COULOMB POTENTIAL FORMULA

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The titles of the two Coulomb wells in the drop down menu should be as follows:
1D Coulomb
Slice of 3D Coulomb (L=0)

For both types of Coulomb wells, the energy levels are given by:

$$E_n = -\frac{m(ke^2)^2}{2\hbar^2 n^2}$$

where $n = 1, 2, 3, ...$

The following formula will be used in calculating the wave functions for both types of Coulomb wells:

$$\psi_{nl}(x) = \frac{1}{\sqrt{4\pi}}|x|^l e^{-|x|/na} \sum_{j=0}^{n-l-1} b_j |x|^j$$

where

$$a = \hbar^2/(mke^2)$$

$$b_0 = 2(na)^{-3/2}$$

$$b_j = \frac{2}{na}\frac{j+l-n}{(j)(j+2l+1)}b_{j-1}$$

For now we should set $l = 0$ in all cases, but I would leave the formula with the l in it in the code, in case we want to set it to something else at some later date or for another simulation. l will always be a positive integer.

For 3D Coulomb,

$$\psi_n(x) = \sqrt{\pi}(na)^{3/2}\psi_{n0}(x)$$

For 1D Coulomb,

$$\psi_n(x) = A_n x \psi_{n0}(x)$$

where A_n is the normalization factor needed to scale the wave function such that the max is at 1. I don't know of an analytic formula for A_n, but since we're only showing the first 10 wave functions, I've calculated the first 10 normalization

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constants, and you can just put them in as an array:

$$A_1 = 1.10851$$
$$A_2 = -1.86636$$
$$A_3 = 2.55958$$
$$A_4 = -3.21387$$
$$A_5 = 3.84064$$
$$A_6 = -4.44633$$
$$A_7 = 5.03504$$
$$A_8 = -5.6096$$
$$A_9 = 6.17208$$
$$A_{10} = -6.72406$$

Important note about probability density:
For BOTH 1D and 3D Coulomb wells, the probability should be given by the absolute square of the **1D** wave function.