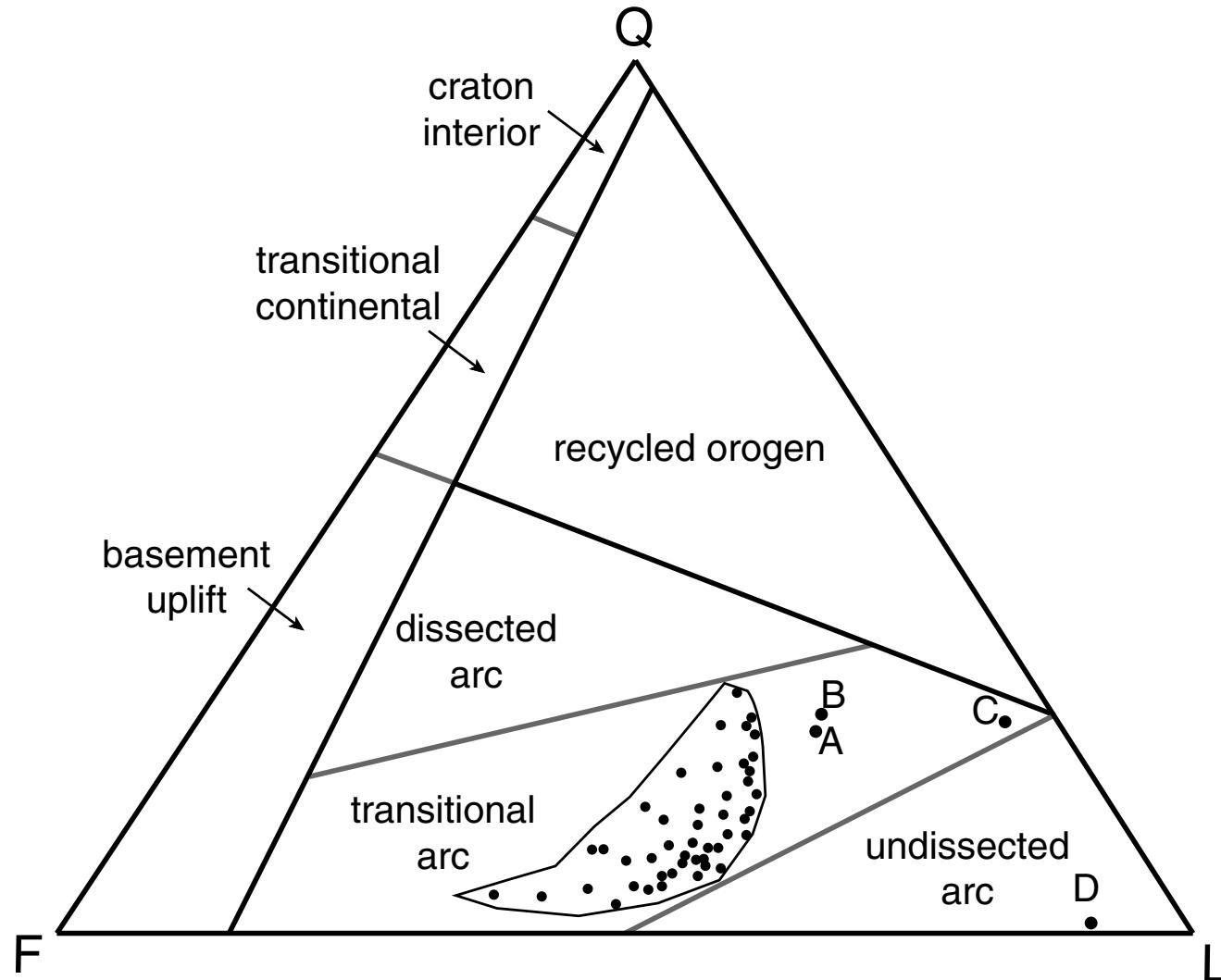


The statistical uncertainties associated with histograms in the Earth Sciences

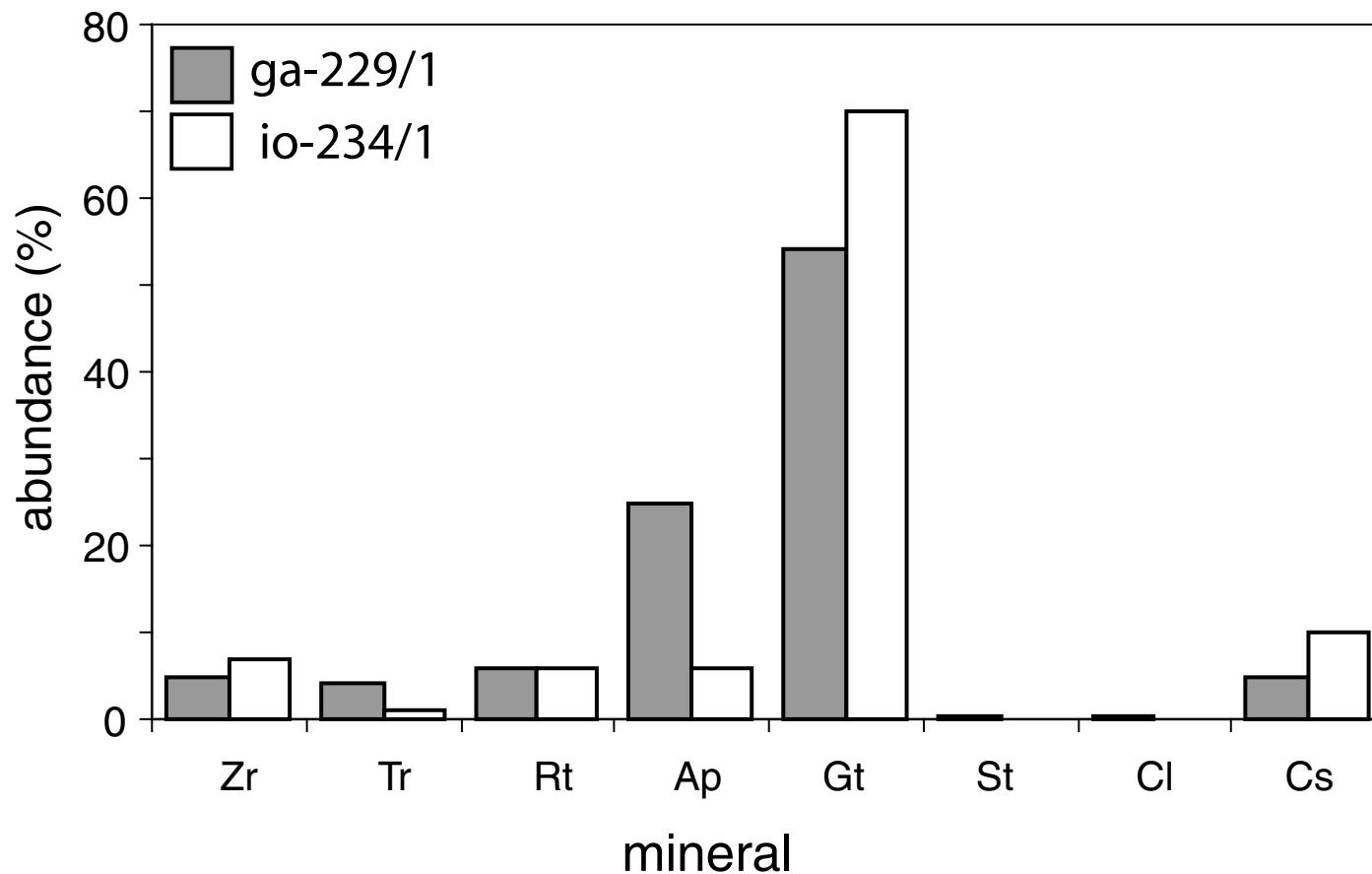
Pieter Vermeesch

November 4, 2004

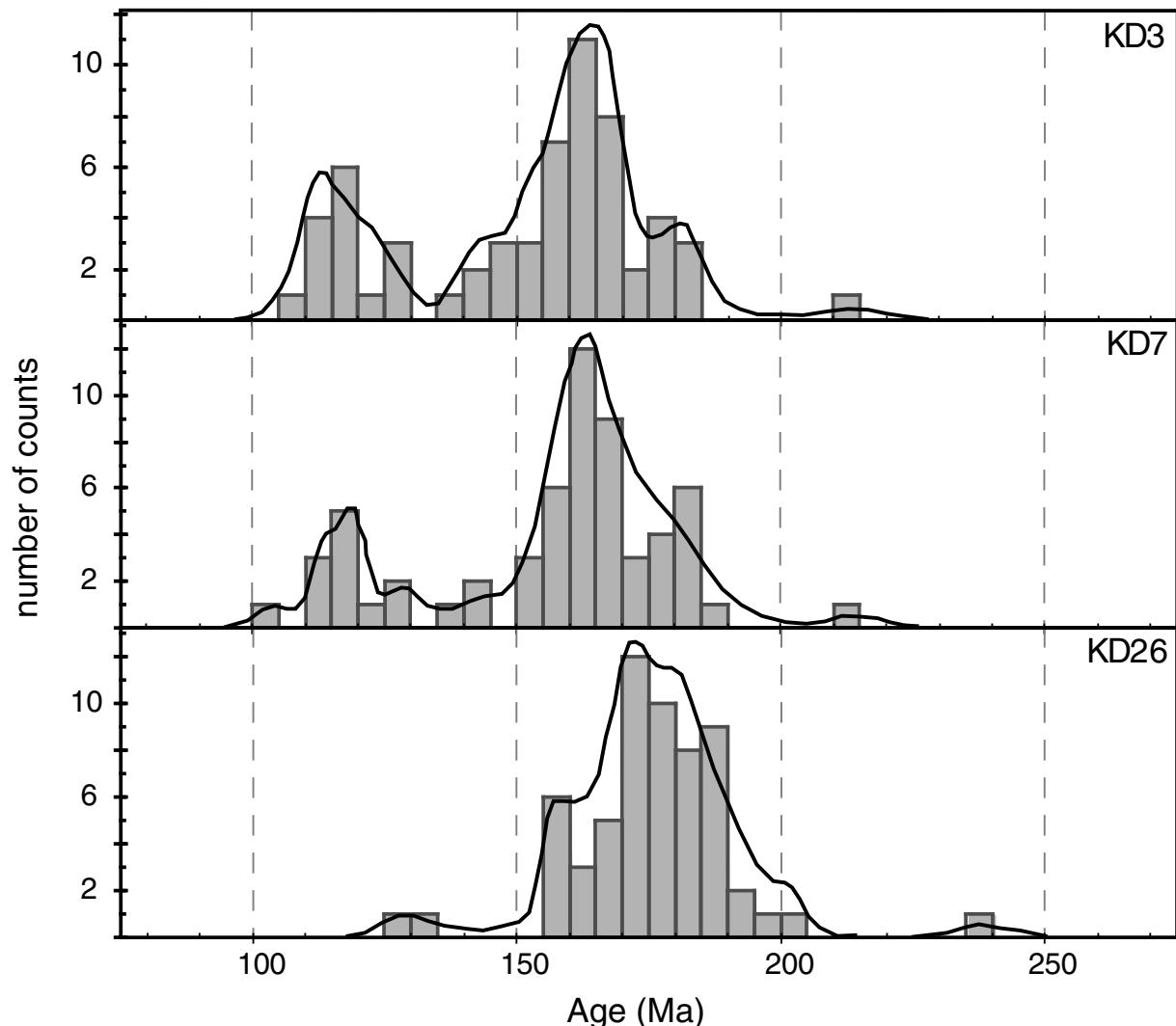
Petrographic ternary diagrams



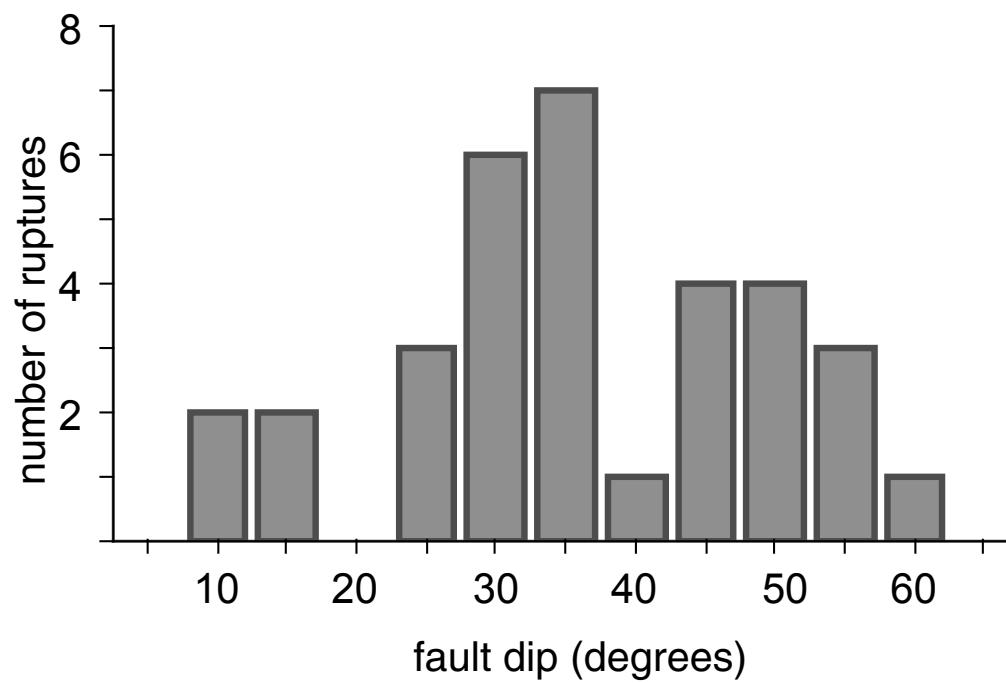
Heavy mineral analysis (Faupl et al, 2002)



Detrital thermochronology (DeGraaff-Surpless et al, 2003)



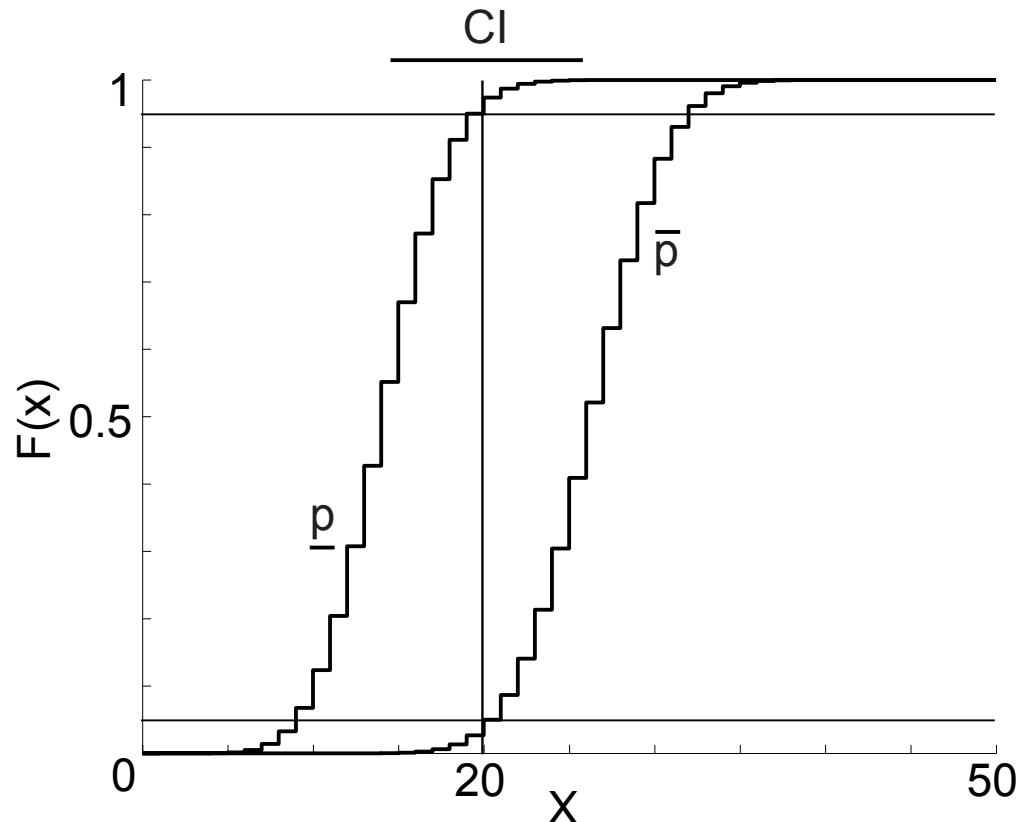
Structural geology (Colletini and Sibson, 2001)



The definition of a $(1-\alpha)\%$ confidence interval for θ

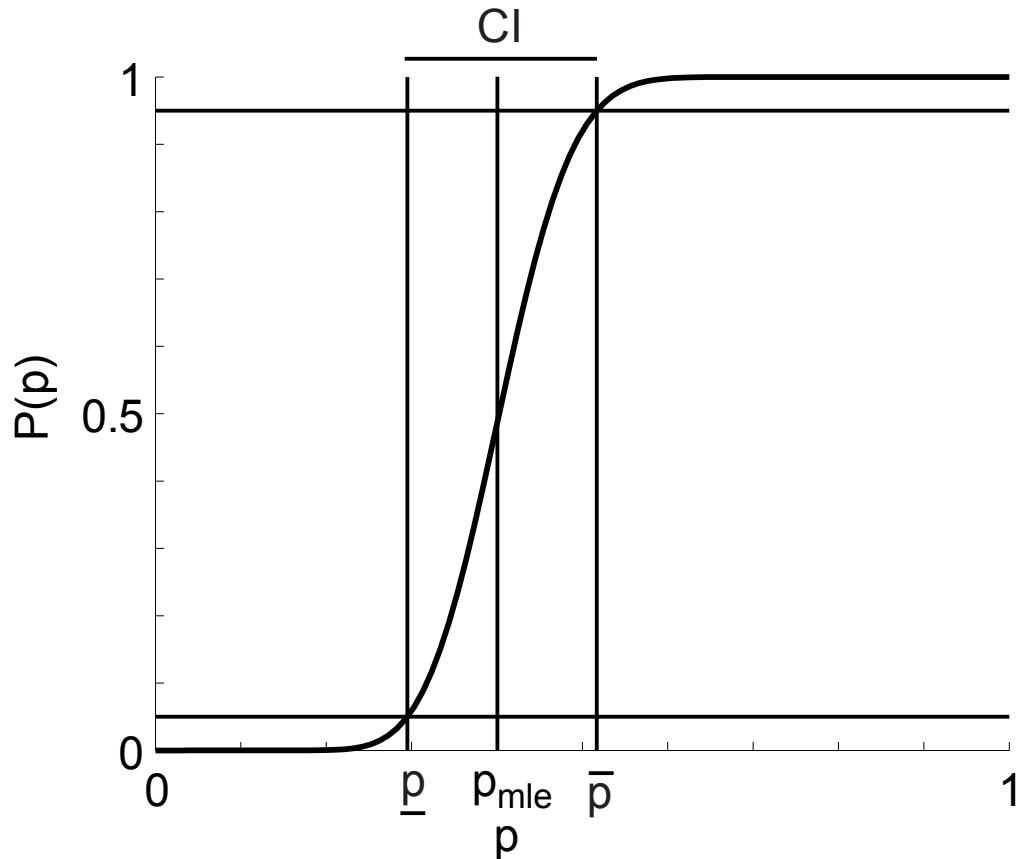
- Frequentist: *all those values of θ_0 for which the null hypothesis $H_0: \theta=\theta_0$ is accepted with more than $\alpha\%$ likelihood*
- Bayesian: *an interval for θ covering $(1-\alpha)\%$ of its posterior distribution*

The frequentist paradigm



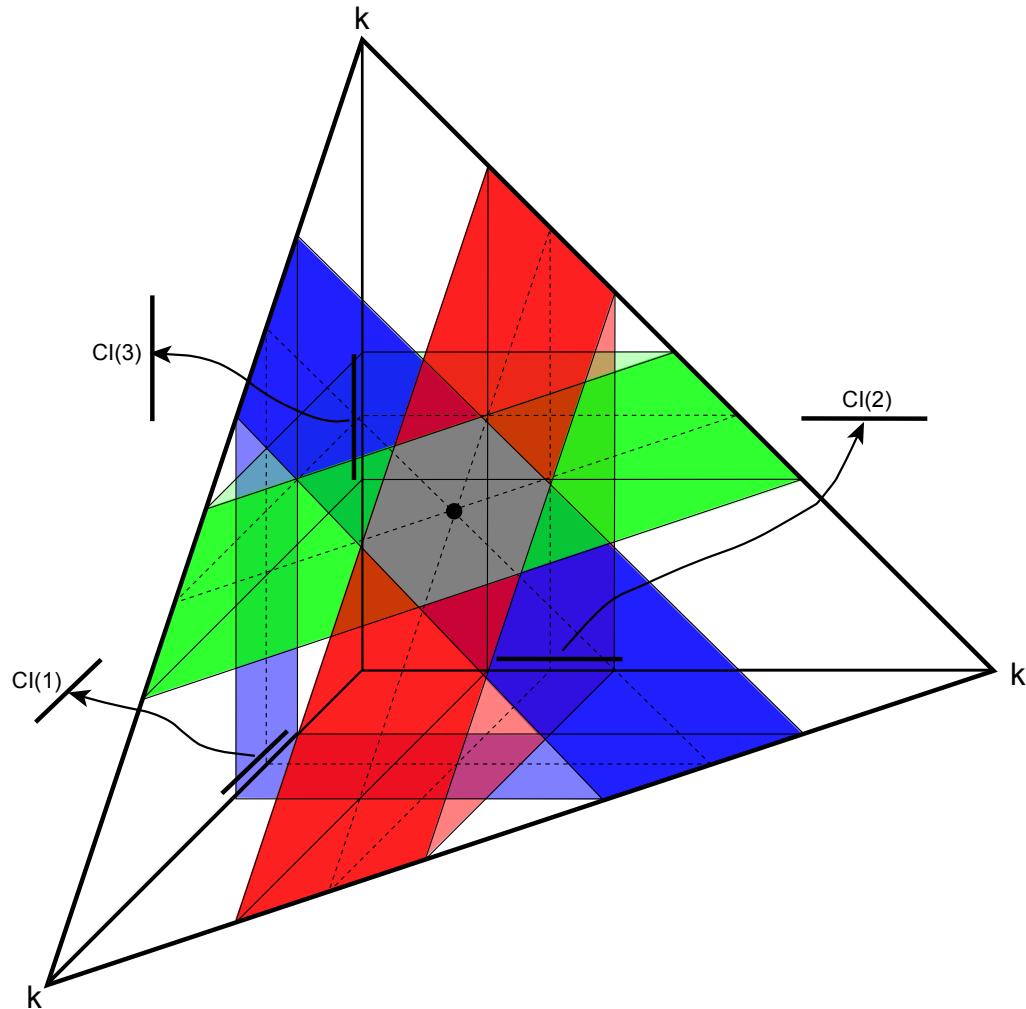
$$\sum_{x=n}^N \binom{N}{x} p_o^x p_o^{N-x} < \frac{\alpha}{2}$$
$$\sum_{x=0}^n \binom{N}{x} p_o^x p_o^{N-x} > \frac{\alpha}{2}$$

The Bayesian paradigm



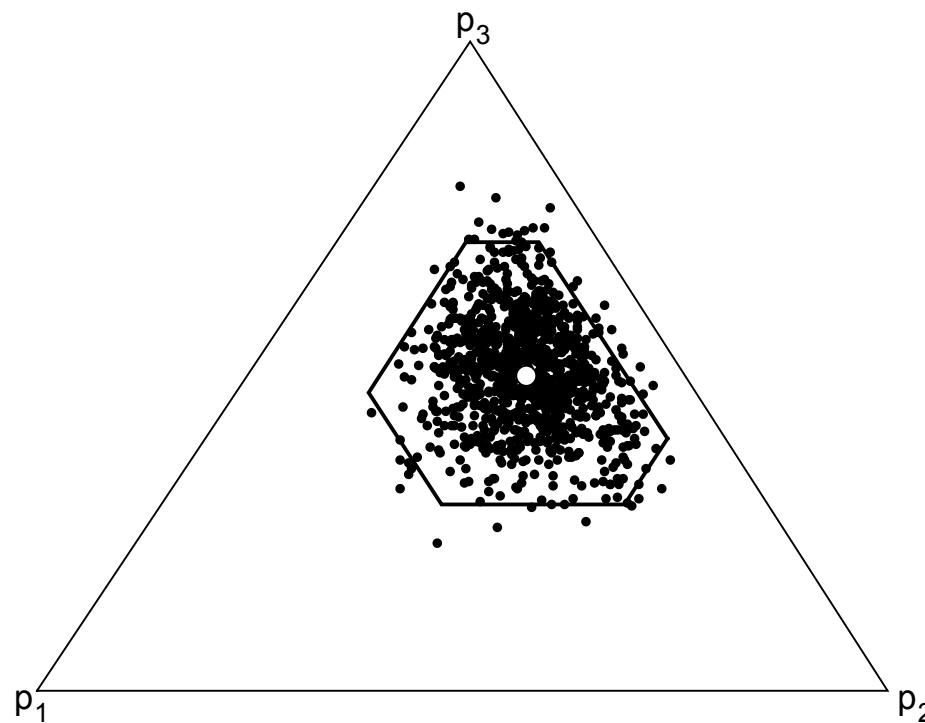
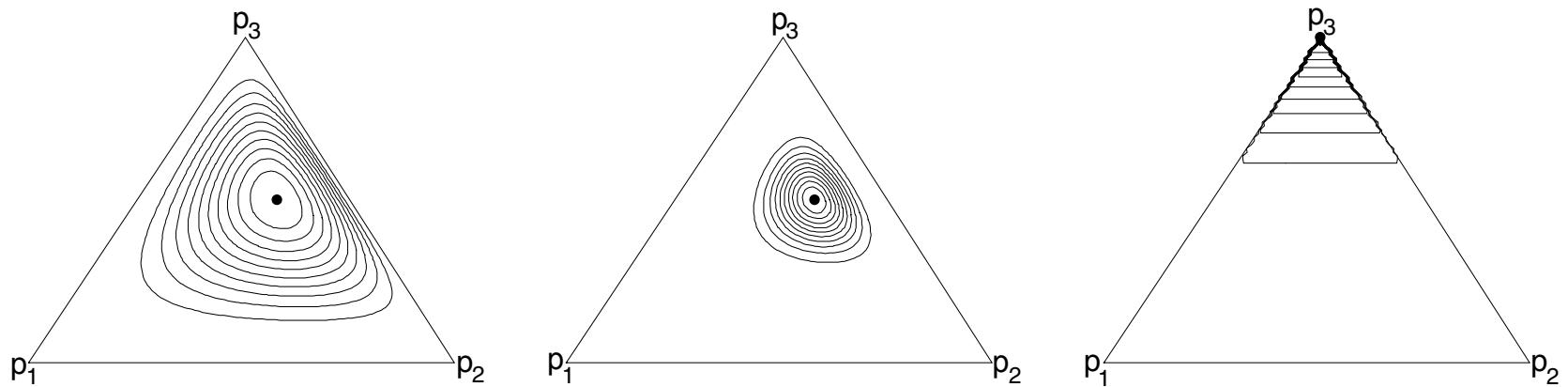
$$P(p|x) \propto P(x|p)P(p)$$

Bayesian posteriors on Δ_2

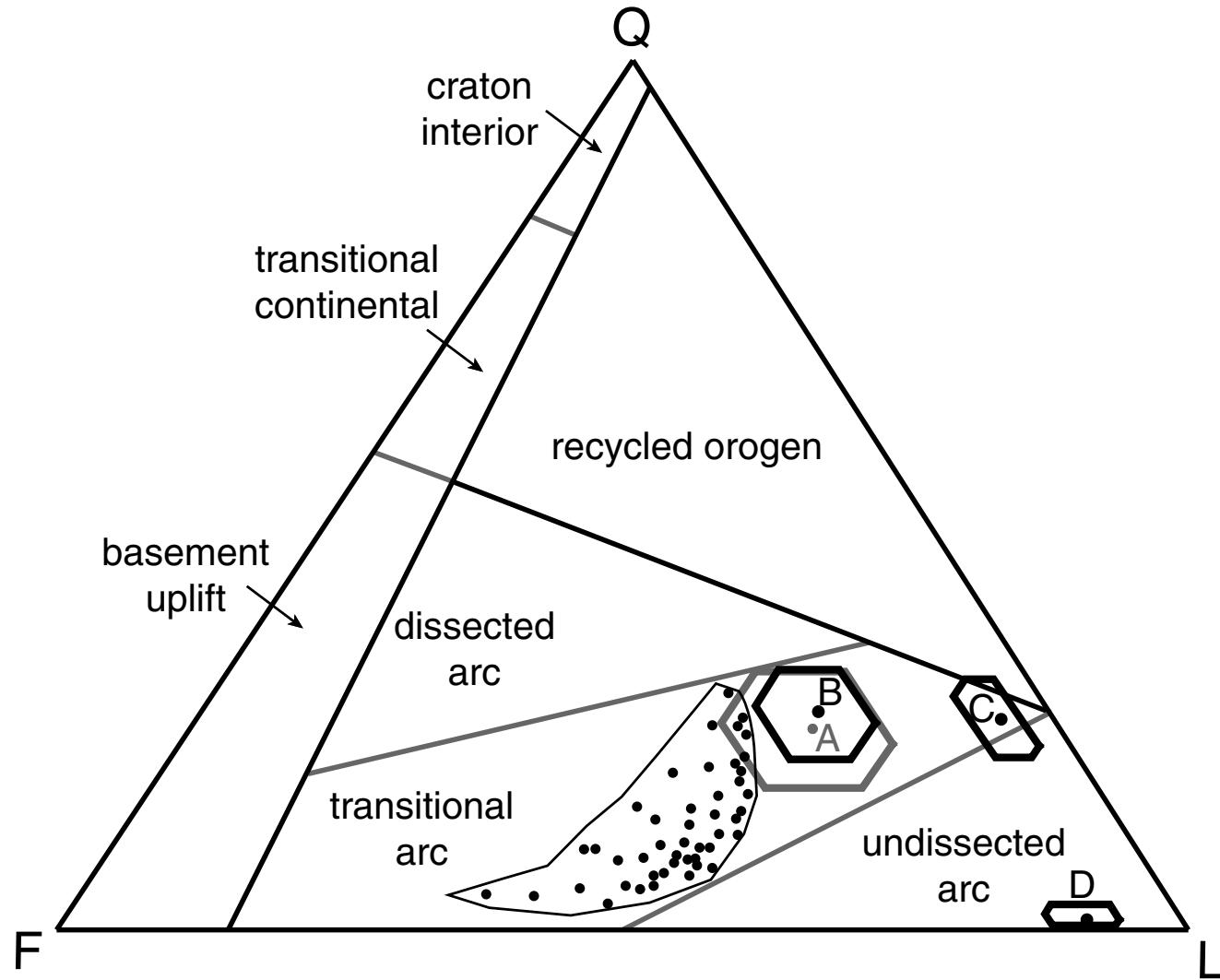


$$D\alpha(p_1, \dots, p_M) = \frac{\Gamma(\sum_{i=1}^M a_i)}{\prod_{i=1}^M \Gamma(a_i)} \prod_{i=1}^{i=M} p_i^{a_i-1}$$

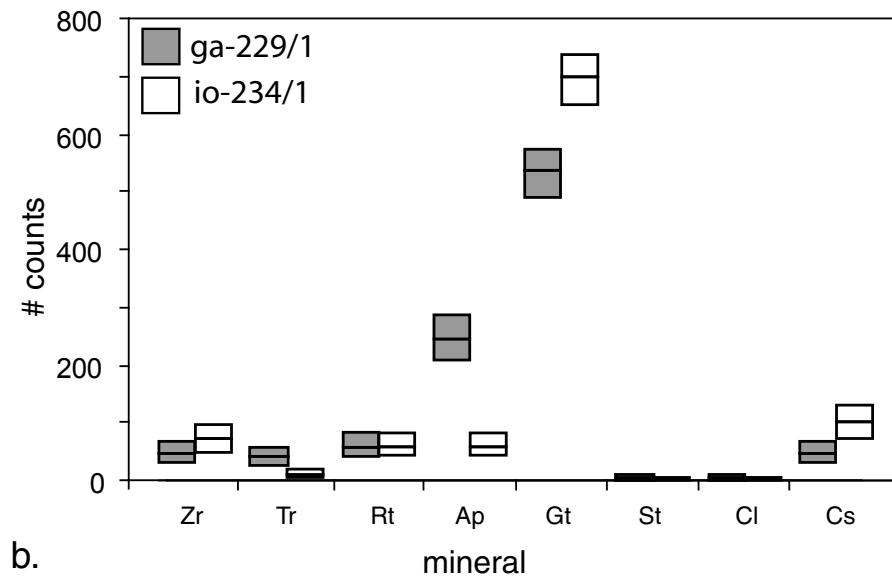
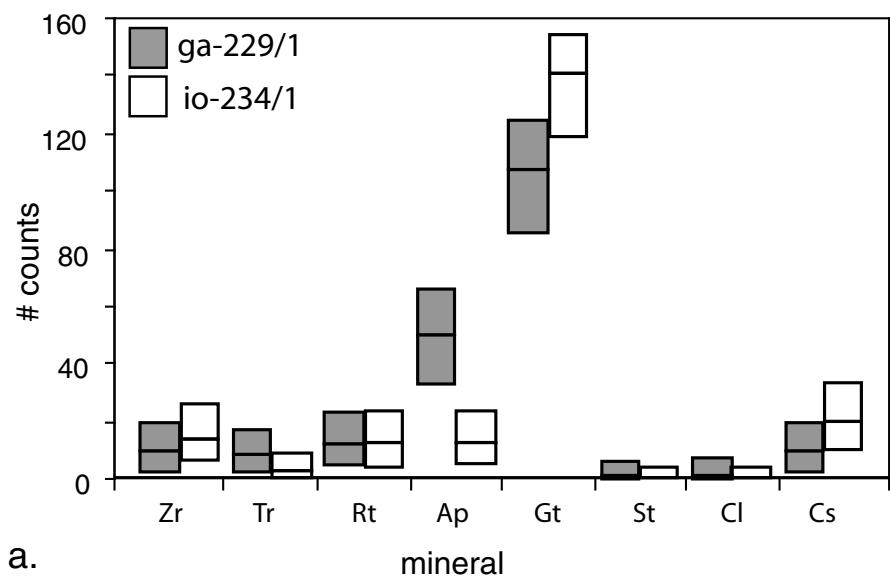
Dirichlet posteriors on Δ_2



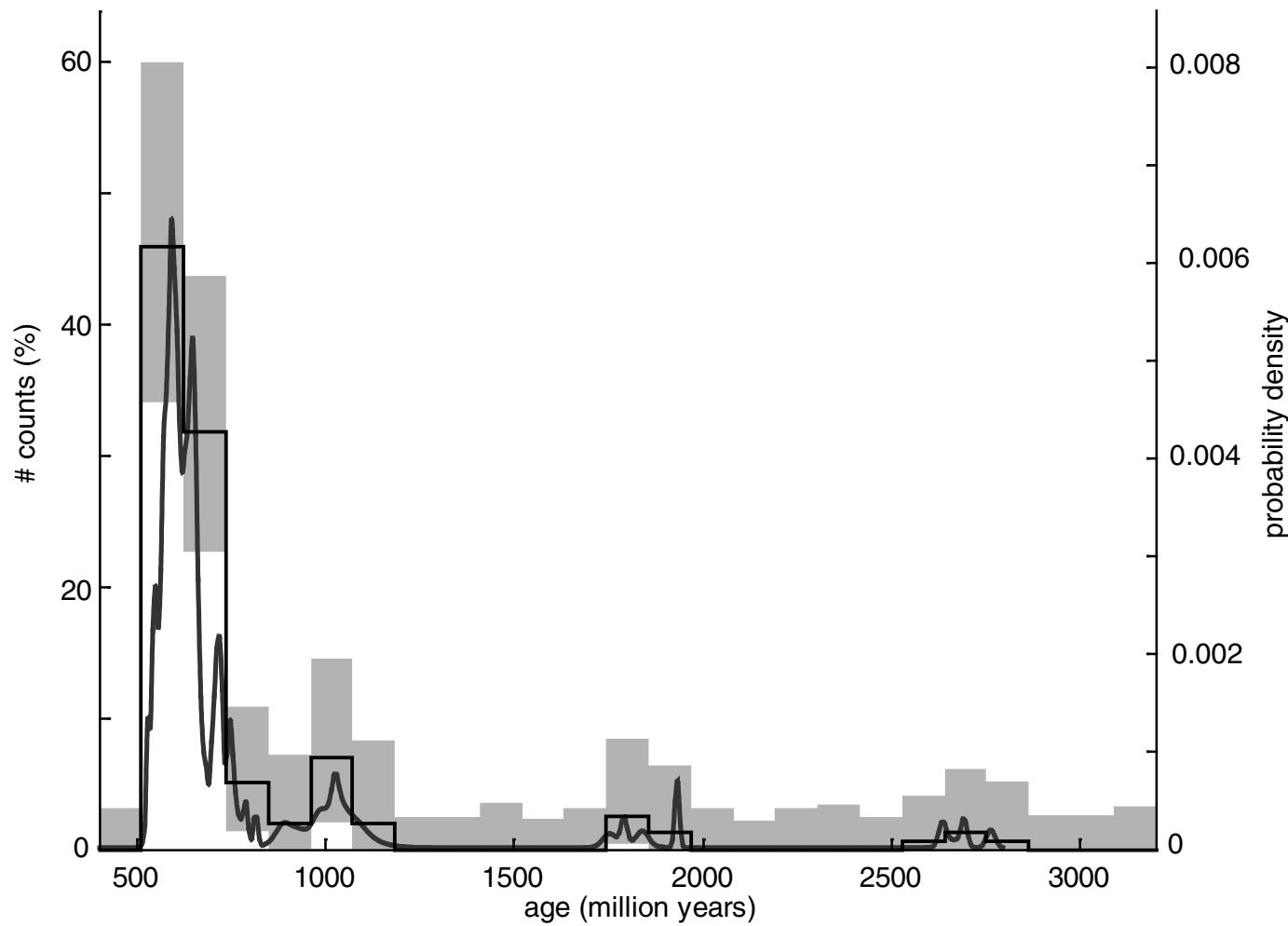
Ternary petrographic diagram revisited



Heavy minerals revisited



The Nubian Sandstone (Avigad et al, 2003)



Histograms of time-series

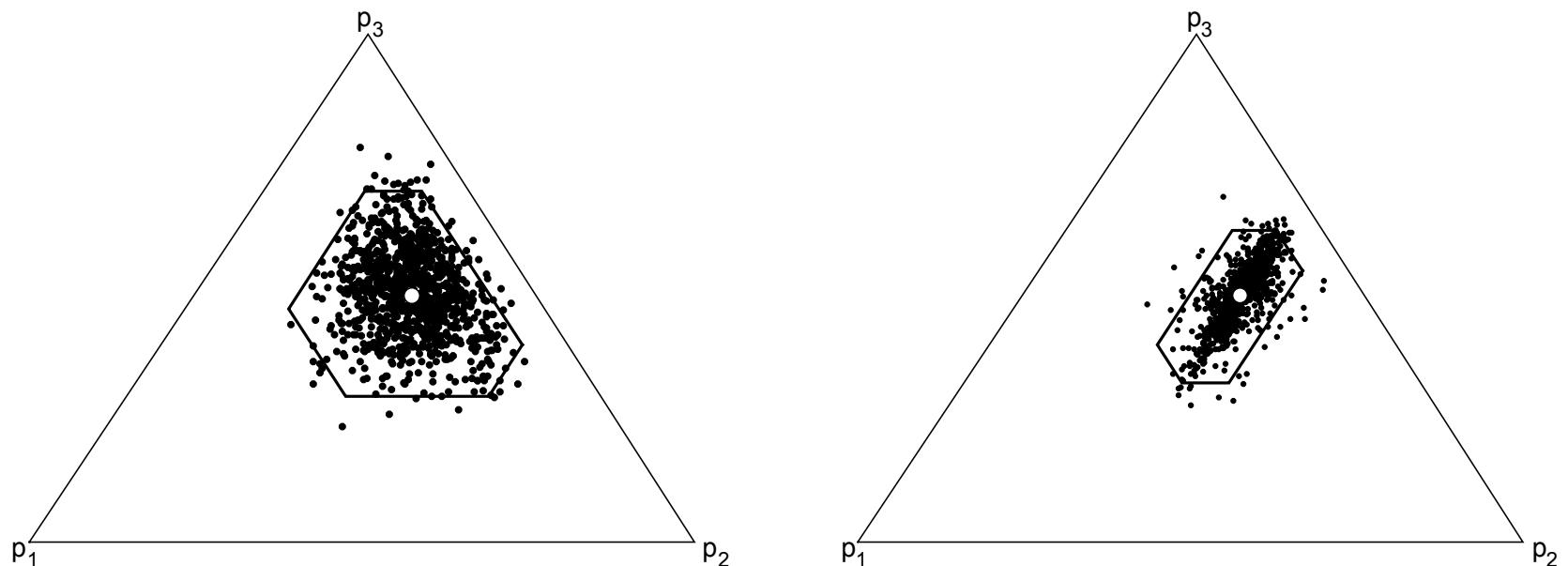
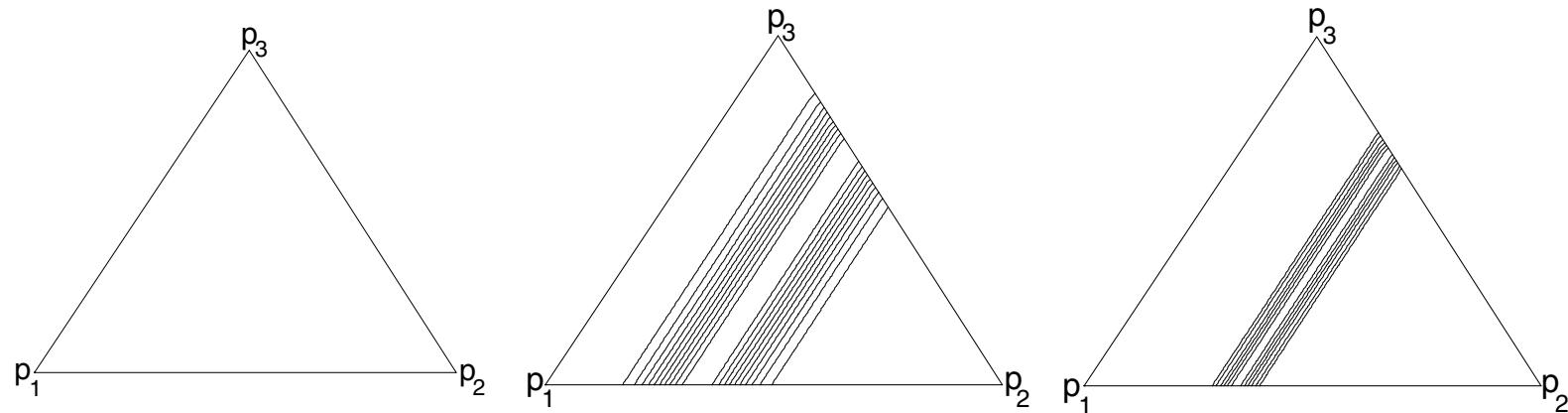
- Adjacent bins are *autocorrelated*
- \Rightarrow Use a *smoothing prior*:

$$r(g(t)) = f \left(\int \left(\frac{d^2 g(t)}{dt^2} \right)^2 dt \right)$$

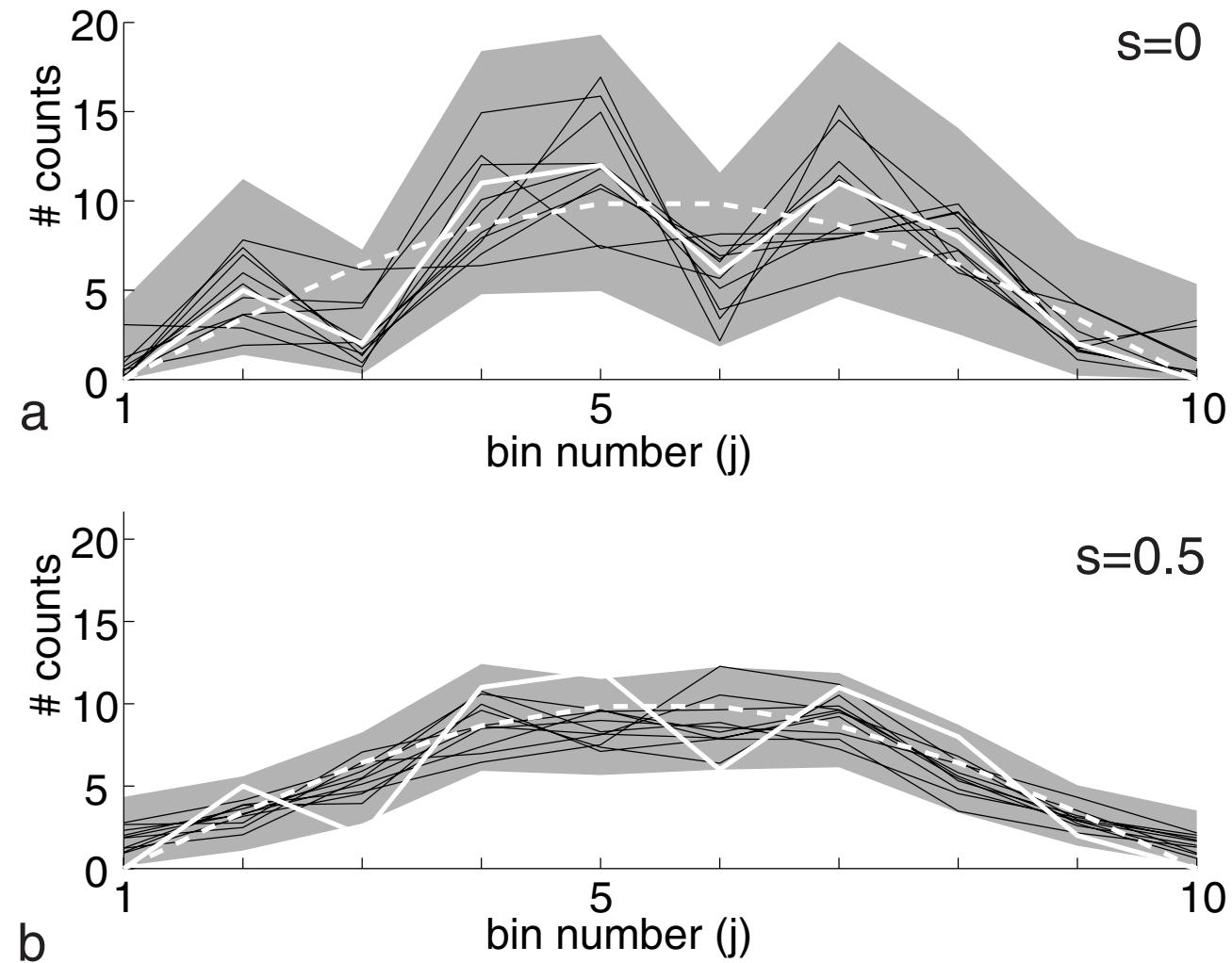
$$r(n) = (n_1 - 2n_2 + n_3)^2$$

$$w = e^{-sr}$$

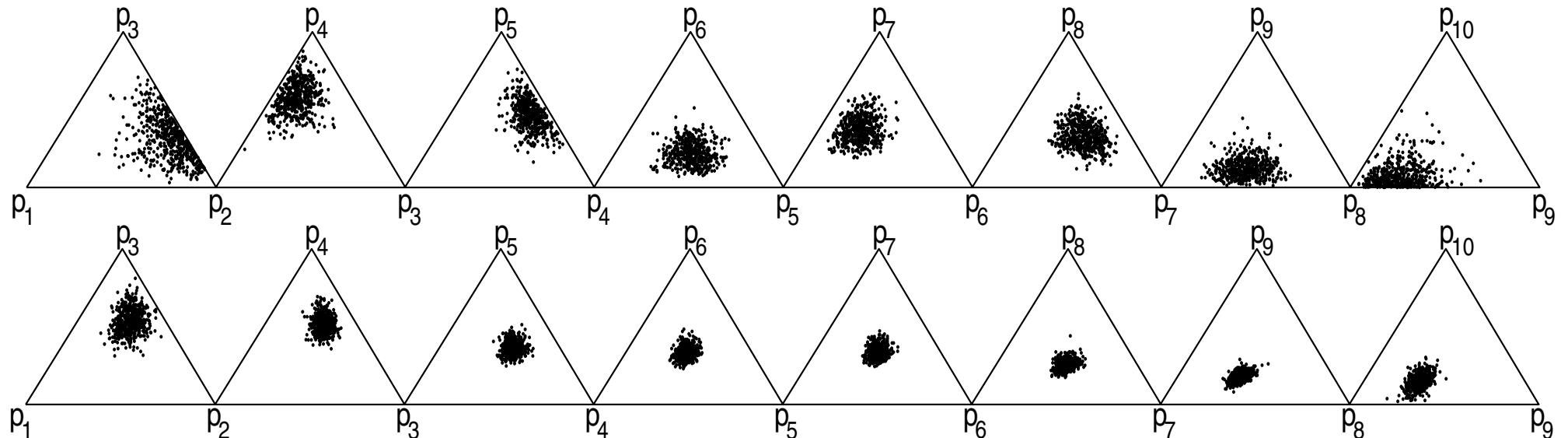
Smoothing priors



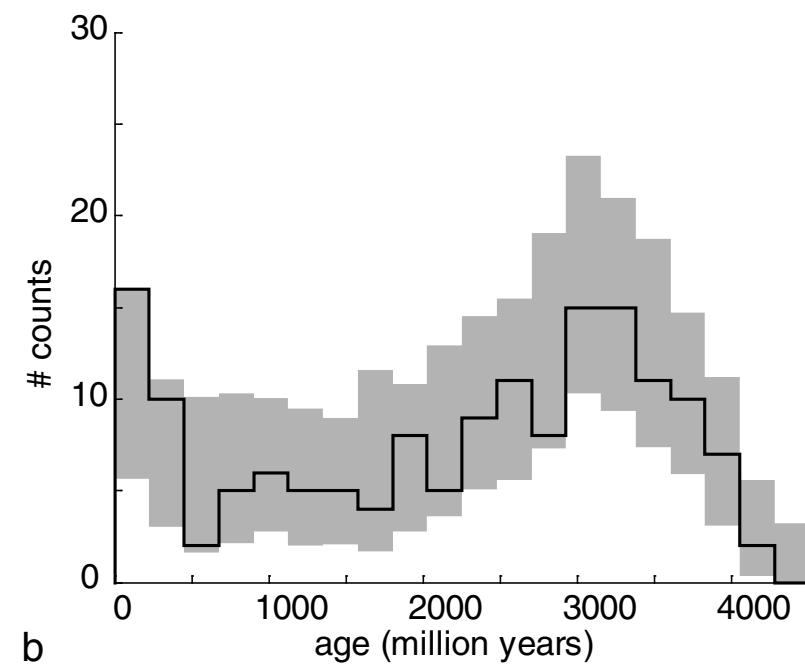
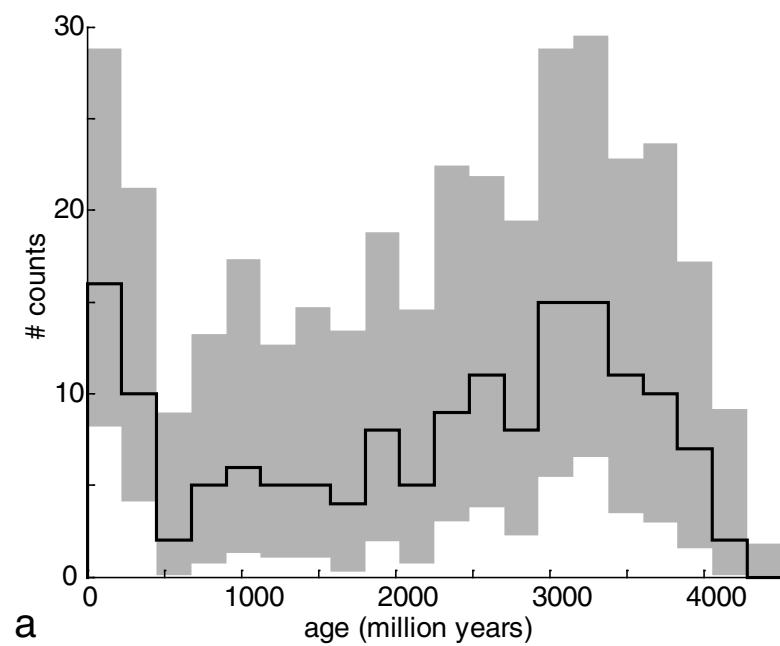
Application to synthetic data



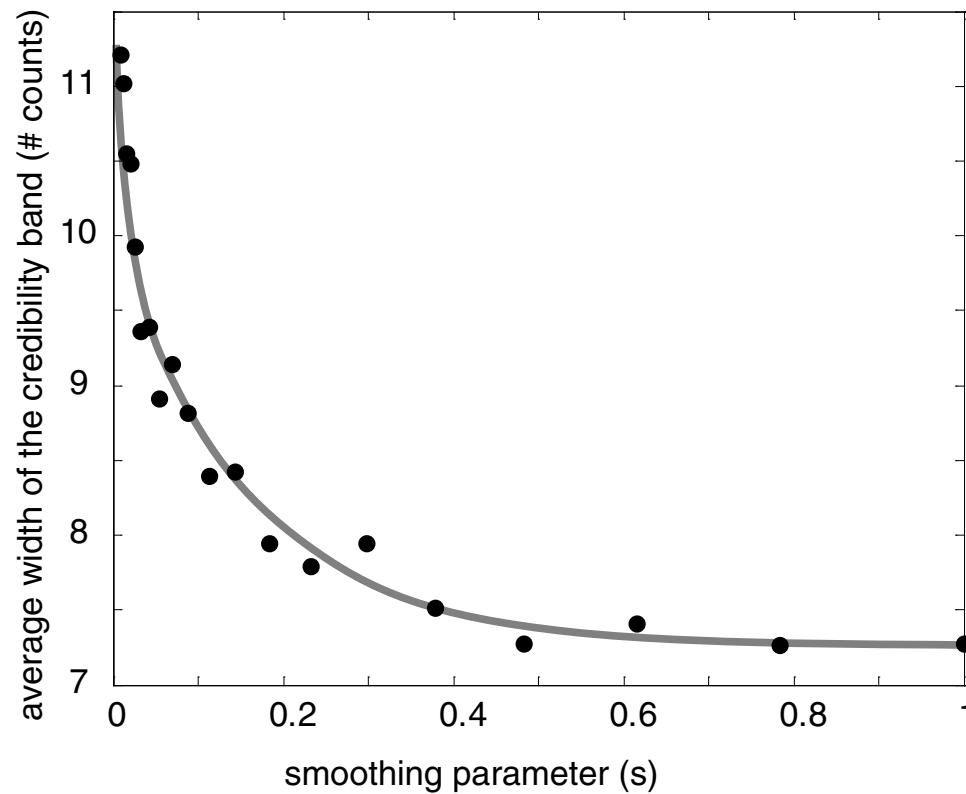
Trinomial “slices” through the posterior



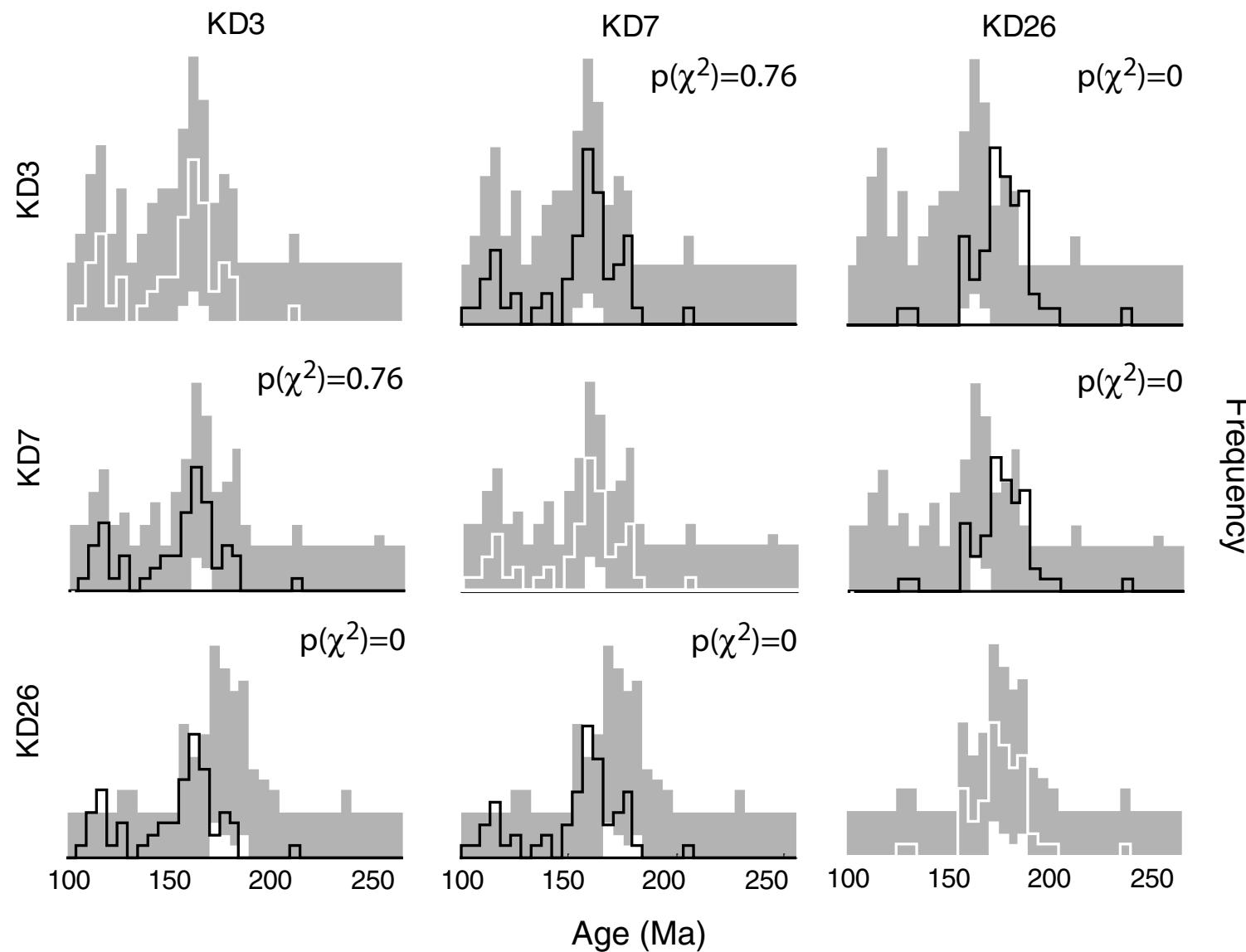
Lunar spherules (Culler et al, 2000)



The importance of the smoothing parameter s



Comparing age histograms



Bimodality test

