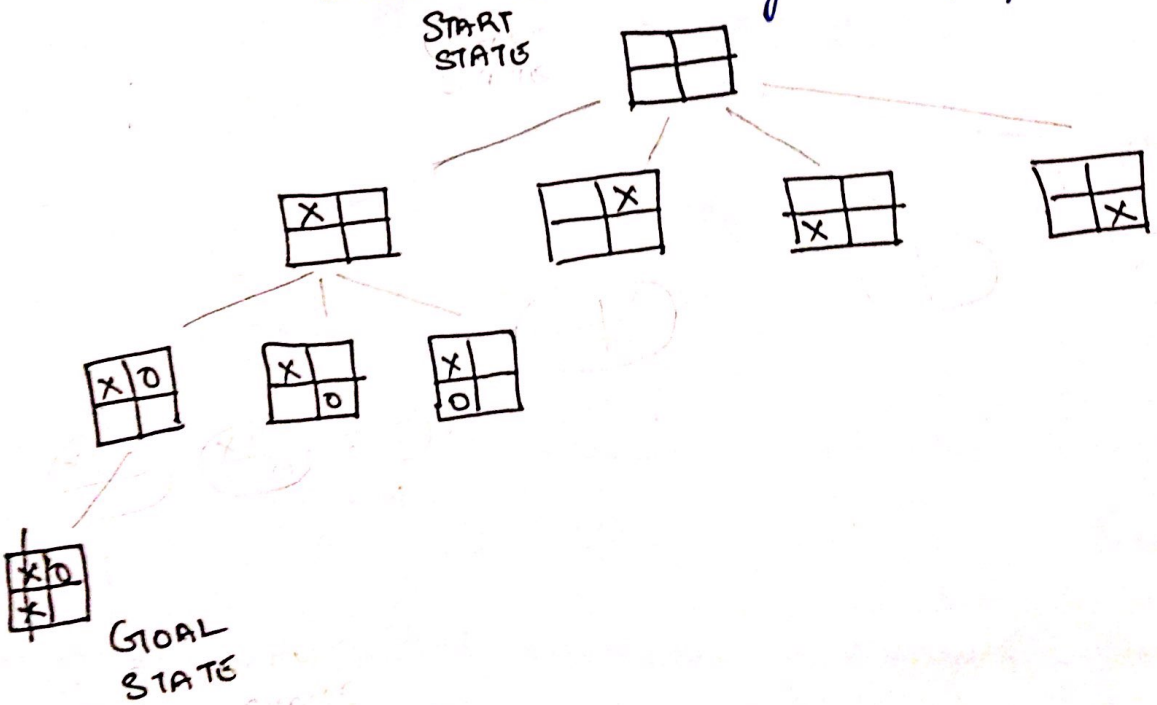
1A. Draw the state graph of 2×2 board where first player to complete a row, column or diagonal loses the game.

Sol TIC-TAC-TOE RULES

1. The 'X' token moves first
2. No player can abstain from moving
3. The game ends when:
   → two horizontal, vertical or diagonal cells are filled with identical tokens
   → this means that the person has lost and other player won, because it is a 2×2 board with only 2 players.



2. HEURISTIC FUNCTION

It is a way to inform the search about the direction to a goal.

A heuristic function is said to be admissible if it never overestimates the cost of reaching the goal, i.e. the cost it estimates to reach the goal is not higher than the lowest possible cost from the current point in the path.

n = node

h = heuristic

$h(n)$ = cost indicated by 'h' to reach a goal from n

$h^*(n)$ = is the actual cost to reach a goal from 'n'.

$h(n)$ is admissible if

$$\forall n,\ h(n) \leq h^*(n)$$

For the given 8 puzzle :-

5		8
4	2	1
7	3	6

START (N)

$\longrightarrow$

1	2	3
4	5	6
7	8	

GOAL STATE

Here $h_1(N)$ = Number of misplaced tiles = 6 is admissible

$h_2(N)$ = Sum of manhattan distances of every tile to its goal position.

$= 2 + 3 + 0 + 1 + 3 + 0 + 3 + 1 = 13$

which is admissible.

$h_3(N)$ = Sum of permutation inversions.

$\Rightarrow n_5 + n_8 + n_4 + n_2 + n_1 + n_7 + n_3 + n_6$

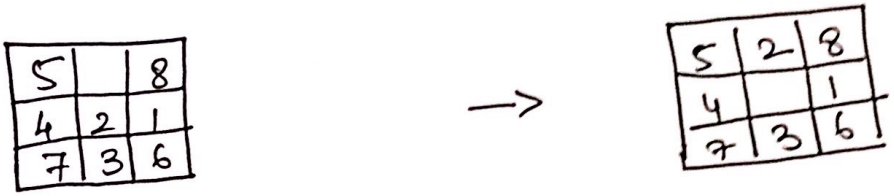
$\Rightarrow 4 + 6 + 3 + 1 + 0 + 2 + 0 + 0$

$\Rightarrow \underline{16}$

This heuristic is not admissible.

PROOF:-

i) Consider moving the empty tile down by one step, then we get the following state:-



Start state

1. Manhattan distance = 13
2. Sum of permutation inversion = 16

Next state

1. Manhattan distance = 12
2. Sum of permutation inversion = 16

From the above states we tried to move the empty tile to one position down, making '2' reach the goal state. This decreases the manhattan distance value but the heuristic value using sum of permutation does not change. Thus this is the over-estimate of the optimal solution by manhattan distance.

Hence Proved that 'h' is not admissible for $h(s)$ = Sum of permutation inversions. as it gives an overestimate of cost.