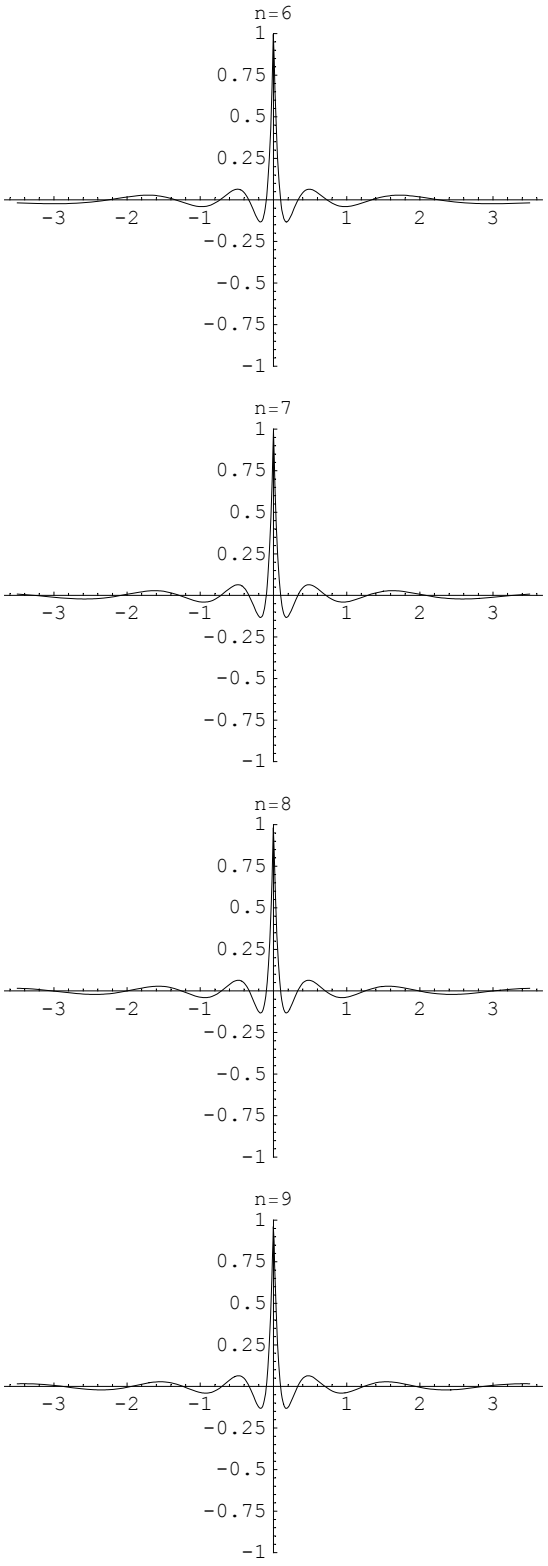


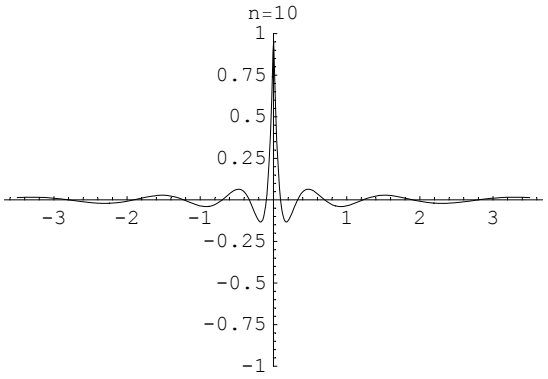

n=2
1
0.75
0.5
0.25
-3
-2
-1
1
2
3
-0.25
-0.5
-0.75
-1


n=3
1
0.75
0.5
0.25
-3
-2
-1
1
2
3
-0.25
-0.5
-0.75
-1

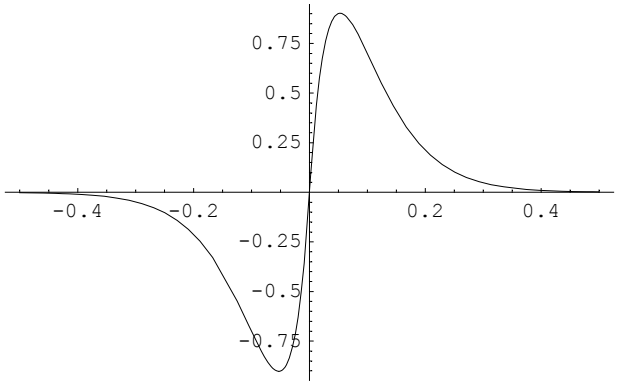

n=4
1
0.75
0.5
0.25
-3
-2
-1
1
2
3
-0.25
-0.5
-0.75
-1


n=5
1
0.75
0.5
0.25
-3
-2
-1
1
2
3
-0.25
-0.5
-0.75
-1


n=6
n=7
n=8
n=9
1
0.75
0.5
0.25
-0.25
-0.5
-0.75
-1
-3
-2
-1
1
2
3



In[73]:= **Plot[Evaluate[ψ1D[1, x]], {x, -0.5, 0.5}, PlotRange → All];**



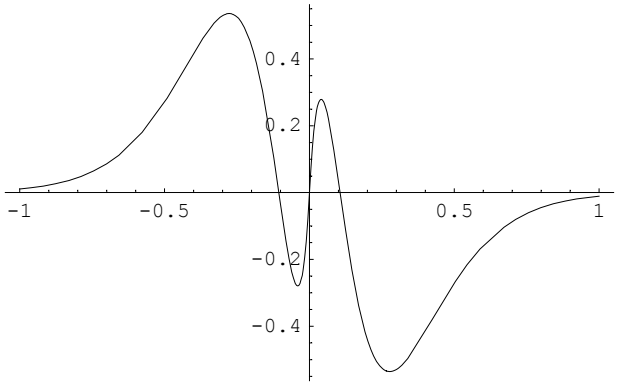
In[79]:= **FindMaximum[ψ1D[1, x], {x, a}]**

```
FindMaximum::lstol :
 The line search decreased the step size to within tolerance specified by AccuracyGoal and
   PrecisionGoal but was unable to find a sufficient increase in the function.  You may need
   more than MachinePrecision digits of working precision to meet these tolerances. More…
```

Out[79]= {0.902111, {x → 0.0529348}}

In[80]:= **A[1] = 1 / 0.9021112566654106`;**

In[77]:= **Plot[Evaluate[ψ1D[2, x]], {x, -1, 1}, PlotRange → All];**

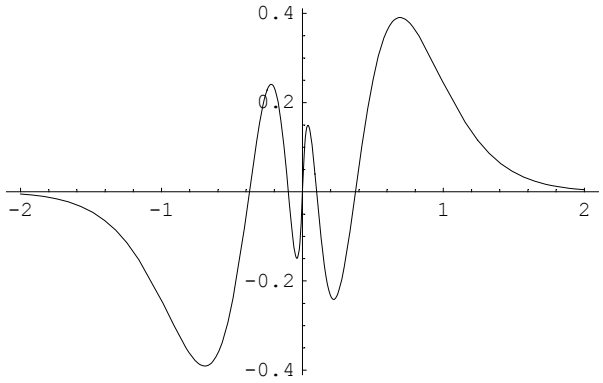


In[69]:= **FindMinimum[ψ1D[2, x] , {x, 4 a}]**

Out[69]= {-0.535803, {x → 0.27717}}

In[81]:= **A[2] = 1 / -0.5358025696564411`;**

In[84]:= **Plot[Evaluate[ψ1D[3, x]], {x, -2, 2}, PlotRange → All];**

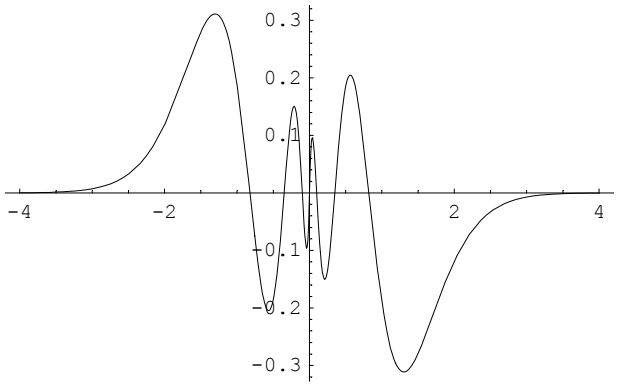


In[86]:= **FindMaximum[ψ1D[3, x] , {x, 9 a}]**

Out[86]= {0.390689, {x → 0.692071}}

In[87]:= **A[3] = 1 / 0.3906888363463391`;**

In[88]:= **Plot[Evaluate[ψ1D[4, x]], {x, -4, 4}, PlotRange → All];**

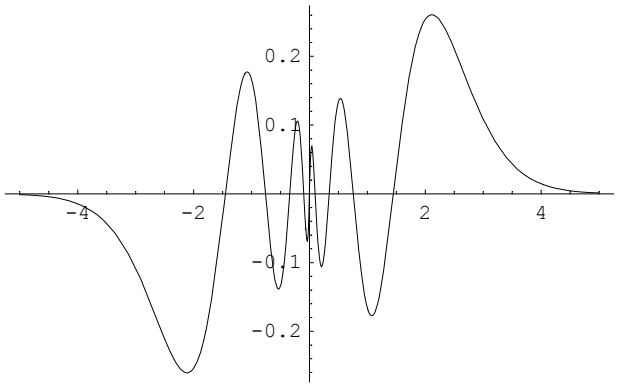


In[89]:= **FindMinimum[ψ1D[4, x] , {x, 16 a}]**

Out[89]= {-0.311151, {x → 1.30315}}

In[90]:= **A[4] = 1 / -0.3111513614361428`;**

In[92]:= **Plot[Evaluate[ψ1D[5, x]], {x, -5, 5}, PlotRange → All];**



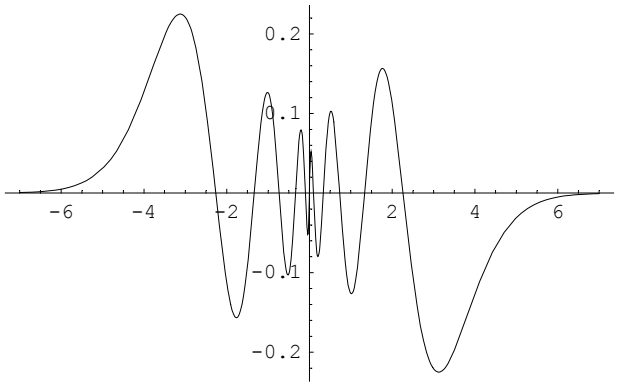
In[94]:= **FindMaximum[ψ1D[5, x] , {x, 25 a}]**

```
FindMaximum::lstol :
 The line search decreased the step size to within tolerance specified by AccuracyGoal and
   PrecisionGoal but was unable to find a sufficient increase in the function.  You may need
   more than MachinePrecision digits of working precision to meet these tolerances. More…
```

Out[94]= {0.260373, {x → 2.11332}}

In[95]:= **A[5] = 1 / 0.26037320634671396`;**

In[97]:= **Plot[Evaluate[ψ1D[6, x]], {x, -7, 7}, PlotRange → All];**



In[98]:= **FindMinimum[ψ1D[6, x] , {x, 36 a}]**

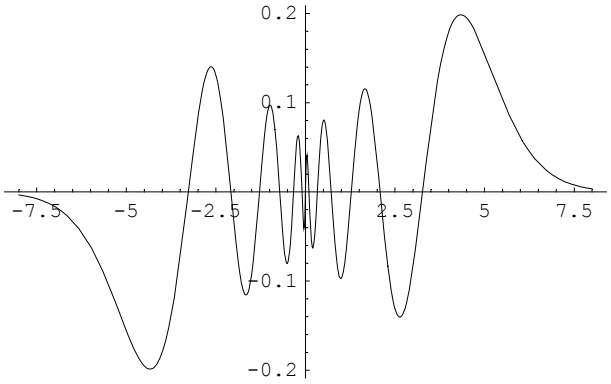
```
FindMinimum::lstol :
 The line search decreased the step size to within tolerance specified by AccuracyGoal and
   PrecisionGoal but was unable to find a sufficient decrease in the function.  You may need
   more than MachinePrecision digits of working precision to meet these tolerances. More…
```

Out[98]= {-0.224905, {x → 3.12442}}

In[99]:= **A[6] = 1 / -0.22490467204036374`;**

In[100]:=

```
Plot[Evaluate[ψ1D[7, x]], {x, -8, 8}, PlotRange → All];
```



In[102]:=

```
FindMaximum[ψ1D[7, x] , {x, 5}]
```

Out[102]=

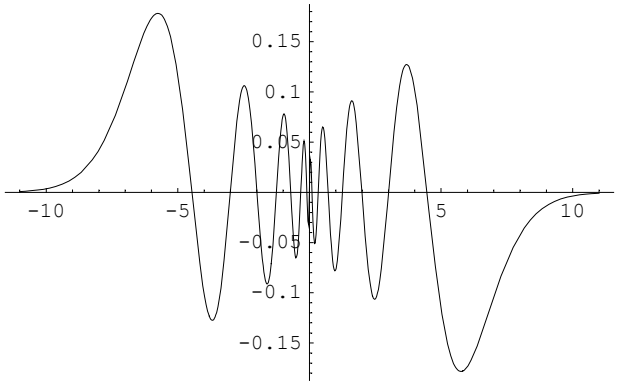
```
{0.198608, {x → 4.33774}}
```

In[110]:=

```
A[7] = 1 / 0.19860828130806119`;
```

In[105]:=

```
Plot[Evaluate[ψ1D[8, x]], {x, -11, 11}, PlotRange → All];
```



In[106]:=

```
FindMinimum[ψ1D[8, x] , {x, 6}]
```

Out[106]=

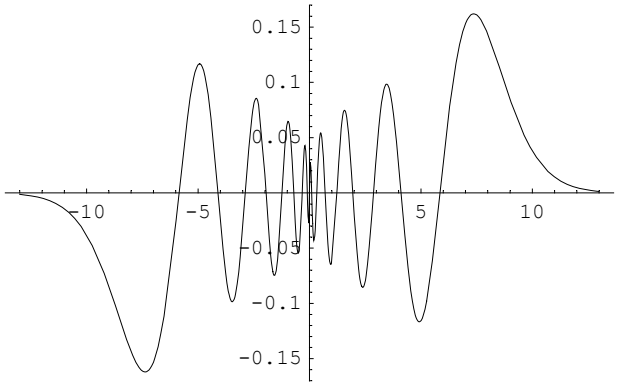
```
{-0.178266, {x → 5.75425}}
```

In[117]:=

```
A[8] = 1 / -0.17826582821193557`;
```

In[107]:=

```
Plot[Evaluate[ψ1D[9, x]], {x, -13, 13}, PlotRange → All];
```



In[108]:=

```
FindMaximum[ψ1D[9, x] , {x, 7}]
```

Out[108]=

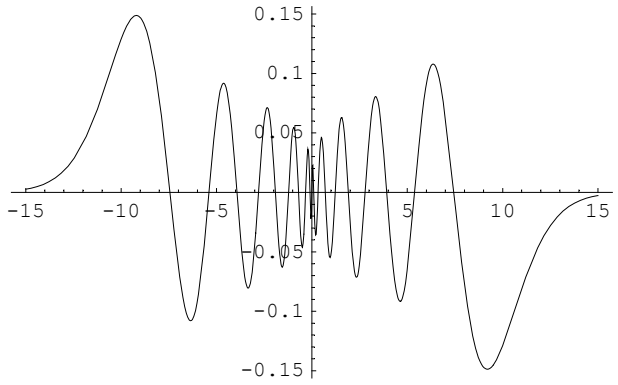
```
{0.16202, {x → 7.37471}}
```

In[109]:=

```
A[9] = 1 / 0.16201982502838655`;
```

In[112]:=

```
Plot[Evaluate[ψ1D[10, x]], {x, -15, 15}, PlotRange → All];
```



In[114]:=

```
FindMinimum[ψ1D[10, x] , {x, 9.1}]
```

```
FindMinimum::lstol :
 The line search decreased the step size to within tolerance specified by AccuracyGoal and
   PrecisionGoal but was unable to find a sufficient decrease in the function.  You may need
   more than MachinePrecision digits of working precision to meet these tolerances. More…
```

Out[114]=

```
{-0.14872, {x → 9.19971}}
```

In[115]:=

```
A[10] = 1 / -0.14871971988580268`;
```

In[118]:=

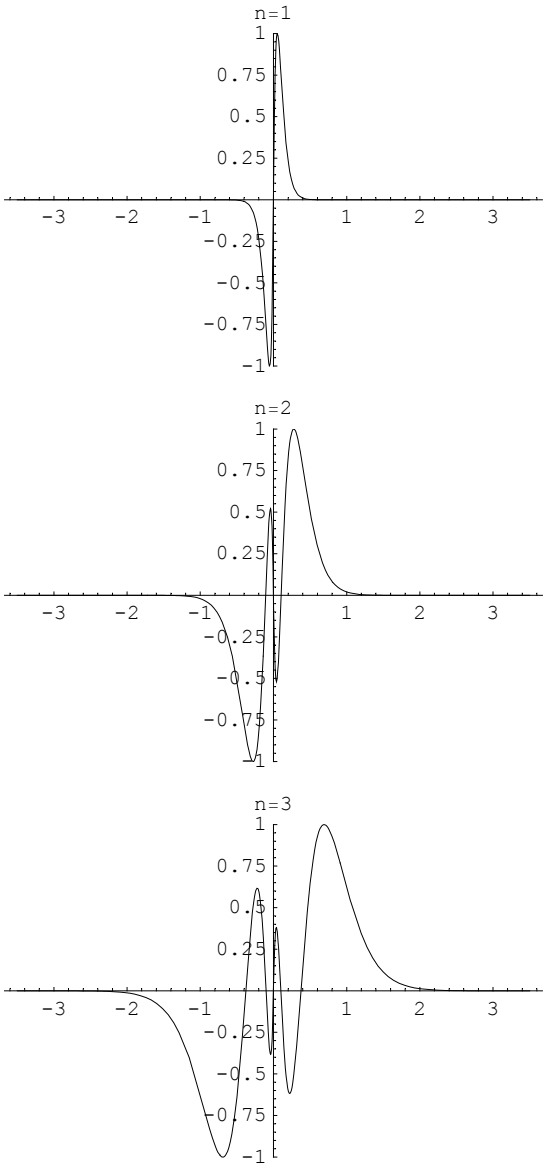
```
Table[A[n], {n, 10}]
```

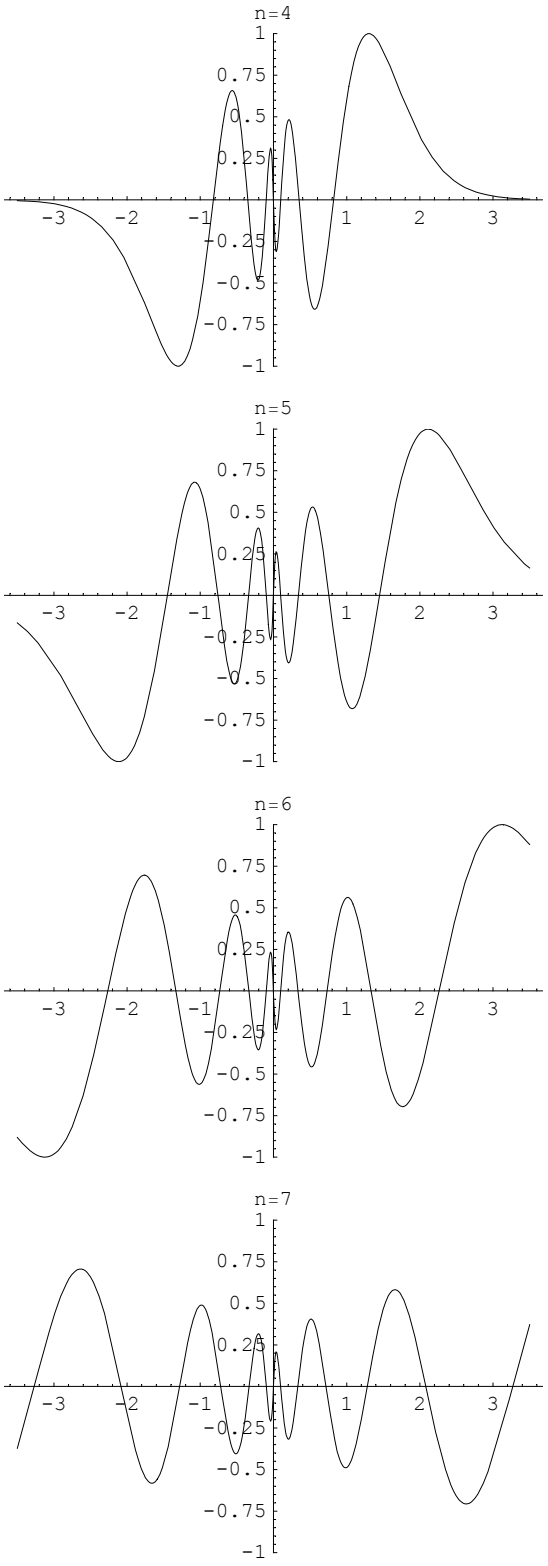
Out[118]=

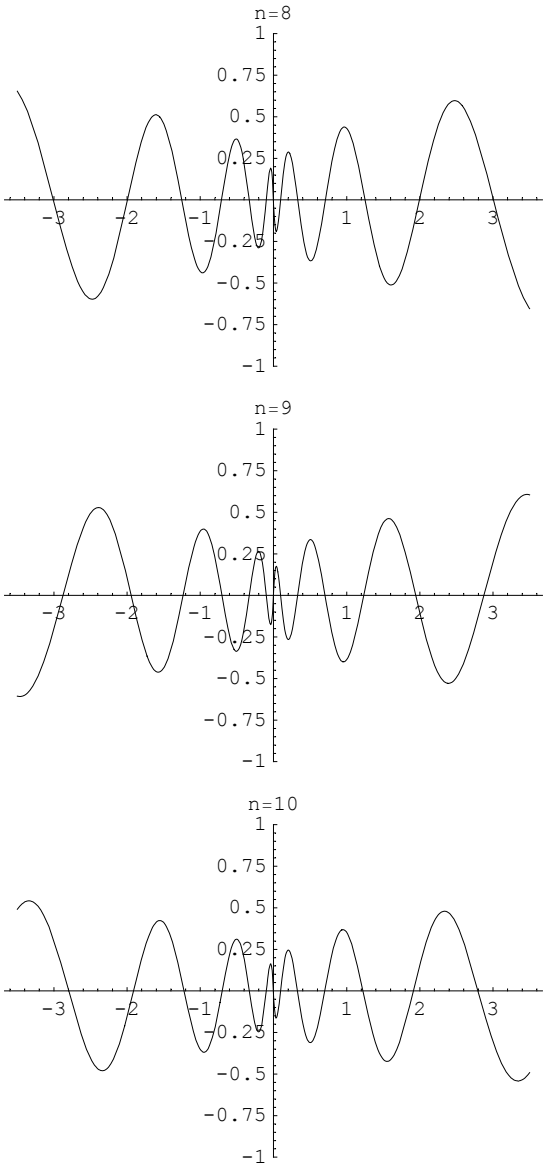
```
{1.10851, -1.86636, 2.55958, -3.21387,
 3.84064, -4.44633, 5.03504, -5.6096, 6.17208, -6.72406}
```

In[119]:=

```
Table[Plot[Evaluate[A[n] ψ1D[n, x]], {x, -3.5, 3.5},
   PlotRange → {-1, 1}, PlotLabel → StringJoin["n=", ToString[n]]], {n, 10}];
```




n=4
n=5
n=6
n=7


n=8
n=9
n=10
1
0.75
0.5
0.25
-0.25
-0.5
-0.75
-1
-3
-2
-1
1
2
3