Additional exercises

This document contains additional exercises that are to be completed during the live sessions on Mondays and Tuesdays. Some of these are advanced exercises that require creative thinking. Completing these exercises will deepen your understanding of the taught material. They illustrate how the tools provided in the notes can be generalised to other chronometers, which we lack the time to discuss in detail. Practical exercises 3 and 4 in Part 2 of this document are quite involved. Do not be afraid to ask for additional tips or hints if you get stuck. I will be happy to share the solution to all the questions with you, but only after you give them a try yourself.

1 Theory

1. A mass spectrometer has a radius of 0.3 m and an acceleration voltage of 10 000 V. The magnetic field is adjusted to the various masses to be measured. Calculate the atomic mass of a single charged ion corresponding to a field of 0.5 T. Note: the elementary charge is 1.60219×10^{-19} C and 1 atomic mass unit (amu) = 1.660538×10^{-27} kg.

2. What types of radioactivity are involved in the following reactions: ${}^{146}_{62}\text{Sm} \rightarrow {}^{142}_{60}\text{Nd}$, ${}^{53}_{25}\text{Mn} \rightarrow {}^{53}_{24}\text{Cr}$, ${}^{230}_{90}\text{Th} \rightarrow {}^{226}_{88}\text{Ra}$?

3. Rb has two isotopes, ^{85}Rb and ^{87}Rb, which exist in a constant ratio in our Solar System: $^{85}\text{Rb}/^{87}\text{Rb} = 2.5933$. The molar mass of Rb is 85.4678. A rock contains 10ppm of Rb. How many mol/g of ^{87}Rb does that correspond to?

4. Following up from the previous question: ^{87}Rb is radioactive with a half-life $t_{1/2} = 48.8$ Gyr. What was the $^{85}\text{Rb}/^{87}\text{Rb}$-ratio 2 billion years ago?

5. The half-life of ^{14}C is 5730 years. The ^{14}C content of the atmosphere is 13.2 disintegrations per minute and per gram (dpm g^{-1}) of carbon (initial activity A_0). We wish to date an Egyptian artefact from approximately 2000 BC. What is the approximate activity (A) of this artefact? If our method can measure 1 dpm, what mass of the (probably precious) sample will have to be destroyed?

6. ^{210}Pb is a short lived ($t_{1/2}$=22.3 yr) intermediate daughter of the ^{238}U decay chain. This natural radioactive lead is incorporated into ice deposited in Greenland by forming successive layers of ice which can be studied like sedimentary strata. The activity of ^{210}Pb is measured at four levels in disintegrations per hour per kilogram of ice.

	1	2	3	4
depth (m)	0	1.0	1.5	2.5
^{210}Pb (dph/kg)	75	34.5	23.4	10.7

   Calculate the sedimentation rate of the ice. Assuming a constant rate and a compaction factor of 5, how much ice will have accumulated in 5000 years? Calculate the ^{210}Pb-content of fresh ice (in mol/kg or atoms/kg).

7. Consider the Rb-Sr composition of three aliquots from the same sample:

	$^{87}Rb/^{86}Sr$	$^{87}Rb/^{86}Rb$
mineral A	0.05	0.70108
mineral B	0.10	0.70215
mineral C	0.20	0.70431

How old is the sample?

8. What is the expected $^{207}Pb/^{206}Pb$-ratio of a 4.57 billion year old meteorite? Assume that the meteorite contained no initial Pb. The half-lives of ^{238}U and ^{235}U are 4.468 and 0.704 Gyr, respectively. The present-day $^{238}U/^{235}U$-ratio is 137.818. What is the expected $^{207}Pb/^{206}Pb$-ratio for modern uranium-ores?

9. Consider the following three aliquots from the same sample:

	$^{206}Pb/^{204}Pb$	$^{207}Pb/^{204}Pb$	$^{238}U/^{204}Pb$
aliquot A	9.00	10.00	0
aliquot B	45.38	14.47	100
aliquot C	56.85	15.12	180
aliquot D	42.56	12.43	200

   a. Calculate the $^{206}Pb/^{238}U$ and $^{207}Pb/^{235}U$ ages

   b. Sketch a geological history for the sample (hint: consider the U-Pb composition in concordia space).

10. Consider the Pb-Pb composition of three aliquots from a meteorite:

	1	2	3
$^{206}Pb/^{204}Pb$	107.0	407	1010
$^{207}Pb/^{204}Pb$	65.8	251	622

Use the following lookup table to estimate the age of the meteorite:

age (Ma)	4450	4475	4500	4525	4550	4575	4600	4625	4650
$^{207}Pb/^{206}Pb$	0.577	0.587	0.597	0.607	0.618	0.629	0.64	0.651	0.662

Hint: plot $^{207}Pb/^{204}Pb$ vs. $^{206}Pb/^{204}Pb$.

11. The silicate Earth weighs approximately 4×10^{24} kg and contains 250 ppm potassium. How much ^{40}Ar is created in 4.5 billion years? Given that ^{40}Ar cannot escape from the Earth and the quantity of ^{40}Ar in the atmosphere is 66000 Gt, what is the “K-Ar age of the Earth's atmosphere”? K has three isotopes with $^{39}K/^{40}K = 7771$ and $^{41}K/^{40}K = 561$. ^{40}K is radioactive and undergoes branched decay to ^{40}Ar (10.72%) and ^{40}Ca (89.28%) with a half-life of 1.248 billion years. The molar mass of K is 39.1 g/mol.

12. ^{129}I is an extinct nuclide of Iodine with a half-life of 15.7 million years. It decays (or rather: decayed!) to the noble gas ^{129}Xe. Although there is no (natural) ^{129}I in the Solar System today, the nuclide did exist during its earliest stages. Iodine-rich meteorites that existed within the first few 10s of millions of years of Solar System history contain measurable amounts of excess of ^{129}Xe.

    Consider two meteorites with $^{129}Xe/^{127}I$ ratios of 13×10^{-5} and 8×10^{-5}, respectively. What is the age *difference* between the two meteorites? Assume that all the ^{129}Xe is radiogenic, and that the early Solar System had a homogeneous $^{129}I/^{127}I$-composition.

13. Given the ^{40}Ar/^{39}Ar-data shown in the following table:

	^{39}Ar/^{36}Ar	^{40}Ar/^{36}Ar
std	100	2536
smp1	10	820
smp2	110	5519
smp3	160	7869

'std' is a 98 Ma reference material whose inherited argon content is of atmospheric origin ($[^{40}\text{Ar}/^{36}\text{Ar}]_{\text{atm}}$=298.5); 'smp1', 'smp2' and 'smp3' are three cogenetic aliquots of a metamorphic sample of unknown age. What are the age and non-radiogenic ^{40}Ar/^{36}Ar-ratio of the sample? Recall that the half-life of ^{40}K is 1248 Myr.

14. Analysis of a granite yields:

^{206}Pb*/^{238}U (zircon)	^{40}Ar*/^{40}K (amphibole)	^{40}Ar*/^{40}K (biotite)	^{87}Sr*/^{87}Rb (muscovite)	^{87}Sr*/^{87}Rb (biotite)
0.084	0.304	0.248	0.00659	0.00645

What is the likely age of intrusion of this granite? How reliable is the result?

15. Age-elevation profiles are a powerful method to determine tectonic exhumation rates: by measuring the U-Th-He (and/or fission track) ages of samples collected at different elevations along a steep mountain transect, it is possible to determine vertical travel times from the closure isotherm ($\sim 60°$C) to the Earth's surface. Consider the following dataset:

elev (m)	He (pmol/g)	s(He)	U (ppm)	s(U)	Th (ppm)	s(Th)
2000	318	10	10	1	20	1
3000	503	10	12	1	15	1
3500	447	10	9	1	12	1
4000	903	10	15	1	25	1

   (a) Convert these data to internally consistent atomic units.

   (b) Plug them into the online version of `IsoplotR` (`http://isoplotr.es.ucl.ac.uk`).

   (c) Calculate the U-Th-He ages.

   (d) Determine the exhumation rate.

16. Consider a rapidly cooled (volcanic) apatite of 10 Ma with an atomic Th/U ratio of 5. What is the ratio of helium atoms to fission tracks in it? Look up the relevant decay constants in the notes.

17. Recent advances in analytical chemistry allow fission track geochronologist to replace the trusted external detector method with an alternative approach, in which the U-content of the sample is measured directly by Laser Ablation Inductively Coupled Mass Spectrometry (LA-ICP-MS), the same technique that is also used for in-situ U-Pb geochronology. Using LA-ICP-MS, it is possible to compute fission track ages using Equation 7.8 of the notes. Calculate the fission track age and propagate its uncertainty for the following apatite data:

parameter	value
fission track track count	49
counting area	5000 μm^2

parameter	value
fission track length	15 µm
U-concentration	10 ppm
density of apatite	3.19 g/cm^3
molar mass of U	238.03 g/mol

18. Consider a rock whose U-Th-He age is 50 Ma. Suppose that the following data were acquired on the same sample:

J	0.005
^{39}Ar/^{36}Ar	100
^{40}Ar/^{36}Ar	862
zeta (yr cm^2)	300
rho$_D$ (cm^{-2})	2500000
N$_s$	13
N$_i$	100

Assuming that the ^{36}Ar is of atmospheric origin (with ^{40}Ar/^{36}Ar$_{atm}$=298.5), and using the notes for decay constants etc., summarise the geological history of the sample.

19. Which of the following pairs of grain sizes and cooling rates are characterised by the same closure temperature?

r (micron)	50	200	100	400	200	100
dT/dt (K/Myr)	40	5	50	5	10	20

20. The ^{234}U/^{238}U activity ratio of sea water rarely exceeds 1.15. However, much higher activity ratios have been measured in fresh waters and cave carbonates (speleothems). Suppose that we have measured a ^{234}U/^{238}U activity ratio of 1.1 in a 1 Ma old speleothem. What was its *initial* ^{234}U/^{238}U-ratio?

21. Although U-series disequilibrium measurements were historically done by α-counting, in recent decades this method has been replaced by mass spectrometry. Consider the following composition of three cogenetic aliquots:

ppm U	ppm Th	^{230}Th/^{232}Th
2	5	0.000005107
4	3	0.000010727
5	3	0.000012734

How old is the rock?

2 Practicals

1. Write a function to simulate radioactive decay, following the recipe shown in slide 13 of the first theory presentation. Compare the numerical approximation with the analytical solution. Explore the effect of different time steps on the accuracy of the simulation.

2. Plot the U-Pb composition of sample 91500 on a Tera-Wasserburg concordia diagram (^{207}Pb/^{206}Pb vs. ^{238}U/^{206}Pb). Use a ^{238}U/^{235}U-ratio of 137.818 for the conversion from Wetherill to Tera-Wasserburg space.

3. Visit `https://github.com/pvermees/geotopes/tree/master/practicals/ArAr/` and download files `blnk.csv`, `stnd.csv` and `smpl1.csv` through `smpl10.csv`. Process these files using your Ar-Ar data reduction script. Plot the results as an Ar-Ar release spectrum, using the following function. `Ar39` and `tx` are both vectors containing the blank corrected ^{39}Ar signals and ^{40}Ar/^{39}Ar age estimates, respectively:

```
spectrum <- function(Ar39,tx){
  Ar39n <- cumsum(c(0,Ar39))/sum(Ar39) # normalise
  plot(x=Ar39n, y=c(tx[1],tx), type='s',
       xlab=expression(''^39*"Ar (%)"), ylab='Ar-Ar age (Ma)')
}
```

4. Visit `https://github.com/pvermees/geotopes/tree/master/practicals/UPb/` and download files `GJ1.csv` (standard) and `smp1.csv` through `smp4.csv` (samples).

   (a) Process these files using your U-Pb data reduction functions and save the atomic ^{207}Pb/^{235}U and ^{206}Pb/^{238}U-ratios and their uncertainties in four vectors.

   (b) Write a function to determine the error correlation between the blank-corrected ^{207}Pb/^{235}U and ^{206}Pb/^{238}U-ratios of the samples.

   (c) Form a matrix with four rows (one for each sample) and five columns with the ^{207}Pb/^{235}U-ratios, the standard errors of the ^{207}Pb/^{235}U-ratios, the ^{206}Pb/^{238}U-ratios, the standard errors of the ^{206}Pb/^{238}U-ratios, and the error correlations obtained from part (b).

   (d) Enter the values in this matrix into the online version of `IsoplotR` (`http://isoplotr.es.ucl.ac.uk`). Note: change to input format 1 in the `Options` menu. Visualise the data on a concordia diagram.

5. Install `IsoplotRgui` on your computer using `install.packages('IsoplotRgui')`. Start the program by typing `IsoplotRgui::IsoplotR()`.

   (a) Open the `RbSr.csv` file from Moodle (first practical) and paste the data into `IsoplotRgui`. Compare with the output of exercise 12.1.12 of the notes.

   (b) Visit `https://github.com/pvermees/IsoplotR/tree/master/inst` and download the following files: `UPb2.csv`, `ArAr3.csv`, `ThU2.csv`.

   (c) Using the insights learned from step (a), plot `UPb2.csv` on a concordia diagram; first using the GUI (`IsoplotRgui`) and then from the command line (`IsoplotR`).

   (d) Plot `ArAr3.csv` as an age spectrum, first using `IsoplotRgui`, then using `IsoplotR`.

   (e) Plot the U-series dataset `ThU2.csv` as an isochron, first using `IsoplotRgui`, then using `IsoplotR`.

   Hint: besides `read.data`, you will also need `IsoplotR`'s `concordia`, `agespectrum` and `isochron` functions. See `?concordia` etc. for help, or download the complete documentation from `https://CRAN.R-project.org/package=IsoplotR`.