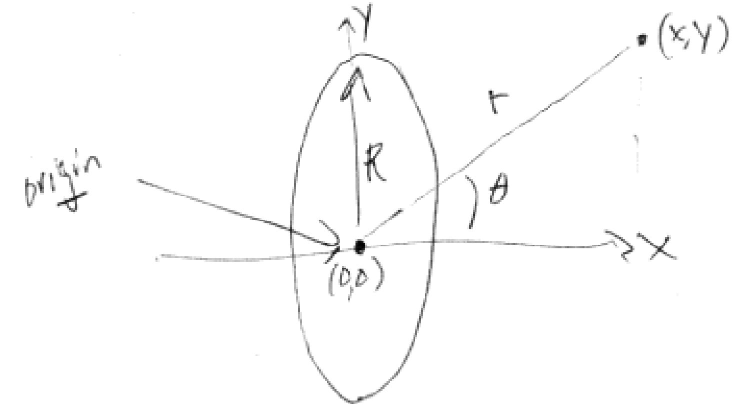
(2005)
-1-

Coil Magnet model
(Mike Dubson 4/14/05)

- works for compact (short squat) coils
- near & far formulas
- circle w/ diameter of coil loop



- constant: M (magnetic moment) describe strength of magnet

  $M = \#\text{loops} * \text{current} * C$

  C = fudge factor, set it so lightbulb lights

- R = radius of loop
- r = distance from origin to point of interest (x, y)

If $r < R$:
(inside)

$$B_x = \frac{2M}{R^3} \qquad B_y = 0$$

If $r \geq R$:
(outside)

$$B_x = \left(\frac{M}{r^3}\right) * \left(3 * \cos(\theta) * \cos(\theta) - 1\right)$$

$$B_y = \left(\frac{M}{r^3}\right) * \left(3 * \cos(\theta) * \sin(\theta)\right)$$

$$\left\{\begin{array}{l} \cos(\theta) = \dfrac{x}{\sqrt{x^2+y^2}} = \dfrac{x}{r} \\ \sin(\theta) = \dfrac{y}{\sqrt{x^2+y^2}} = \dfrac{y}{r} \\ r = \sqrt{x^2+y^2} \end{array}\right.$$

- will fix strength issue, should behave proportionally
- field lines should no longer wrap inside coil
- disadvantage 1: glitch in B-field when you cross the circle off x-axis, may cause sudden emf

(over) →

(2005 -2-)

disadvantage 2:

For a single loop, this model is not very accurate.

Set current to fixed value in calculating M.

Rewriting in terms of (x,y):

outside:

$$Bx = \frac{m}{(x^2+y^2)^{1.5}}\left(\frac{3\cdot x\cdot x}{x^2+y^2} - 1\right)$$

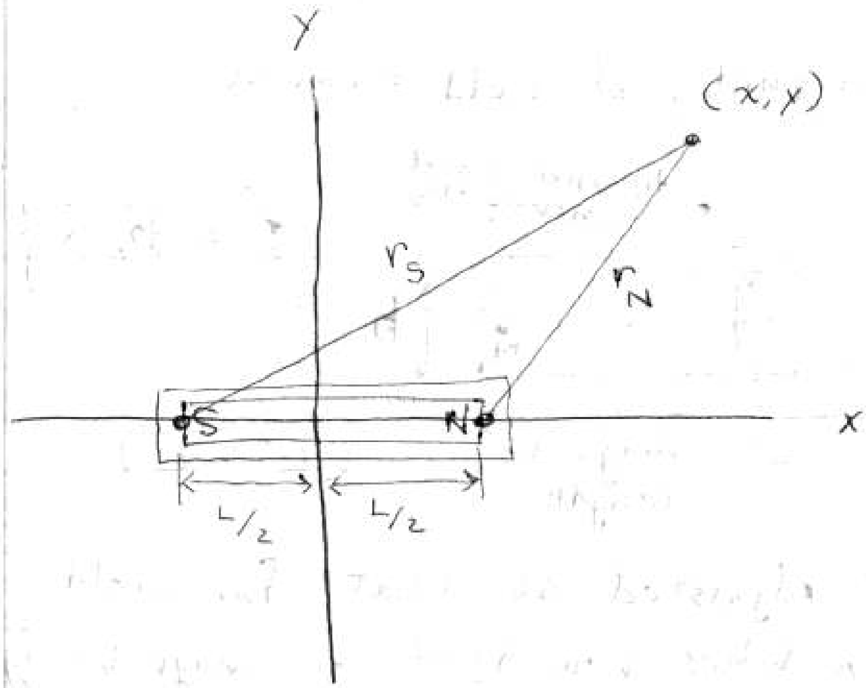
$$By = \frac{m}{(x^2+y^2)^{1.5}}\left(\frac{3\cdot x\cdot y}{x^2+y^2}\right)$$

Rewrite m in terms of magnet strength: (my contribution)

Inside the magnet, Bx = magnet strength.

So, $m = \frac{(\text{magnet strength}) * R^3}{2}$.

(Dipole) B-field of a bar magnet



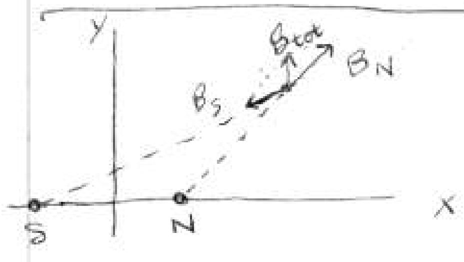
$$r_N = \sqrt{(x - L/2)^2 + y^2}, \quad r_S = \sqrt{(x + \tfrac{L}{2})^2 + y^2}$$

$$\vec{B}_{total} = \vec{B}_S + \vec{B}_N = \frac{-C}{r_S^3}\left[(x + \tfrac{L}{2}), y\right]$$

$$+ \frac{C}{r_N^3}\left[(x - \tfrac{L}{2}), y\right]$$

$$\vec{B}_{total} = [B_x, B_y] = C\left[\frac{-(x + \frac{L}{2})}{r_S^3} + \frac{(x - \frac{L}{2})}{r_N^3}, y\left(\frac{1}{r_N^3} - \frac{1}{r_S^3}\right)\right]$$

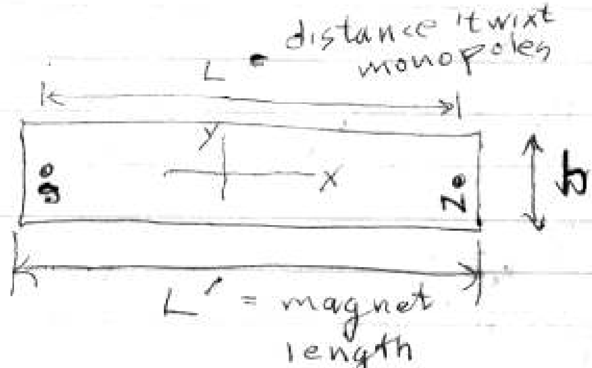
C = arbitrary (positive) fudge-factor



over

Near field vs. far-field B-field

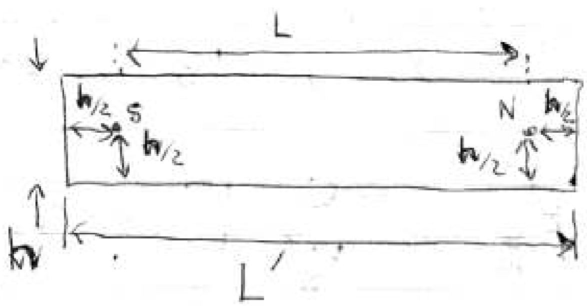
inside magnet, B-field = const



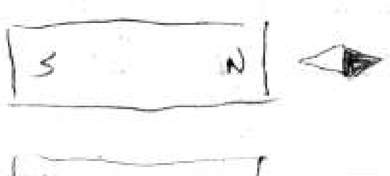
$$\vec{B} = B_0 \hat{x} \Big\} \begin{array}{l} |x| < L'/2 \\ |y| < h/2 \end{array}$$

$(L' > L)$

L, L' adjusted so that far-field
≈ matches near-field at magnet edges



$$q = B^{a} \leftarrow \text{fudge}$$



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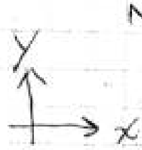
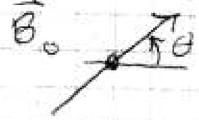
Needle w/ mass in field.





moment of Inertia I

magnet moment mu

(alpha) angular acceleration

(omega) angular velocity

$$\omega = \dot{\theta} = \frac{d\theta}{dt}, \quad \alpha = \ddot{\theta} = \frac{d\omega}{dt}$$

drag coeff

$$\text{torque } \vec{\tau} = \vec{r} \times \vec{F} = \underbrace{\vec{\mu} \times \vec{B}}_{\vec{\mu} \text{ aligns w/ } \vec{B}} - \underbrace{b\,\vec{\omega}}_{\text{friction}} = I\vec{\alpha}$$

$$\vec{\mu} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \mu_x & \mu_y & 0 \\ B_x & B_y & 0 \end{vmatrix} = \hat{k}(\mu_x B_y - \mu_y B_x).$$

$$(\vec{\mu} \times \vec{B})_z - b\omega = I\dot{\omega} \qquad (\text{sign?})$$

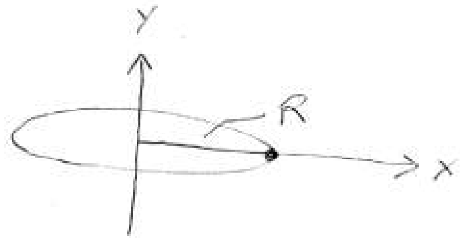
$$\alpha = \frac{[\mu_x B_y - \mu_y B_x] - b \cdot \omega}{I}, \qquad \mu_x = mu \cdot \cos\theta, \quad \mu_y = mu \cdot \sin\theta$$

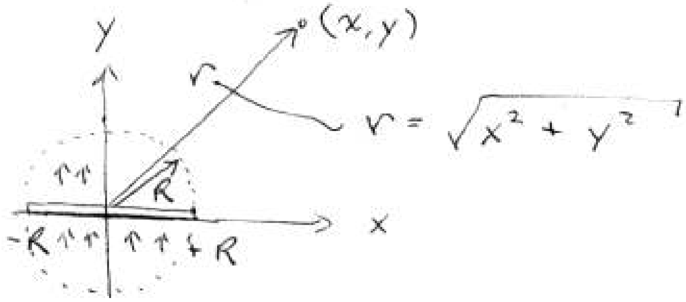
$$\left.\begin{aligned} &② \quad \omega_{n+1} = \omega_n + \alpha_n \cdot \Delta t \\ &① \quad \theta_{n+1} = \theta_n + \omega_n \cdot \Delta t + \tfrac{1}{2}\alpha_n \cdot \Delta t^2 \\ &③ \quad \alpha_{n+1} = \alpha[\omega_{n+1}, \theta_{n+1}] \end{aligned}\right\} \text{loop}$$

Adjustable variables

$\cancel{I,\ mu\,(=\mu)},\ b,\ \Delta t \Rightarrow \alpha = A(\cos\theta\, B_y - \sin\theta\, B_x) - B\omega$

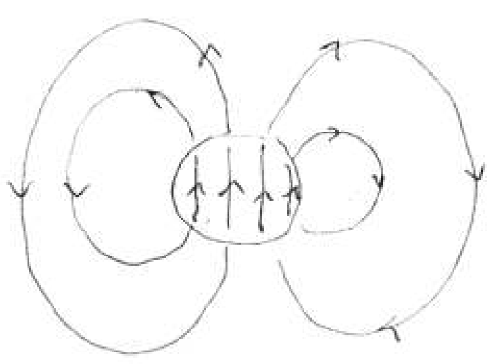
$A = \frac{mu}{I}, \quad \frac{b}{I} = B$

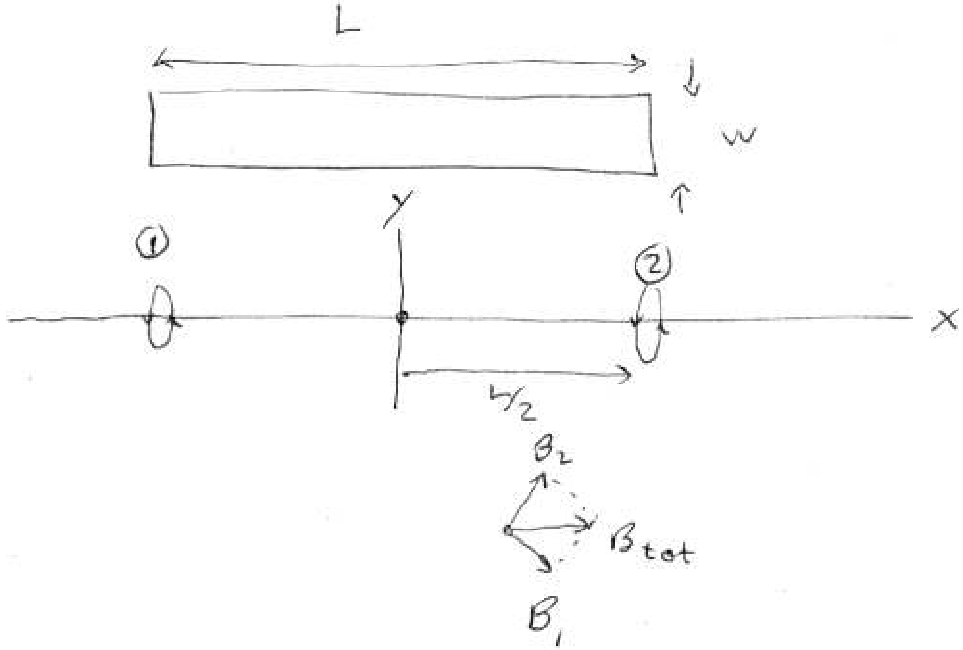




$$\vec{B}\begin{cases}\vec{B}_{near} = C \cdot I \;\hat{y}\,, \quad (r < R)\\[2ex] \vec{B}_{far} = \dfrac{C \cdot I}{2}\left(\dfrac{R}{r}\right)^2 \Big[\underbrace{\dfrac{3xy}{r^2}}_{\hat{x}}\,,\;\underbrace{\left(\dfrac{3y^2}{r^2} - 1\right)}_{\hat{y}}\Big]\,, \quad (r > R)\end{cases}$$

$$B_y = \frac{C\,I}{2}\left(\frac{R}{r}\right)^2\left(\frac{3y^2}{r^2} - 1\right)$$

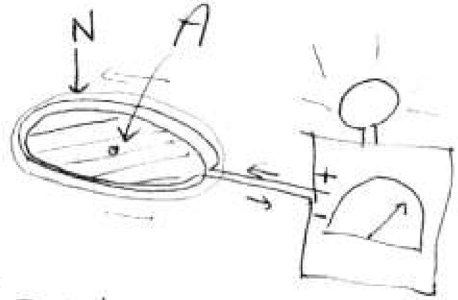



L
w
y
①
②
x
L/2
B_2
B_{tot}
B_1

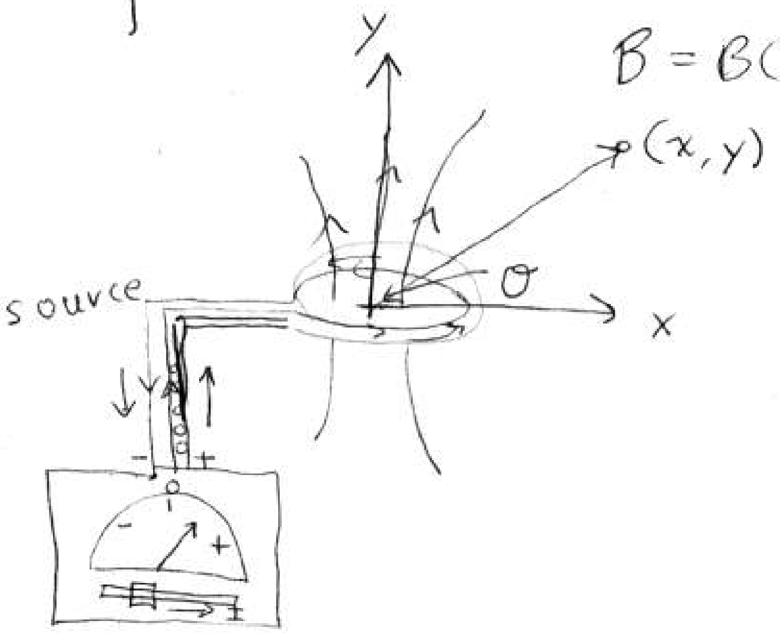
$\hat{y}$

$$|\mathcal{E}| = \frac{d\,\Phi}{dt} = \underbrace{N\frac{d}{dt}(BA)}$$

N = # Loop



$$B = B(x,y)$$



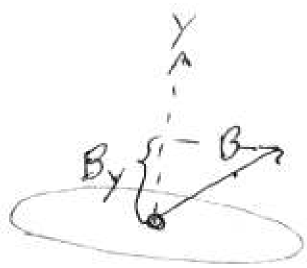
$$\Phi = N \cdot A \cdot \underline{\underline{B_{avg}}}$$

$$B_{avg} = B_{center\ loop}\ ✓$$

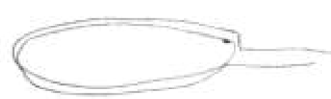
$$\neq \frac{B_{center} + B_{edge}}{2}$$

$$\Phi = N A B_y$$

y-component of B



N = 1 

N = 2 

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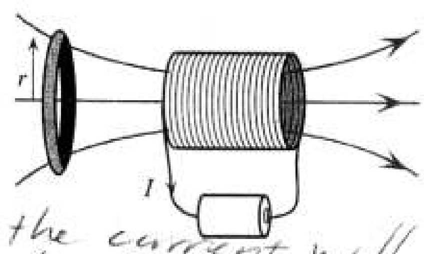
2. A copper wire loop is constructed so that its radius, r, can change. It is held near a solenoid that has a constant current through it.

   a. Suppose that the radius of the loop were increasing. Use Lenz' law to explain why there would be an induced current through the wire. Indicate the direction of that current.



Increasing radius = more flux through loop

Must fight change, therefore the current will flow clockwise up the front the front and back down the back.

   b. Check your answer regarding the direction of the induced current by considering the magnetic force that is exerted on the charge in the wire of the loop.

It is correct because the magnetic moment points to the left.

   c. Find:

   • the direction of the magnetic moment of the loop and

$\leftarrow \mu$ Points to the left.

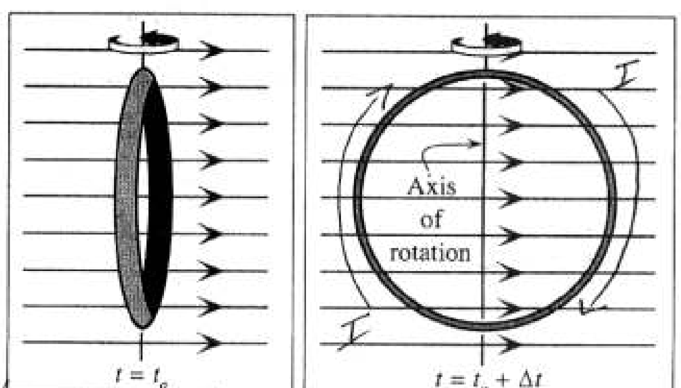
   • the direction of the force exerted on the loop by the solenoid.

F The force points to the left

why?

3. A copper wire loop is initially at rest in a uniform magnetic field. Between times $t = t_o$ and $t = t_o + \Delta t$ the loop is rotated about a vertical axis as shown.

   Will current flow through the wire of the loop during this time interval? If so, indicate the direction of the induced current and explain your reasoning. If not, explain why not.



Current flows during the interval since the flux through the loop gets reduced. We must fight the the change, therefore the induced current will flow clockwise as shown in the diagram.