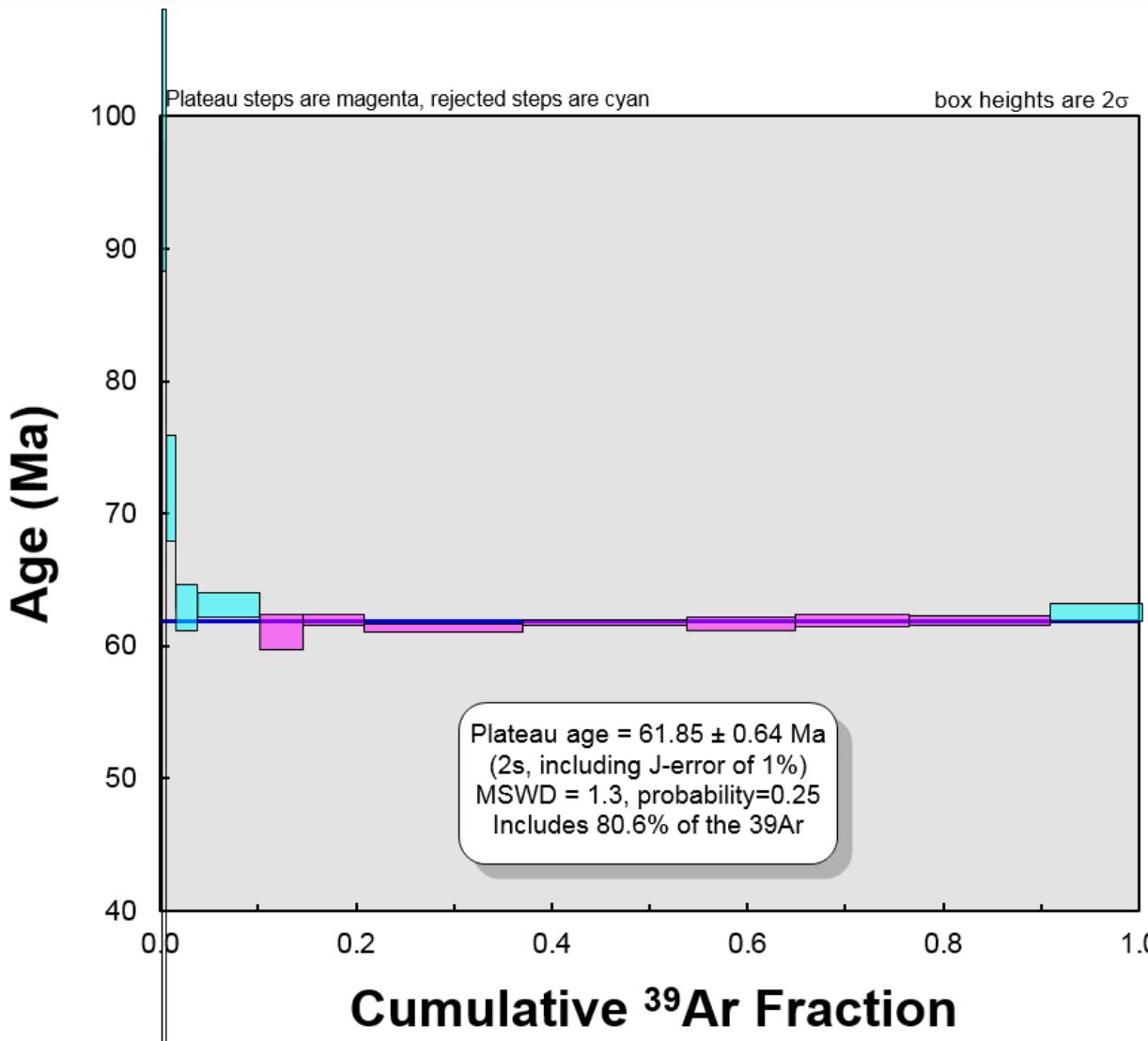


Beyond **Isoplot**: new software for better geochronology

Pieter Vermeesch

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Isoplot 3 is not compatible with either Excel 2007 or Excel 2010. If you must use Excel 2007 or Excel 2010, request a copy Isoplot 4, which is compatible with Excel 2007/2010 only. Isoplot 4 has no more functionality than Isoplot 3.7, and somewhat less flexibility. Expect a speed penalty of up to a factor of 10 when using Excel 2007/2010.

There are no versions of Isoplot compatible with native Macintosh versions of Excel, including Excel 2011. To run Isoplot on the Macintosh, you will need to acquire a Windows-enabling program (Fusion will work, but not Crossover nor Parallels). Despite expectations on my part, Excel 2011 is not Isoplot-compatible



***IsoplotR*: a free and extendable toolbox for geochronology**

Introduction Online Offline Command Line News

Home U-Th-He helioplot Options Help IsoplotR

He s[He] U s[U] Th s[Th] Sm s[Sm] (C) J K L M N

1	1.401	0.211	66.02	3.85	50.65	3.95								
2	2.096	0.315	138.51	6	99.49	8.8								
3	0.63	0.095	33.89	2	18.01	1.5								
4	0.765	0.115	52.26	3.35	22.82	1.9								
5	1.379	0.208	94.01	6.1	53.94	5.05								
6	0.383	0.058	26.67	1.35	11.51	0.75								
7	1.178	0.181	84.36	4.95	62.16	5.3								
8	0.309	0.047	23.33	1.2	14.07	1.05								
9	2.226	0.342	86.29	6.5	85.72	6.7								
10	0.778	0.117	34.22	3.25	19.51	2.2								
11	0.828	0.134	55.25	3.9	65.09	6.25								
12														
13														
14														
15														
16														
17														
18														
19	pvermeesch@pieter-laptop: ~/Dropbox/Programming/R/IsoplotR													
20	> library(IsoplotR)													
21	> data(examples)													
22	> concordia(examples\$UPb)													
23	>													
24														
25														
26														
27														
28														
29														
30														
31														

central age = 11.27 ± 0.55 [1.15 | 1.21 Ma] n= 11
MSWD = 5.4, p(χ²) = 5.4e-14

10 Ma

0.038 0.040

$^{208}\text{Pb}/^{232}\text{U}$

0.260 0.265 0.270 0.275 0.280 0.285

$\log(\text{U}/\text{He})$

4.0 4.5

Defaults Clear Open Save PLOT PDF

IsoplotR is a free and open-source substitute for Kenneth Ludwig's popular *Isoplot* add-in to Microsoft Excel. *IsoplotR* is programmed in R and can be run in three different modes:

1. Online: A user-friendly Graphical User Interface (GUI) that runs in a web browser on any internet-connected device.
2. Offline: The GUI can be run natively on any computer that has R installed on it. R is free software that is available on Windows, Mac and Linux/Unix.
3. Command Line: Advanced users can access the full functionality of *IsoplotR* from R's command line. This enables *IsoplotR* to be extended and incorporated into automation scripts.

Citable reference:
Vermeesch, P., 2018, IsoplotR: a free and open toolbox for geochronology. *Geoscience Frontiers*, v.9, p.1479-1493, doi: 10.1016/j.gsf.2018.04.001.

pieter-vermeesch.es.ucl.ac.uk/shiny/IsoplotRshiny/R - Chromium
pieter-vermeesch.es.ucl.ac.uk

Home U-Pb concordia Options Help IsoplotR

	8/6	s[8/6]	7/6	s[7/6]	(rho)	(C)	(omit)	H	I	J	K	L	M
1	25.094	0.025	0.05131	0.00004									
2	25.126	0.025	0.05128	0.00016									
3	25.138	0.063	0.05131	0.00008									
4	25.151	0.032	0.05129	0.00009									
5	25.176	0.025	0.05139	0.00006									
6	25.183	0.063	0.05134	0.00007									
7	25.208	0.025	0.05143	0.00011									
8	25.214	0.038	0.05139	0.00007									
9	25.164	0.025	0.0514	0.00006									
10	27.724	0.038	0.05135	0.00004									
11													
12													
13													
14													
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35													
36													
37													
38													

IsoplotR is an R implementation of Ken Ludwig's popular Isoplot add-in to Microsoft Excel that was designed to be free, flexible and future-proof. The program implements functions for U-Pb, Pb-Pb, $^{40}\text{Ar}/^{39}\text{Ar}$, Rb-Sr, Sm-Nd, Lu-Hf, Re-Os, U-Th-He, fission track and U-series disequilibrium dating as well as detrital geochronology.

This website provides easy to use point-and-click access to IsoplotR's most commonly used functions (see the [tutorial](#) for details). Alternatively, the same functions (and more) can also be accessed from the command-line through the [IsoplotR package](#) on [CRAN](#).

Citable reference:
Vermeesch, P., 2018, IsoplotR: a free and open toolbox for geochronology. *Geoscience Frontiers*, v.9, p.1479-1493, doi:10.1016/j.gsf.2018.04.001.

Defaults Clear Open Save PLOT PDF

R 127.0.0.1:4565 x +

Home U-Pb concordia Options Help IsoplotR

	8/6	s[8/6]	7/6	s[7/6]	(rho)	(C)	(omit)	H	I	J	K
1	25.094	0.025	0.05131	0.00004							
2	25.126	0.025	0.05128	0.00016							
3	25.138	0.063	0.05131	0.00008							
4	25.151	0.032	0.05129	0.00009							
5	25.176	0.025	0.05139	0.00006							
6	25.183	0.063	0.05134	0.00007							
7	25.208	0.025	0.05143	0.00011							
8	25.214	0.038	0.05139	0.00007							
9	25.164	0.025	0.0514	0.00006							
10	27.724	0.038	0.05135	0.00004							
11											
12											
13	pvermees@pieter-laptop: ~/Dropbox/Programming/R/IsoplotR\$ R										
14	Type 'demo()' for some demos, 'help()' for on-line hel										
15	p, or										
16	'help.start()' for an HTML browser interface to help.										
17	Type 'q()' to quit R.										
18	> library(IsoplotRgui)										
19	> IsoplotR()										
20	Loading required package: shiny										
21	Listening on http://127.0.0.1:4565										
22											
23											
24											
25											
26											
27											
28											

IsoplotR is an R implementation of Ken Ludwig's popular Isoplot add-in to Microsoft Excel that was designed to be free, flexible and future-proof. The program implements functions for U-Pb, Pb-Pb, $^{40}\text{Ar}/^{39}\text{Ar}$, Rb-Sr, Sm-Nd, Lu-Hf, Re-Os, U-Th-He, fission track and U-series disequilibrium dating as well as detrital geochronology.

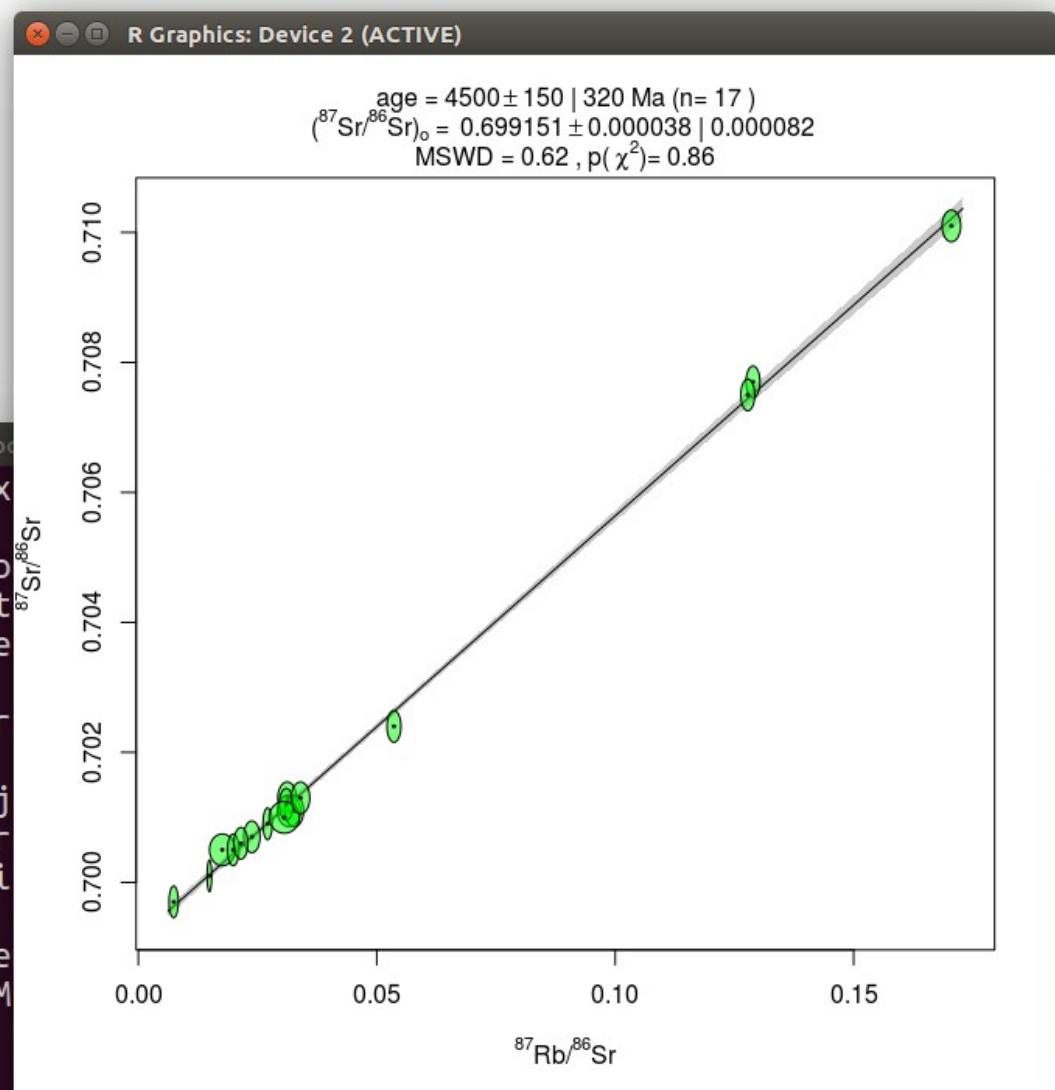
Website provides easy to use point-and-click access to IsoplotR's most commonly used functions (see the [tutorial](#) for details). Alternatively, the same functions (and more) can also be accessed from the command-line through the [IsoplotR](#) package on [CRAN](#).

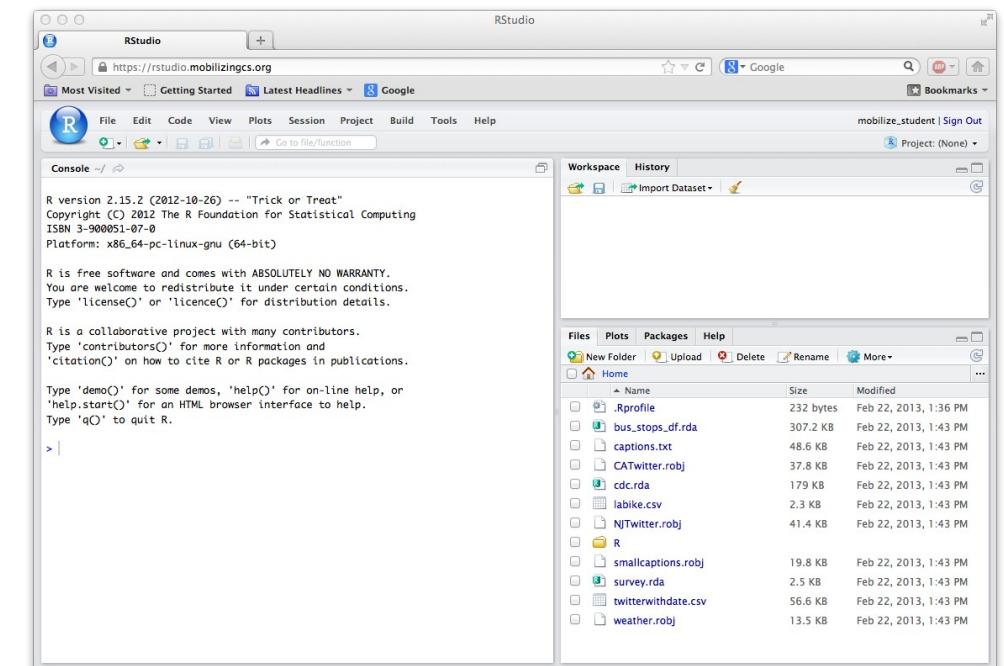
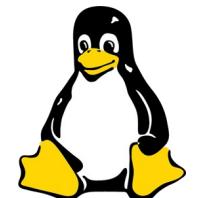
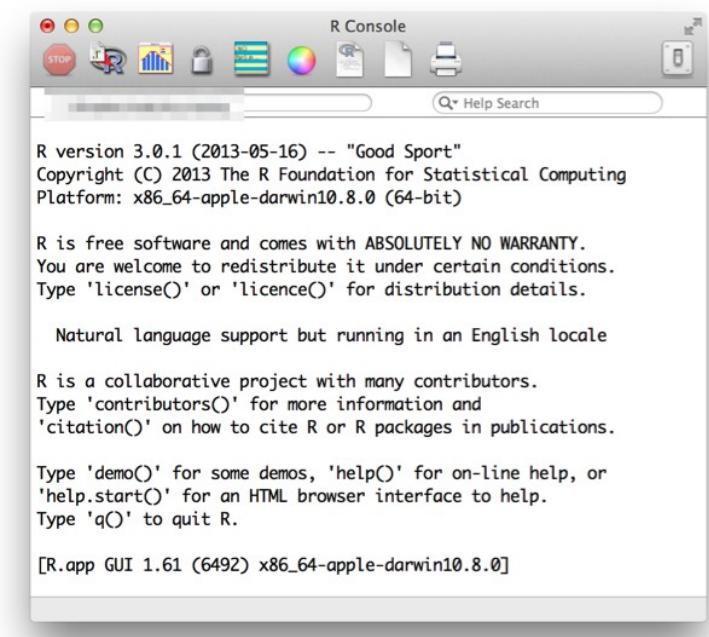
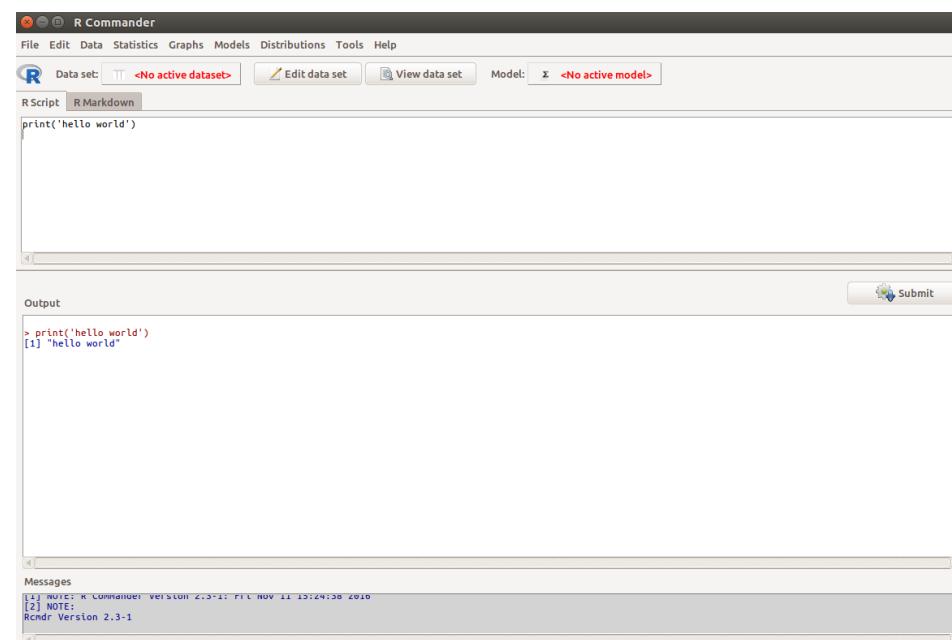
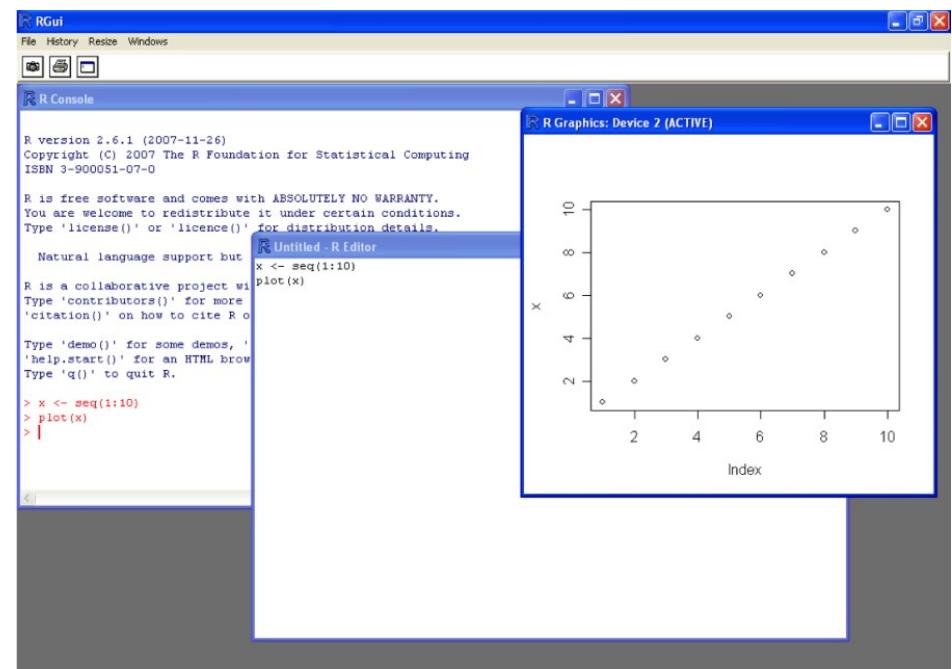
Citable reference:

Vermeesch, P., 2018, IsoplotR: a free and open toolbox for geochronology. *Geoscience Frontiers*, v.9, p.1479-1493, doi:10.1016/j.gsf.2018.04.001.

Defaults Clear Open Save PLOT PDF

```
pvermees@pieter-laptop: ~/Dropbox/geo/geochem/Rb-Sr/iso  
Platform: x86_64-pc-linux  
  
R is free software and co  
You are welcome to redist  
Type 'license()' or 'lice  
  
Natural language suppor  
  
R is a collaborative proj  
Type 'contributors()' for  
'citation()' on how to ci  
  
Type 'demo()' for some de  
'help.start()' for an HTM  
Type 'q()' to quit R.  
  
> library(IsoplotR)  
> RbSr <- read.data('RbSr1.csv',method='Rb-Sr',format=1)  
> isochron(RbSr)  
> □
```





Secure | https://cran.r-project.org



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FAQs
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ISLR
ISM
ismeye
isni
isnullptr
Iso
IsoCI
isocir
ISOcodes
IsoGene
isopam
isopat
isoph
IsoplotR
ISOpureR
IsoR

```
pvermees@pieter-laptop: ~/Dropbox/Programming/R/IsoplotR/inst
R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

> install.packages('IsoplotR')
Installing package into '/usr/local/lib/R/site-library'
(as 'lib' is unspecified)
--- Please select a CRAN mirror for use in this session ---
itertools2: Functions creating iterators for efficient
looping
```

GitHub, Inc. [US] | https://github.com/pvermees/IsoplotR

pvermees / IsoplotR

An R version of Isoplot

257 commits 2 branches 0 releases 1 contributor

Branch: master New pull request Create new file Upload files Find file Clone or download

pvermees Increased robustness by replacing all instances of log(det(x)) with d... 6 days ago

IsoplotR.Rcheck Confidence envelopes for model-2 discordia lines in 3D 8 days ago

R Increased robustness by replacing all instances of log(det(x)) with d... 6 days ago

data 5 months ago

inst 6 months ago

man 10 days ago

DESCRIPTION 2 months ago

IsoplotR.pdf 10 days ago

NAMESPACE 8 months ago

Secure CRAN mirrors

Brazil (SP 1) [https] 10 days ago

Brazil (SP 2) [https] 2 months ago

Bulgaria [https] 10 days ago

Chile [https] 8 months ago

China (Hong Kong) [https] 7 months ago

China (Lanzhou) [https] 7 months ago

China (Shanghai) [https] 7 months ago

Colombia (Cali) [https] 7 months ago

Czech Republic [https] 7 months ago

Denmark [https] 7 months ago

Ecuador (Cuenca) [https] 7 months ago

Ecuador (Quito) [https] 7 months ago

Estonia [https] 7 months ago

France (Lyon 1) [https] 7 months ago

France (Lyon 2) [https] 7 months ago

France (Marseille) [https] 7 months ago

France (Montpellier) [https] 7 months ago

Germany (Erlangen) [https] 7 months ago

Germany (Göttingen) [https] 7 months ago

Germany (Münster) [https] 7 months ago

Germany (Regensburg) [https] 7 months ago

Greece [https] 7 months ago

Hungary [https] 7 months ago

to install IsoplotR from GitHub, you also need R command line prompt:

n.r-project.org/package=IsoplotR and can be

OK Cancel

lgc.es.ucl.ac.uk/shiny/IsoplotRshiny/R - Chromium
lgc.es.ucl.ac.uk/shiny/IsoplotRshiny/R - +
Not secure | lgc.es.ucl.ac.uk/shiny/IsoplotRshiny/R/

Home U-Pb concordia Options Help IsoplotR

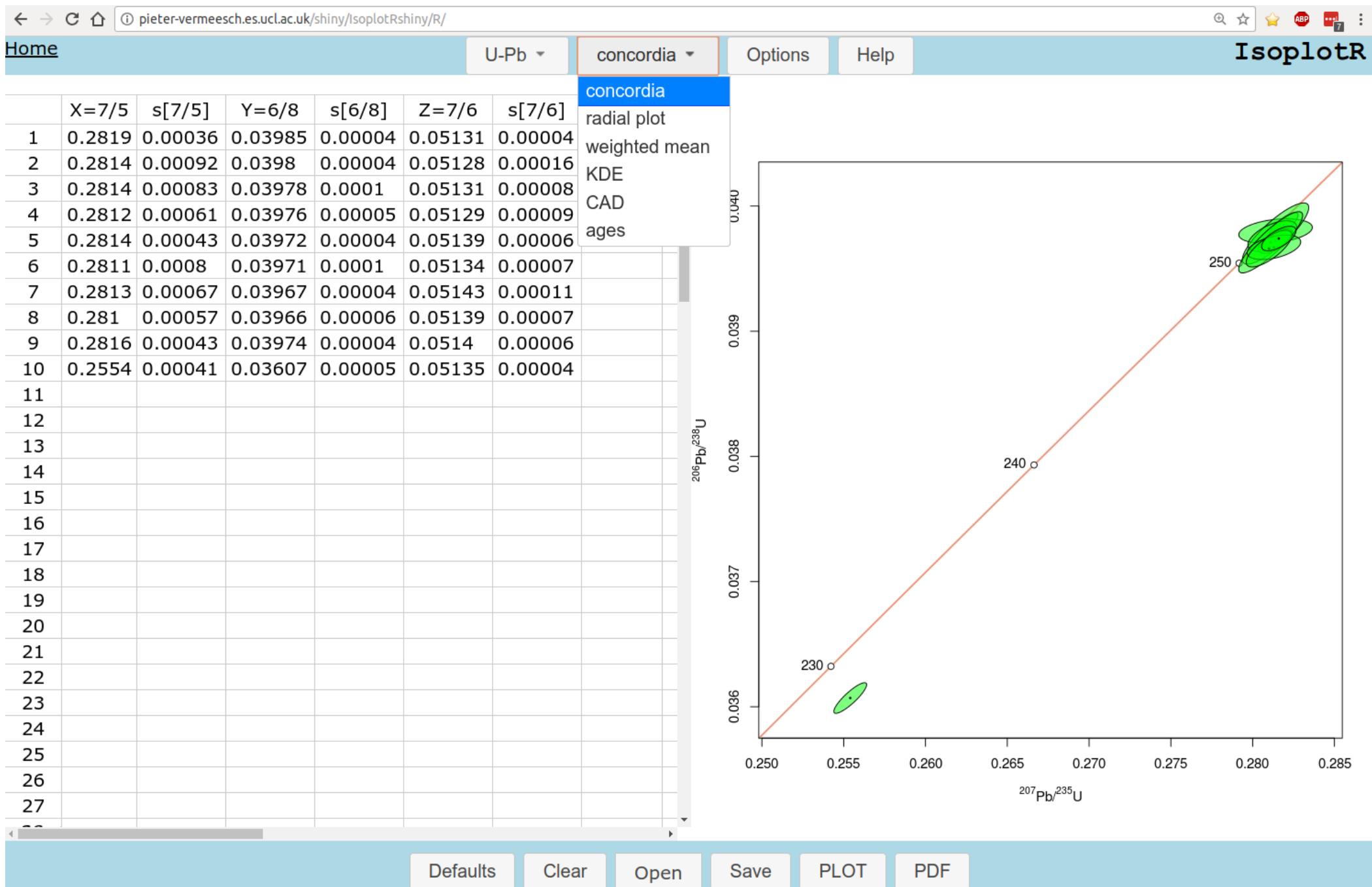
	8/6	s[8/6]	7/6	s[7/6]	(rho)	(C)	(omit)	
1	25.094	0.025	0.05131	0.00004				Pb-Pb
2	25.126	0.025	0.05128	0.00016				Ar-Ar
3	25.138	0.063	0.05131	0.00008				K-Ca
4	25.151	0.032	0.05129	0.00009				Rb-Sr
5	25.176	0.025	0.05139	0.00006				Sm-Nd
6	25.183	0.063	0.05134	0.00007				Re-Os
7	25.208	0.025	0.05143	0.00011				Lu-Hf
8	25.214	0.038	0.05139	0.00007				U-Th-He
9	25.164	0.025	0.0514	0.00006				fission tracks
10	27.724	0.038	0.05135	0.00004				U-series
11								detritals
12								other
13								
14								
15								
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30								

IsoplotR is an R implementation of Ken Ludwig's popular Isoplot add-in to Microsoft Excel that was designed to be free, flexible and future-proof. The program implements functions for U-Pb, Pb-Pb, $^{40}\text{Ar}/^{39}\text{Ar}$, Rb-Sr, Sm-Nd, Lu-Hf, Re-Os, U-Th-He, fission track and U-series disequilibrium dating as well as detrital geochronology.

This website provides easy to use point-and-click access to IsoplotR's most commonly used functions (see the [tutorial](#) for details). Alternatively, the same functions (and more) can also be accessed from the command-line through the [IsoplotR package on CRAN](#).

Citable reference:
Vermeesch, P., 2018, IsoplotR: a free and open toolbox for geochronology. *Geoscience Frontiers*, v.9, p.1479-1493, doi:10.1016/j.gsf.2018.04.001.

Defaults Clear Open Save PLOT PDF



pieter-vermeesch.es.ucl.ac.uk/shiny/IsoplotRshiny/R/

Home U-Pb concordia Options Help IsoplotR

	X=7/5	s[7/5]	Y=6/8	s[6/8]	Z=7/6	s[7/6]	(rhoXY)	(rhoYZ)	(C)	J
1	0.2819	0.00036	0.03985	0.00004	0.05131	0.00004				
2	0.2814	0.00092	0.0398	0.00004	0.05128	0.00016				
3	0.2814	0.00083	0.03978	0.0001	0.05131	0.00008				
4	0.2812	0.00061	0.03976	0.00005	0.05129	0.00009				
5	0.2814	0.00043	0.03972	0.00004	0.05139	0.00006				
6	0.2811	0.0008	0.03971	0.0001	0.05134	0.00007				
7	0.2813	0.00067	0.03967	0.00004	0.05143	0.00011				
8	0.281	0.00057	0.03966	0.00006	0.05139	0.00007				
9	0.2816	0.00043	0.03974	0.00004	0.0514	0.00006				
10	0.2554	0.00041	0.03607	0.00005	0.05135	0.00004				
11										
12										
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27										
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29										
30										
31										
32										
33										
34										
35										
36										
37										
38										

Input:

7/5: Measured $^{207}\text{Pb}/^{235}\text{U}$ -ratios.
s[7/5]: Standard error of the $^{207}\text{Pb}/^{235}\text{U}$ -ratios.
6/8: Measured $^{206}\text{Pb}/^{238}\text{U}$ -ratios.
s[6/8]: Standard error of the $^{206}\text{Pb}/^{238}\text{U}$ -ratios.
7/6: Measured $^{207}\text{Pb}/^{206}\text{Pb}$ -ratios.
s[7/6]: Standard error of the $^{207}\text{Pb}/^{206}\text{Pb}$ -ratios.
(rhoXY): Error correlation between the $^{207}\text{Pb}/^{235}\text{U}$ - and $^{206}\text{Pb}/^{238}\text{U}$ -ratios (optional).
(rhoYZ): Error correlation between the $^{238}\text{U}/^{206}\text{Pb}$ - and $^{207}\text{Pb}/^{206}\text{Pb}$ -ratios (optional). The same assumption applies as for rhoYZ
(rhoXZ): Error correlation between the $^{238}\text{U}/^{206}\text{Pb}$ - and $^{207}\text{Pb}/^{206}\text{Pb}$ -ratios (optional).
(C): Optional variable to be used as fill colours in concordia and radial plots.

If rhoxy, rhoxz or rhoyz are omitted, then their value will be inferred from the redundancy between the three sets of standard errors (s[x], s[y] and s[z]). This inference assumes that these errors are based on the same number of data points. If some data has been rejected from one ratio but not another, because of isobaric interferences, instrument issues, outlier rejection, or any other reason, then this method will not work.

Output:

The concordia diagram is a graphical means of assessing the internal consistency of U-Pb data. It sets out the measured $^{206}\text{Pb}/^{238}\text{U}$ - and $^{207}\text{Pb}/^{235}\text{U}$ -ratios against each other ('Wetherill' diagram) or, equivalently, the $^{207}\text{Pb}/^{206}\text{Pb}$ - and $^{207}\text{Pb}/^{235}\text{U}$ -ratios ('Tera-Wasserburg' diagram). The space of concordant isotopic compositions is marked by a curve, the 'concordia line'. Isotopic ratio measurements are shown as 100(1- α)% confidence ellipses, where α is set in the Options menu. Concordant samples plot near to, or overlap with, the concordia line. They represent the pinnacle of geochronological robustness. Samples that plot away from the concordia line but are aligned along a linear trend form an isochron (or 'discordia' line) that can be used to infer the composition of the non-radiogenic ('common') lead or to constrain the timing of prior lead loss.

References:

Ludwig, K.R., 1998. On the treatment of concordant uranium-lead ages. *Geochimica et Cosmochimica Acta*, 62(4), pp.665-676.

Defaults Clear Open Save PLOT PDF

Home U-Pb concordia Options Help IsoplotR

Input format: [7/5], [6/8], [7/6]

Common Pb correction: none

$^{238}\text{U}/^{235}\text{U}$ ratio: 137.818 \pm 0.0225

^{238}U decay constant: 0.000155125 \pm 8.3e-8 Myr $^{-1}$

^{235}U decay constant: 0.00098485 \pm 6.7e-7 Myr $^{-1}$

Plot as Tera-Wasserburg?

Calculate no age

Minimum t: auto

Maximum t: auto

Minimum X: auto

Maximum X: auto

Minimum Y: auto

Maximum Y: auto

Probability cutoff: 0.05

Significant digits: 2

Ellipse colour 1: rgb(0,1,0,0.5)

Ellipse colour 2: rgb(1,0,0,0.5)

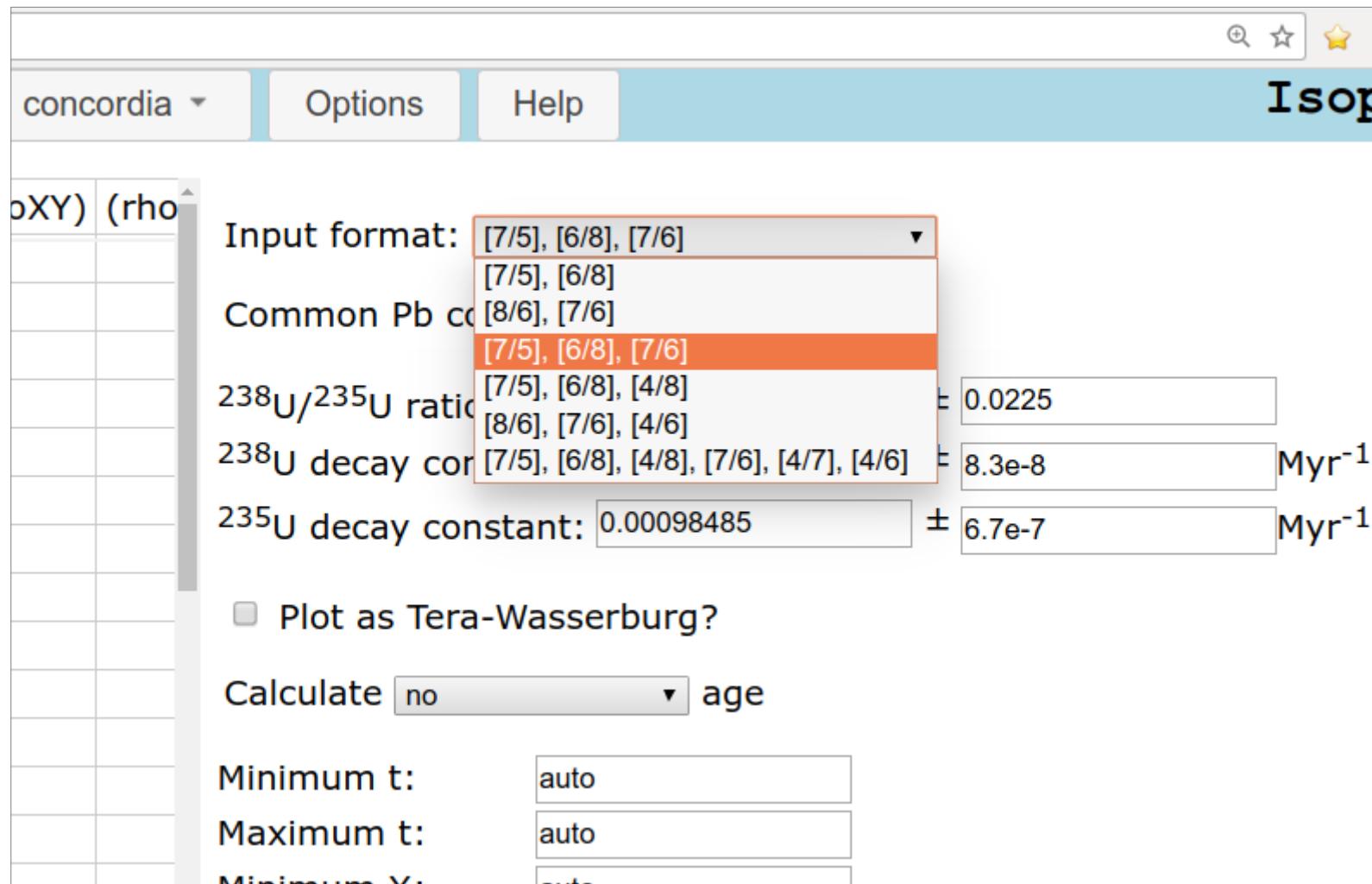
Colour label:

Show decay constant and $^{238}\text{U}/^{235}\text{U}$ -uncertainties?

Add sample numbers to error ellipses?

	X=7/5	s[7/5]	Y=6/8	s[6/8]	Z=7/6	s[7/6]	(rhoXY)	(rho)
1	0.2819	0.00036	0.03985	0.00004	0.05131	0.00004		
2	0.2814	0.00092	0.0398	0.00004	0.05128	0.00016		
3	0.2814	0.00083	0.03978	0.0001	0.05131	0.00008		
4	0.2812	0.00061	0.03976	0.00005	0.05129	0.00009		
5	0.2814	0.00043	0.03972	0.00004	0.05139	0.00006		
6	0.2811	0.0008	0.03971	0.0001	0.05134	0.00007		
7	0.2813	0.00067	0.03967	0.00004	0.05143	0.00011		
8	0.281	0.00057	0.03966	0.00006	0.05139	0.00007		
9	0.2816	0.00043	0.03974	0.00004	0.0514	0.00006		
10	0.2554	0.00041	0.03607	0.00005	0.05135	0.00004		
11								
12								
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24								
25								
26								
27								
28								

Defaults Clear Open Save PLOT PDF



Z=7/6	s[7/6]	(rhoXY)	(rhoZ)
0.05131	0.00004		
0.	0.	Input format:	[7/5], [6/8], [7/6]
0.	0.		x
help			
Choose one of six input formats:			
1. 7/5 s[7/5] 6/8 s[6/8] rho			
where rho is the error correlation between 7/5 and 6/8			
2. 8/6 s[8/6] 7/6 s[7/6] (rho)			
where the error correlation is optional			
3. 7/6 s[7/6] 6/8 s[6/8] 7/5 s[7/5]			
in which the error correlations are calculated from the redundancies			
between the three sets of uncertainties.			
4. X=7/5 s[7/5] Y=6/8 s[6/8] Z=4/8 s[4/8] rho[X,Y] rho[X,Z] rho[Y,Z]			
5. X=8/6 s[8/6] Y=7/6 s[7/6] Z=4/6 s[4/6] rho[X,Y] rho[X,Z] rho[Y,Z]			
6. 7/5 s[7/5] 6/8 s[6/8] 4/8 s[4/8] 7/6 s[7/6] 4/7 s[4/7] 4/6 s[4/6]			
in which the error correlations are calculated from the redundancies			
between the six sets of uncertainties.			

Maximum Y: auto

± 0.0225
± 8.3e-8
± 6.7e-7

官方网站

U-Pb

concordia

选项

帮助

中文

IsoplotR

	X=38/06	err[X]	Y=07/06	err[Y]	(rho)	(C)	(omit)	H	I	J	输入误差:
1	25.094	0.025	0.05131	0.00004							1se (abs) ▾
2	25.126	0.025	0.05128	0.00016							输入格式: [38/06], [07/06] ▾
3	25.138	0.063	0.05131	0.00008							常规铅校正: none ▾
4	25.151	0.032	0.05129	0.00009							<input type="checkbox"/> 不平衡校正?
5	25.176	0.025	0.05139	0.00006							$^{238}\text{U}/^{235}\text{U}$ 比: 137.818 \pm 0.0225
6	25.183	0.063	0.05134	0.00007							^{238}U 裂变常数: 0.000155125 \pm 8.3e-8 Myr ⁻¹
7	25.208	0.025	0.05143	0.00011							^{235}U 裂变常数: 0.00098485 \pm 6.7e-7 Myr ⁻¹
8	25.214	0.038	0.05139	0.00007							
9	25.164	0.025	0.0514	0.00006							
10	27.724	0.038	0.05135	0.00004							
11											谐和线类型: Wetherill ▾
12											计算 空 ▾ 年龄
13											
14											最小值 t: auto
15											最大值 t: auto
16											最小值 X: auto
17											最大值 X: auto
18											最小值 Y: auto
19											最大值 Y: auto
20											概率截止: 0.05
21											有效数字: 2
22											椭圆颜色: c(rgb(0,1,0,0.5),rgb(1,0,0,0.5))
23											字体放大: 1
24											颜色标签:
25											年龄限制: auto
26											
27											
28											<input type="checkbox"/> 传播系统的不确定性?
29											

重置设置

清空数据

打开

保存文件

生成投图

PDF

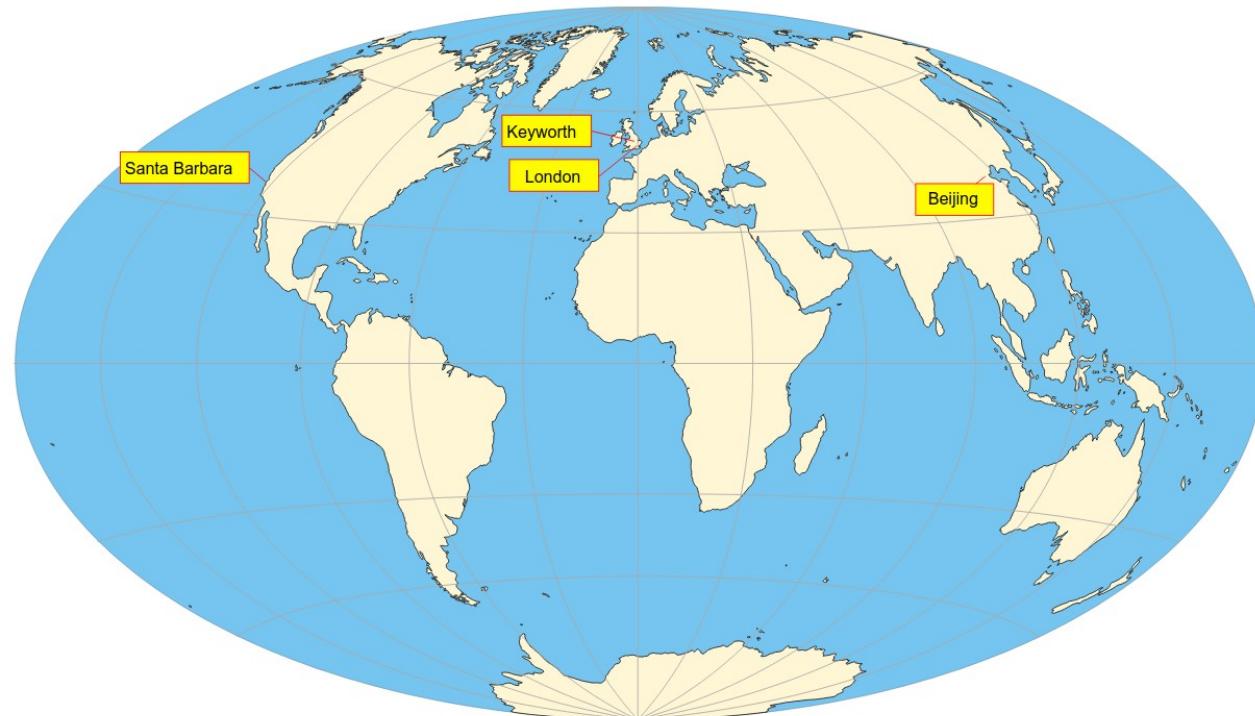
IsoplotR: a free and extendable toolbox for geochronology

[Introduction](#)[Online](#)[Offline](#)[Command Line](#)[News](#)[Contribute](#)

An online GUI is hosted by the following international collaborators:

1. [Beijing](#): Qiuye Yu (China University of Geosciences Beijing)
2. [Keyworth](#): Ian Millar (British Geological Survey)
3. [London](#): Pieter Vermeesch (University College London)
4. [Santa Barbara](#): John Cottle (University of California)

Click on one of the yellow boxes below to select a nearby mirror. Otherwise you will be automatically directed to the London server in **2 seconds** ...

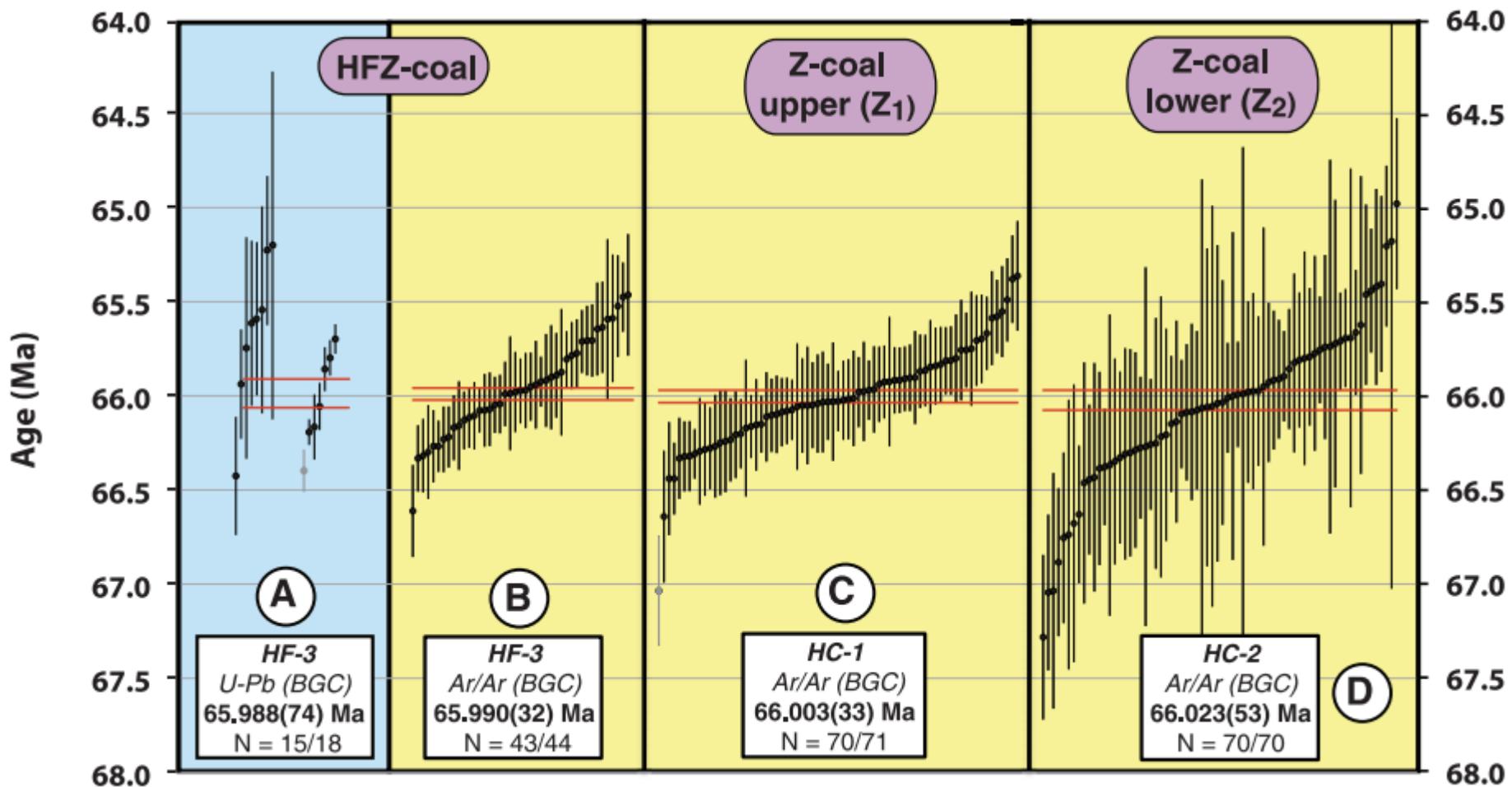


If you would like to make your own IsoplotR mirror available to the world via this website, then please get in touch with me at [p.vermeesch \[at\] ucl.ac.uk](mailto:p.vermeesch[at]ucl.ac.uk).

Synchronizing Rock Clocks of Earth History

K. F. Kuiper,^{1,2} A. Deino,³ F. J. Hilgen,¹ W. Krijgsman,¹ P. R. Renne,^{3,4} J. R. Wijbrans²

25 APRIL 2008 VOL 320 SCIENCE



Time Scales of Critical Events Around the Cretaceous-Paleogene Boundary

Paul R. Renne,^{1,2*} Alan L. Deino,^{1†} Frederik J. Hilgen,^{3†} Klaudia F. Kuiper,^{4†} Darren F. Mark,^{5†} William S. Mitchell III,^{2,6†} Leah E. Morgan,^{5†} Roland Mundil,^{1†} Jan Smit^{4†}

8 FEBRUARY 2013 VOL 339 SCIENCE

Geochimica et Cosmochimica Acta 171 (2015) 325–337



Revised error propagation of $^{40}\text{Ar}/^{39}\text{Ar}$ data, including covariances

Pieter Vermeesch

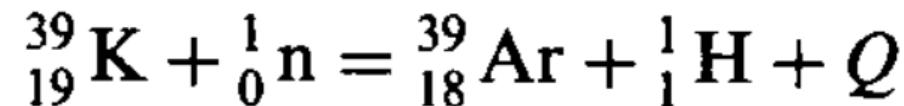
Department of Earth Sciences, University College London, United Kingdom

Received 14 May 2015; accepted in revised form 14 September 2015; available online 25 September 2015

**Geochimica et
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Acta**

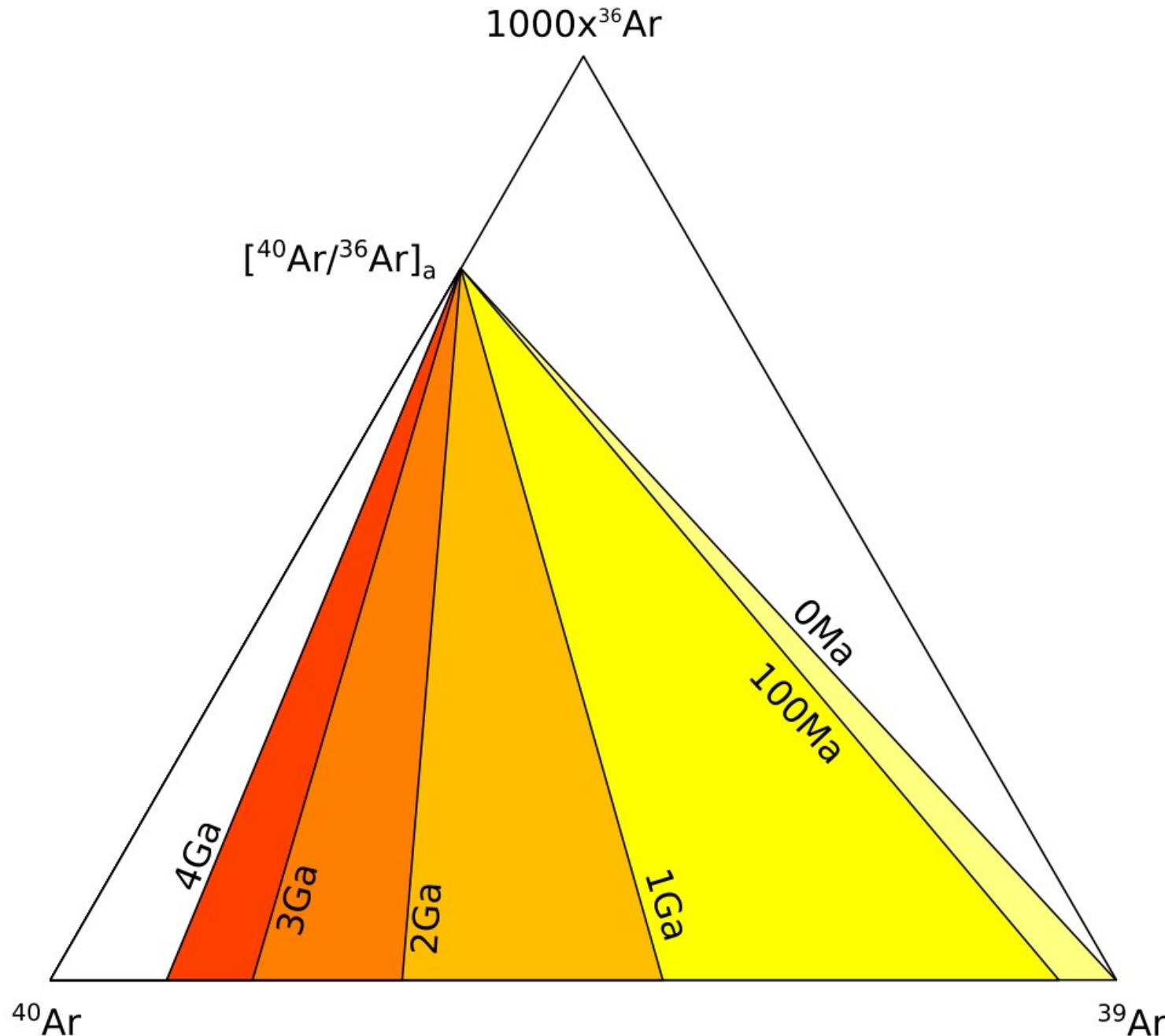
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- 1. Isotopic measurements are compositional data**
- 2. Error correlations are commonplace in geochronology**



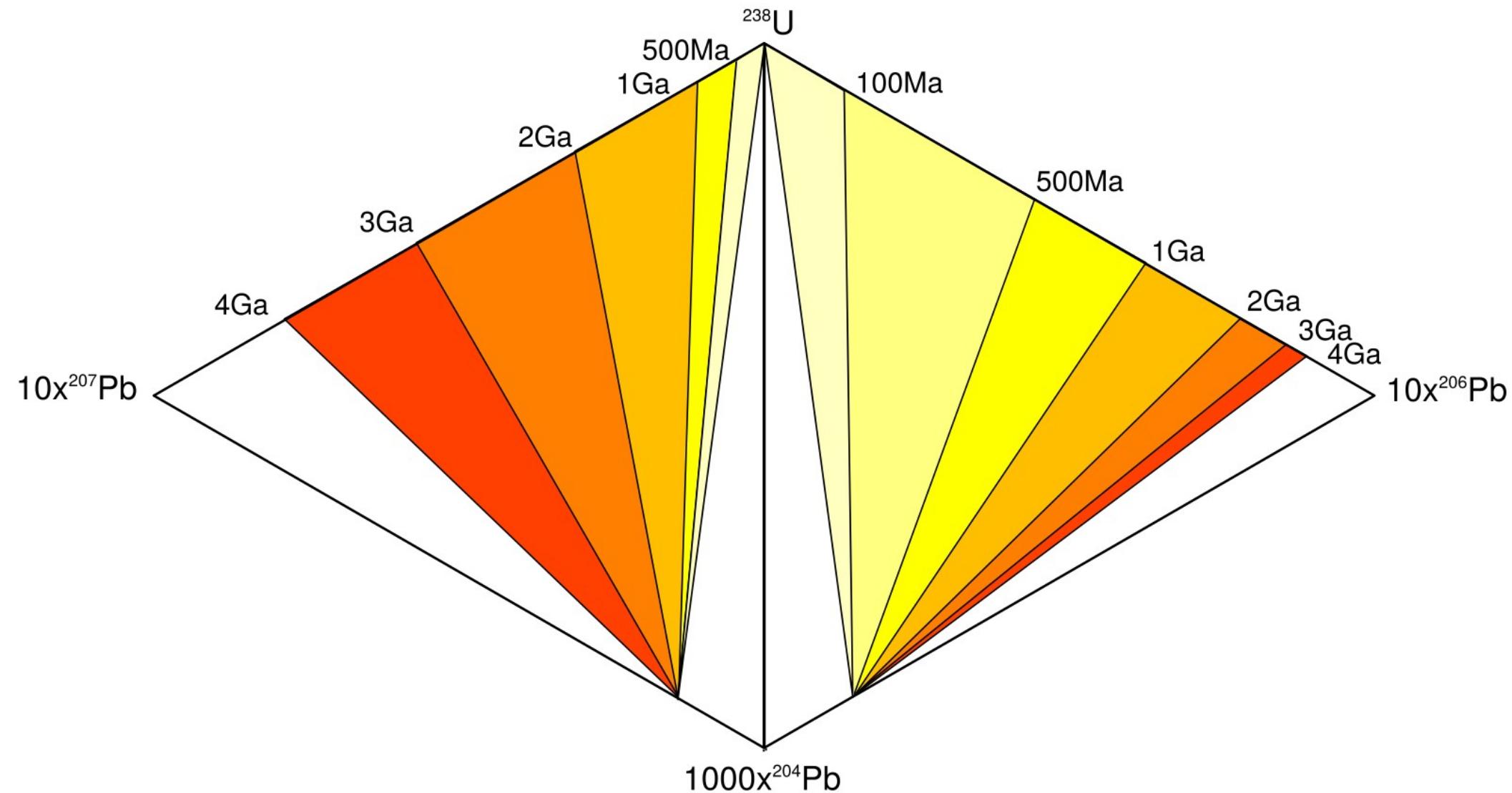
stable	radioactive			
^{38}Ca Z=20,N=18 $t_{1/2} = 440\text{ms}$	^{39}Ca Z=20,N=19 $t_{1/2} = 860\text{ms}$	^{40}Ca Z=20,N=20 $t_{1/2} = \infty$	^{41}Ca Z=20,N=21 $t_{1/2} = 10\text{kyr}$	^{42}Ca Z=20,N=22 $t_{1/2} = \infty$
^{37}K Z=19,N=18 $t_{1/2} = 1.2\text{s}$	^{38}K Z=19,N=19 $t_{1/2} = 7.6\text{m}$	^{39}K Z=19,N=20 $t_{1/2} = \infty$	^{40}K Z=19,N=21 $t_{1/2} = 1.25\text{Gyr}$	^{41}K Z=19,N=22 $t_{1/2} = \infty$
^{36}Ar Z=18,N=18 $t_{1/2} = \infty$	^{37}Ar Z=18,N=19 $t_{1/2} = 35\text{d}$	^{38}Ar Z=18,N=20 $t_{1/2} = \infty$	^{39}Ar Z=18,N=21 $t_{1/2} = 269\text{yr}$	^{40}Ar Z=18,N=22 $t_{1/2} = \infty$

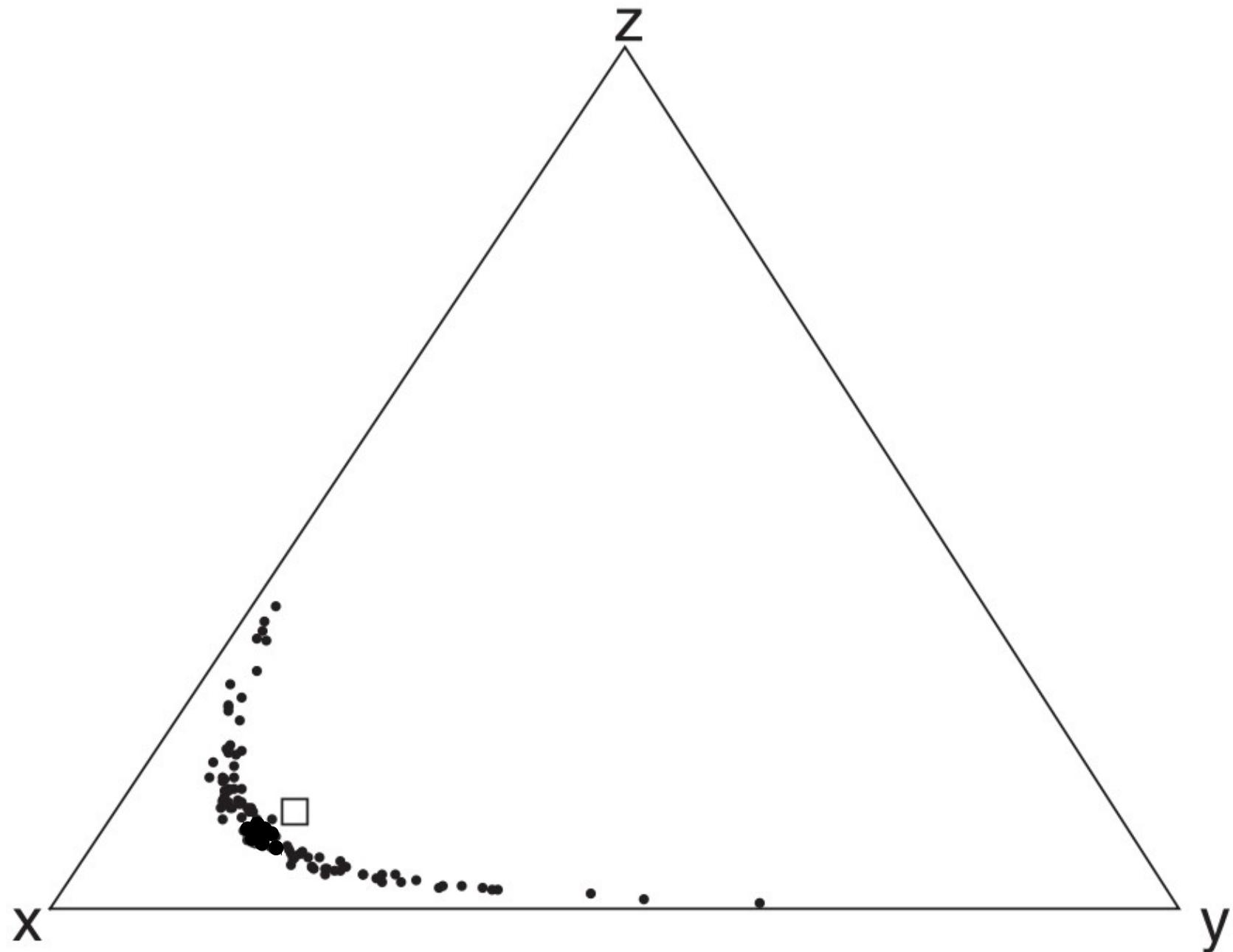
$$t = \frac{1}{\lambda} \ln \left[1 + J \left(\frac{^{40}\text{Ar}}{^{39}\text{Ar}} - 298.56 \frac{^{36}\text{Ar}}{^{39}\text{Ar}} \right) \right]$$

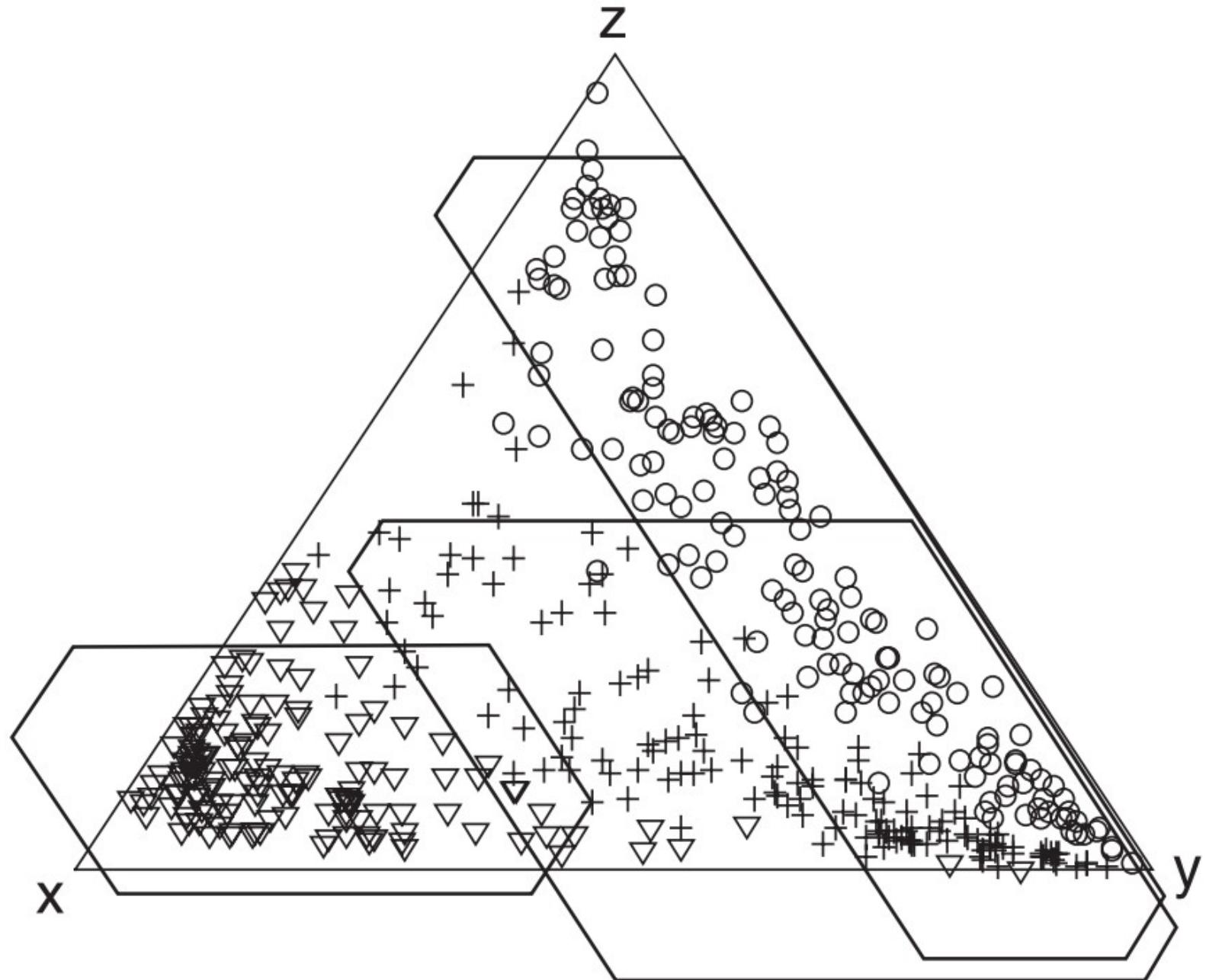


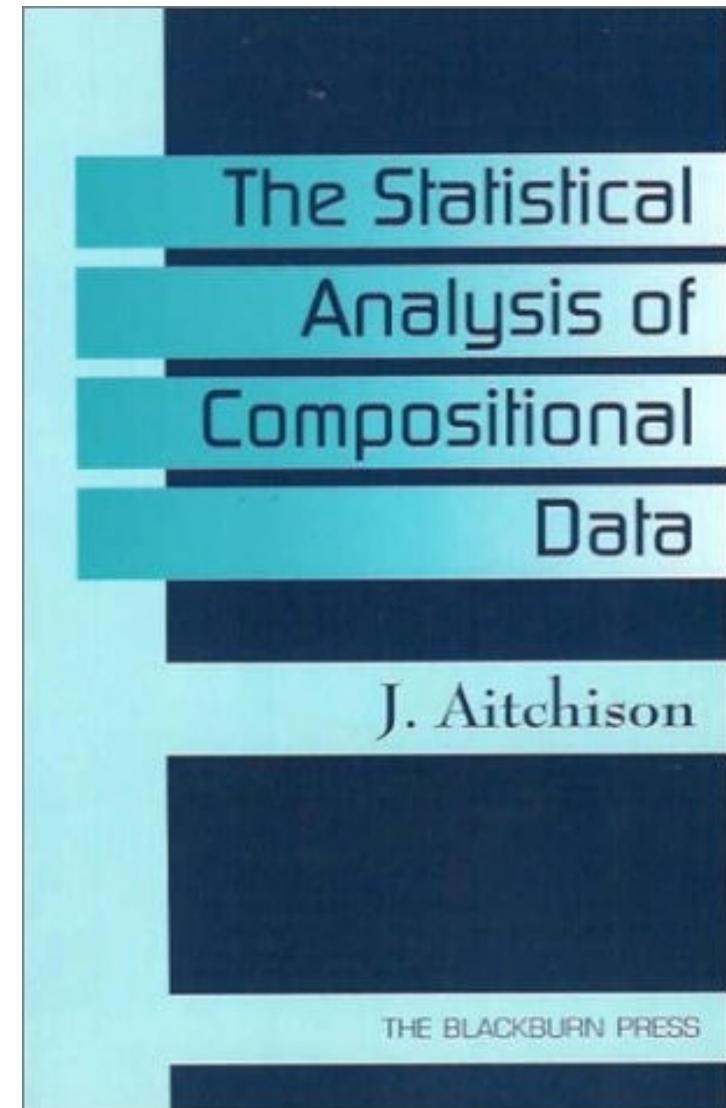
$$t = \frac{1}{\lambda_{238}} \ln \left(\frac{\left(\frac{^{206}Pb}{^{204}Pb} \right) - \left(\frac{^{206}Pb}{^{204}Pb} \right)_o}{\frac{^{238}U}{^{204}Pb}} + 1 \right)$$

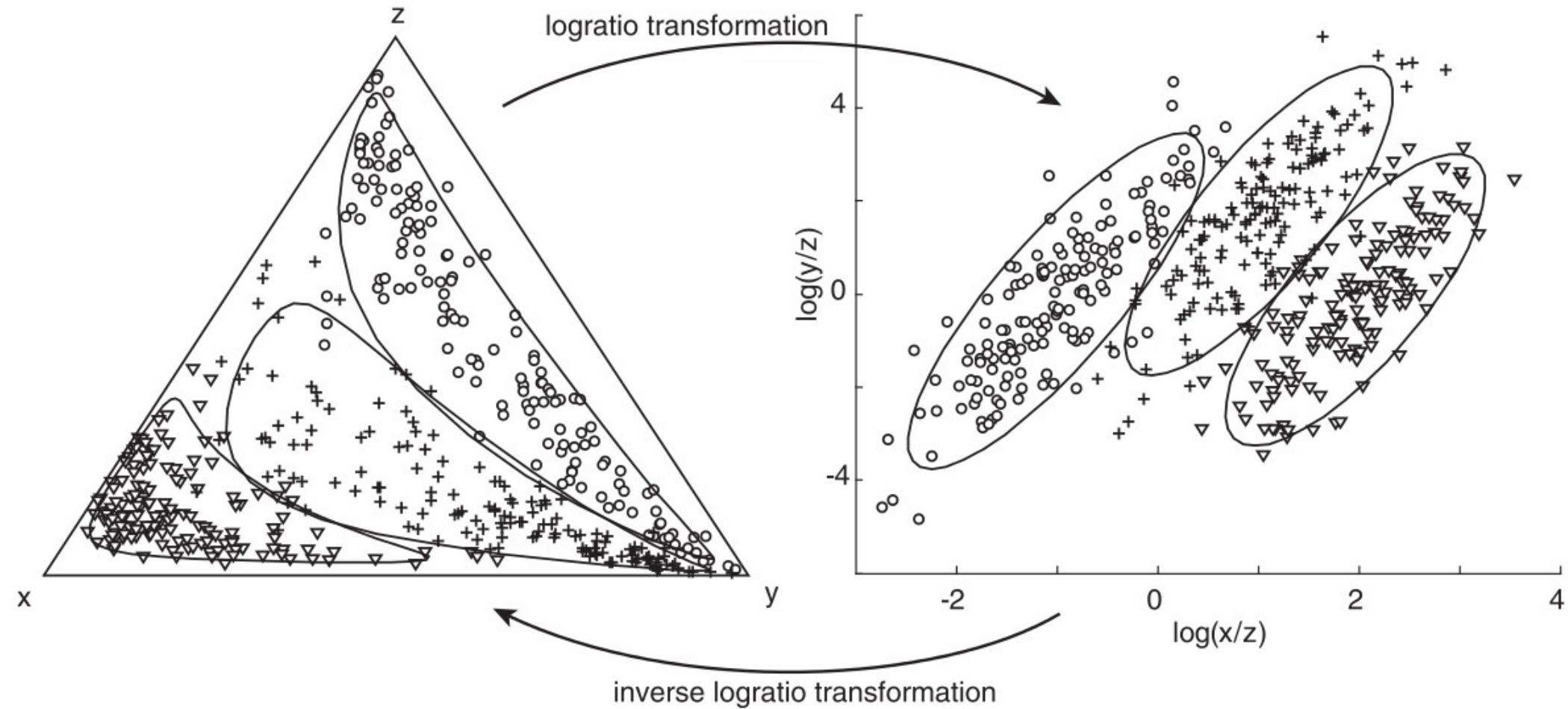
$$t = \frac{1}{\lambda_{235}} \ln \left(\frac{\left(\frac{^{207}Pb}{^{204}Pb} \right) - \left(\frac{^{207}Pb}{^{204}Pb} \right)_o}{137.818 \frac{^{238}U}{^{204}Pb}} + 1 \right)$$

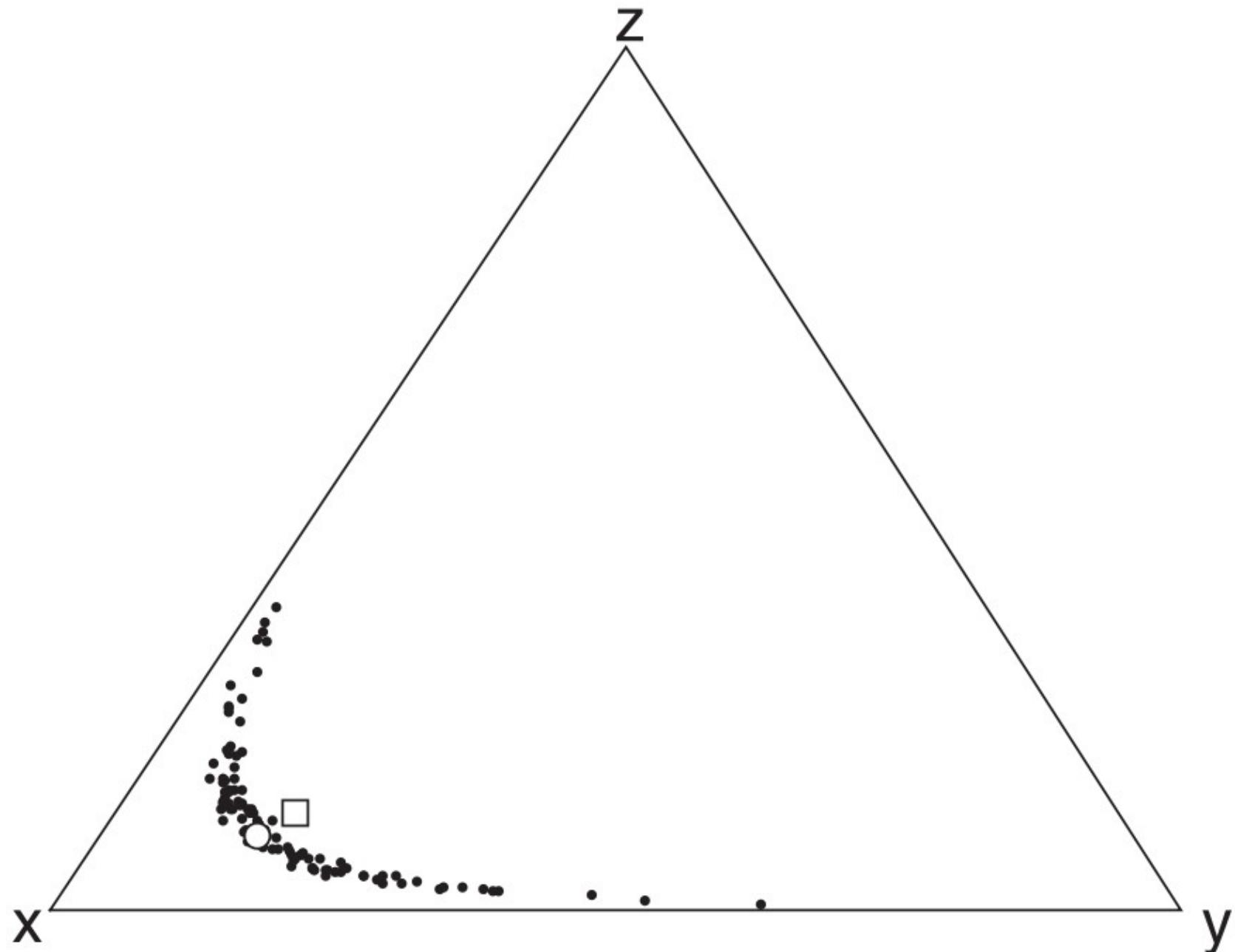


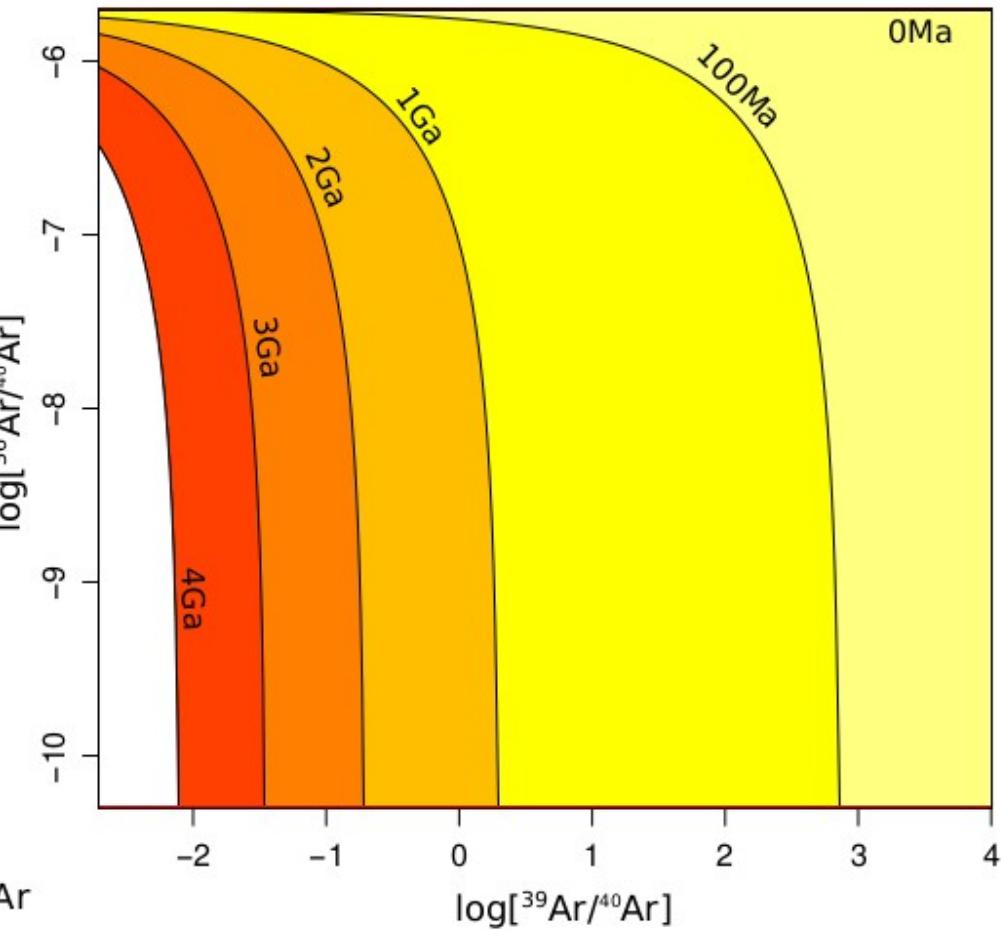
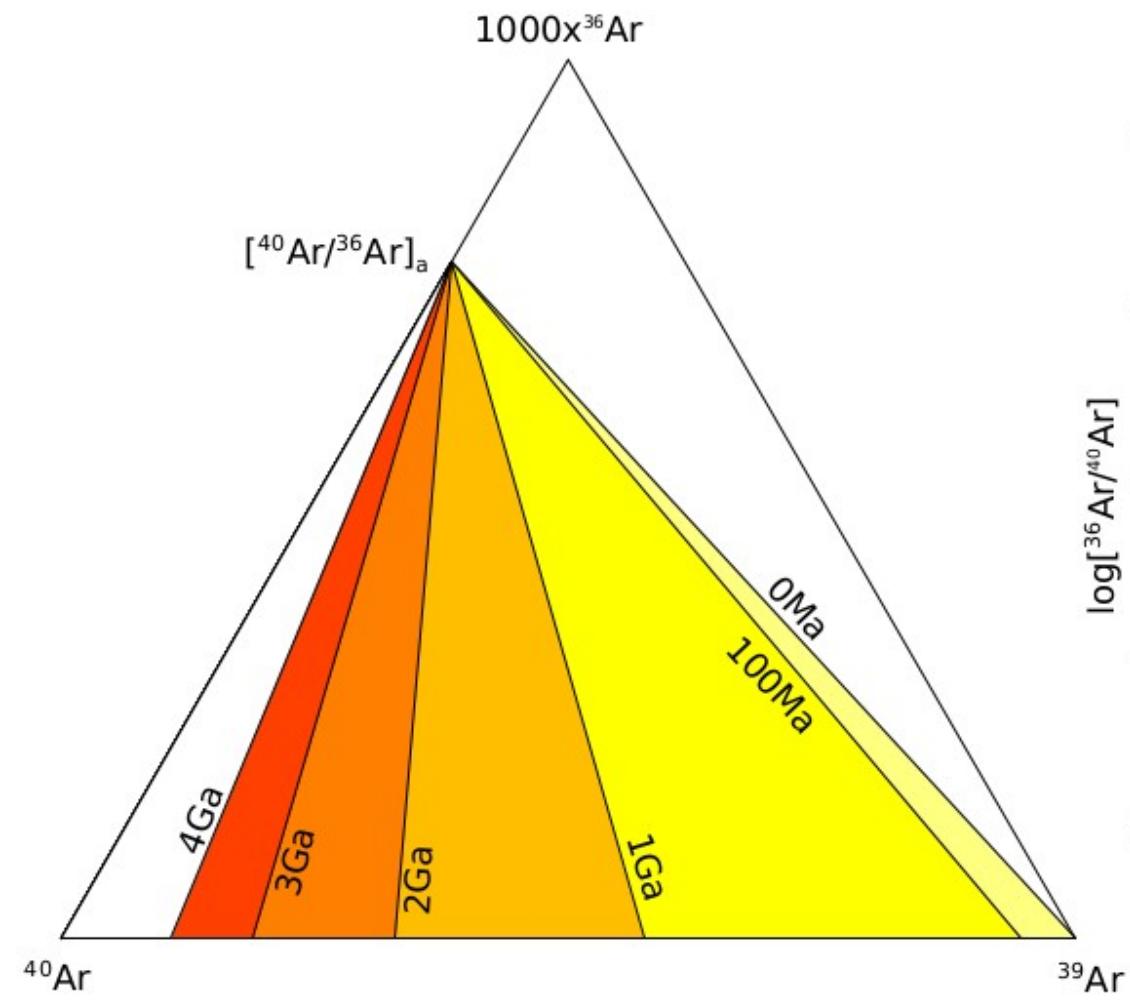


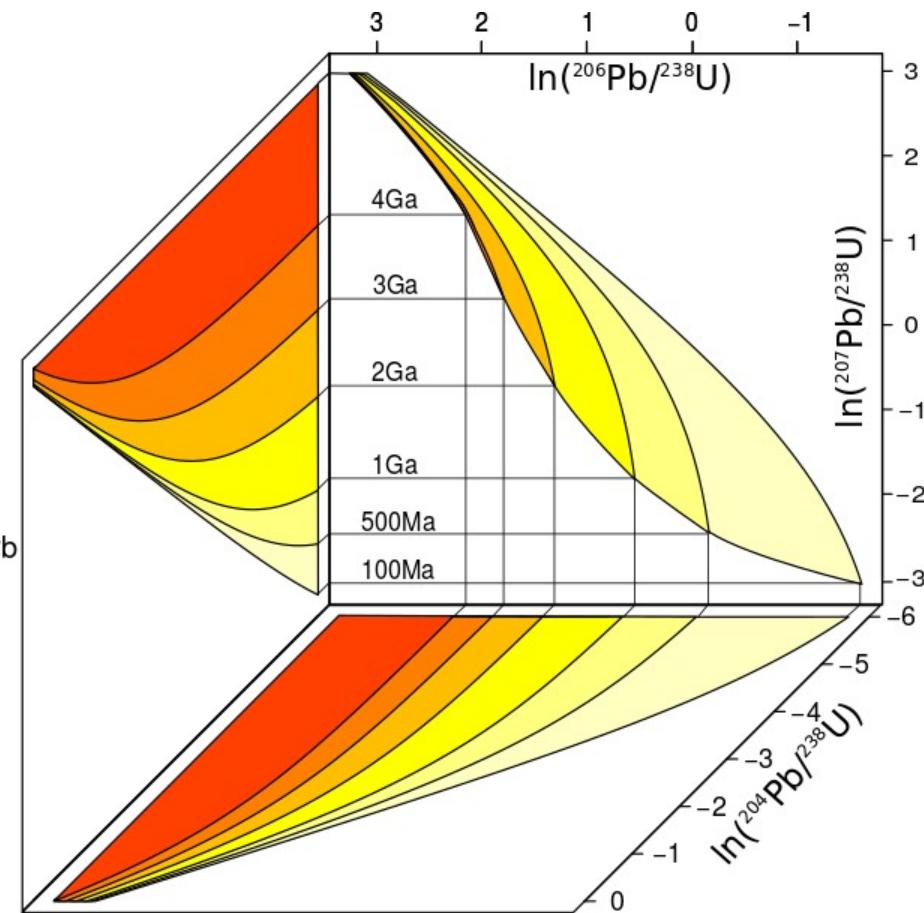
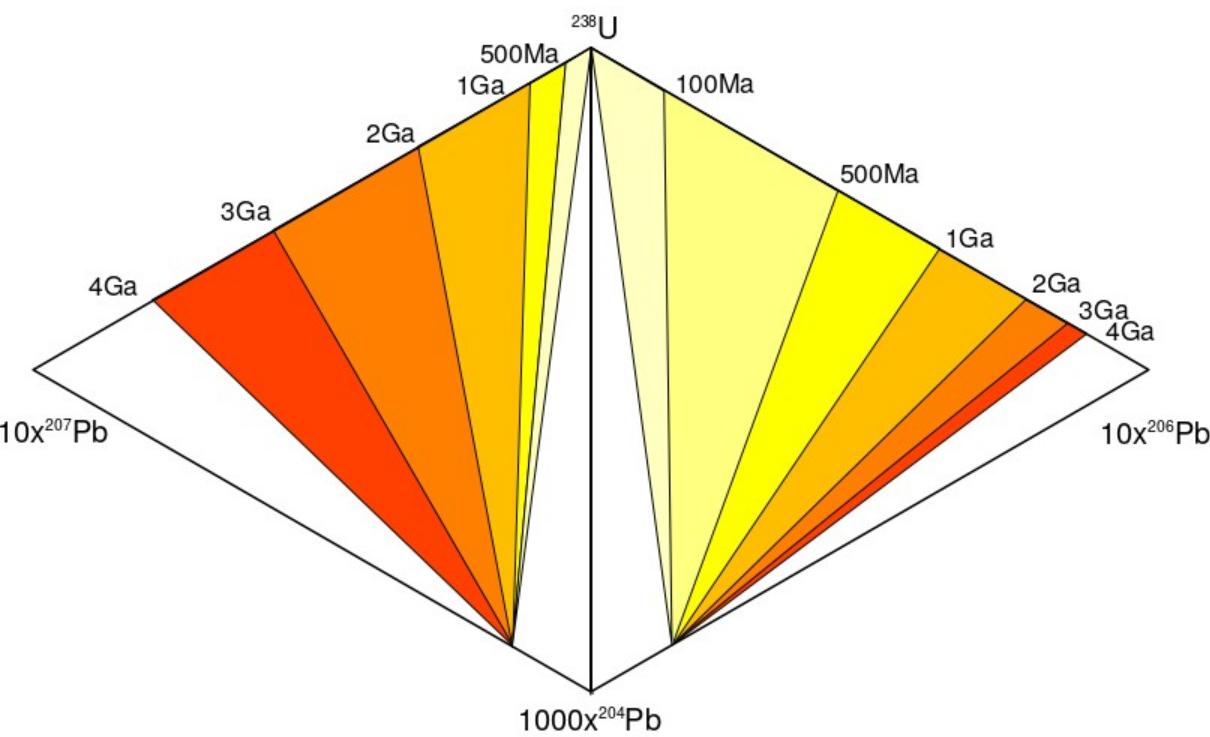




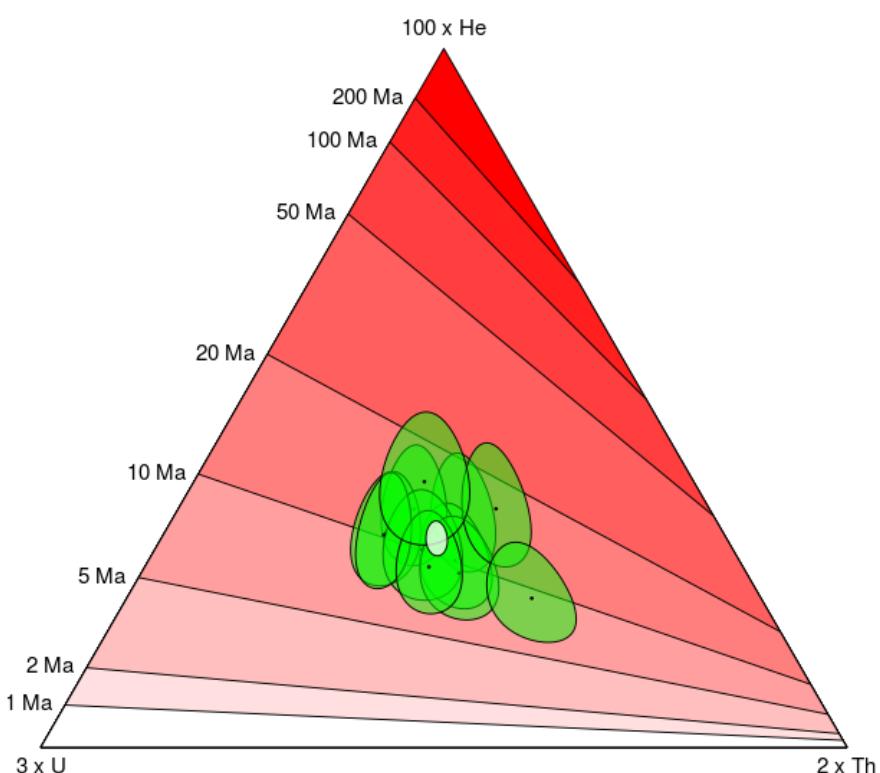








	He	err[He]	U	err[U]	Th	err[Th]	Sm	err[Sm]	(C)	(omit)
1	1.401	0.211	66.02	3.85	50.65	3.95				
2	2.096	0.315	138.51	6.0	99.49	9.0				
3	0.63	0.095	33.89	2.0	18.01	1.5				
4	0.765	0.115	52.26	3.35	22.82	1.9				
5	1.379	0.208	94.01	6.0	53.94	5.05				
6	0.383	0.060	26.67	1.35	11.51	0.75				
7	1.178	0.181	84.36	4.95	62.16	5.5				
8	0.309	0.047	23.33	1.2	14.07	1.05				
9	2.226	0.342	86.29	6.5	85.72	6.5				
10	0.778	0.117	34.22	3.25	19.51	2.2				
11	0.828	0.134	55.25	3.9	65.09	6.25				
12										
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Defaults

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PLOT

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Geochimica et Cosmochimica Acta 171 (2015) 325–337

Revised error propagation of $^{40}\text{Ar}/^{39}\text{Ar}$ data, including covariances

Pieter Vermeesch

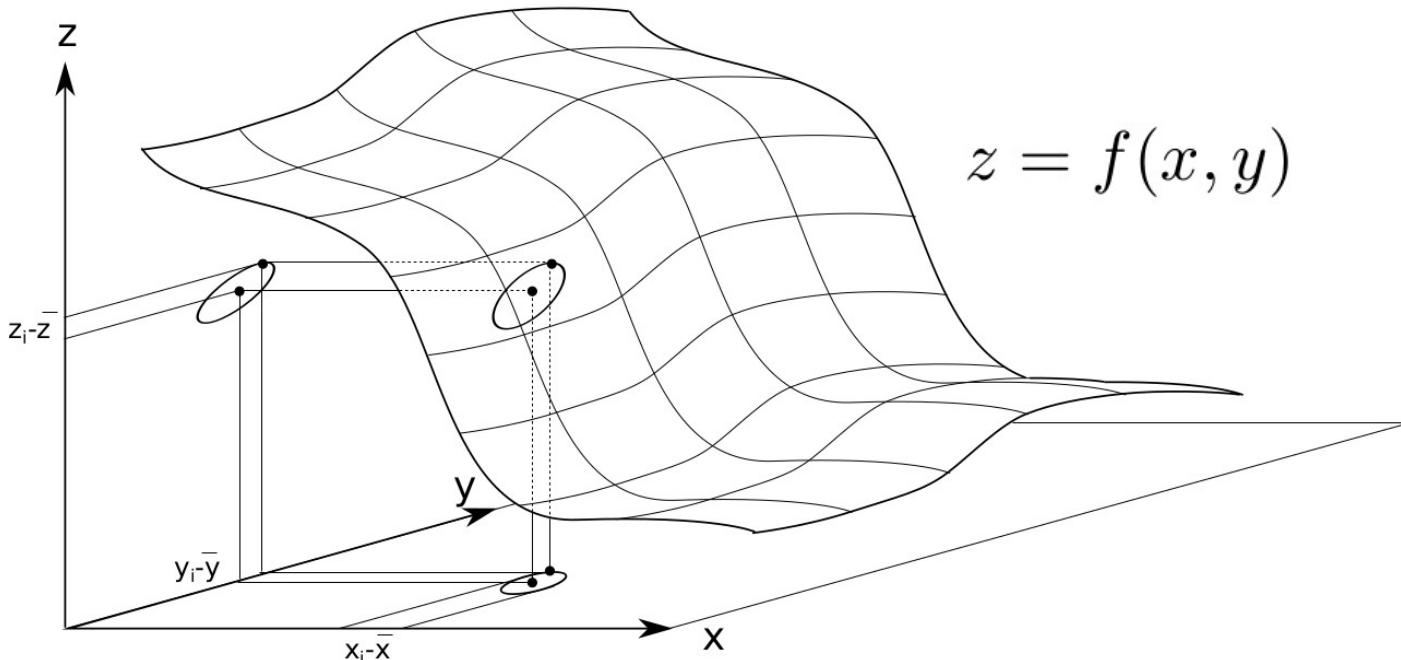
Department of Earth Sciences, University College London, United Kingdom

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Acta**

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1. Isotopic measurements are compositional data
2. **Error correlations are commonplace in geochronology**



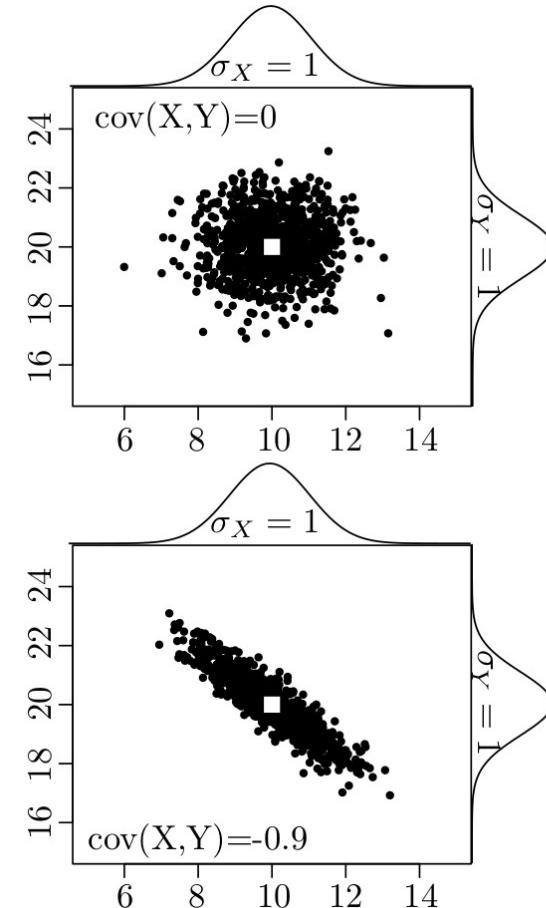
$$\sigma_z^2 = \left(\frac{\partial f}{\partial x} \right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y} \right)^2 \sigma_y^2 + 2 \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} cov(x, y)$$

$$\sigma_z^2 = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \begin{bmatrix} \sigma_x^2 & cov(x, y) \\ cov(x, y) & \sigma_y^2 \end{bmatrix} \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Jacobian matrix

Covariance matrix

Transpose of the
Jacobian matrix



$$z = f(x, y)$$

$$\sigma_z^2 = \left(\frac{\partial f}{\partial x} \right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y} \right)^2 \sigma_y^2 + 2 \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \cancel{cov(x, y)}$$

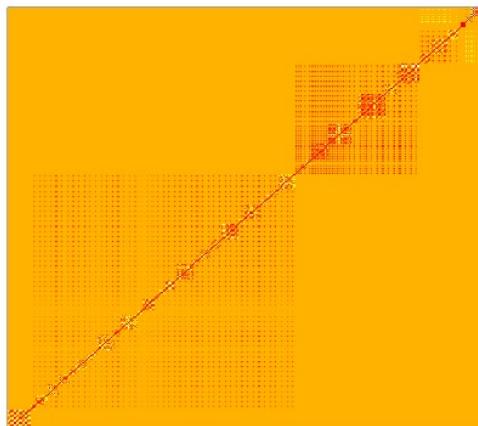
$$\sigma_z^2 = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$t = \frac{1}{\lambda} \ln(1 + JR)$$

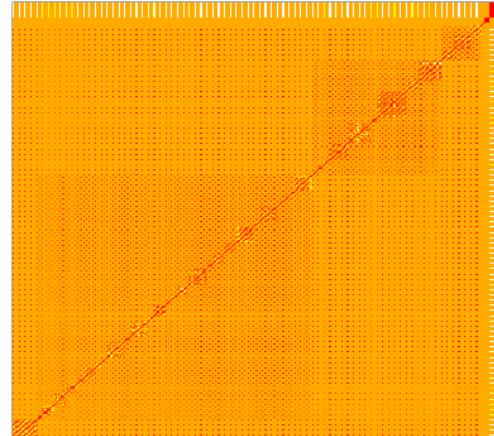
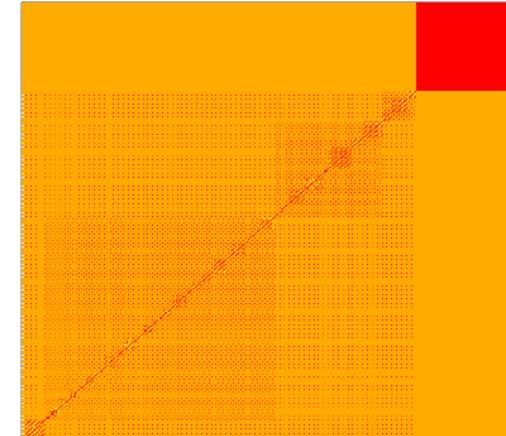
$$\sigma_t^2 = \frac{J^2 \sigma_R^2 + R^2 \sigma_J^2}{\lambda^2 (1 + RJ)}$$

(e.g., McDougall & Harrison, 1999)

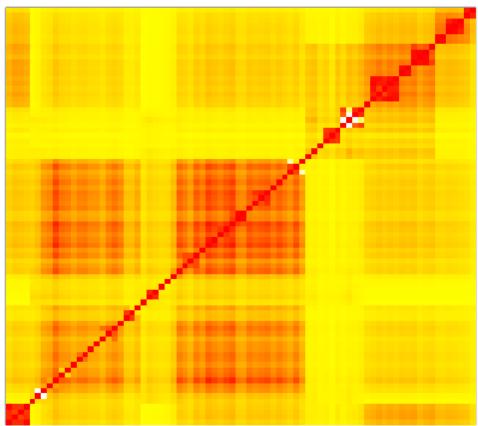
$$\sigma_z^2 = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \begin{bmatrix} \sigma_x^2 & \text{cov}(x, y) \\ \text{cov}(x, y) & \sigma_y^2 \end{bmatrix} \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$



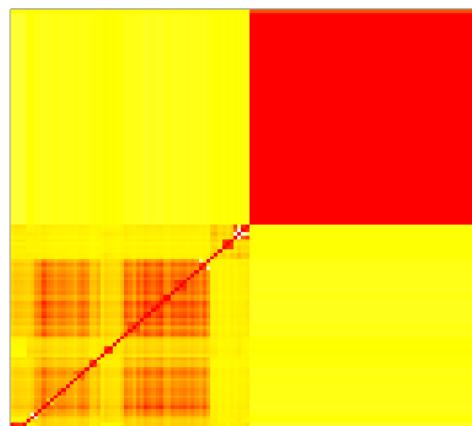
Blank correction

Regression to t_0 

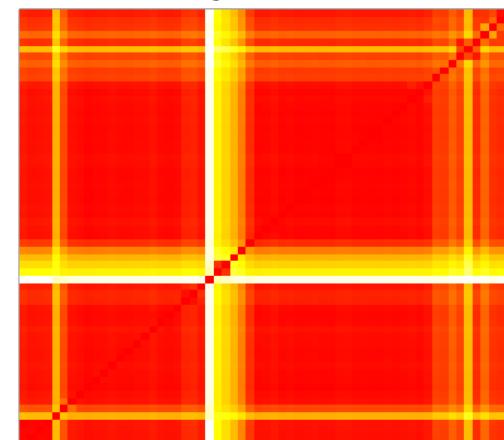
Decay correction



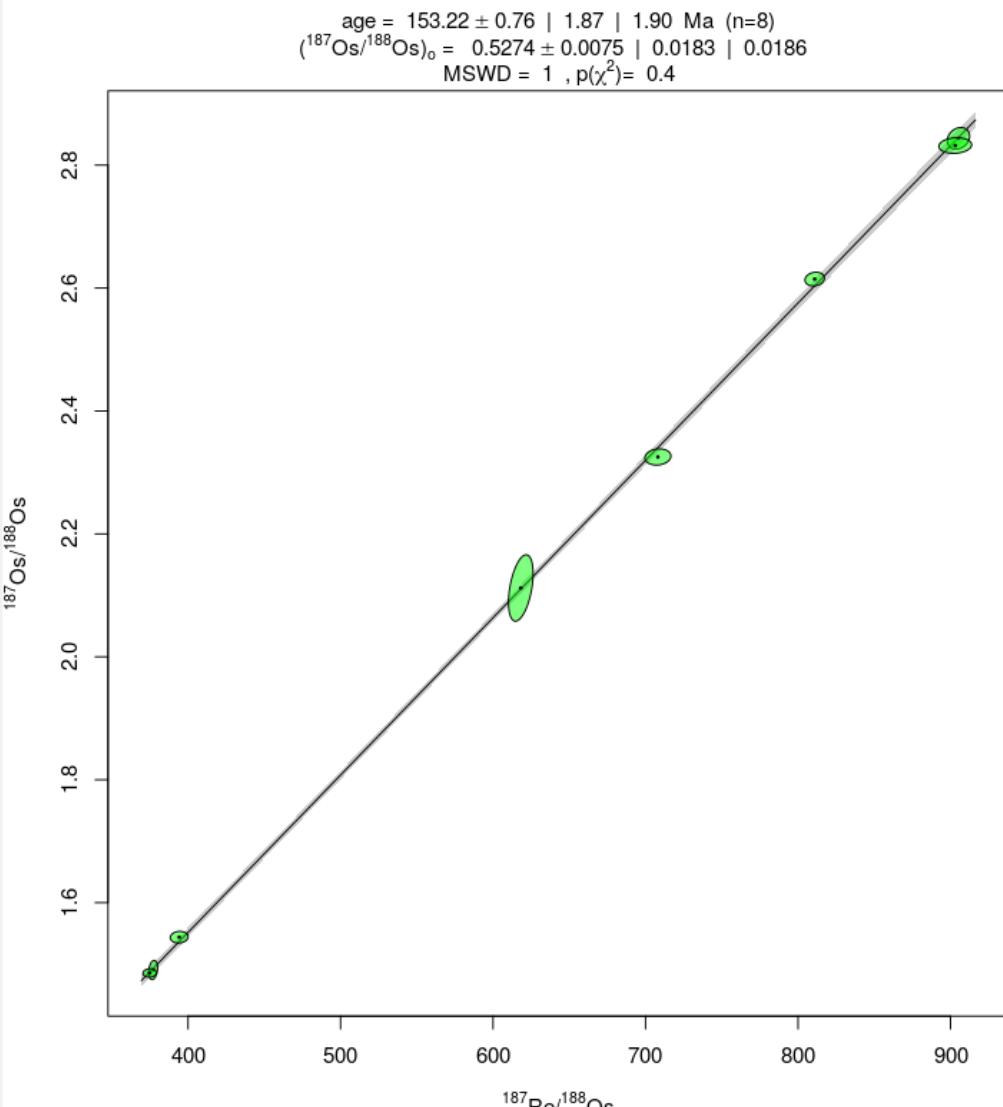
Interference correction



J-factor

 $^{40}\text{Ar}^*/^{39}\text{Ar}$

	Re187/Os188	err[Re]	Os187/Os188	err[Os18] (rho)	(C)	(omit)	H
1	394.2	2.4	1.5438	0.0039	0.073		
2	377.1	1.2	1.4904	0.0064	0.23		
3	374.8	1.8	1.4856	0.0028	0.067		
4	708.1	3.5	2.3252	0.0054	0.113		
5	618.2	3.3	2.112	0.0221	0.434		
6	810.9	2.6	2.6146	0.0046	0.143		
7	905.2	3	2.8436	0.0072	0.215		
8	903.1	4.4	2.8319	0.0052	0.104		
9							
10							
11							
12							
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Defaults

Clear

Open

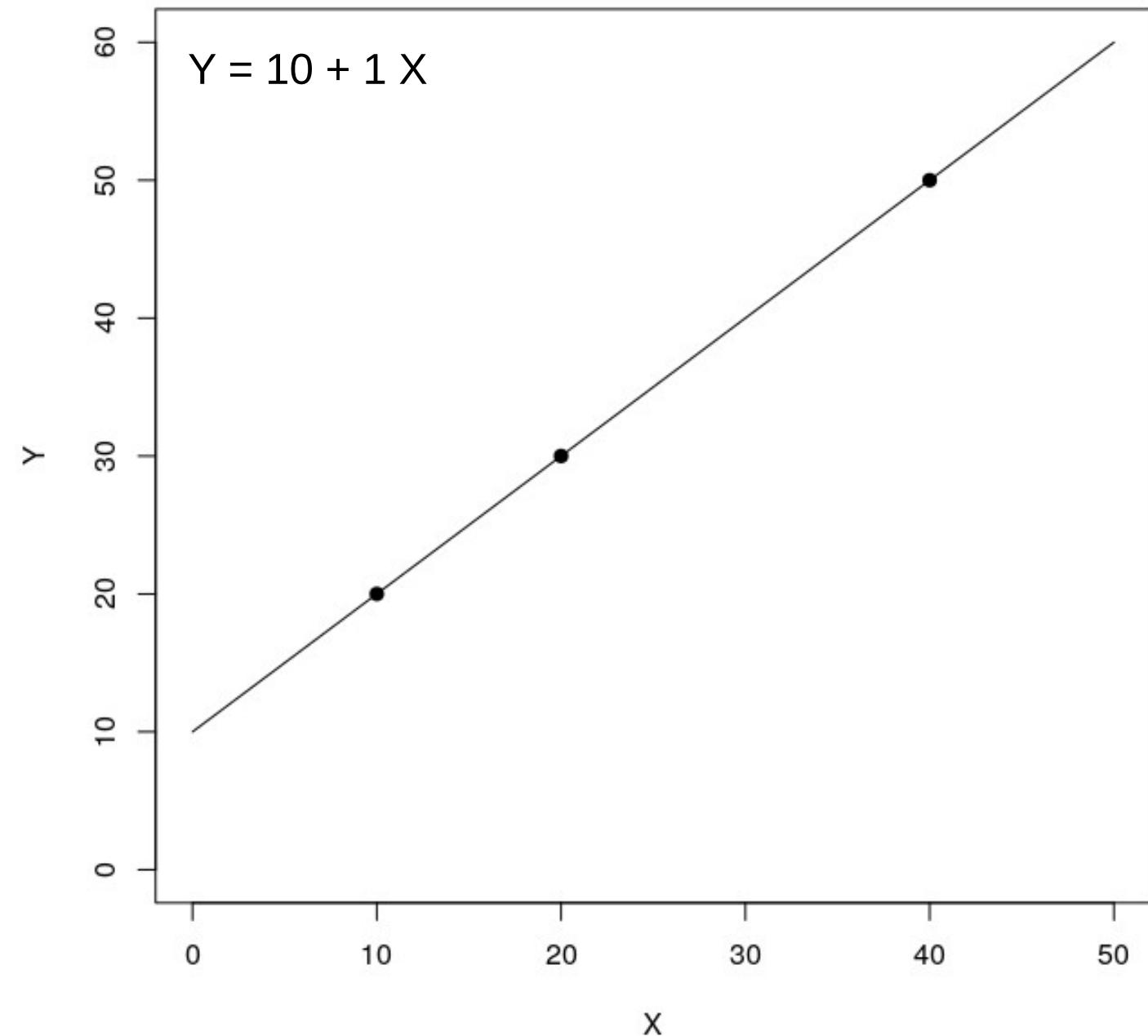
Save

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$$Y = a + b X$$

X	Y
10	20
20	30
40	50

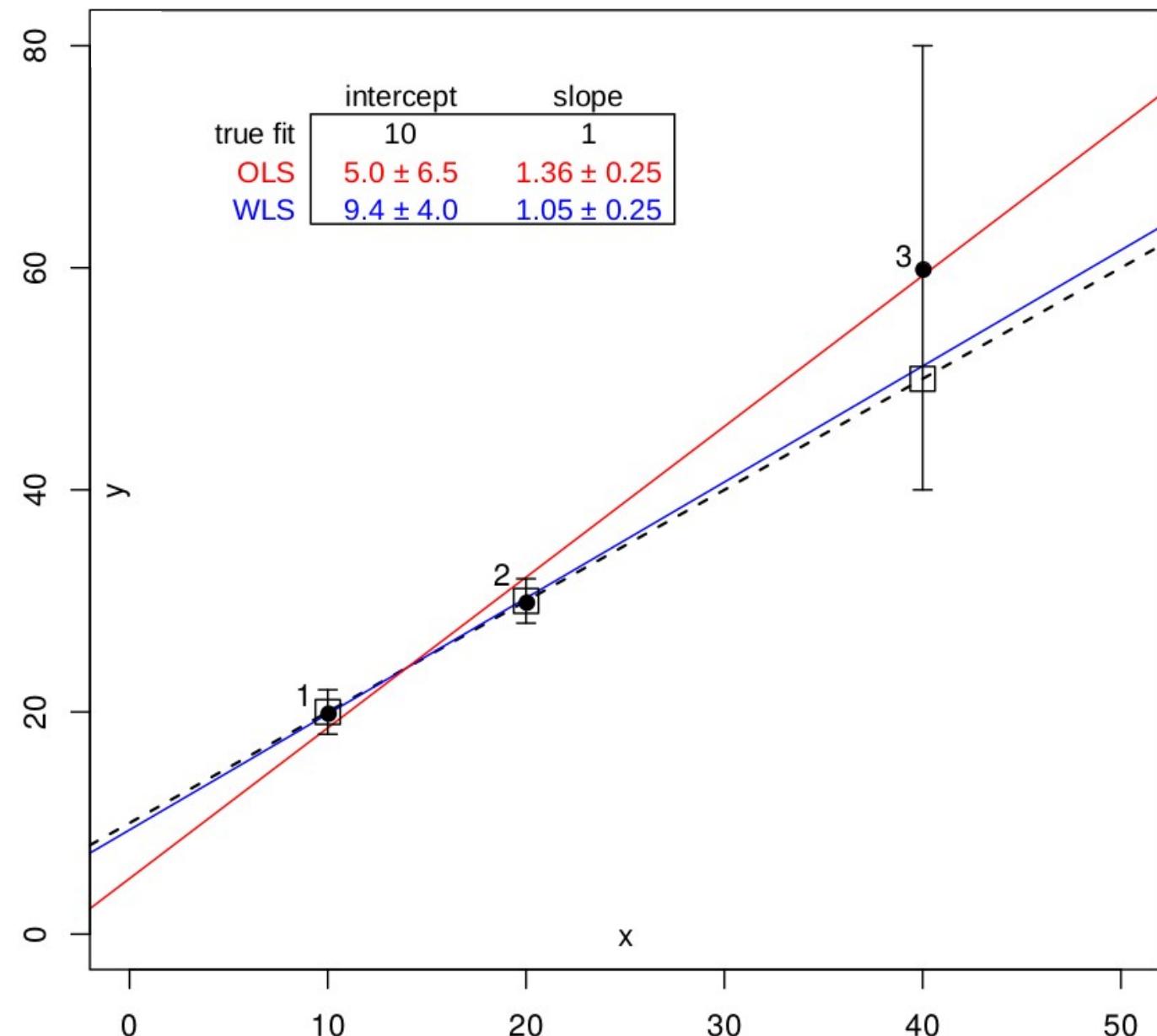


true values (squares):

x	y	s[y]
10	20	1
20	30	1
40	50	10

measurements (circles):

X	Y	s[Y]
10	20	1
20	30	1
40	60	10

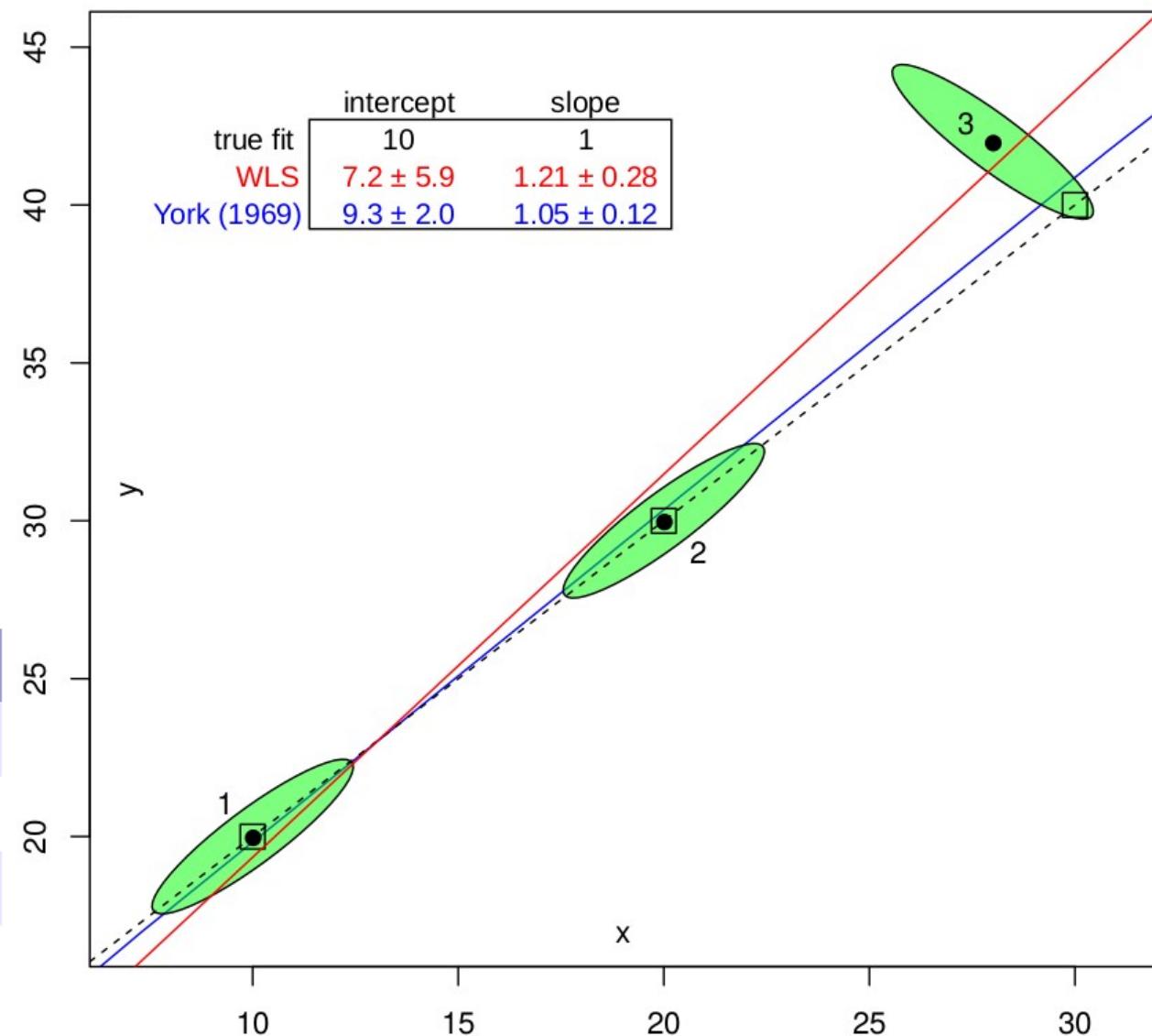


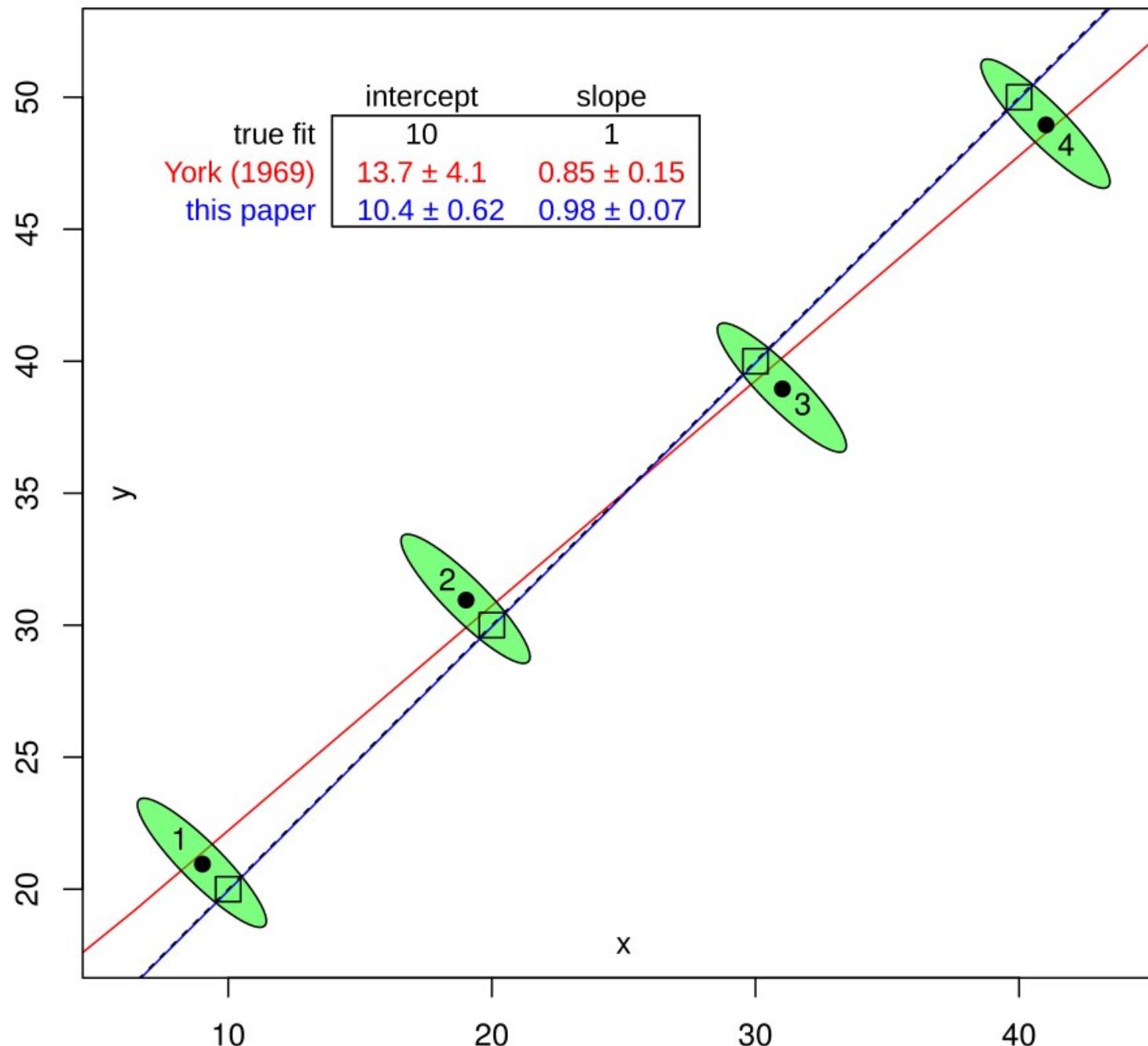
true values (squares):

x	s[x]	y	s[y]	cov[x,y]
10	1	20	1	0.9
20	1	30	1	0.9
30	1	40	1	-0.9

measurements (circles):

X	s[X]	X	s[Y]	cov[X,Y]
10	1	20	1	0.9
20	1	30	1	0.9
28	1	42	1	-0.9





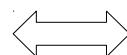
true values		measurements		covariance matrix							
		X1	X2	X3	X4	Y1	Y2	Y3	Y4		
x1	10	X1	9	1	0.99	0	0	-0.9	-0.9	0	0
x2	20	X2	19	0.99	1	0	0	-0.9	-0.9	0	0
x3	30	X3	31	0	0	1	0.99	0	0	-0.9	-0.9
x4	40	X4	41	0	0	0.99	1	0	0	-0.9	-0.9
y1	20	Y1	21	-0.9	-0.9	0	0	1	0.99	0	0
y2	30	Y2	31	-0.9	-0.9	0	0	0.99	1	0	0
y3	40	Y3	39	0	0	-0.9	-0.9	0	0	1	0.99
y4	50	Y4	49	0	0	-0.9	-0.9	0	0	0.99	1

flat table

X	Y	s[Y]
10	20	1
20	30	1
40	60	10

vector
of
means

		X1	X2	X3	Y1	Y2	Y3
X1	10	0	0	0	0	0	0
X2	20	0	0	0	0	0	0
X3	40	0	0	0	0	0	0
Y1	20	0	0	0	1	0	0
Y2	30	0	0	0	0	1	0
Y3	60	0	0	0	0	0	100



X	s[X]	X	s[Y]	cov[X, Y]
10	1	20	1	0.9
20	1	30	1	0.9
28	1	42	1	-0.9



		X1	X2	X3	Y1	Y2	Y3
X1	20	1	0	0	0.9	0	0
X2	30	0	1	0	0	0.9	0
X3	43	0	0	1	0	0	-0.9
Y1	20	0.9	0	0	1	0	0
Y2	30	0	0.9	0	0	1	0
Y3	42	0	0	-0.9	0	0	1

$^{40}\text{Ar}/^{39}\text{Ar}$

Ar40/Ar39[1]	a1	s[a1]^2	s[a1,b1]	s[a1,a2]	s[a1,b2]	...	s[a1,an]	s[a1,bn]	s[a1,l4]	s[a1,l8]	s[a1,l5]
Ar40/Ar36[1]	b1	s[b1,a1]	s[b1]^2	s[b1,a2]	s[b1,b2]	...	s[b1,an]	s[b1,bn]	s[b1,l4]	s[b1,l8]	s[b1,l5]
Ar40/Ar39[2]	a2	s[a2,a1]	s[a2,b1]	s[a2]^2	s[a2,b2]	...	s[a2,an]	s[a2,bn]	s[a2,l4]	s[a2,l8]	s[a2,l5]
Ar40/Ar36[2]	b2	s[b2,a1]	s[b2,b1]	s[b2,a2]	s[b2]^2	...	s[b2,an]	s[b2,bn]	s[b2,l4]	s[b2,l8]	s[b2,l5]
...
Ar40/Ar39[n]	an	s[an,a1]	s[an,b1]	s[an,a2]	s[an,b2]	...	s[an]^2	s[an,bn]	s[an,l4]	s[an,l8]	s[an,l5]
Ar40/Ar36[n]	bn	s[bn,a1]	s[bn,b1]	s[bn,a2]	s[bn,b2]	...	s[a1,an]	s[bn]^2	s[bn,l4]	s[bn,l8]	s[bn,l5]
Lambda[40]	l4	s[l4,a1]	s[l4,b1]	s[l4,a2]	s[l4,b2]	...	s[l4,an]	s[l4,bn]	s[l4]^2	s[l4,l8]	s[l4,l5]
Lambda[238]	l8	s[l8,a1]	s[l8,b1]	s[l8,a2]	s[l8,b2]	...	s[l8,an]	s[l8,bn]	s[l8,l4]	s[l8]^2	s[l8,l5]
Lambda[235]	l5	s[l5,a1]	s[l5,b1]	s[l5,a2]	s[l5,b2]	...	s[l5,an]	s[l5,bn]	s[l5,l4]	s[l5,l8]	s[l5]^2

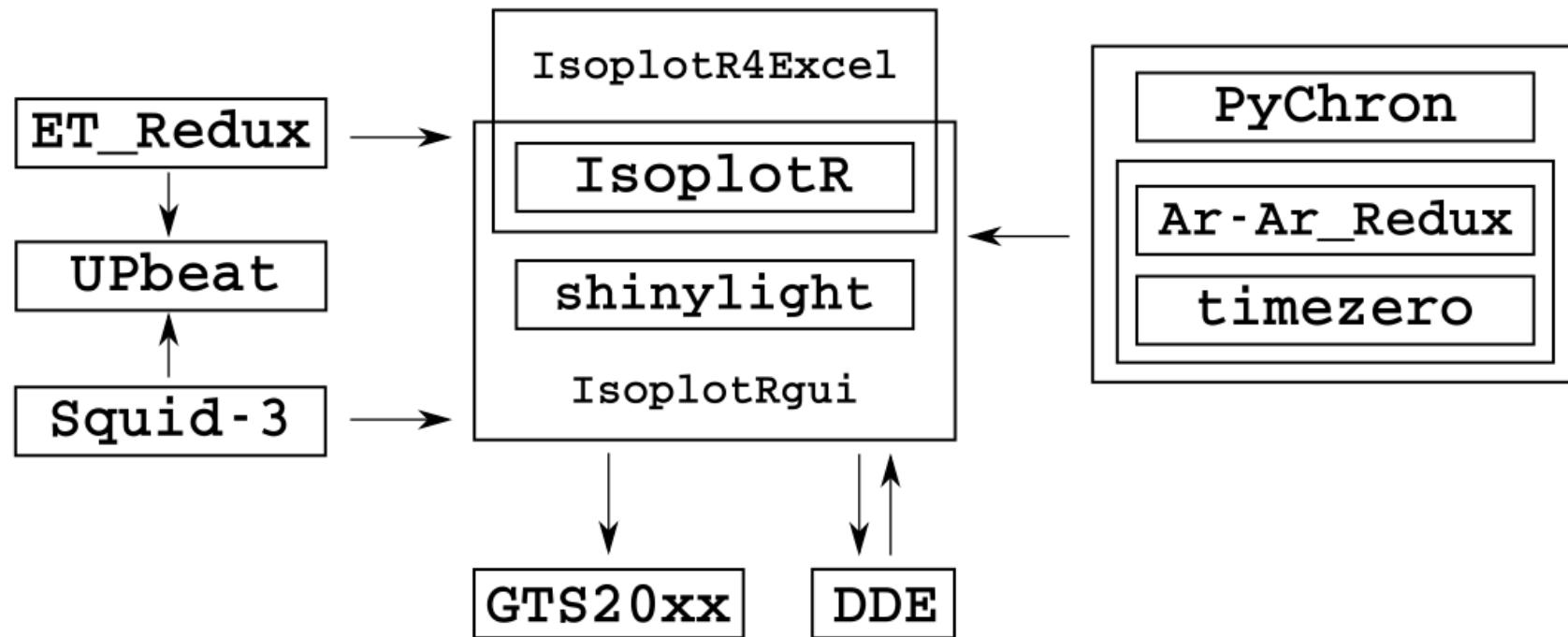
U-Pb

Lambda[40]	l4	s[l4]^2	s[l4,l8]	s[l4,l5]	s[l4,c1]	s[l4,d1]	s[l4,c2]	s[l4,d2]	...	s[l4,cn]	s[l4,dn]
Lambda[238]	l8	s[l8,l4]	s[l8]^2	s[l8,l5]	s[l8,c1]	s[l8,d1]	s[l8,c2]	s[l8,d2]	...	s[l8,cn]	s[l8,dn]
Lambda[235]	l5	s[l5,l4]	s[l5,l8]	s[l5]^2	s[l5,c1]	s[l5,d1]	s[l5,c2]	s[l5,d2]	...	s[l5,cn]	s[l5,dn]
Pb206/U238[1]	c1	s[c1,l4]	s[c1,l8]	s[c1,l5]	s[c1]^2	s[c1,d1]	s[c1,c2]	s[c1,d2]	...	s[c1,cn]	s[c1,dn]
Pb207/U235[1]	d1	s[d1,l4]	s[d1,l8]	s[d1,l5]	s[d1,c1]	s[d1]^2	s[d1,c2]	s[d1,d2]	...	s[d1,cn]	s[d1,dn]
Pb206/U238[2]	c2	s[c2,l4]	s[c2,l8]	s[c2,l5]	s[c2,c1]	s[c2,d1]	s[c2]^2	s[c2,d2]	...	s[c2,cn]	s[c2,dn]
Pb207/U235[2]	d2	s[d2,l4]	s[d2,l8]	s[d2,l5]	s[d2,c1]	s[d2,d1]	s[d2,c2]	s[d2]^2	...	s[d2,cn]	s[d2,dn]
...
Pb206/U238[m]	cm	s[cm,l4]	s[cm,l8]	s[cm,l5]	s[cm,c1]	s[cm,d1]	s[cm,c2]	s[cm,d2]	...	s[cm]^2	s[cm,dn]
Pb207/U235[m]	dm	s[dm,l4]	s[dm,l8]	s[dm,l5]	s[dm,c1]	s[dm,d1]	s[dm,c2]	s[dm,d2]	...	s[c1,cn]	s[dm]^2

$^{40}\text{Ar}/^{39}\text{Ar}$

Ar40/Ar39[1]	a1	s[a1]^2	s[a1,b1]	s[a1,a2]	s[a1,b2]	...	s[a1,an]	s[a1,bn]	s[a1,l4]	s[a1,l8]	s[a1,l5]	s[a1,c1]	s[a1,d1]	s[a1,c2]	s[a1,d2]	...	s[a1,cn]	s[a1,dn]
Ar40/Ar36[1]	b1	s[b1,a1]	s[b1]^2	s[b1,a2]	s[b1,b2]	...	s[b1,an]	s[b1,bn]	s[b1,l4]	s[b1,l8]	s[b1,l5]	s[b1,c1]	s[b1,d1]	s[b1,c2]	s[b1,d2]	...	s[b1,cn]	s[b1,dn]
Ar40/Ar39[2]	a2	s[a2,a1]	s[a2,b1]	s[a2]^2	s[a2,b2]	...	s[a2,an]	s[a2,bn]	s[a2,l4]	s[a2,l8]	s[a2,l5]	s[a2,c1]	s[a2,d1]	s[a2,c2]	s[a2,d2]	...	s[a2,cn]	s[a2,dn]
Ar40/Ar36[2]	b2	s[b2,a1]	s[b2,b1]	s[b2,a2]	s[b2]^2	...	s[b2,an]	s[b2,bn]	s[b2,l4]	s[b2,l8]	s[b2,l5]	s[b2,c1]	s[b2,d1]	s[b2,c2]	s[b2,d2]	...	s[b2,cn]	s[b2,dn]
...
Ar40/Ar39[n]	an	s[an,a1]	s[an,b1]	s[an,a2]	s[an,b2]	...	s[an]^2	s[an,bn]	s[an,l4]	s[an,l8]	s[an,l5]	s[an,c1]	s[an,d1]	s[an,c2]	s[a2,d2]	...	s[an,cn]	s[an,dn]
Ar40/Ar36[n]	bn	s[bn,a1]	s[bn,b1]	s[bn,a2]	s[bn,b2]	...	s[a1,an]	s[bn]^2	s[bn,l4]	s[bn,l8]	s[bn,l5]	s[bn,c1]	s[bn,d1]	s[bn,c2]	s[bn,d2]	...	s[bn,cn]	s[bn,dn]
Lambda[40]	l4	s[l4,a1]	s[l4,b1]	s[l4,a2]	s[l4,b2]	...	s[l4,an]	s[l4,bn]	s[l4]^2	s[l4,l8]	s[l4,l5]	s[l4,c1]	s[l4,d1]	s[l4,c2]	s[l4,d2]	...	s[l4,cn]	s[l4,dn]
Lambda[238]	l8	s[l8,a1]	s[l8,b1]	s[l8,a2]	s[l8,b2]	...	s[l8,an]	s[l8,bn]	s[l8]^2	s[l8,l8]	s[l8,l5]	s[l8,c1]	s[l8,d1]	s[l8,c2]	s[l8,d2]	...	s[l8,cn]	s[l8,dn]
Lambda[235]	l5	s[l5,a1]	s[l5,b1]	s[l5,a2]	s[l5,b2]	...	s[l5,an]	s[l5,bn]	s[l5]^2	s[l5,l8]	s[l5,l5]	s[l5,c1]	s[l5,d1]	s[l5,c2]	s[l5,d2]	...	s[l5,cn]	s[l5,dn]
Pb206/U238[1]	c1	s[c1,a1]	s[c1,b1]	s[c1,a2]	s[c1,b2]	...	s[c1,an]	s[c1,bn]	s[c1,l4]	s[c1,l8]	s[c1,l5]	s[c1]^2	s[c1,d1]	s[c1,c2]	s[c1,d2]	...	s[c1,cn]	s[c1,dn]
Pb207/U235[1]	d1	s[d1,a1]	s[d1,b1]	s[d1,a2]	s[d1,b2]	...	s[d1,an]	s[d1,bn]	s[d1,l4]	s[d1,l8]	s[d1,l5]	s[d1,c1]	s[d1]^2	s[d1,c2]	s[d1,d2]	...	s[d1,cn]	s[d1,dn]
Pb206/U238[2]	c2	s[c2,a1]	s[c2,b1]	s[c2,a2]	s[c2,b2]	...	s[c2,an]	s[c2,bn]	s[c2,l4]	s[c2,l8]	s[c2,l5]	s[c2,c1]	s[c2,d1]	s[c2]^2	s[c2,d2]	...	s[c2,cn]	s[c2,dn]
Pb207/U235[2]	d2	s[d2,a1]	s[d2,b1]	s[d2,a2]	s[d2,b2]	...	s[d2,an]	s[d2,bn]	s[d2,l4]	s[d2,l8]	s[d2,l5]	s[d2,c1]	s[d2,d1]	s[d2,c2]	s[d2]^2	...	s[d2,cn]	s[d2,dn]
...
Pb206/U238[m]	cm	s[cm,a1]	s[cm,b1]	s[cm,a2]	s[cm,b2]	...	s[cm,an]	s[cm,bn]	s[cm,l4]	s[cm,l8]	s[cm,l5]	s[cm,c1]	s[cm,d1]	s[cm,c2]	s[cm,d2]	...	s[cm]^2	s[cm,dn]
Pb207/U235[m]	dm	s[dm,a1]	s[dm,b1]	s[dm,a2]	s[dm,b2]	...	s[dm,an]	s[dm,bn]	s[dm,l4]	s[dm,l8]	s[dm,l5]	s[dm,c1]	s[dm,d1]	s[dm,c2]	s[dm,d2]	...	s[dm,cn]	s[dm]^2

U-Pb



X_1	$s[X_1]^2$	$cov[X_1, X_2]$	\dots	$cov[X_1, X_n]$	$cov[X_1, Y_1]$	\dots	$cov[X_1, Y_n]$	$cov[X_1, Z_1]$	\dots	$cov[X_1, Z_n]$
X_2	$cov[X_2, X_1]$	$s[X_2]^2$	\dots	$cov[X_2, X_n]$	$cov[X_2, Y_1]$	\dots	$cov[X_2, Y_n]$	$cov[X_2, Z_1]$	\dots	$cov[X_2, Z_n]$
\vdots	\vdots	\vdots	\ddots	\vdots	\ddots	\vdots	\vdots	\vdots	\ddots	\vdots
X_n	$cov[X_n, X_1]$	$cov[X_n, X_2]$	\dots	$s[X_n]^2$	$cov[X_n, Y_1]$	\dots	$cov[X_n, Y_n]$	$cov[X_n, Z_1]$	\dots	$cov[X_n, Z_n]$
Y_1	$cov[Y_1, X_1]$	$cov[Y_1, X_2]$	\dots	$cov[Y_1, X_n]$	$s[Y_1]^2$	\dots	$cov[Y_1, Y_n]$	$cov[Y_1, Z_1]$	\dots	$cov[Y_1, Z_n]$
\vdots	\vdots	\vdots	\ddots	\vdots	\ddots	\vdots	\vdots	\vdots	\ddots	\vdots
Z_n	$cov[Z_n, X_1]$	$cov[Z_n, X_2]$	\dots	$cov[Z_n, X_n]$	$cov[Z_n, Y_1]$	\dots	$cov[Z_n, Y_n]$	$cov[Z_n, Z_1]$	\dots	$s[Z_n]^2$