



TRƯỜNG ĐẠI HỌC
SƯ PHẠM KỸ THUẬT TP. HỒ CHÍ MINH

HCMC University of Technology and Education

Faculty of Information Technology

Informed Search Algorithms (ARTIFICIAL INTELLIGENCE)

Instructor: Assoc. Prof. PhD. Hoàng Văn Dũng

Email: dunghv@hcmute.edu.vn

(slides adapted and revised from Dan Klein, Pieter Abbeel, Anca Dragan, et al)

Content

- Informed Search
 - Best first search
 - Heuristics
 - A* Search

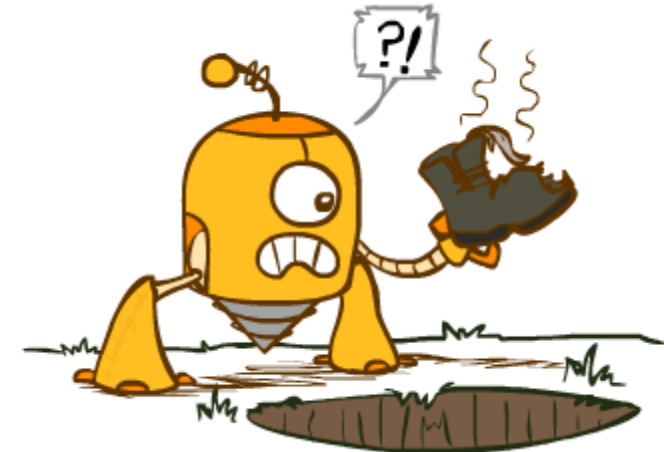
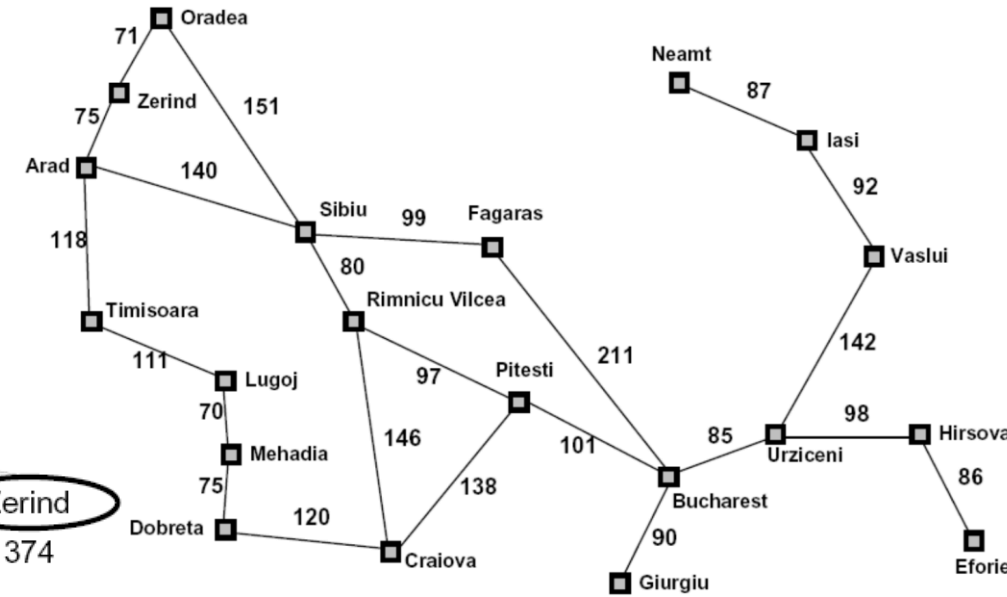
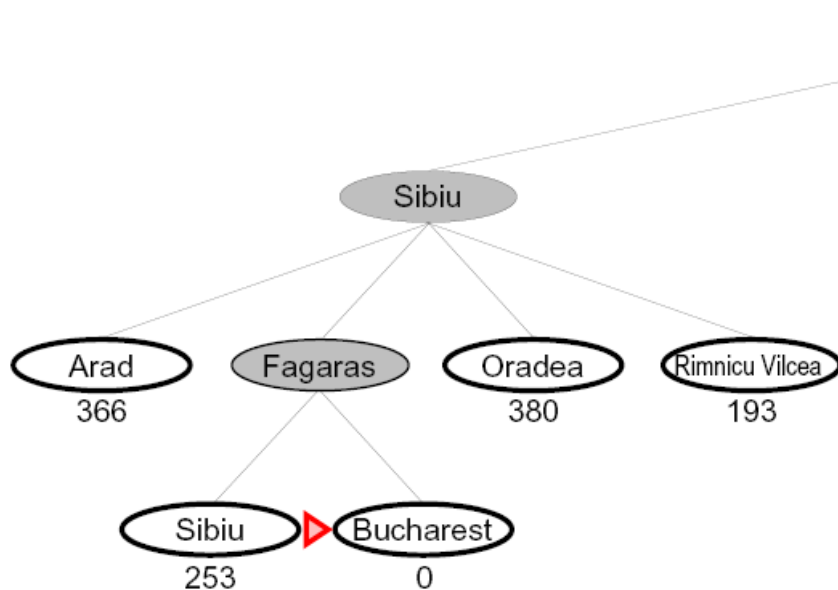


Greedy Search



Greedy Search

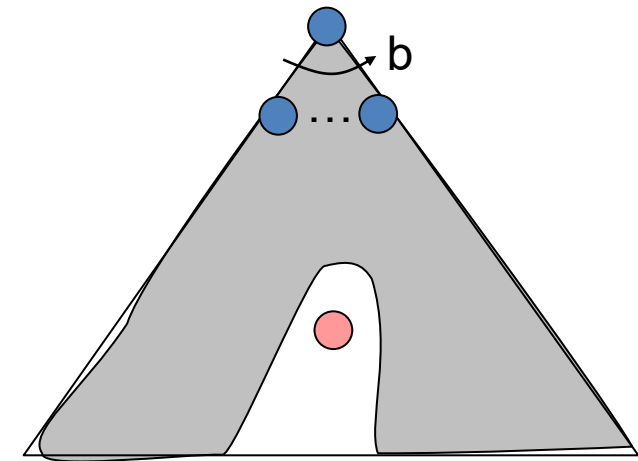
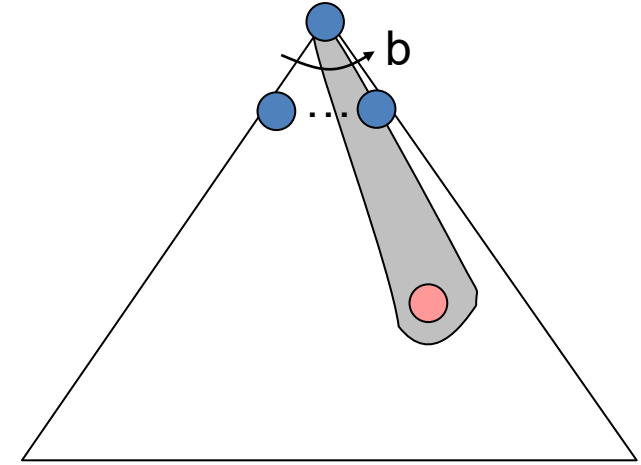
- Expand the node that seems closest...



- What can go wrong?

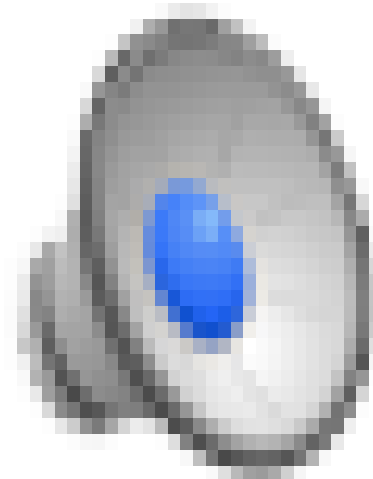
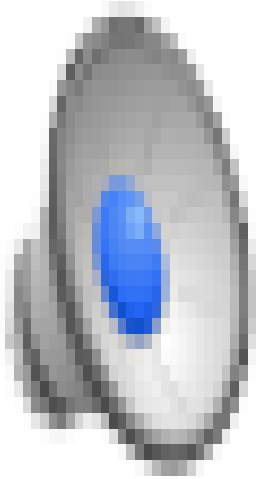
Greedy Search

- Strategy: expand a node that you think is closest to a goal state
 - Heuristic: estimate of distance to nearest goal for each state
- A common case:
 - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS



Greedy Search

- Video of Demo Greedy



A* Search



A* Search



UCS



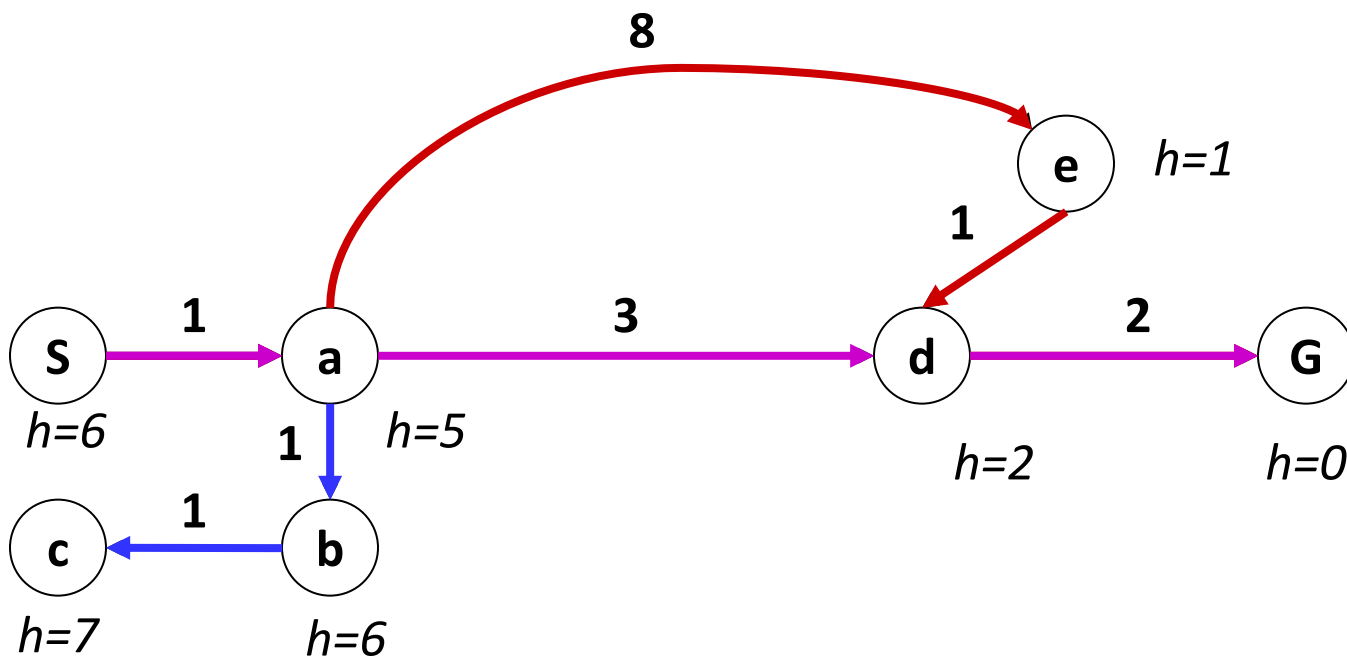
Greedy



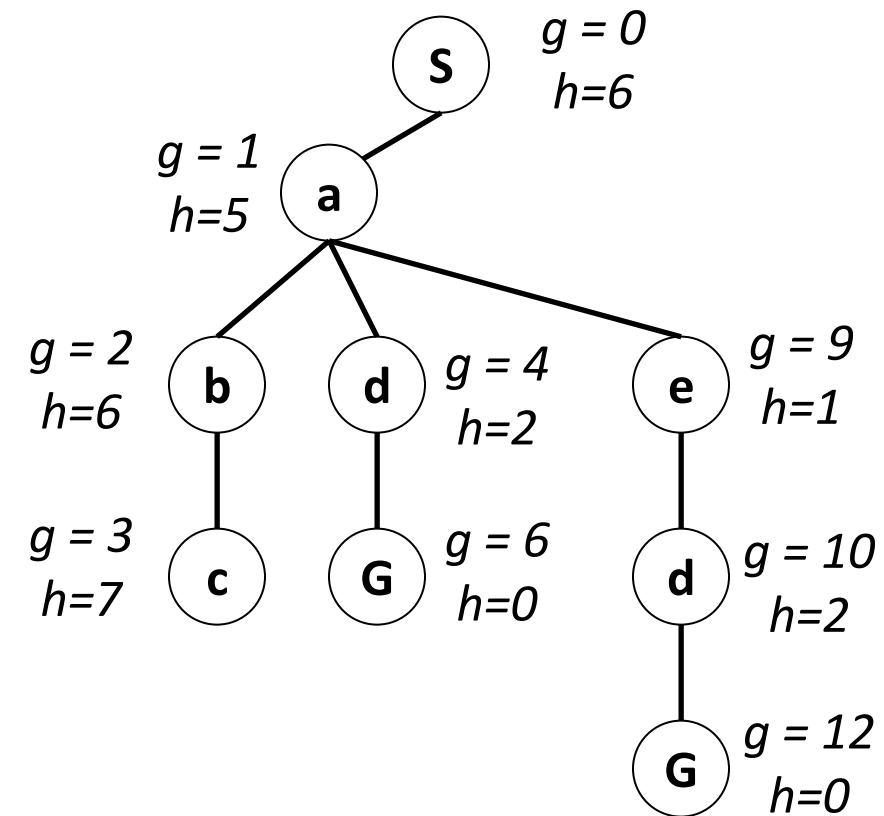
A*

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* $g(n)$
- Greedy orders by goal proximity, or *forward cost* $h(n)$



- A* Search orders by the sum: $f(n) = g(n) + h(n)$

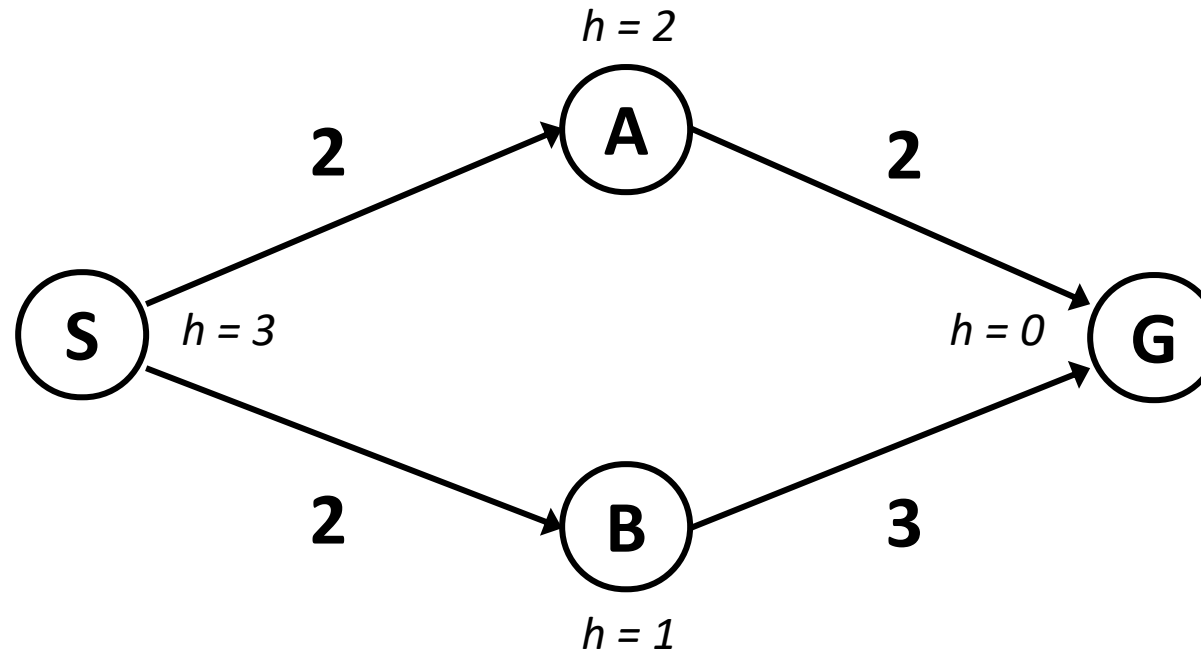


Example: Teg Grenager

A* Search

When should A* terminate?

- Should we stop when we enqueue a goal?

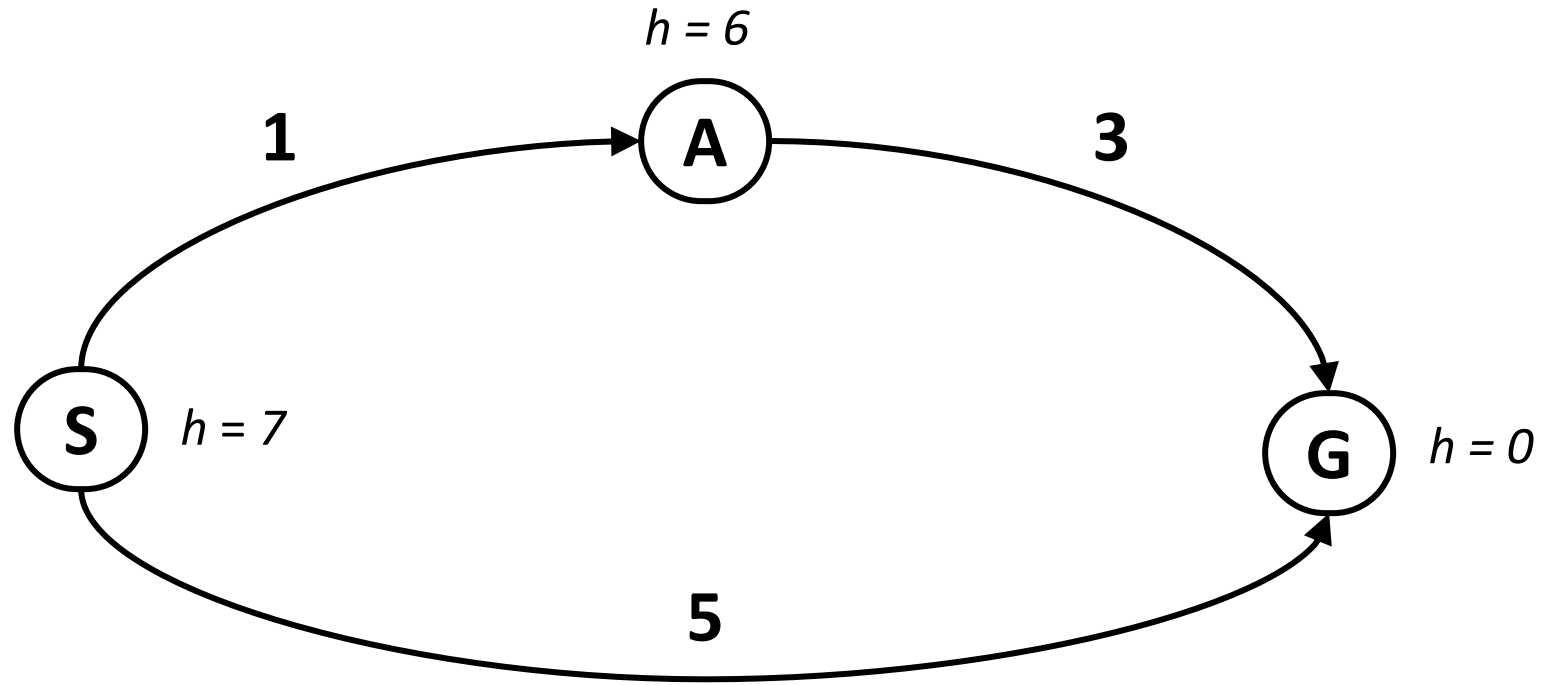


- No: only stop when we dequeue a goal

$h = 1$

A* Search

Is A* Optimal?



- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

A* Search

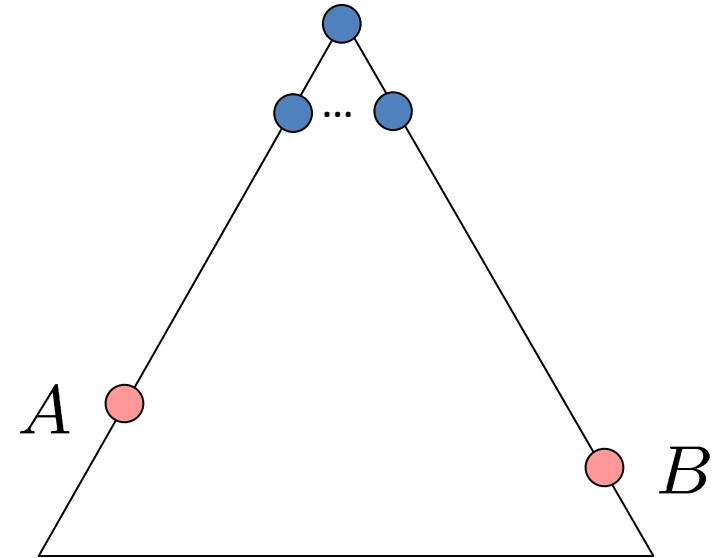
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

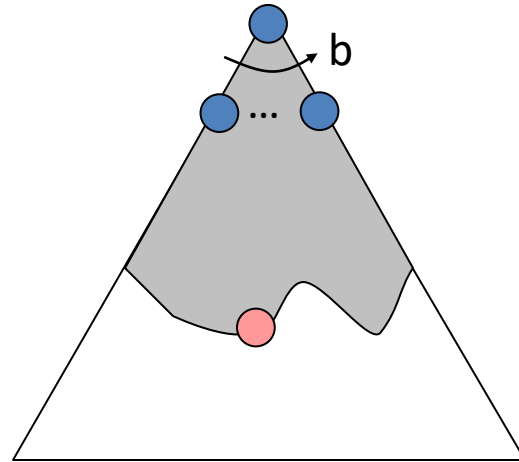
Claim:

- A will exit the fringe before B

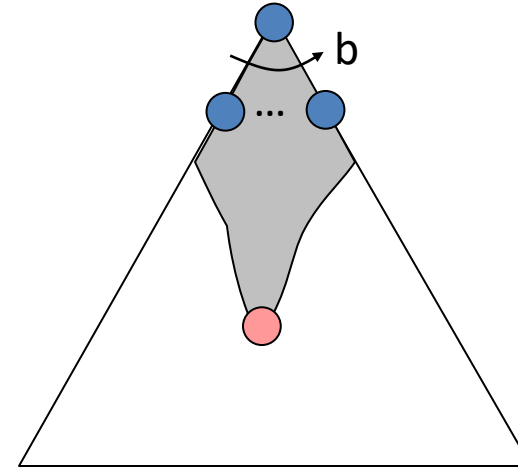


Properties of A*

Uniform-Cost

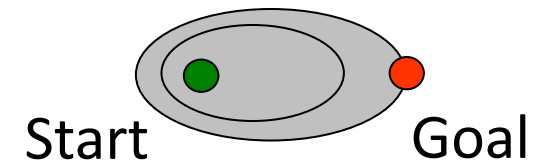
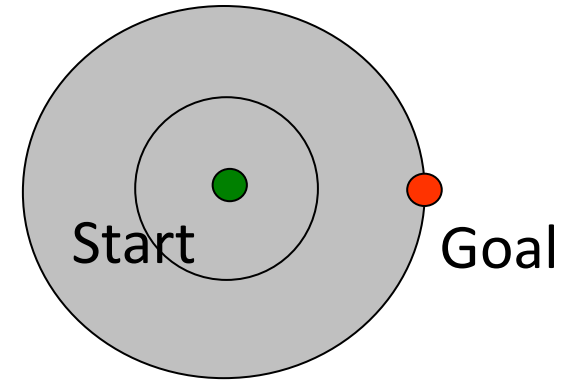


A*



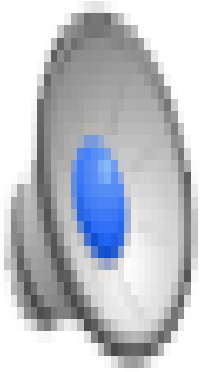
UCS vs A* Contours

- Uniform-cost expands equally in all “directions”
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality

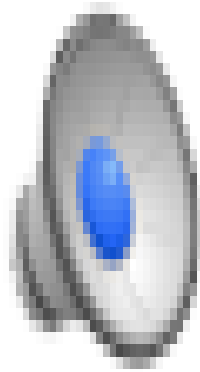


Comparison

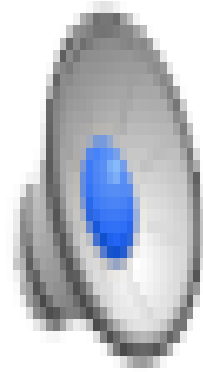
- Video of Demo Contours (Empty)



UCS



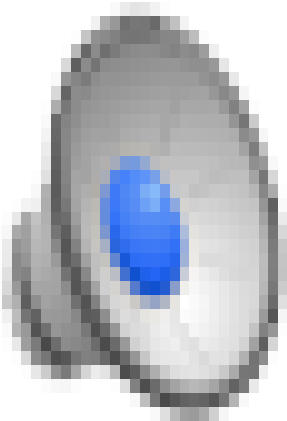
Greedy



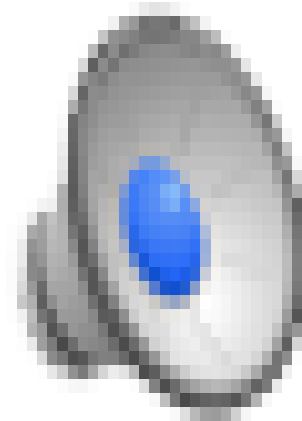
A*

Comparison

- Video of Demo Contours (Pacman Small Maze)



Greedy



A*

Comparison



Greedy



Uniform Cost



A*

A* search

A* Applications

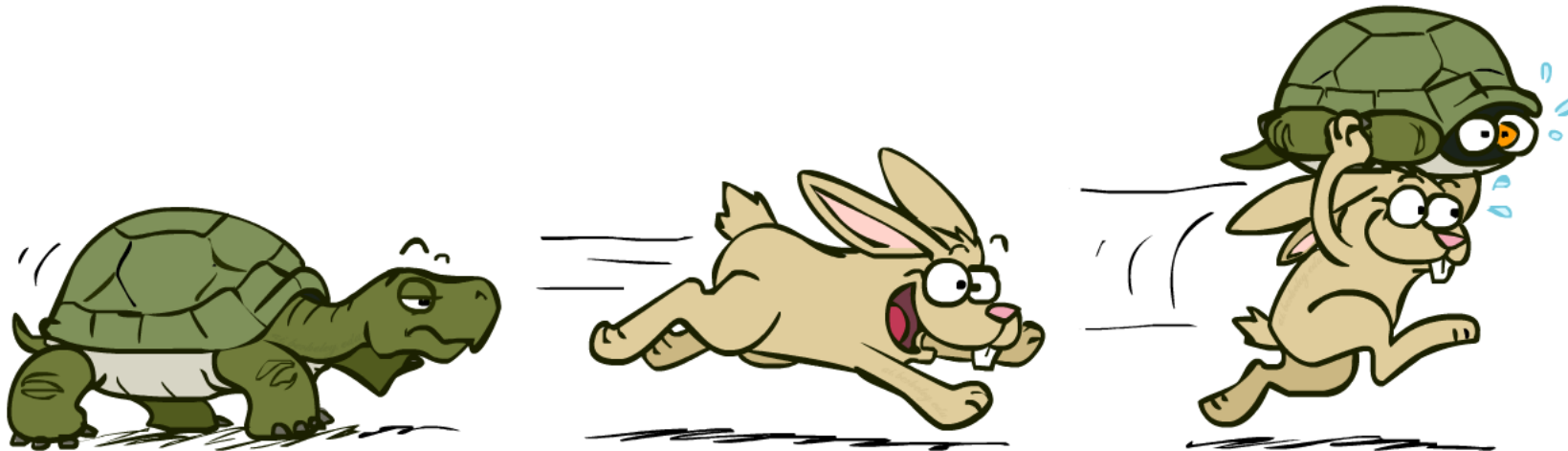
- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- ...



A* search

A* Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



Search Heuristics

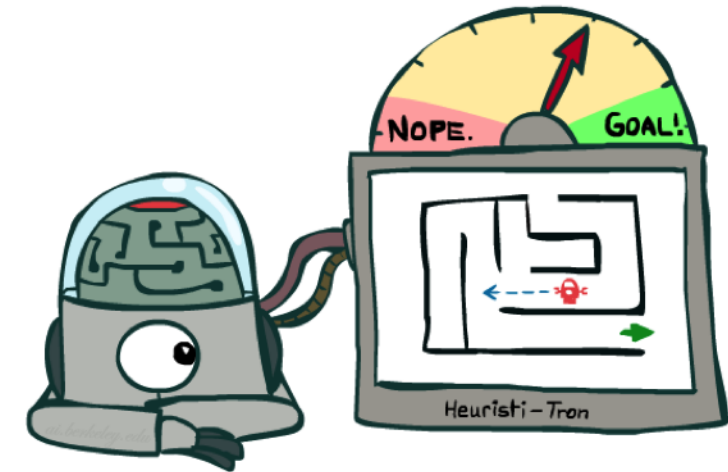
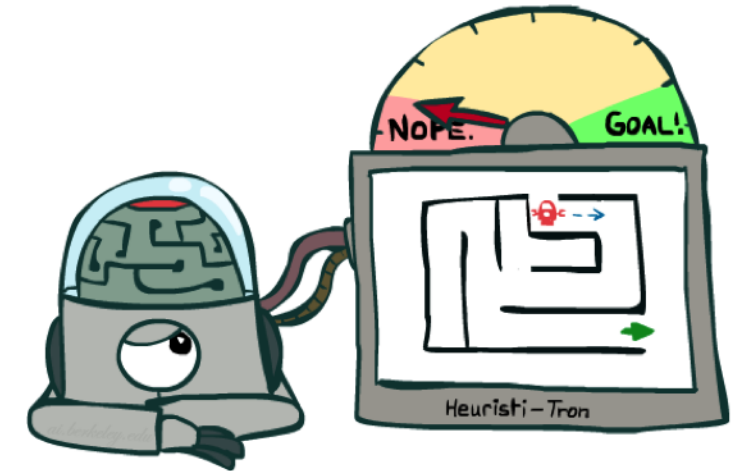
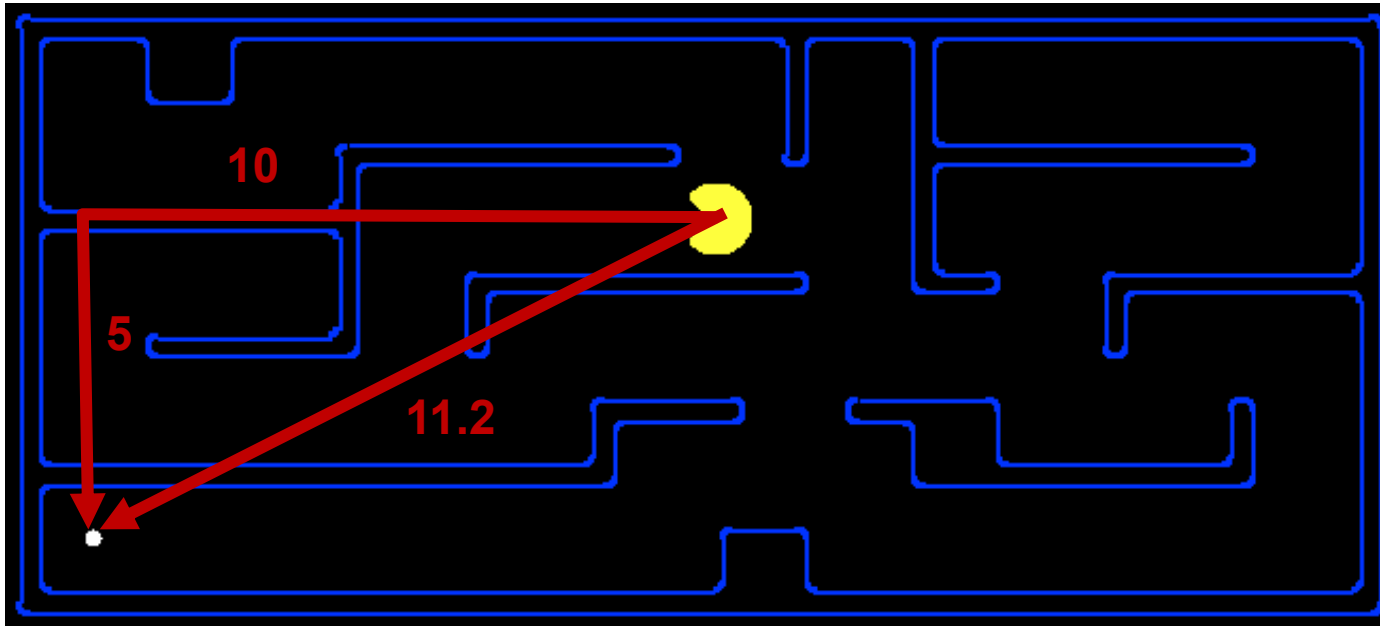
- Heuristic is:

- A function that *estimates* how close a state is to a goal

Function $h(s)$ with s is a state

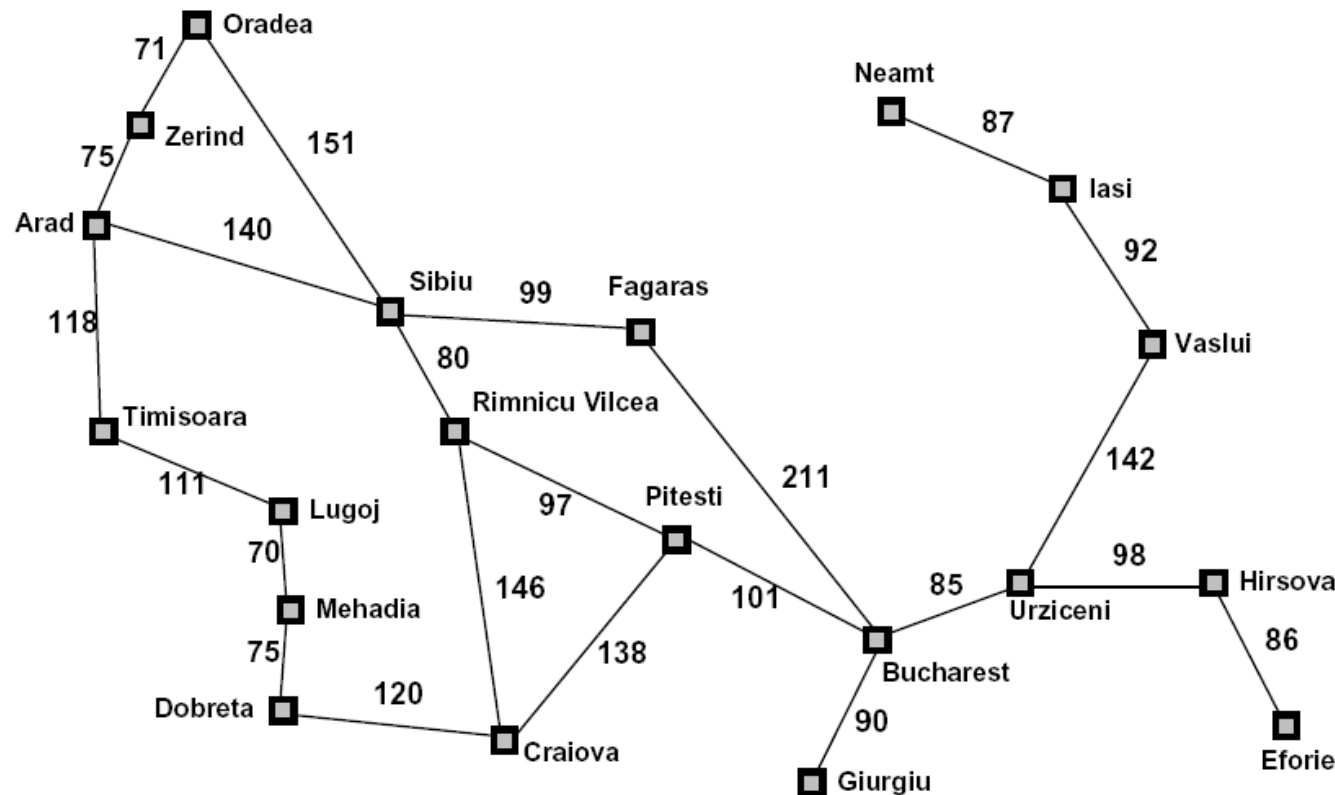
- Designed for a particular search problem

Examples: Manhattan distance, Euclidean distance for pathing



Search Heuristics

- Example: Heuristic Function



Straight-line distance
to Bucharest

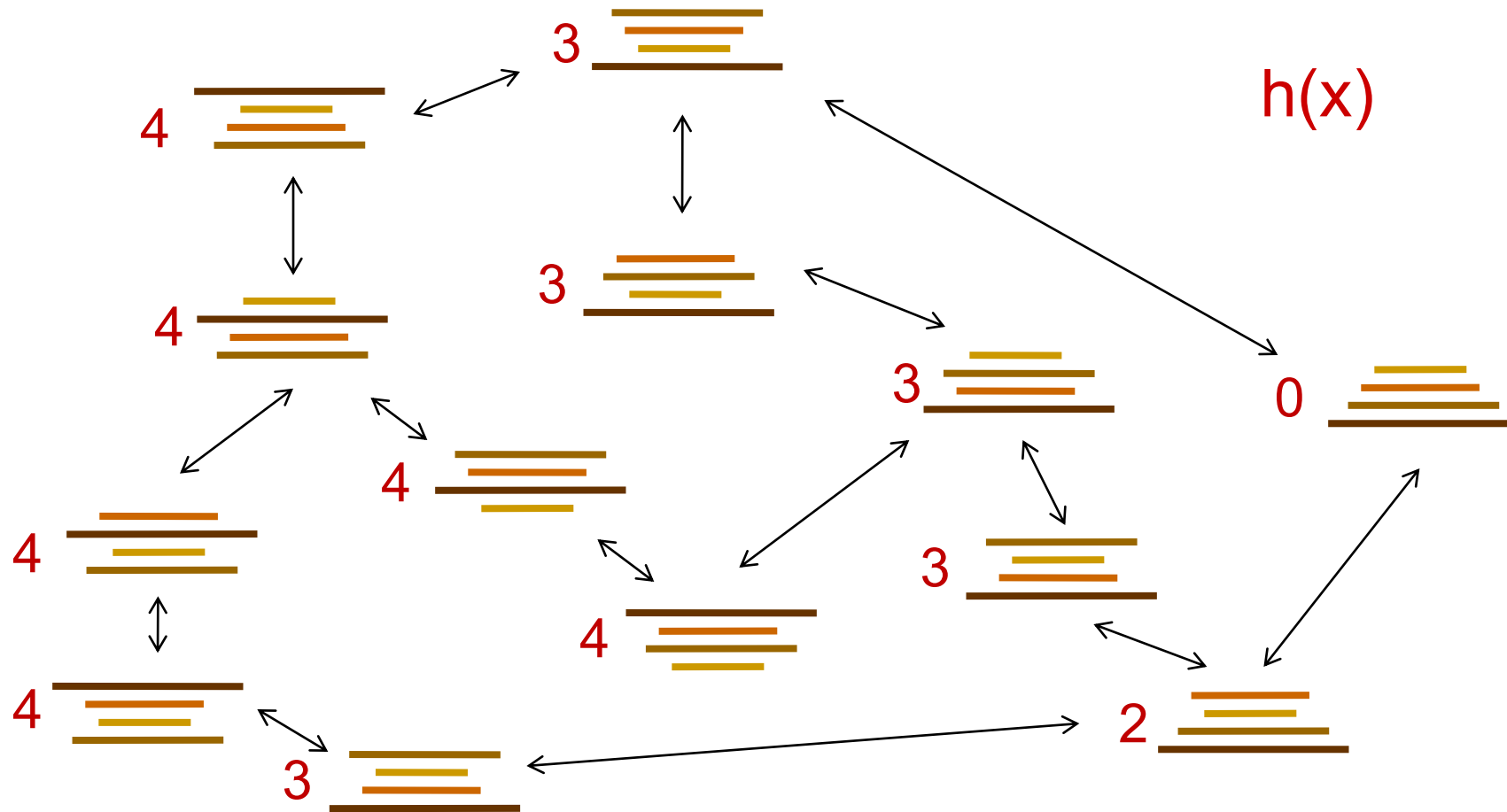
| | |
|----------------|-----|
| Arad | 366 |
| Bucharest | 0 |
| Craiova | 160 |
| Dobreta | 242 |
| Eforie | 161 |
| Fagaras | 178 |
| Giurgiu | 77 |
| Hirsova | 151 |
| Iasi | 226 |
| Lugoj | 244 |
| Mehadia | 241 |
| Neamt | 234 |
| Oradea | 380 |
| Pitesti | 98 |
| Rimnicu Vilcea | 193 |
| Sibiu | 253 |
| Timisoara | 329 |
| Urziceni | 80 |
| Vaslui | 199 |
| Zerind | 374 |

$h(x)$

Search Heuristics

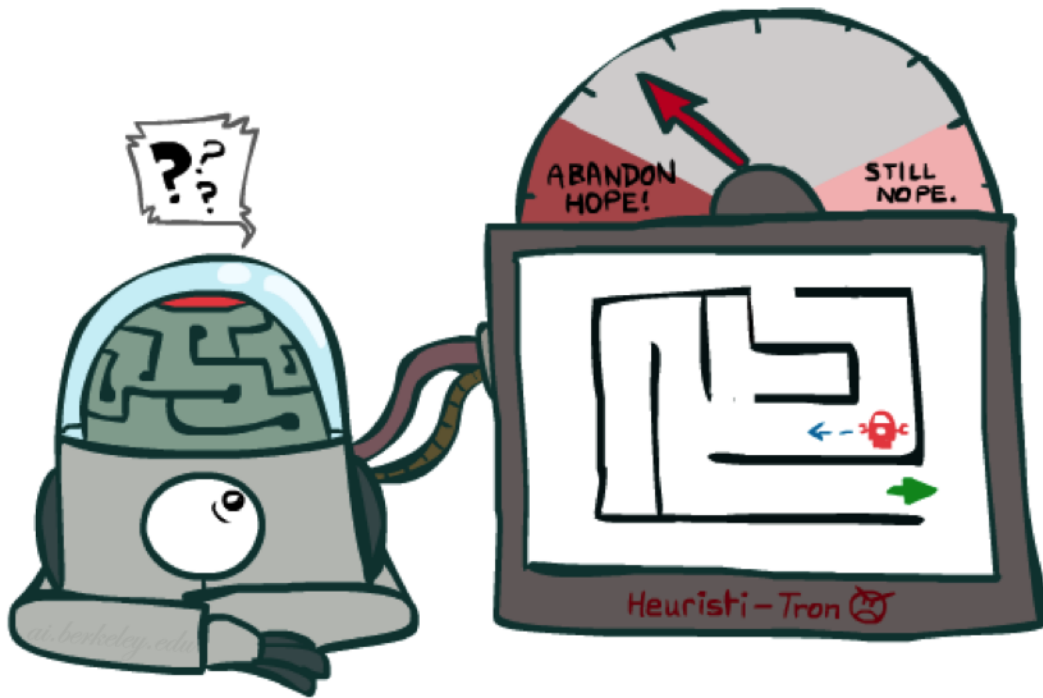
- Example: Heuristic Function

Heuristic: the number of the largest pancake that is still out of place

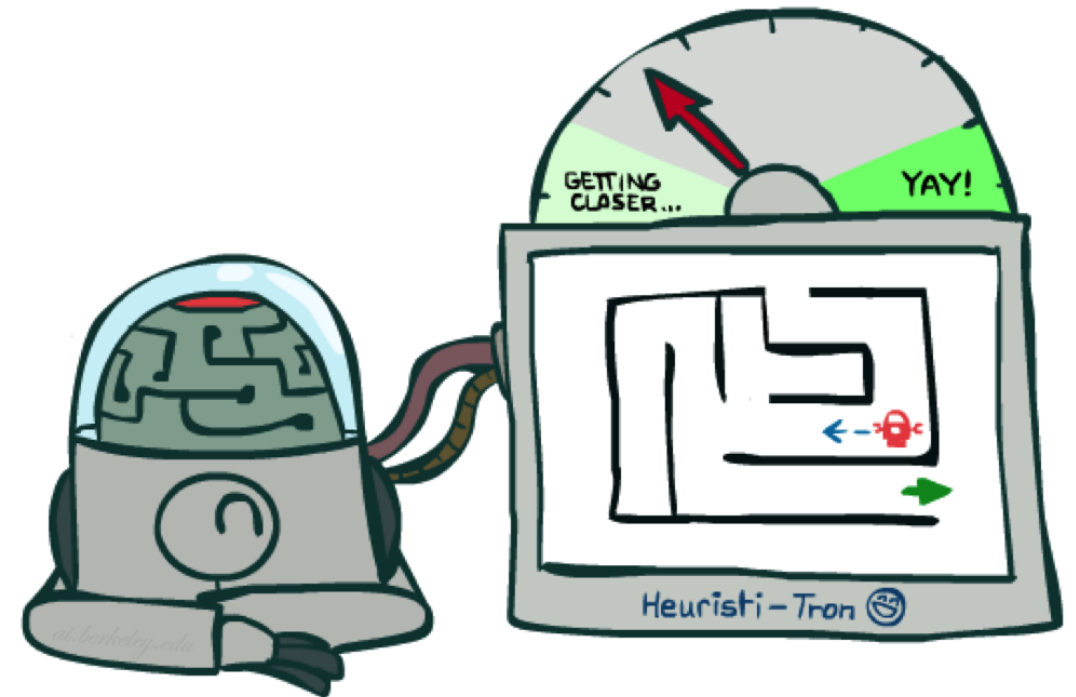


Admissible Heuristics

- Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

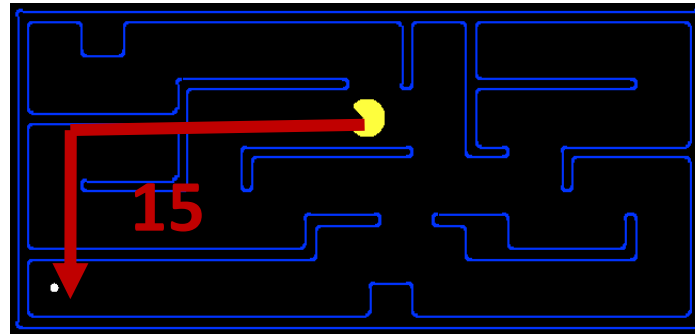
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:



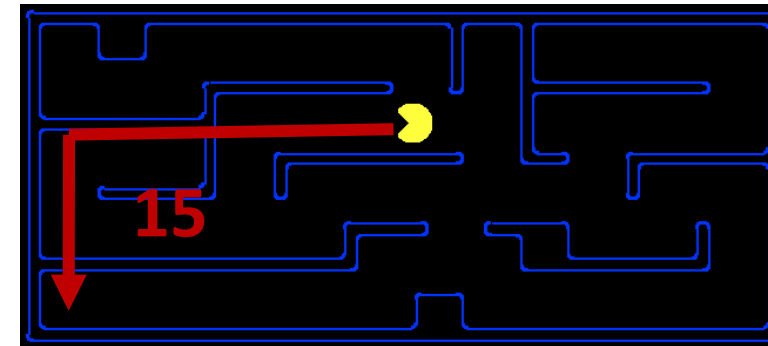
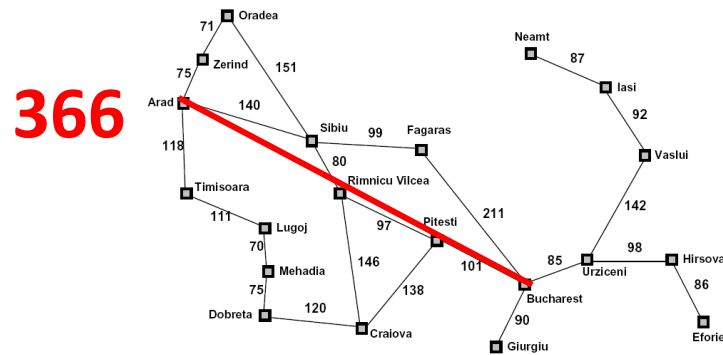
4



- Coming up with admissible heuristics is most of what's involved in using A^* in practice.

Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

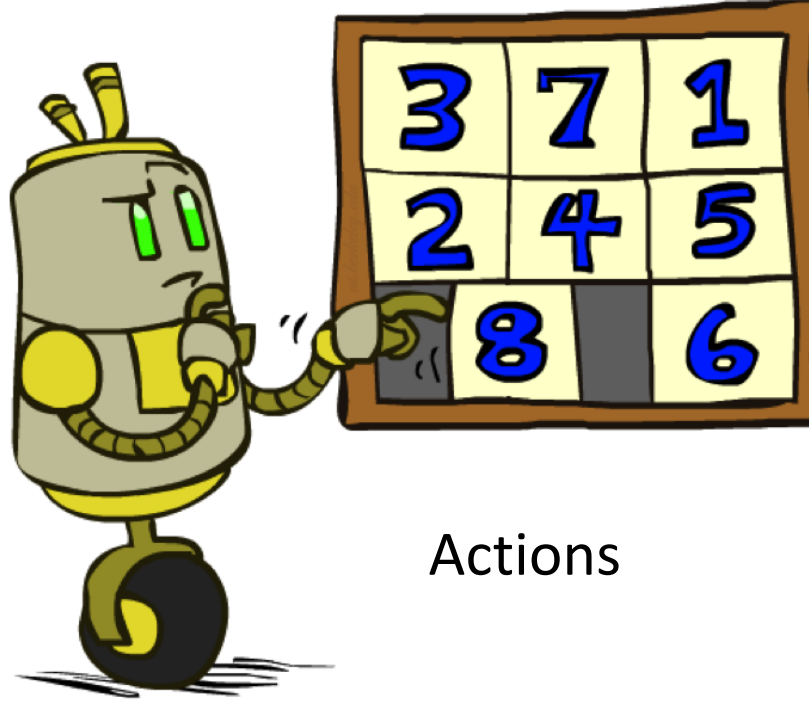


- Inadmissible heuristics are often useful too

Example: 8 Puzzle

| | | |
|---|---|---|
| 7 | 2 | 4 |
| 5 | | 6 |
| 8 | 3 | 1 |

Start State



Actions

| | | |
|---|---|---|
| | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |

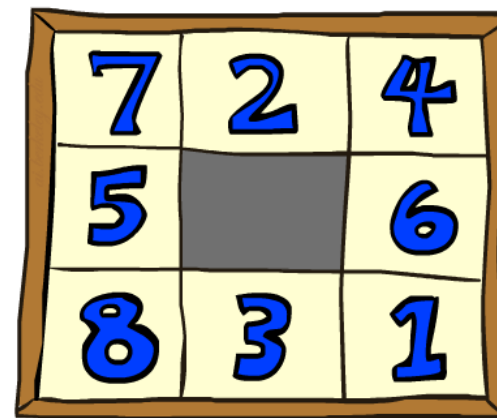
Goal State

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

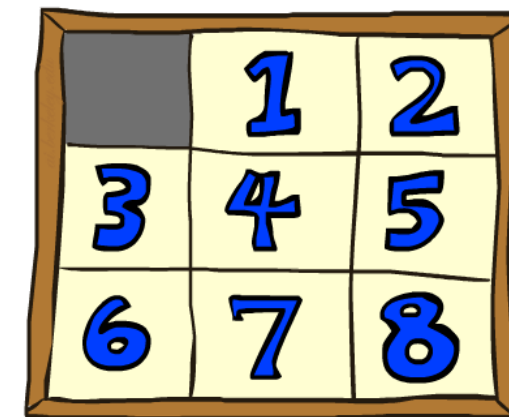
8 Puzzle heuristics

8 Puzzle I

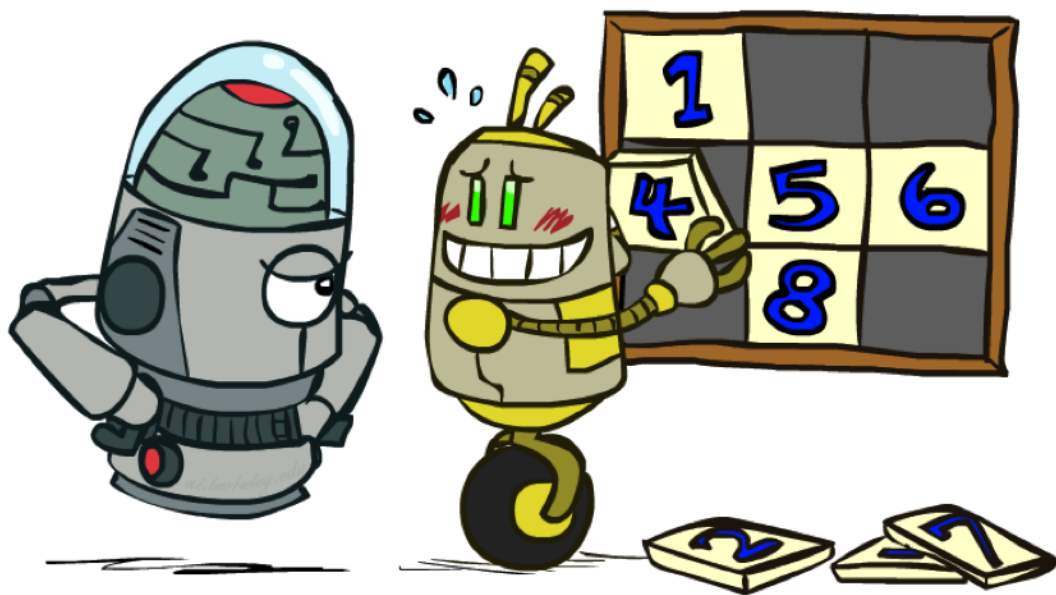
- Heuristic: Number of tiles misplaced
- Why is it admissible?
- $h_1(\text{start}) = 8$
- This is a *relaxed-problem* heuristic



Start State



Goal State



Average nodes expanded
when the optimal path has...

| | ...4 steps | ...8 steps | ...12 steps |
|-------|------------|------------|-------------------|
| UCS | 112 | 6,300 | 3.6×10^6 |
| TILES | 13 | 39 | 227 |

8 Puzzle heuristics

| | | |
|----------|----------|----------|
| 4 | 6 | 5 |
| 7 | 2 | |
| 1 | 3 | 8 |

admissible?

| | | |
|----------|----------|----------|
| 3 | 1 | 8 |
| 7 | 4 | 2 |
| 6 | 5 | |

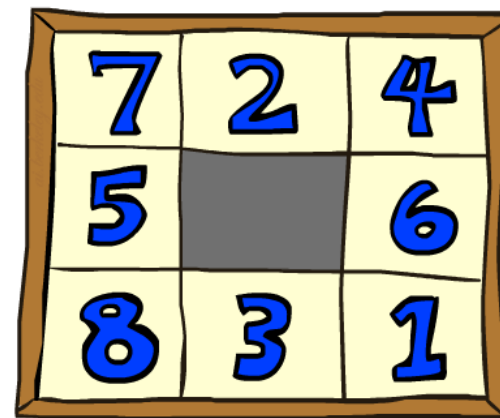
tiles misplaced

| Tile | n | Goal |
|-----------------|----------|-------------|
| 1 | 1 | 0 |
| 2 | 1 | 0 |
| 3 | 1 | 0 |
| 4 | 1 | 0 |
| 5 | 1 | 0 |
| 6 | 1 | 0 |
| 7 | 0 | 0 |
| 8 | 1 | 0 |
| h1(n) | 7 | |
| h1(goal) | 0 | |

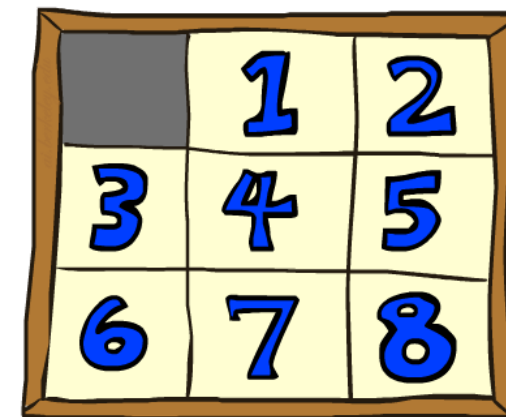
8 Puzzle heuristics

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- **Total *Manhattan* distance**
- Why is it admissible?
- $h(\text{start}) = 3 + 1 + 2 + \dots = 18$



Start State



Goal State

| Average nodes expanded when the optimal path has... | | | |
|---|------------|------------|-------------|
| | ...4 steps | ...8 steps | ...12 steps |
| TILES | 13 | 39 | 227 |
| MANHATTAN | 12 | 25 | 73 |

8 Puzzle heuristics

| | | |
|----------|----------|----------|
| 4 | 6 | 5 |
| 7 | 2 | |
| 1 | 3 | 8 |

admissible?

| | | |
|----------|----------|----------|
| 3 | 1 | 8 |
| 7 | 4 | 2 |
| 6 | 5 | |

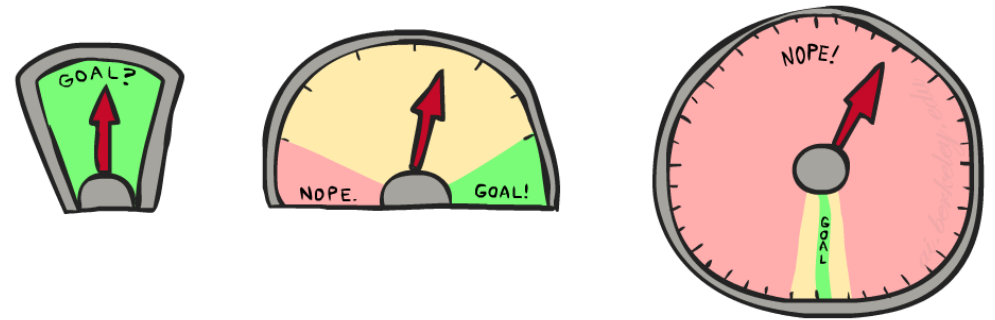
Manhattan distance

| Tile | n | Goal |
|-------------------------------|-----------|-------------|
| 1 | 3 | |
| 2 | 1 | |
| 3 | 3 | |
| 4 | 2 | |
| 5 | 3 | |
| 6 | 3 | |
| 7 | 0 | |
| 8 | 2 | |
| $h_2(n)$ | 17 | |
| $h_2(goal)$ | 0 | |

8 Puzzle heuristics

8 Puzzle III

- How about using the *actual cost* as a heuristic?
 - Would it be admissible?
 - Would we save on nodes expanded?
 - What's wrong with it?



- With A*: a trade-off between quality of estimate and work per node
 - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

8 Puzzle with A*

- Solving 8 Puzzle with A* using tree search

| | | |
|---|---|---|
| 3 | 1 | 8 |
| 4 | 2 | |
| 7 | 6 | 5 |

start

| | | |
|---|---|---|
| 3 | 1 | 8 |
| 7 | 4 | 2 |
| 6 | 5 | |

goal

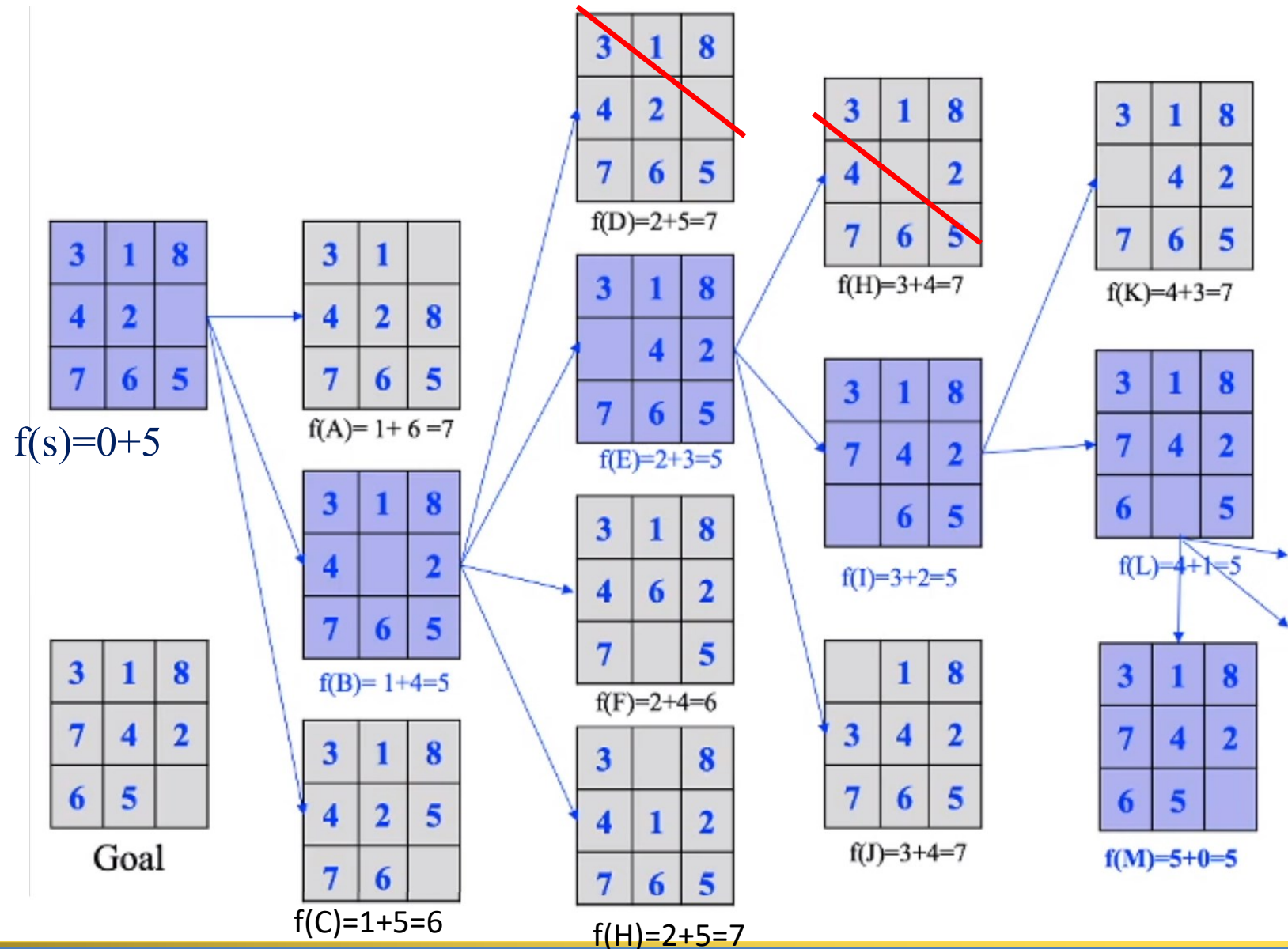
- Heuristic function $h(n)$ = the number of misplaced tiles.
- Steps of A*

Application of A*

8 Puzzle with A*

- Heuristic function $h(n)$ = the number of misplaced tiles.

A* Search orders by the sum: $f(n) = g(n) + h(n)$



Assignment

- Sử dụng A* để tìm lời giải cho bài toán 8-puzzle với hàm h tính theo số miếng số sai vị trí, và g tính theo số bước dịch chuyển.

| | | |
|---|---|---|
| 3 | 1 | 8 |
| 7 | 4 | 2 |
| 6 | 5 | |

start

| | | |
|---|---|---|
| 4 | 6 | 5 |
| 7 | 2 | |
| 1 | 3 | 8 |

goal

Assignment

- Sử dụng A* để tìm lời giải cho bài toán 8-puzzle với hàm h tính theo khoảng cách Manhattan miềng số sai vị trí và g tính theo số bước dịch chuyển.

| | | |
|----------|----------|----------|
| 3 | 1 | 8 |
| 7 | 4 | 2 |
| 6 | 5 | |

start

| | | |
|----------|----------|----------|
| 4 | 6 | 5 |
| 7 | 2 | |
| 1 | 3 | 8 |

goal

Thanks for your attention!

Q&A
