10/18/2015

BSS1 Assignment 3: Probability

Part 1:- Written Problems:

1. Probability that no two adjacent parakeets are the same color:-

Sol:- Given 8 parakeets, four green and four blue.

$$\Rightarrow \frac{2 \times 4! \times 4!}{8!} \Rightarrow \frac{2 \times 4 \times 3 \times 2 \times 1}{8 \times 7 \times 6 \times 5} \Rightarrow \frac{1}{35}$$

2. a) Probability that a given C.P.U. will have 8 functioning compute cores:-

C.P.U will have no defect which is

$1 - 0.3 = 0.7$

One Picking one which is not defective:- 0.7.

We want all 8 are not defective $= (0.7)^8$

$= 0.05764$

b) Probability of manufacturing 8 functioning cores - Extreme model:-

→ From the previous question we found that if all 8 cores are functioning, then the probability would be $\Rightarrow (0.7)^8 = 0.05764$

For the probability to find the advanced model manufacture.

→ Advanced model has at least 4 functioning cores:-

⇒ 4 functioning × 4 non functioning +
3 non functioning × 5 ~~non~~ functioning +
2 non functioning × 6 ~~non~~ functioning +

+ 1 non-functioning × 7 functioning

$\Rightarrow (0.7)^4 (0.3)^4 + (0.7)^5 (0.3)^3 + (0.7)^6 (0.3)^2 + (0.7)^7 (0.3)$

$\Rightarrow 0.00194481 + 0.00453789 + 0.01058 + 0.024706$

$\Rightarrow \underline{0.0417774}$

$P_{(Advanced)} = 0.0417774$

→ Great model = 1 functioning × 7 non-functioning +
2 functioning × 6 non functioning +
3 functioning × 5 non functioning

$\Rightarrow (0.7)^1 (0.3)^7 + (0.7)^2 (0.3)^6 + (0.7)^3 (0.3)^5$

$\Rightarrow 0.00015309 + 0.00035721 + 0.00083349$

$P_{(Great)} = 0.00134379.$

If the company has to make 1000 C.P.U.s

Number of great C.P.US = 13

Number of advanced C.P.US = 417

Number of Extreme C.P.Us = 570.

3.(a) Judge J1 has voted guilty.

$$P(\text{person} = G \mid J_1 = G) = \frac{P(J_1 = G \mid \text{Person} = G)\, P(\text{Person} = G)}{P(J_1 = G)}$$

$\Rightarrow P(J_1 = G) = P(J_1 = G \mid \text{Person} = G) . P(\text{Person} = G) + P(J_1 = \bar{G} \mid \text{Person} = \bar{G}) . P(\text{Person} = \bar{G})$

$\Rightarrow P(J_1 = G) = (0.7 \times 0.7) + (0.2 \times 0.3)$

$= \underline{0.55}$

(3)

$P(\text{Person} = G \mid J1 = G) = \dfrac{0.7 \times 0.7}{0.55}$

$\Rightarrow \underline{\underline{0.8909}}$

b.) Probability that accused person is infact guilty given all three judges voted guilty :-

$$\frac{P(J_1 = G, J_2 = G, J_3 = G \mid P = G)\ P(P = G)}{P(J1 = G, J2 = G, J3 = G)}$$

$\Rightarrow P(J1 = G, J2 = G, J3 = G) = P(J1 = G, J2 = G, J3 = G \mid P = G)\, P(P = G)$
$+ P(J1 = \bar{G}, J2 = \bar{G}, J3 = \bar{G} \mid P = \bar{G})\, P(P$

$\Rightarrow (0.7)^4 + (0.2)^{3}(0.3)$

0.2417

Probability that person is guilty $= \dfrac{(0.7)^4}{0.2417} = \underline{\underline{0.9937}}$

c) $P(J_3 = G \mid J_1 = I, J2 = I) = \dfrac{P(J1 = I, J2 = I \mid J3 = G) \cdot P(J3 = G)}{P(J1 = I, J2 = I)}$

→ ~~Since~~ all the judges are ~~voting~~ independent of each other all terms involving J1 and J2 can be discarded

$$P(J3=G)\,P(P=G) + P(J3=G)\,P(P=I)$$

$$= (0.7)(0.7) + (0.2)(0.3)$$

$$= 0.55$$

Q. c) Expected Revenue => 50x Number of great + 100x Number of advanced + 1000 Extreme models

$$\Rightarrow 50 \times 13 + 100 \times 417 + 570 \times 1000$$

$$\Rightarrow 650 + 41700 + 570000$$

$$\Rightarrow 612350$$

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4. (b) Let D1 = Day 1
D2 = Day 2
D3 = Day 3
D4 = Day 4

We have to find $P(D_4 = \text{rain} \mid D1 = \text{rain})$

$$= D_1 \longrightarrow D_2 \longrightarrow D_3 \longrightarrow D_4$$

$$\Rightarrow \sum_{D1}\sum_{D2}\sum_{D3} P(D1, D_2, D3, D_4)$$

$$\Rightarrow \sum_{D1}\sum_{D_2}\sum_{D3} P(D)\,P(D2|D1)\,P(D3|D_2)\,P(D_4|D3)$$

$$\Rightarrow \sum_{D_3} P(D_4|D_3) \sum_{P3} P(D_3|D_2) \underbrace{\sum_{P_1} P(D1)\,P(D_2|D_1)}$$

$$\phi_{01} = \sum_{D1} P(D1)\, P(D2|D1)$$

$$= P(D1=rain)\, P(D2=rain \mid D1=rain) + P(D1=\overline{rain})\, P(D_2=rain \mid D1=\overline{rain})$$

$$= 0.25$$

$$\Rightarrow \sum_{D3} P(D4|D3) \cdot \underbrace{\sum_{D2} P(D3|D2)}_{\phi_{02}}$$

$$\phi_{02} = \sum_{D_2} P(D_3|D_2)\, \phi_{01}$$

$$= (0.25)\, P(D_3=rain \mid D2=rain) + (0.75)\, P(D3=rain \mid D_2=\overline{rain})$$

$$= (0.25)(0.65) + (0.75)(0.25)$$

$$= 0.1625 + 0.1875$$

$$= 0.35$$

$$\sum_{D_3} P(D_4|D_3)\, \phi_{02}$$

$$\Rightarrow 0.35\, P(D_4=rain \mid D_3=rain) + (0.65)\, P(D_4=rain \mid D_3=\overline{rain})$$

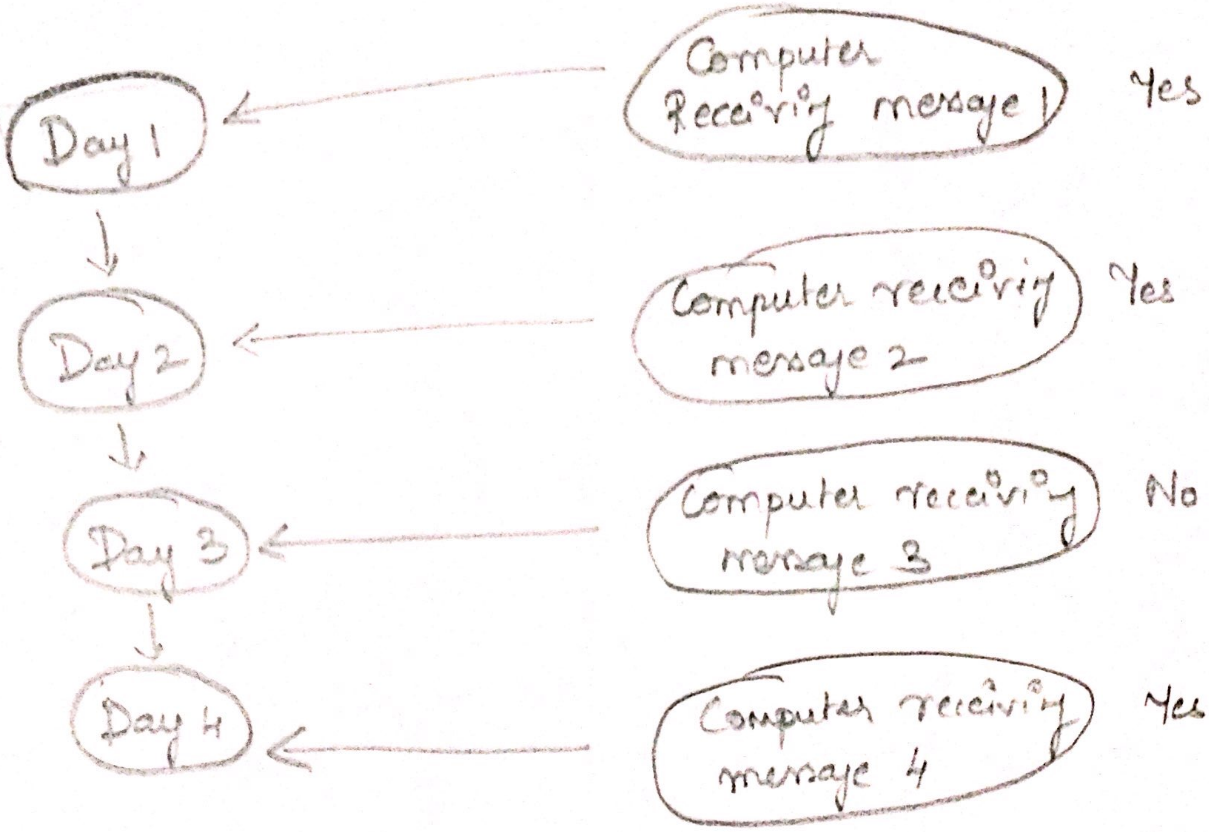
$$\Rightarrow (0.35)(0.65) + (0.65)(0.25)$$

$$= 0.2275 + 0.1625$$

$$\Rightarrow 0.39$$

Probability that it rained on the fourth day using a variable elimination algorithm is $\underline{\underline{0.39}}$

4(a) Bayes Network to model this system in terms of 4 unobserved variables and 4 observed variables.



	$P(R)$	$P(\overline{R})$
Y	0.6	0.8
N	0.4	0.2

$P(RDy)$	$P(RDt)$	$P(RDt \mid RDy)$
Y	Y	0.65
Y	N	0.35
N	Y	0.25
N	N	0.75

$P(RDy) \Rightarrow$ Probability that it rained yesterday

$P(RDt) \Rightarrow$ Probability that it rained today

Y $\Rightarrow$ Yes message   N $\Rightarrow$ No message.