Quantum Tunneling

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New Changes 11/15/05

- Add configure button so advanced users can enter exact numbers for energies in textboxes
- Move energy legend above graph and move textboxes into configure window.
- We will solve Schrodinger equation using Richardson algorithm (http://www.neti.no/java/sgi_java/WaveSim.html) for wave packets, but use closed form solutions for plane waves. This means we can show incident and reflected waves separately for plane waves, but not for wave packets.
- Measure button is always enabled
- Pressing the Measure button in plane wave mode causes wave function to disappear
- Restart button is always enabled
- pressing Restart does not change the Play/Pause state of the simulation
- the time display is in units of "fs" with integer precision, resets to "0 fs" when the Restart button is pressed
- the sim has a "Save/Load configuration" feature
- range of x-axis (position) on all graphs: 0 to 20 nm
- range of y-axis on energy graph: -1 to +10 eV (is this OK for wells?)
- range of wave packet width: 0.1 to 4 nm (default 0.5), width slider shows value to 1 decimal place, ticks at 1 nm intervals, labels at min & max
- range of wave packet center: 0 to 20 nm (default 1.5), center slider shows value to 1 decimal place, ticks at 5 nm intervals, labels at min & max

Learning Goals

- To be able to visualize wave functions for constant, step, and barrier potentials.
- To be able to visualize both plane wave and wave packet solutions to the Schrodinger equation and understand how they relate to each other.
- To understand and distinguish the real part, imaginary part, and absolute value of the wave function, as well as the probability density.
- To understand how the probability of reflection and transmission of a wave are related to the energy of the wave, the energy of the step or barrier, and the width of the barrier.
- To understand that a wave packet is composed of a range of energies but that this range does NOT account for reflection and transmission.

Similar Existing Simulations

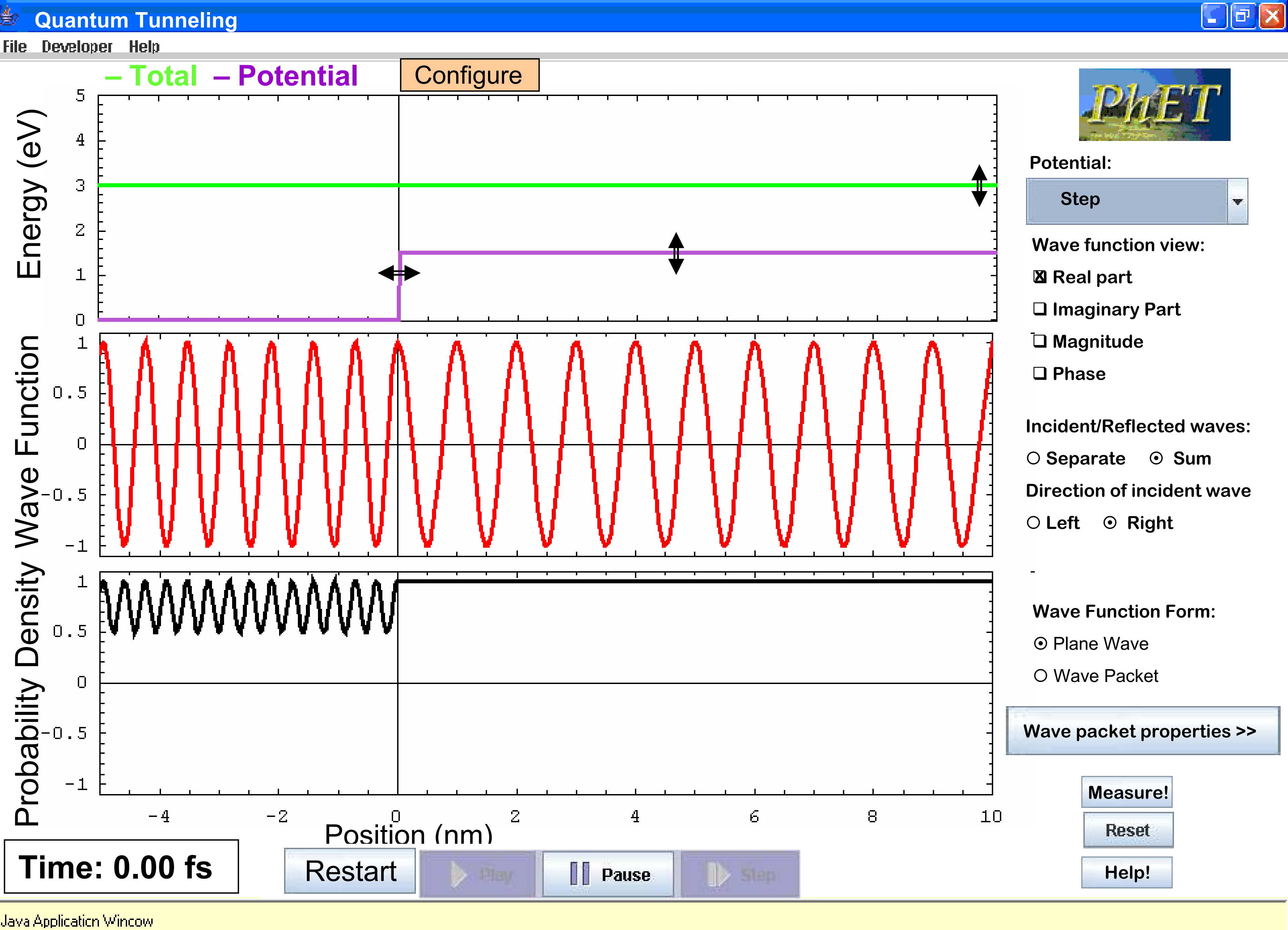
- http://www.phys.ksu.edu/perg/vqmorig/programs/java/qumotion/quantum_motion.html
- http://www.quantum-physics.polytechnique.fr/, panels 1.3 & 1.5
- http://www.neti.no/java/sgi_java/WaveSim.html
- http://www.sc.ehu.es/sbweb/fisica/ondas/refraccion/refraccion.html

What can our sim do that others don’t?

- Display range of energies for wave packet.
  - Other sims show single energy for wave packet, leading to misconception that a wave packet represents a single energy. Since misconception that range of energies causes classically forbidden tunneling/reflection, it is especially important to show range, so that students can see these effects even when entire range is classically forbidden.
- Display wave function and energy on separate graphs.
  - Research shows that the common practice of displaying wave function and energy on the same graph leads students to confuse the two.
- Allow user to change energies continuously by clicking and dragging directly on energy graph.
  - For students to explore how changing energies and widths affect reflection and transmission, it’s important to make it as easy as possible to change these continuously.
- Display multiple representations of wave function.
  - Students are often confused by the representation of the wave function, and exploring many different representations will allow them to confront this confusion.
- Show real units for time as well as space and energy.
  - This allows students to make a connection between sim and physical reality.

Plane Wave Mode:

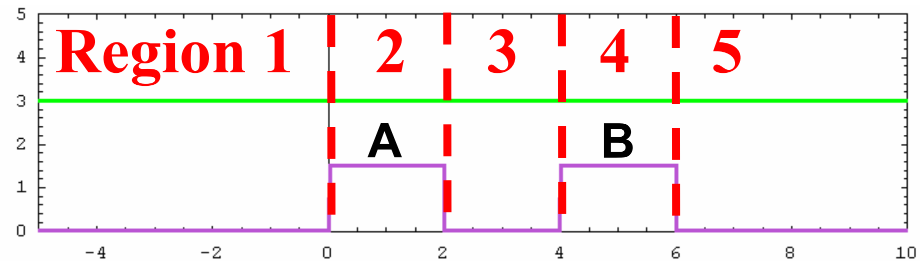
- A plane wave is given by the equation: $\Psi(x,t)=L\exp[ikx-Et/\hbar] + R\exp[-ikx-Et/\hbar]$ where $k=\mathrm{sqrt}[2m(E-V)/\hbar^2]$ and L and R are the amplitudes of the waves traveling to the left and to the right.



Quantum Tunneling
File Developer Help
– Total – Potential
Configure
Energy (eV)
Wave Function
Probability Density
Position (nm)
Time: 0.00 fs
Restart
Play
Pause
Step
PhET
Potential:
Step
Wave function view:
Real part
Imaginary Part
Magnitude
Phase
Incident/Reflected waves:
Separate
Sum
Direction of incident wave
Left
Right
Wave Function Form:
Plane Wave
Wave Packet
Wave packet properties >>
Measure!
Reset
Help!
Java Application Window

Configure Button

- Pressing the configure button produces a pop-up window like this one with a bunch of textboxes so that you can enter numbers for energies, widths, and positions to 2 decimal places.
- The graph at the top of pop-up window responds immediately to typing in changes, but the graph in the main sim doesn't change until you press "Apply" button.



Total Energy: eV

Potential Energy:

Region I: eV

Region II: eV

Region III: eV

Region IV: eV

Region V: eV

Barrier width:

Barrier A: nm

Barrier B: nm

Barrier position (left edge):

Barrier A: nm

Barrier B: nm

Apply

Defaults

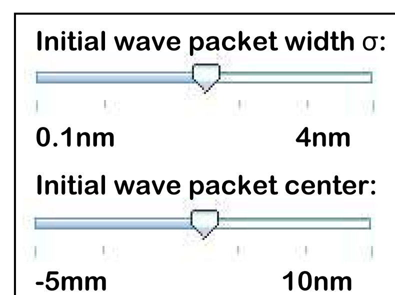
- Defaults should be as shown on previous slide, but with potential set to barrier instead of step. Also, the range on the wave function graph should be -2 to 2 and the range on the probability density graph should be 0 to 4.

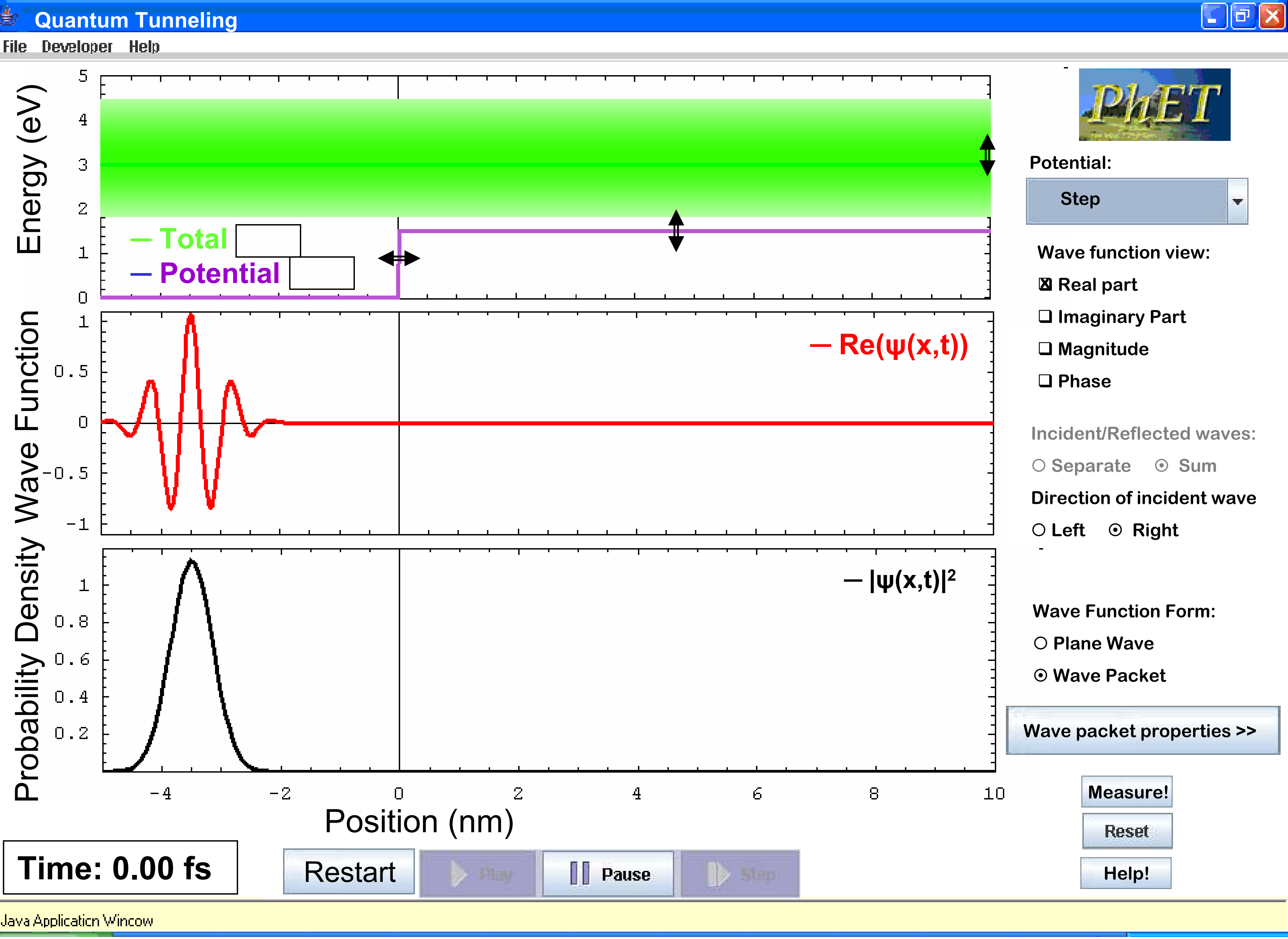
Wave Packet Mode:

- A wave packet is given by the equation:

$$\Psi(x,t) = \frac{\sqrt{\sigma}}{\pi^{1/4}\sqrt{\sigma^2 + i\hbar t/m}} e^{-\frac{(x-\hbar k_0 t/m)^2}{2(\sigma^2 + i\hbar t/m)}} e^{ik_0(x-\hbar k_0 t/2m)}$$

  Where k_0=sqrt[2m(E-V)/$\hbar^2$] for average energy E, x_0 is initial position of the center of packet, and σ is initial width of packet.
- When “wave packet properties” button is pressed, the following options are available on the control panel:

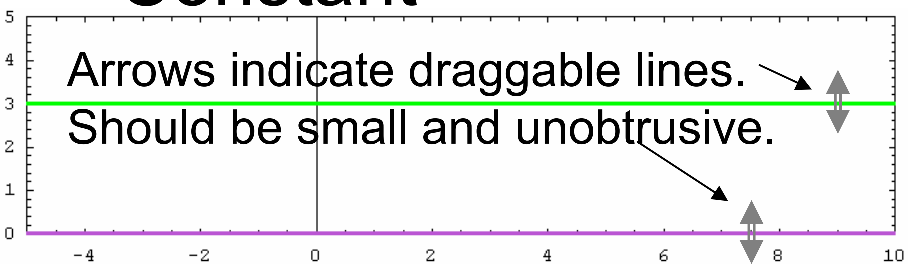




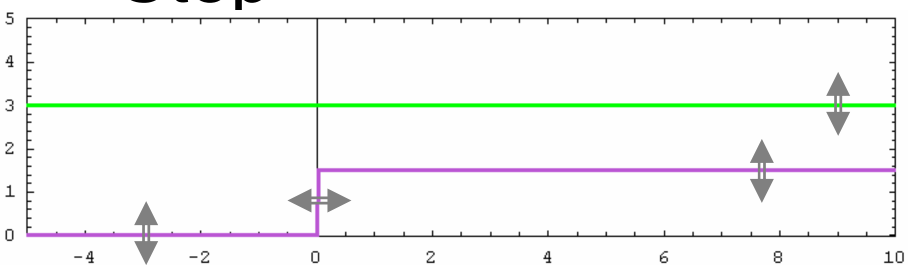
Quantum Tunneling
File Developer Help
Energy (eV)
Total
Potential
Wave Function
Re(ψ(x,t))
Probability Density
|ψ(x,t)|²
Position (nm)
Potential:
Step
Wave function view:
Real part
Imaginary Part
Magnitude
Phase
Incident/Reflected waves:
Separate
Sum
Direction of incident wave
Left
Right
Wave Function Form:
Plane Wave
Wave Packet
Wave packet properties >>
Measure!
Reset
Help!
Time: 0.00 fs
Restart
Play
Pause
Step
Java Application Window

Options for Potential

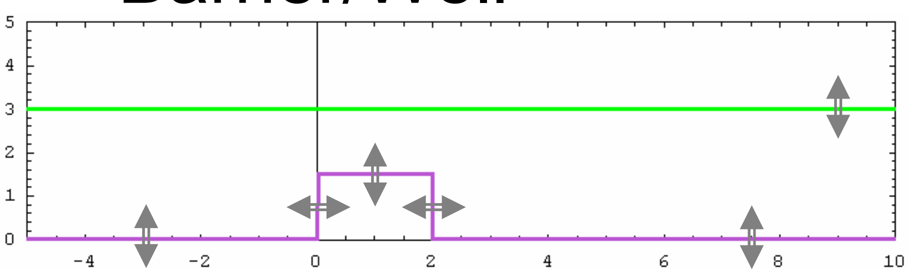
- Constant



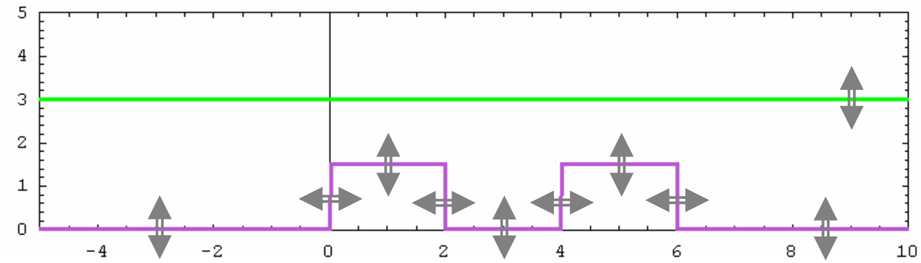
- Step



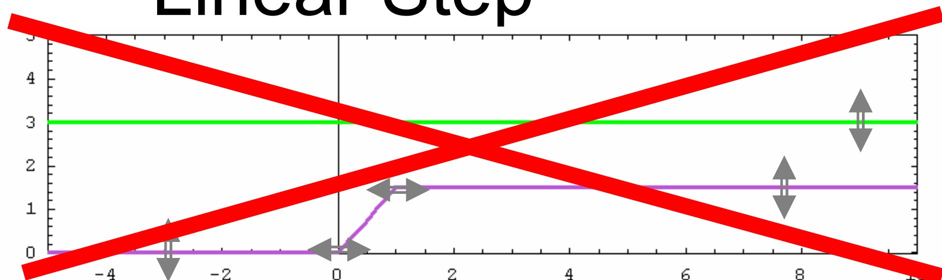
- Barrier/Well



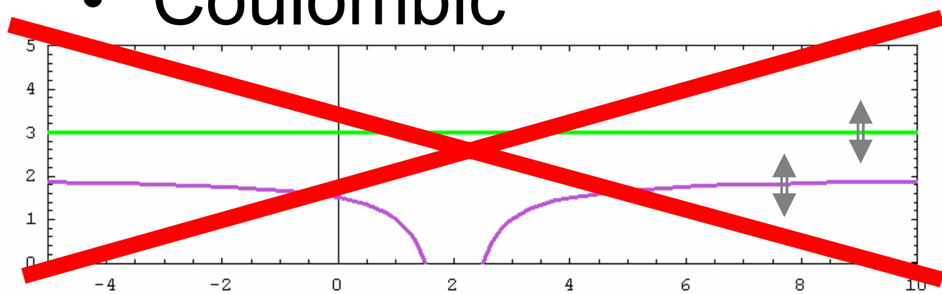
- Double Barrier/Well



- Linear Step

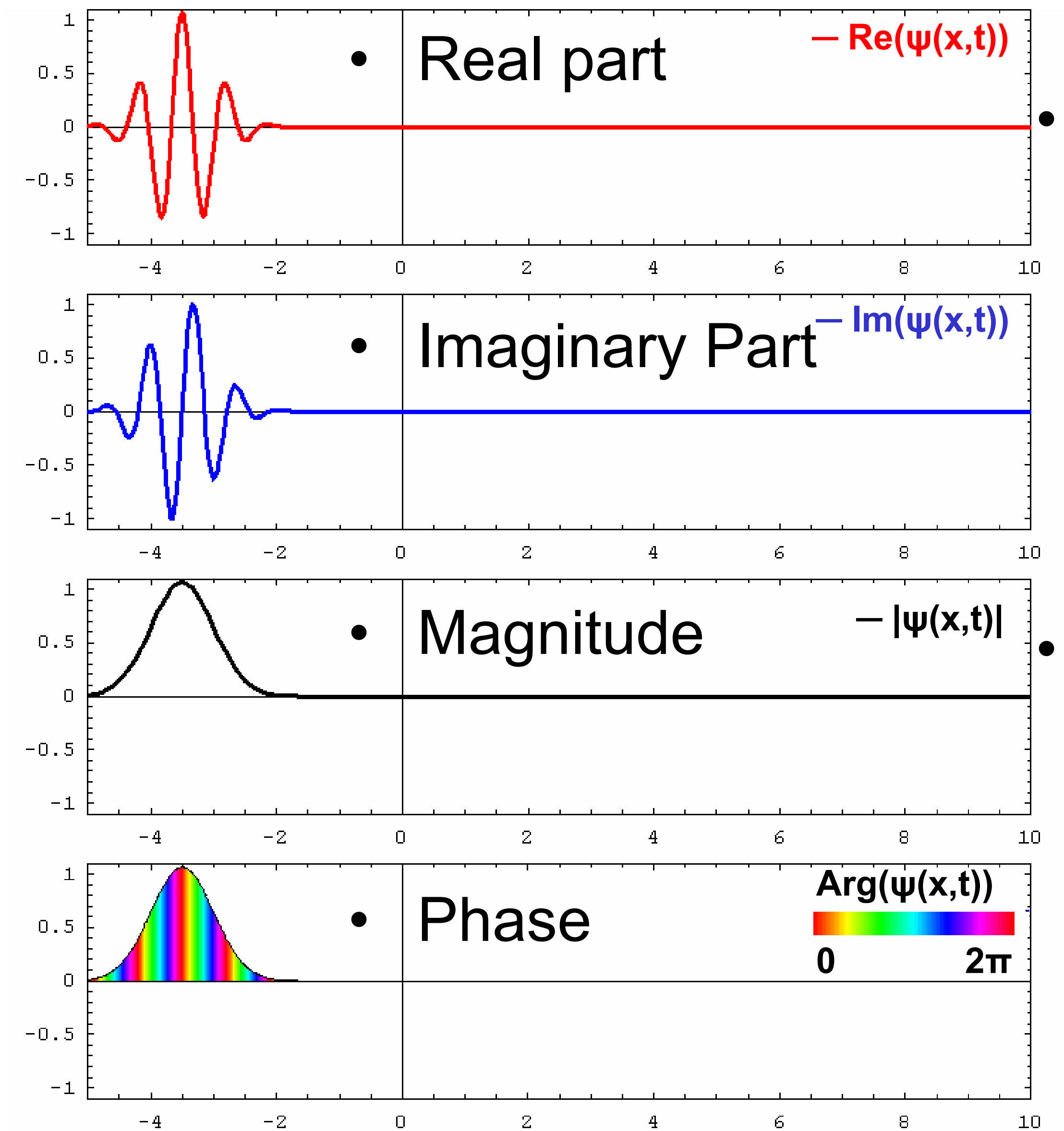


- Coulombic



Don’t implement last two options

Options for Wave Function View



- Real part
- Imaginary Part
- Magnitude
- Phase

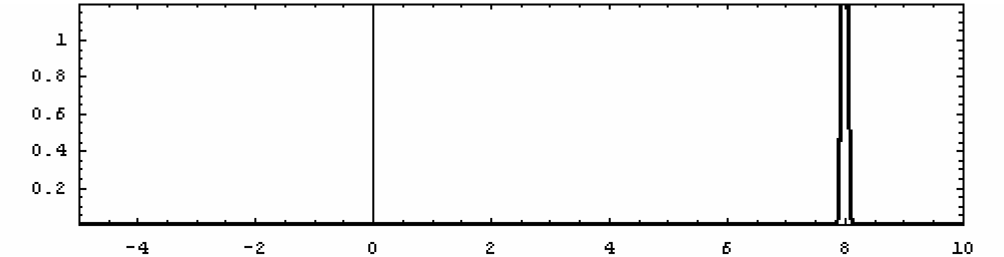
- These options should be checkboxes, not radio buttons, so that the user can choose only one of them, or several at the same time.
- If more than one is selected, they should be plotted on top of each other.

Restart, or what to do when wave packet goes off the screen.

- A wave packet will start in one place, and move across the screen and eventually off the screen, after which nothing will happen unless the user presses restart, in which case the wave packet will reappear at its initial position.
- Whenever the user adjust any of the energies, wave packet properties, or direction of motion, the simulation should respond as if the restart button was pressed.
- If the user changes the wave function view or the view of the incident/reflected waves, these changes should be made without restarting wave packet (since these affect display, not physical situation).
- Restart resets time to t=0 fs.

Measurements

- If you hit the “Measure!” button, the probability of finding the wave function in each 0.1nm region of space is calculated (it’s the area under the probability density curve), and one region of space is chosen randomly, based on these probabilities.
- The wave packet then disappears and is replaced by a narrow wave packet localized in the selected region, which then spreads according to the Schrodinger equation as shown in the movie below:



Click on graph to show movie again!

- In plane wave mode, the measure button should always make the entire wave function disappear (Since a particle described by a plane wave is equally likely to be found anywhere in space, the probability of finding it in the region shown on this graph is essentially zero.)

Constants, Units, and Formulas

Throughout this simulation, the units are:

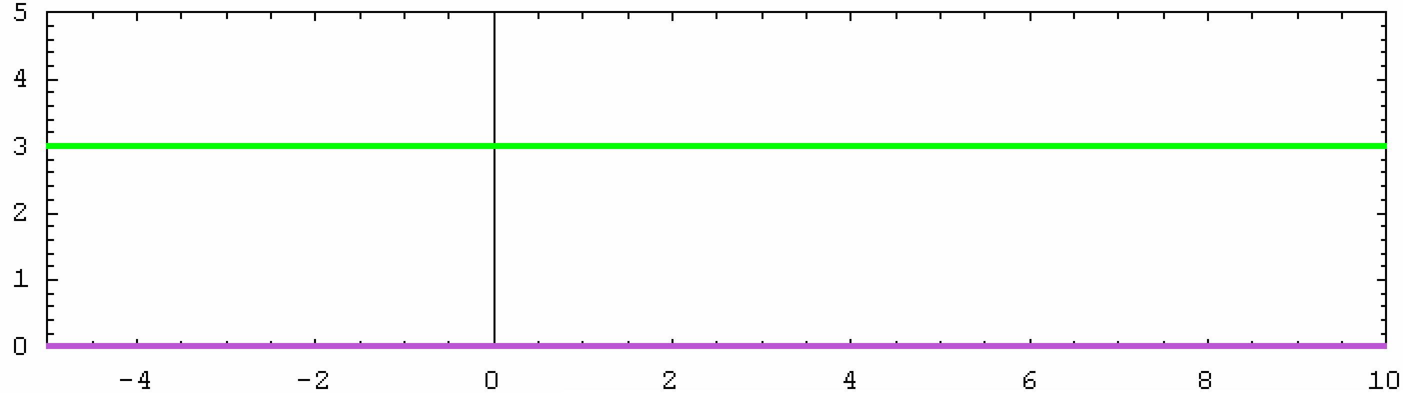
- Energy – eV
- Distance – nm
- Time – fs
- Mass – eV/c^2

To give correct units, the constants used in equations should be:

- $\hbar = 0.658$
- m = 5.7

The following slides give the formulas to use for $\Psi(x,t)$ for plane waves.

Constant Potential



- Wave function: $\Psi(x,t) = e^{ikx} e^{-iEt/\hbar}$

where $k = \sqrt{2m(E-V)/\hbar^2}$

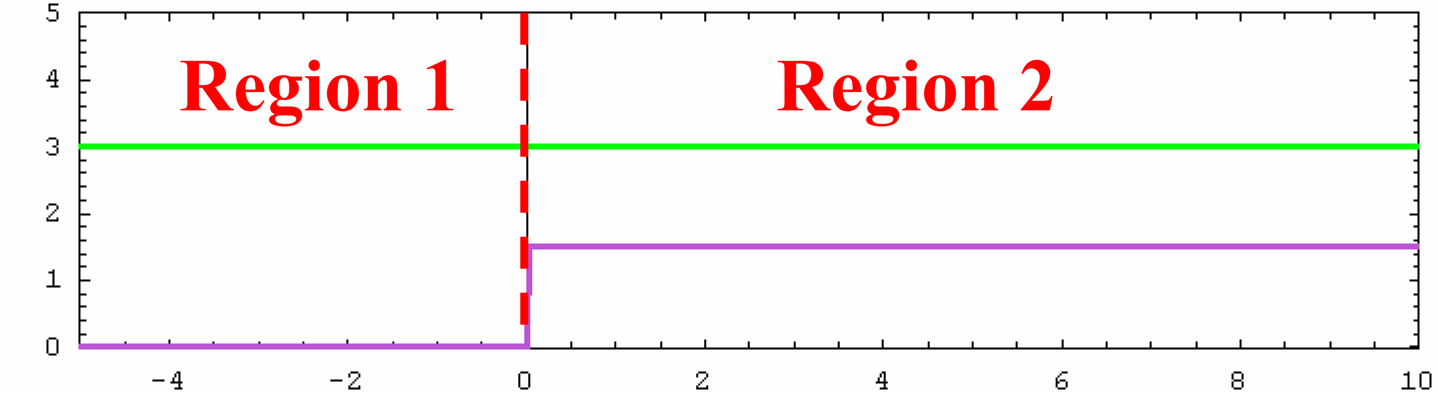
E = total energy (green line)

V = potential energy (purple line)

(Note that if E<V, k is imaginary.)

If direction of incident wave reversed, wf becomes: $\Psi(x,t) = e^{-ikx} e^{-iEt/\hbar}$

Step Potential



Region 1: $\Psi_1(x,t) = \left(e^{ik_1 x} + Be^{-ik_1 x}\right)e^{-iEt/\hbar}$

Region 2: $\Psi_2(x,t) = Ce^{ik_2 x}e^{-iEt/\hbar}$

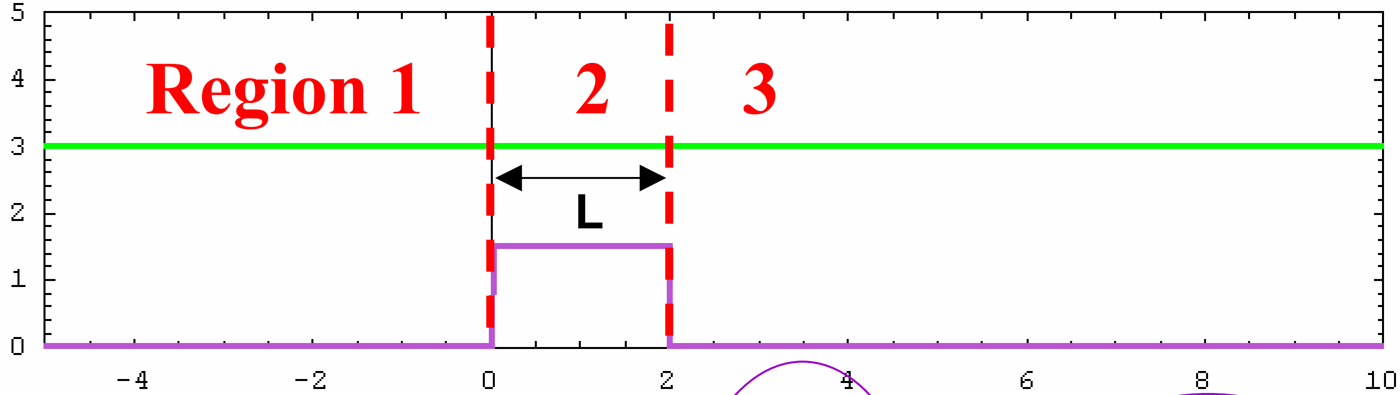
Left-moving wave

Right-moving waves

$$B = \frac{k_1 - k_2}{k_1 + k_2}$$

$$C = \frac{2k_1}{k_1 + k_2} \qquad k = \sqrt{2m(E-V)/\hbar^2}$$

Barrier Potential



Region 1: $\Psi_1(x,t) = \left(e^{ik_1 x} + Be^{-ik_1 x}\right)e^{-iEt/\hbar}$

Region 2: $\Psi_2(x,t) = \left(Ce^{ik_2 x} + De^{-ik_2 x}\right)e^{-iEt/\hbar}$

Region 3: $\Psi_3(x,t) = Fe^{ik_3 x}e^{-iEt/\hbar}$

Right-moving waves

Left-moving waves

$$k = \sqrt{2m(E-V)/\hbar^2}$$

Coefficients for Barrier Potential

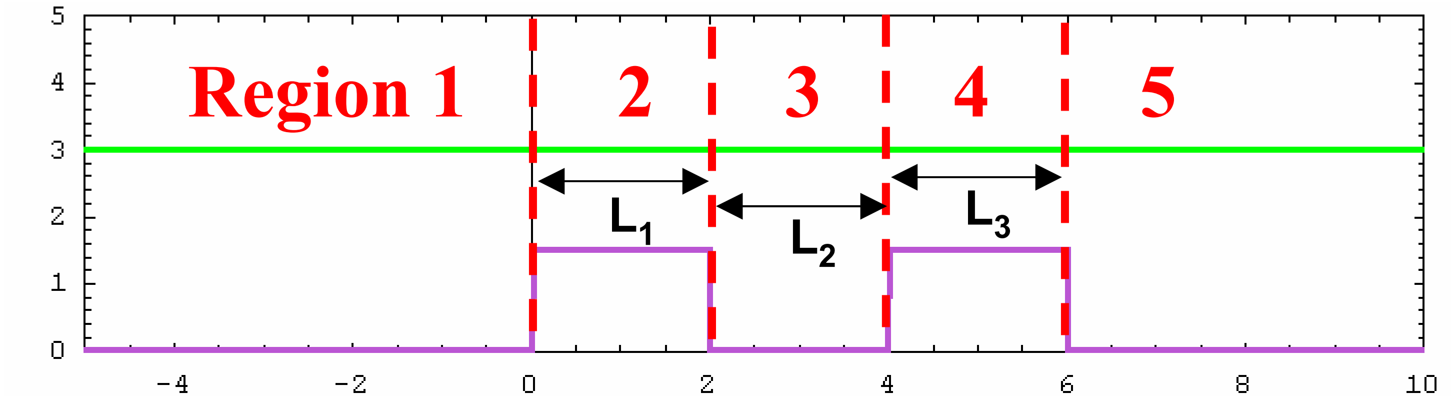
$$B = \frac{e^{2ik_2L}(k_1+k_2)(k_2-k_3)+(k_1-k_2)(k_2+k_3)}{e^{2ik_2L}(k_1-k_2)(k_2-k_3)+(k_1+k_2)(k_2+k_3)}$$

$$C = \frac{2k_1(k_2+k_3)}{e^{2ik_2L}(k_1-k_2)(k_2-k_3)+(k_1+k_2)(k_2+k_3)}$$

$$D = \frac{2e^{2ik_2L}k_1(k_2-k_3)}{e^{2ik_2L}(k_1-k_2)(k_2-k_3)+(k_1+k_2)(k_2+k_3)}$$

$$F = -\frac{4e^{i(k_2-k_3)L}k_1k_2}{e^{2ik_2L}(k_2-k_1)(k_2-k_3)-(k_1+k_2)(k_2+k_3)}$$

Double Barrier Potential



Region 1: $\Psi_1(x,t) = (e^{ik_1 x} + Be^{-ik_1 x})e^{-iEt/\hbar}$

Region 2: $\Psi_2(x,t) = (Ce^{ik_2 x} + De^{-ik_2 x})e^{-iEt/\hbar}$

Region 3: $\Psi_3(x,t) = (Fe^{ik_3 x} + Ge^{-ik_3 x})e^{-iEt/\hbar}$

Region 4: $\Psi_4(x,t) = (He^{ik_4 x} + Ie^{-ik_4 x})e^{-iEt/\hbar}$

Region 5: $\Psi_5(x,t) = Je^{ik_5 x}e^{-iEt/\hbar}$

Left-moving waves

Right-moving waves

$$k = \sqrt{2m(E-V)/\hbar^2}$$

$$
\begin{aligned}
B= -\Big(& e^{2i(k_2 L_1+k_3 L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)+e^{2i(k_3 L_1+k_4(L_1+L_2+L_3))}(k_1-k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+\\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4 L_3)}(-k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2)+k_4 L_3)}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+\\
& e^{2i(k_3+k_4)(L_1+L_2)}(-k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+\\
& e^{2i((k_2+k_3+k_4)L_1+k_4 L_2)}(k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)+e^{2i(k_3 L_1+k_4(L_1+L_2))}(k_1-k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)\Big/\\
\Big(& e^{2i(k_2 L_1+k_3 L_1+k_4(L_1+L_2+L_3))}(-k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+\\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4 L_3)}(k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2)+k_4 L_3)}(-k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+\\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2))}(-k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+\\
& e^{2i((k_2+k_3+k_4)L_1+k_4 L_2)}(-k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
C= -\Big(2k_1\Big(& e^{2i(k_3 L_1+k_4(L_1+L_2+L_3))}(k_2+k_3)(k_3-k_4)(k_4-k_5)-e^{2i((k_3+k_4)(L_1+L_2)+k_4 L_3)}(-k_2+k_3)(k_3+k_4)(k_4-k_5)-\\
& e^{2i(k_3+k_4)(L_1+L_2)}(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_3 L_1+k_4(L_1+L_2))}(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)\Big)\Big/\\
\Big(& e^{2i(k_2 L_1+k_3 L_1+k_4(L_1+L_2+L_3))}(-k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+\\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4 L_3)}(k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2)+k_4 L_3)}(-k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+\\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2))}(-k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+\\
& e^{2i((k_2+k_3+k_4)L_1+k_4 L_2)}(-k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
D= -\Big(2e^{2ik_2 L_1}k_1\Big(& -e^{2i(k_3 L_1+k_4(L_1+L_2+L_3))}(-k_2+k_3)(k_3-k_4)(k_4-k_5)+e^{2i((k_3+k_4)(L_1+L_2)+k_4 L_3)}(k_2+k_3)(k_3+k_4)(k_4-k_5)+\\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_2+k_3)(k_3-k_4)(k_4+k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2))}(-k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)\Big)\Big/\\
\Big(& e^{2i(k_2 L_1+k_3 L_1+k_4(L_1+L_2+L_3))}(-k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+\\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4 L_3)}(k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2)+k_4 L_3)}(-k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+\\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2))}(-k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+\\
& e^{2i((k_2+k_3+k_4)L_1+k_4 L_2)}(-k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
F= -\Big(& 4e^{i((k_2+k_3)L_1+2k_4(L_1+L_2))}k_1 k_2\Big(e^{2ik_4 L_3}(k_3-k_4)(k_4-k_5)+(k_3+k_4)(k_4+k_5)\Big)\Big)\Big/\\
\Big(& e^{2i(k_2 L_1+k_3 L_1+k_4(L_1+L_2+L_3))}(-k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+\\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4 L_3)}(k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2)+k_4 L_3)}(-k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+\\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2 L_1+(k_3+k_4)(L_1+L_2))}(-k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+\\
& e^{2i((k_2+k_3+k_4)L_1+k_4 L_2)}(-k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)-e^{2i(k_3 L_1+k_4(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
G= & -\left(4 e^{i((k_2+3k_3+2k_4)L_1+2(k_3+k_4)L_2)} k_1 k_2 \left(e^{2 i k_4 L_3}(k_3+k_4)(k_4-k_5)+(k_3-k_4)(k_4+k_5)\right)\right) / \\
& \Big(e^{2i(k_2L_1+k_3L_1+k_4(L_1+L_2+L_3))}(-k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)-e^{2i(k_3L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+ \\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4L_3)}(k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2L_1+(k_3+k_4)(L_1+L_2)+k_4L_3)}(-k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+ \\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2L_1+(k_3+k_4)(L_1+L_2))}(-k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+ \\
& e^{2i((k_2+k_3+k_4)L_1+k_4L_2)}(-k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)-e^{2i(k_3L_1+k_4(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
H= & -\left(8 e^{i((k_2+2k_3+k_4)L_1+(k_3+k_4)L_2)} k_1 k_2 k_3 (k_4+k_5)\right) / \\
& \Big(e^{2i(k_2L_1+k_3L_1+k_4(L_1+L_2+L_3))}(-k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)-e^{2i(k_3L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+ \\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4L_3)}(k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2L_1+(k_3+k_4)(L_1+L_2)+k_4L_3)}(-k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+ \\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2L_1+(k_3+k_4)(L_1+L_2))}(-k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+ \\
& e^{2i((k_2+k_3+k_4)L_1+k_4L_2)}(-k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)-e^{2i(k_3L_1+k_4(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)
\end{aligned}
$$

$$
\begin{aligned}
I= & -\left(8 e^{i(k_2L_1+3k_4(L_1+L_2)+k_3(2L_1+L_2)+2k_4L_3)} k_1 k_2 k_3 (k_4-k_5)\right) / \\
& \Big(e^{2i(k_2L_1+k_3L_1+k_4(L_1+L_2+L_3))}(-k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)-e^{2i(k_3L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+ \\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4L_3)}(k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2L_1+(k_3+k_4)(L_1+L_2)+k_4L_3)}(-k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+ \\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2L_1+(k_3+k_4)(L_1+L_2))}(-k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+ \\
& e^{2i((k_2+k_3+k_4)L_1+k_4L_2)}(-k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)-e^{2i(k_3L_1+k_4(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)
\end{aligned}
$$

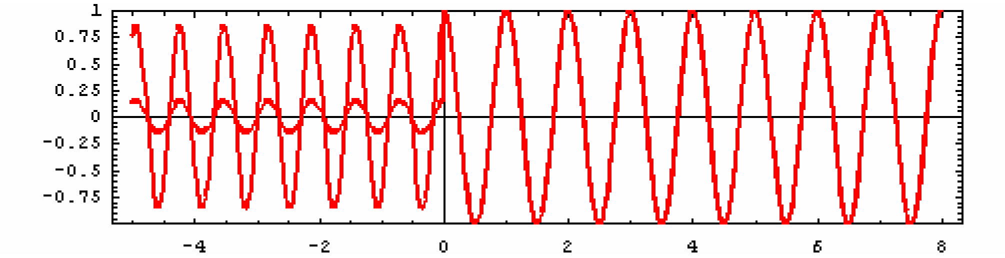
$$
\begin{aligned}
J= & -\left(16 e^{i(k_2L_1+(2k_4-k_5)(L_1+L_2)+k_3(2L_1+L_2)+(k_4-k_5)L_3)} k_1 k_2 k_3 k_4\right) / \\
& \Big(e^{2i(k_2L_1+k_3L_1+k_4(L_1+L_2+L_3))}(-k_1+k_2)(k_2-k_3)(k_3-k_4)(k_4-k_5)-e^{2i(k_3L_1+k_4(L_1+L_2+L_3))}(k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4-k_5)+ \\
& e^{2i((k_3+k_4)(L_1+L_2)+k_4L_3)}(k_1+k_2)(-k_2+k_3)(k_3+k_4)(k_4-k_5)+e^{2i(k_2L_1+(k_3+k_4)(L_1+L_2)+k_4L_3)}(-k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4-k_5)+ \\
& e^{2i(k_3+k_4)(L_1+L_2)}(k_1+k_2)(-k_2+k_3)(k_3-k_4)(k_4+k_5)+e^{2i(k_2L_1+(k_3+k_4)(L_1+L_2))}(-k_1+k_2)(k_2+k_3)(k_3-k_4)(k_4+k_5)+ \\
& e^{2i((k_2+k_3+k_4)L_1+k_4L_2)}(-k_1+k_2)(k_2-k_3)(k_3+k_4)(k_4+k_5)-e^{2i(k_3L_1+k_4(L_1+L_2))}(k_1+k_2)(k_2+k_3)(k_3+k_4)(k_4+k_5)\Big)
\end{aligned}
$$

Note about reversing direction of motion & displaying waves separately

- In all these equations, terms that look like e^{ikx} represent waves traveling right and terms that look e^{-ikx} like represent waves traveling left.
- If the option is chosen to display incident and reflected waves separately, wave function graph should display these terms on top of each other, rather than adding them together. (see next slide)
- If the direction of motion of incident wave is chosen to be left rather than right, every k in all equations for $\Psi(x,t)$ should be replaced by a –k.

Incident/Reflected Waves

- Separate View:



- Sum View: