(8)

1 A.) Z = set of integers = $\{\ldots,-3,-2,-1,0,1,2,3\ldots\}$

(a) Both addition and multiplication are commutative

$$\forall x,y \in Z\left[(x+y)=(y+x) \wedge (x \times y)=(y \times x)\right]$$

(b) Z is closed under addition and multiplication

$$\forall x,y \in Z\left[(x+y \in Z) \wedge (x \times y \in Z)\right]$$

(d) Multiplication and addition are associative. ~~distributes over addition~~

$$\forall x,y,z \in Z\left[\left[(x+y)+z = x+(y+z)\right] \wedge \left((x\times y)\times z = (x \times (y \times z))\right)\right]$$

(c) Multiplication distributes over addition ~~and addition are associative~~

$$\forall x,y,z \in Z\left[x \times (y+z) = (x \times y) + (x \times z)\right]$$

(e) multiplication & addition have the identity proper

$$\forall x \in Z\left[\left((x+0)=x\right) \wedge \left((x \times 1)=x\right)\right]$$

2.) Consider the following:-

$\alpha = \forall x (P(x) \vee Q(x))$ and

$\beta = \forall x \{ P(x) \vee \forall x Q(x)$.

Does $\alpha \models \beta$?

Sol: From first statement:-

$\alpha = \forall x [P(x) \vee Q(x)]$

I am considering the statement :

1. All characters in english are either vowels or consonants.

$\alpha = \forall c [Vowel(c) \vee Consonant(c)]$

where c = characters $\in \{A, B, \dots Z\}$

2. For 'β', not all characters are vowels or all characters are consonants.

that is:-

$\beta \neq \forall c\ Vowels(c) \vee \forall c\ Consonant(c)$

Thus, α is true in this model or statement but 'β' is not.

3.) (a) $Owns(H, M)$

(b) $Unhappy(P)$

(c) $Loves(P, H)$

(d) $\forall x : Owns(x, M) \vee Unhappy(x) \rightarrow Visits(x, O)$

(e) $\forall x : Visits(x, O) \rightarrow Wise(x)$

(f) $\forall x : Owns(x, M) \wedge Visits(x, O) \rightarrow Teaches(O, x, L)$

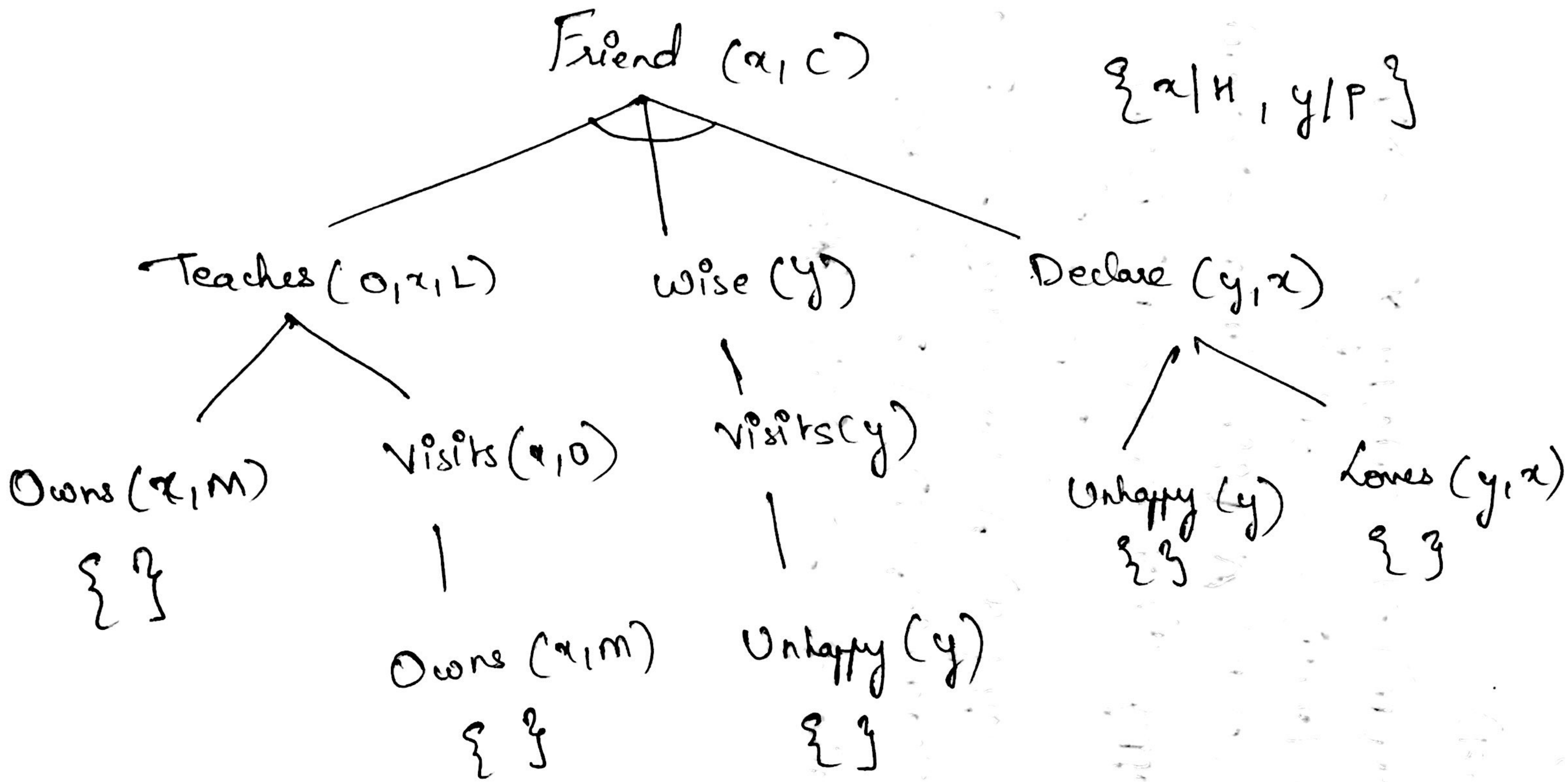
(g) $\forall x : Unhappy(x) \vee Owns(x, M) \wedge Teaches(O, x, L) \rightarrow Joins(x, R)$

(h) $\forall x \forall y : Unhappy(x) \wedge Loves(x, y) \rightarrow Declare(x, y)$

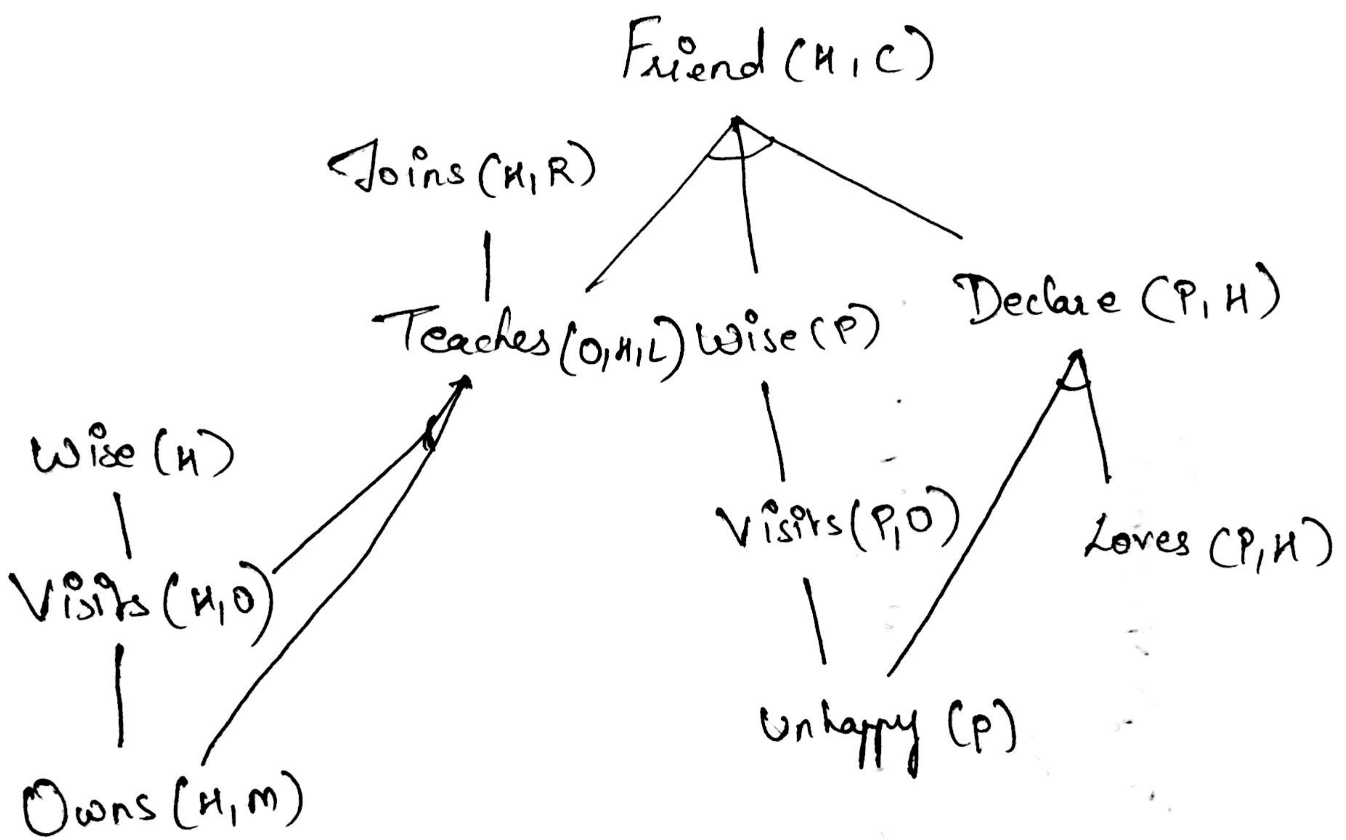
(i) $\forall x, y : Teaches(O, x, L) \wedge Declares(y, x) \wedge Wise(y) \rightarrow Friend(x, C)$

ii)

Backward Chaining :



Forward Chaining:



11/5/2015

CS-B551 Assignment-4

1.

4.) Converting the clause form to first order logic statements:-

a.) $A(Z) \wedge W(Z) \longrightarrow P(E)$

b) $\neg A(Z) \rightarrow I(Z)$

c) $\neg W(Z) \rightarrow M(Z)$

d) $\neg P(E)$

e.) $Exists(Z) \longrightarrow \neg I(\cancel{Z}) \wedge \neg M(Z)$

Z : Zeus   A(Z) : Zeus were able

E : Evil   W(Z) : Zeus were willing.

P(E) : Zeus prevents evil

I(Z) : Impotent Zeus

M(Z) : malevolent zeus

~~TP(E)~~ Using Demorgans Law to eliminate the implication

a.) $\neg A(Z) \vee \neg W(Z) \vee P(E)$

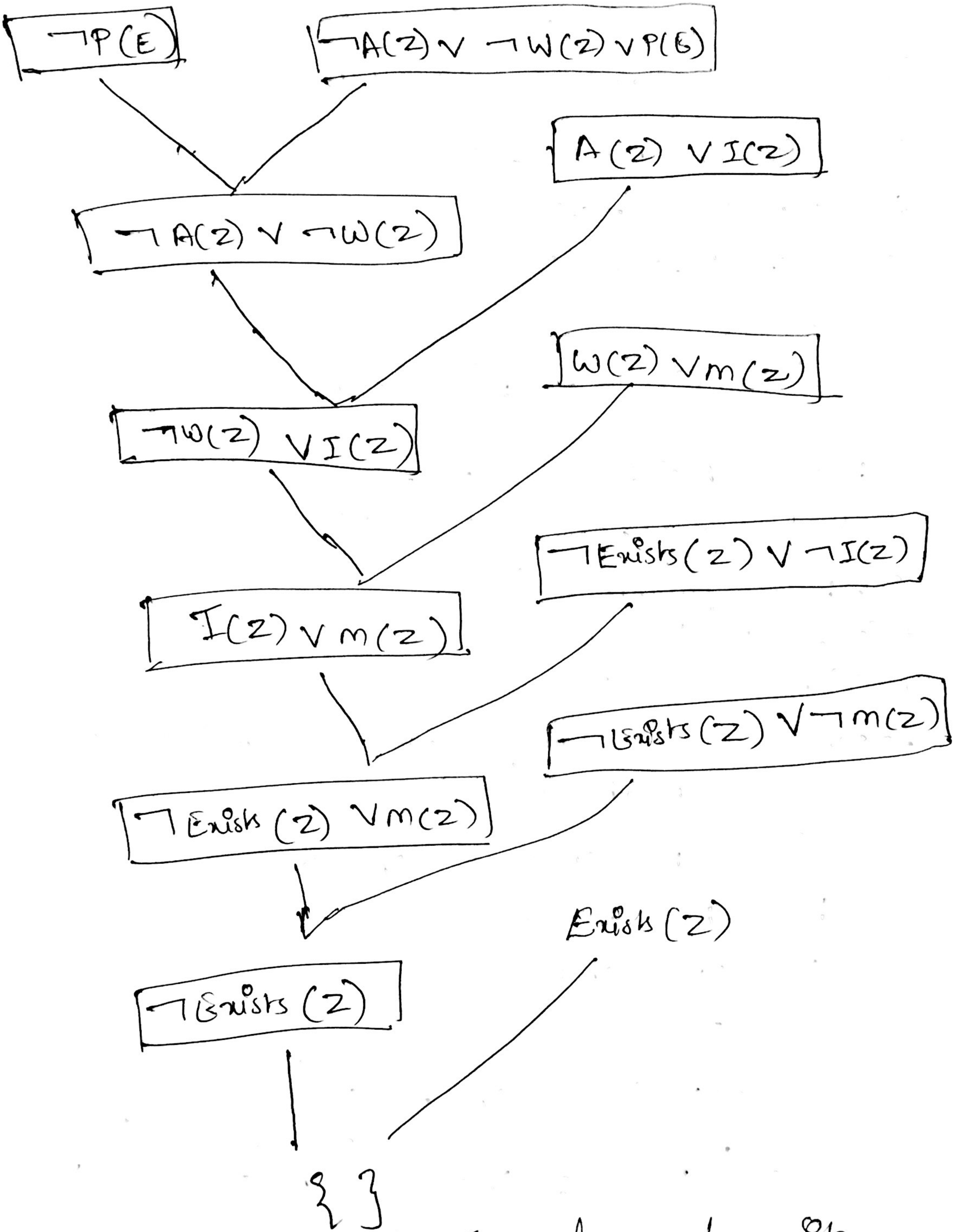
b.) $A(Z) \vee I(Z)$

c) $W(Z) \vee M(Z)$

d) $\neg P(E)$

e) $\neg Exists(Z) \vee [\neg I(Z) \wedge \neg M(Z)]$

$\Rightarrow [\neg Exists(Z) \vee \neg I(Z)] \wedge [\neg Exists(Z) \vee \neg M(Z)]$



Hence proved that Zeus does not exist

5.

a. $\forall x\,(P(x) \Rightarrow P(x))$

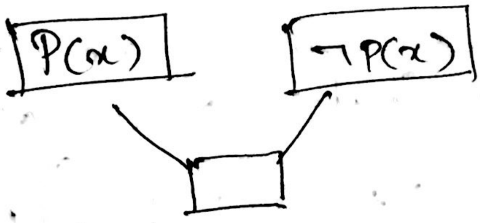
Converting to Conjunctive Normal Form:

$$\forall x\,(P(x) \Rightarrow P(x)) = \forall x\,(\neg P(x) \vee P(x))$$

$$= \neg P(x) \vee P(x)$$

Proving the sentence is valid by showing that it's negation leads to a contradiction by resolution.

Negation: $P(x) \wedge \neg P(x)$



Thus, negation leads to an empty set by resolution which is a contradiction.

Hence we say that.

$\forall x\,(P(x) \Rightarrow P(x))$ is valid.

b) $(\neg \exists x\, P(x)) \Rightarrow (\forall x\, \neg P(x))$

Converting to CNF :-

$$\boxed{\alpha \Rightarrow \beta = \neg \alpha \vee \beta}$$

$(\cancel{\exists x}\ (\neg \exists x\, P(x)) \Rightarrow (\forall x\, \neg P(x))$

$\Rightarrow \neg(\neg \exists x\, P(x)) \vee \forall x\, \neg P(x)$

$\Rightarrow \exists x\, P(x) \vee \forall x\, \neg P(x)$

We can show that the sentence is valid by showing that resolution of negation of sentence leads to contradiction.

negation: $\neg(\exists x P(x) \vee \forall x \neg P(x))$

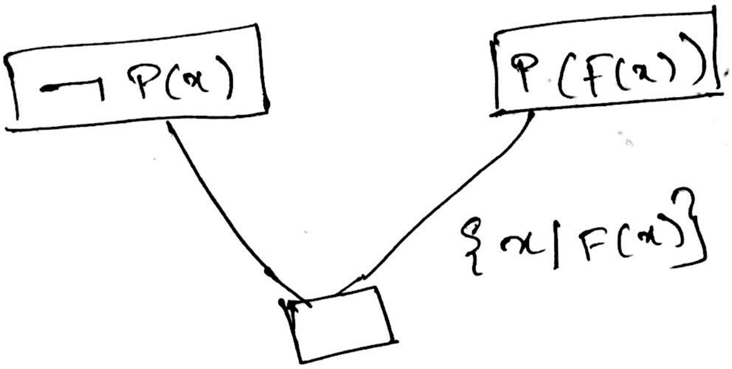
$\Rightarrow \neg \exists x P(x) \wedge \neg \forall x \neg P(x)$

$\Rightarrow \forall x \neg P(x) \wedge \exists x P(x)$

Now we shall perform skolemization to eliminate the existential quantifier $(\exists x)$

$\Rightarrow \forall x \neg P(x) \wedge \exists x P(x)$

$\Rightarrow \neg P(x) \wedge P(F(x))$



Therefore, negation leads to an empty set, which means that the sentence

$\neg \exists x P(x) \Rightarrow \forall x \neg P(x)$ is valid

c.) $(\forall x (P(x) \vee Q(x))) \Rightarrow ((\forall x P(x)) \vee (\exists x Q(x)))$

Converting to CNF:

$\neg(\forall x (P(x) \vee Q(x))) \vee ((\forall x P(x)) \vee (\exists x Q(x)))$

$\neg \forall x (P(x) \vee Q(x)) \vee ((\forall x P(x)) \vee (\exists x Q(x)))$

$\exists x (\neg P(x) \wedge \neg Q(x)) \vee ((\forall x P(x)) \vee (\exists x Q(x)))$

We prove that the sentence is valid by showing that negation leads to contradiction by resolution

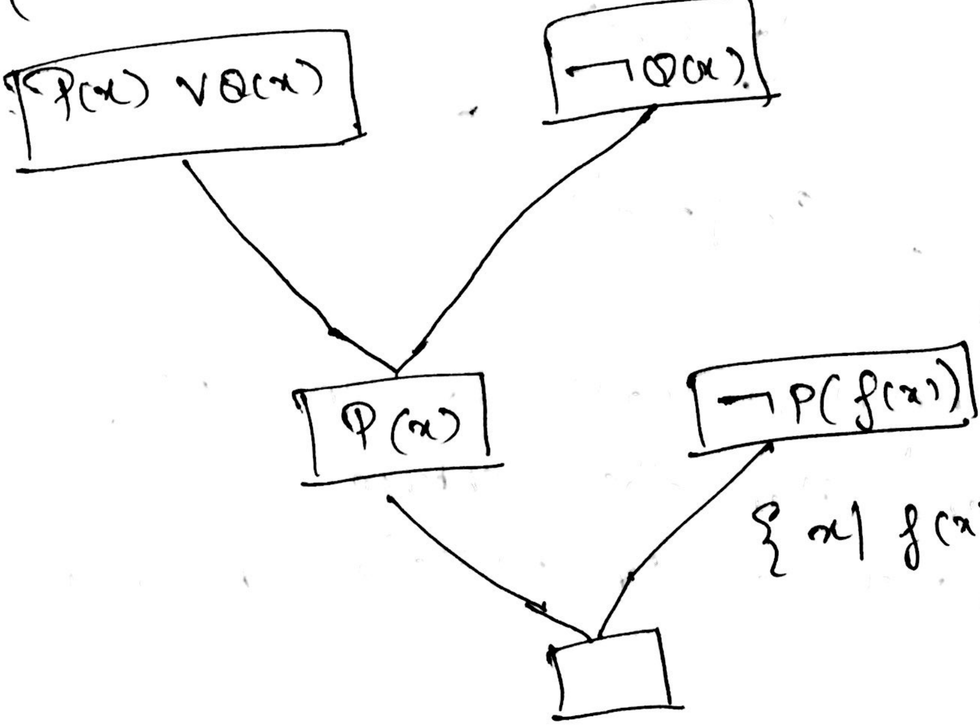
Negation : $\neg (\exists x (\neg P(x) \wedge \neg Q(x))) \wedge$

$\neg ((\forall x P(x)) \vee \exists x Q(x))$

$\forall x (P(x) \vee Q(x)) \wedge (\neg (\forall x P(x)) \wedge \neg (\exists x Q(x)))$

$(P(x) \vee Q(x)) \wedge ((\exists x \neg P(x)) \wedge (\forall x \neg Q(x)))$

$(P(x) \vee Q(x)) \wedge (\neg P(f(x)) \wedge \neg Q(x))$



Therefore negation leads to an empty set by resolution, which is a contradiction.

Hence $\forall x (P(x) \vee Q(x)) \Rightarrow ((\forall x P(x)) \vee (\exists x Q(x)))$ is valid